粲介子含轻衰变的格点QCD研究

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提纲

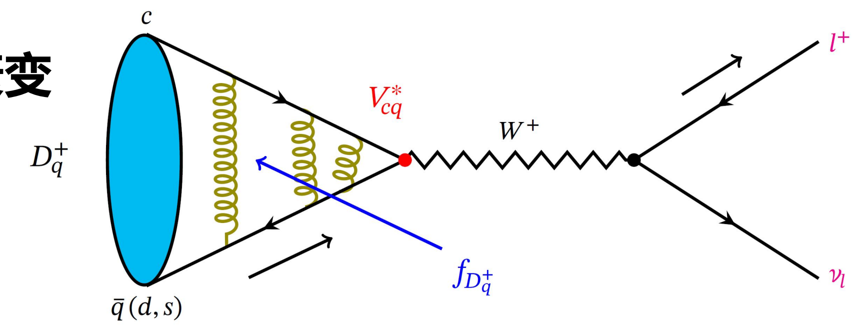
- 粲物理与格点QCD
- 粲介子衰变常数（纯轻衰变）
- 粲介子形状因子（半轻衰变）
- 小结

粲物理与LQCD

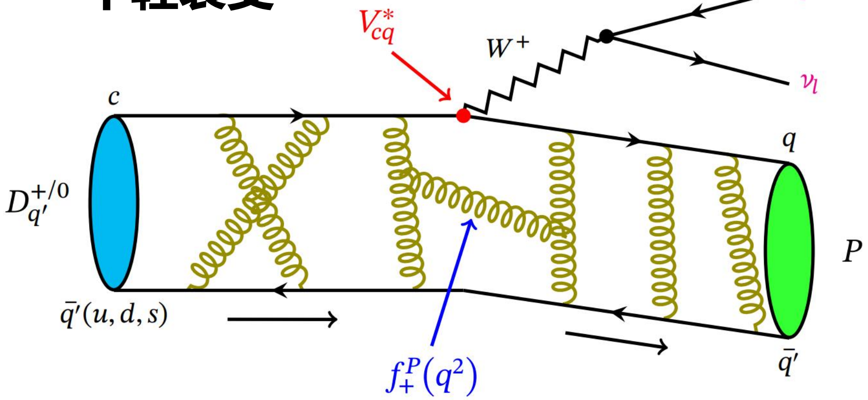
- LQCD can calculate form factors and meson decay constants appearing in weak decays of hadrons
- Combined with experiments, they can give us CKM matrix elements
- Test the SM (is the CKM matrix unitary?)
- Or use V_{ab} from elsewhere to compare QCD/SM results with experiments

$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$	$B \to \pi\ell\nu$
	$K \to \pi\ell\nu$	
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$	$D_s \to \ell\nu$	$B \to D\ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$	$B \to D^*\ell\nu$
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$B_d \leftrightarrow \overline{B}_d$	$B_s \leftrightarrow \overline{B}_s$	

纯轻衰变



半轻衰变



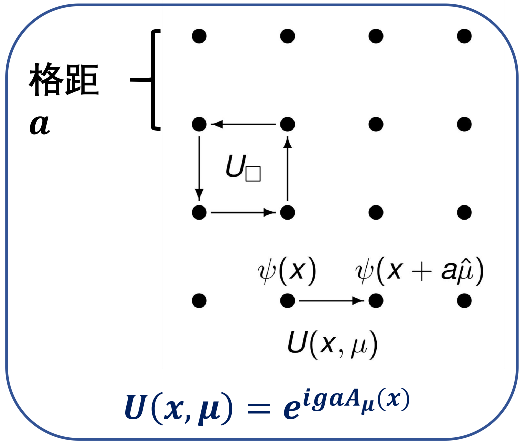
courtesy arXiv:2103.00908

CKM矩阵元 LQCD计算

例如: $$\Gamma(P \to \ell\nu) = \frac{G_F^2 |V_{q_1 q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$$

实验测量

格点QCD (1973, Wilson; 1979, Creutz) 用数值模拟研究QCD的非微扰性质



- 4维闵氏时空 → 4维欧氏空间（$\tau = it$）
- $\langle O \rangle = \dfrac{\int DA_\mu D\bar{\psi}D\psi O[A,\bar{\psi},\psi] e^{-\int \mathcal{L}_{QCD}\, d^4x}}{\int DA_\mu D\bar{\psi}D\psi e^{-\int \mathcal{L}_{QCD}\, d^4x}}$，$\mathcal{L}_{QCD} = \bar{\psi} M[A] \psi + \mathcal{L}_G$

$$M = \gamma \cdot D + m_q$$

- $\langle O \rangle = \dfrac{\int DU_\mu O[U, M^{-1}[U]] \mathrm{Det}[M[U]] e^{-S_G}}{\int DU_\mu \mathrm{Det}[M[U]] e^{-S_G}} \sim \dfrac{\int dx\, f(x)\, \rho(x)}{\int dx\, \rho(x)}$ ➡ $\sim \dfrac{1}{N}\sum_{n=1}^{N} f(x_n)$

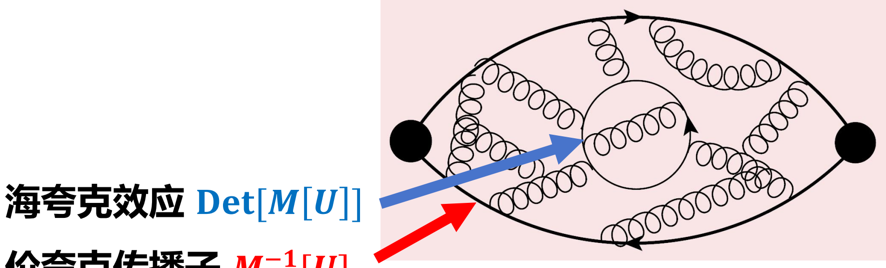
离散的x_n按$\rho(x_n)$分布

上式为带权重$\mathrm{Det}[M[U]]e^{-S_G}$的平均，类似Boltzmann系综平均

- 在有限体积4维超立方格子上，自由度个数可数，路径积分具有良好定义
- 巨大高维积分，无法直接计算；用重点抽样按权重分布产生U（组态）
- 路径积分变为对组态的统计平均：$\langle O \rangle = \frac{1}{N}\sum_{i=1}^{N} O_i$

  N有限，统计误差$\sim 1/\sqrt{N}$

输入若干实验测量值，确定自由参数：
格距 a 和夸克质量 m_q， 预言其他结果



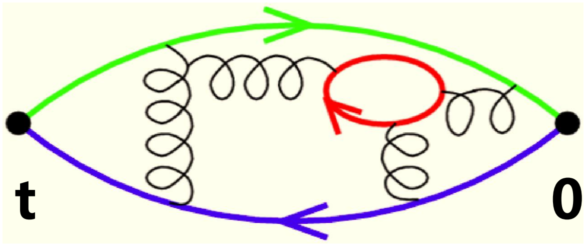
介子两点关联函数

强子矩阵元的格点计算

$$C(t)=\langle\Omega|O(t)O^\dagger(0)|\Omega\rangle \xrightarrow{t\to\infty} |\langle\Omega|O|P\rangle|^2 e^{-m_P t}\equiv Ae^{-m_P t}$$

- 介子衰变常数从两点函数抽取，例如$O=\bar{q}\gamma_0\gamma_5 c$

$$\langle 0|\bar{q}(0)\gamma_\mu\gamma_5 c(0)|P(p)\rangle = if_P p_\mu,\quad q=d,s$$

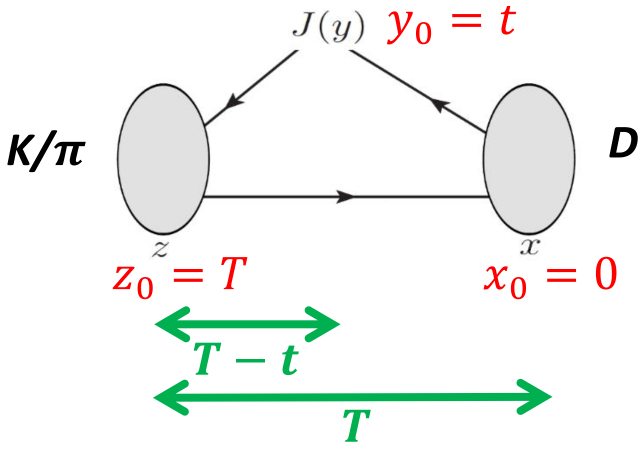


两点关联函数

- 结合两点和三点函数可抽取半轻过程强子矩阵元（形状因子）：

$$C_3(\vec{p},\vec{p}',T,t)=\sum_{\vec{z}}\sum_{\vec{y}}\langle 0|O_P(\vec{z},T)J(\vec{y},t)O_D^\dagger(\vec{x},0)|0\rangle e^{-i\vec{p}\cdot\vec{z}}e^{i\vec{q}\cdot\vec{y}}$$

$$\xrightarrow[(T-t)\to\infty]{t\to\infty,\ T\to\infty}\langle 0|O_P|P\rangle\langle P|J|D\rangle\langle D|O_D^\dagger|0\rangle e^{-m_D t}e^{-m_P(T-t)}$$



三点关联函数

- 算符的重正化常数
  - 由于离散效应，格子上的局域(轴)矢量流 $\bar{q}\gamma_\mu c$ ($\bar{q}\gamma_\mu\gamma_5 c$) 需要归一化常数 $Z_{V,A}$
  - 标量及张量流算符随能标跑动，$Z_{S,T}$
- 使用手征格点费米子有时可避免重正化常数的计算（$Z_S Z_m=1$）
  - PCAC：$(m_q+m_c)\langle 0|\bar{q}(0)\gamma_5 c(0)|P(p)\rangle = f_P m_{PS}^2$
- PCVC：$\langle K|S|D\rangle = f_0^{D\to K}(q^2)\dfrac{M_D^2-M_K^2}{m_c-m_s}$

粲介子衰变常数

- $f_{D^{(*)}_{(s)}}$

$$\langle 0|\bar{q}(0)\gamma_\mu\gamma_5 c(0)|P(p)\rangle = i f_P p_\mu, \quad q = d, s$$

$$\langle 0|\bar{q}(0)\gamma^\mu q'(0)|V(p,\lambda)\rangle = f_V m_V e^\mu_\lambda$$

- $f_V^T/f_V(\overline{\text{MS}}, 2\text{ GeV})$ $\quad \langle 0|\left(\bar{q}(0)\sigma^{\mu\nu}q'(0)\right)(\mu)|V(p,\lambda)\rangle = i f_V^T(\mu)(e^\mu_\lambda p^\nu - e^\nu_\lambda p^\mu)$ (需要张量流重整化常数$Z_T^{\overline{\text{MS}}}$)

实验测量 ← 抽取CKM矩阵元 → 衰变常数可由LQCD给出

$$\Gamma(P\to \ell\nu) = \frac{G_F^2|V_{q_1q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1-\frac{m_\ell^2}{M_P^2}\right)^2$$

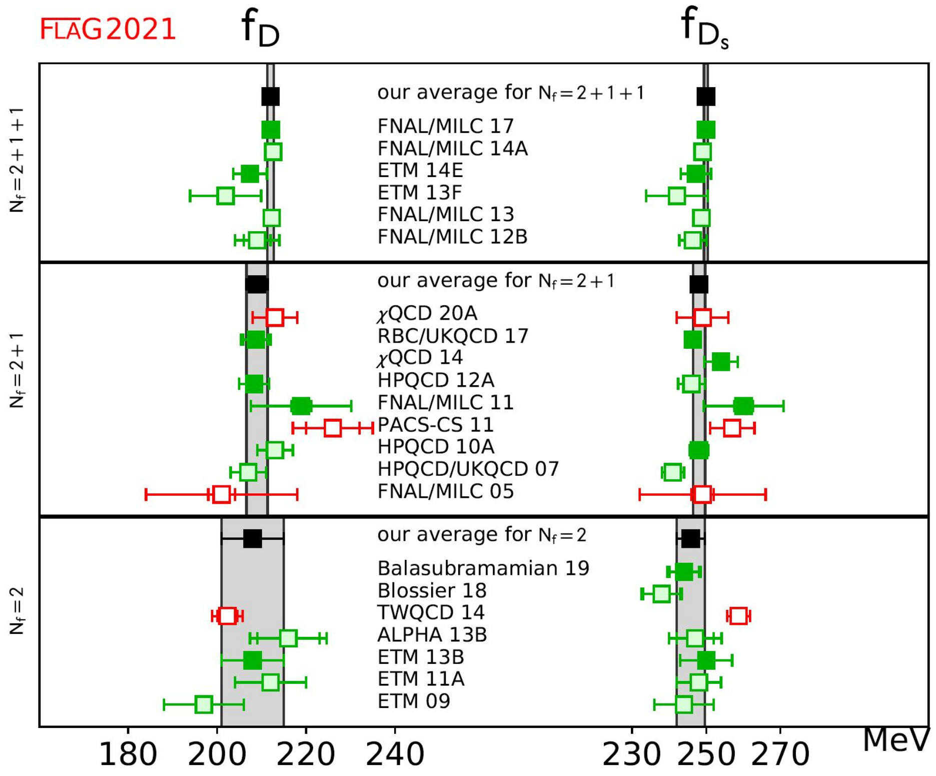
$$\Gamma_{(D_s^*\to\ell\nu)} = \frac{G_F^2}{12\pi}|V_{cs}|^2 f_{D_s^*}^2 M_{D_s^*}^3\left(1-\frac{m_\ell^2}{M_{D_s^*}^2}\right)^2\left(1+\frac{m_\ell^2}{2M_{D_s^*}^2}\right)$$

$\text{Br} = 3.4(1.4)\times 10^{-5}$ [HPQCD, PRL 112, 212002 (2014)]

- Determine CKM elements
- f_V not easy to be measured
  - Leptonic decay BRs are small; $D_s^*\to l\nu$ reported by BESIII (PRL131.141802(2023))
- Test the accuracy of Heavy Quark Effective Theory: $f_V/f_P = 1 + \mathcal{O}(1/m_Q)$
- f_V^T/f_V for D^* and D_s^* are inputs for LCSR in calculations of $B\to V$ form factors at low q^2
- Input parameters for QCD factorization in studies of nonleptonic B decays, e.g., $B\to D^{(*)}M$

粲介子衰变常数格点结果

- 赝标粲介子衰变常数的LQCD计算精度已达到 $\lesssim$1%
- 进一步提高精度需要考虑同位旋破缺效应



FLAG Review 2021, Eur. Phys. J. C (2022) 82, 869

χQCD, PRD92.034517 (2015), arXiv:1410.3343

χQCD, CPC45.023109 (2021), arXiv:2008.05208

- $N_f = 2+1$:

$$f_{D_s} = 248.0(1.6)\ \text{MeV}$$
$$f_D = 209.0(2.4)\ \text{MeV}$$

- $N_f = 2+1+1$:

$$f_{D_s} = 249.9(0.5)\ \text{MeV}$$
$$f_D = 212.0(0.7)\ \text{MeV}$$

- PDG2016 (CPC40):

$$f_{D_s^+}^{exp} = 257.8(4.1)\ \text{MeV}$$

格点结果与实验在2σ之内一致

PDG2020 [PTEP2022.083C01]:

$$f_D|V_{cd}| = 46.2(1.2)\ \text{MeV}$$
$$f_{D_s}|V_{cs}| = 243.5(2.7)\ \text{MeV}$$

BESIII:

$f_{D_s} = 251.1(2.4)(3.0)$ MeV [PRL127.171801.2021], 6.32 fb^{-1}

$f_{D_s^+} = 255.0(4.0)(3.2)(1.0)$ MeV [PRD108.092014.2023], 7.33 fb^{-1}

- **2021年之后的格点QCD计算**

A. Bussone et al. (Alpha Collab.), 2309.14154

2+1 味，twisted mass fermion on improved Wilson fermion

$$f_D = 211.3(1.9)(0.6)\ \text{MeV}$$
$$f_{D_s} = 247.0(1.9)(0.7)\ \text{MeV}$$
$$f_{D_s}/f_D = 1.177(15)(5)$$

S. Kuberski et al. (RQCD & Alpha), 2405.04506

2+1 味，improved Wilson fermion

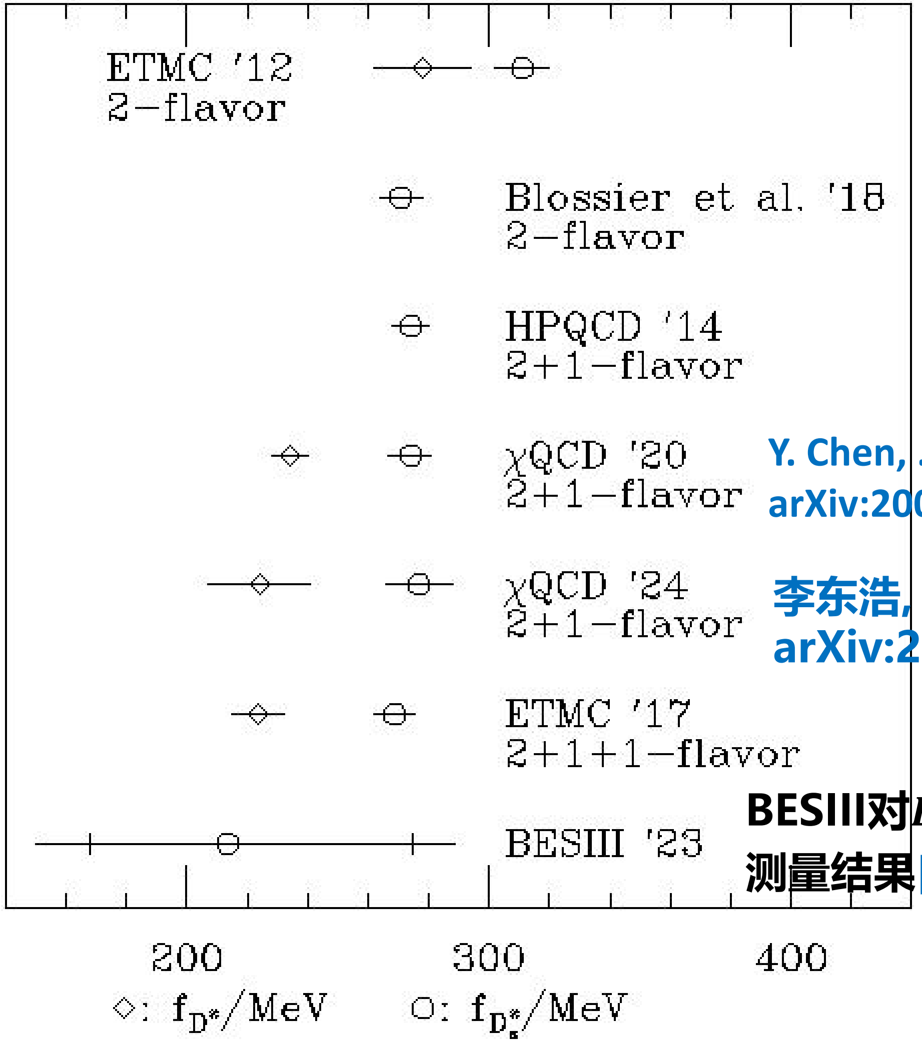
$$f_D = 208.4(1.5)\ \text{MeV}$$
$$f_{D_s} = 246.8(1.3)\ \text{MeV}$$
$$f_{D_s}/f_D = 1.1842(36)$$

- **STCF预期统计精度好于0.15%**

[STCF CDR, Front. Phys. 19(1), 14701, arXiv:2303.15790]

矢量粲介子衰变常数

- 矢量粲介子衰变常数的格点计算相对较少



Y. Chen, ..., ZL et al., CPC45 (2021), arXiv:2008.05208

李东浩, ..., 王庭霄 et al., arXiv:2407.03697

BESIII对$D_s^{*+} \to e^+\nu_e$的首个测量结果[2304.12159, PRL]

衰变常数/MeV	D^*	D_s^*	2+1味
chiQCD '20	234(3)(5)	274(5)(5)	$a^2 \sim 0.012\ \mathrm{fm}^2$
chiQCD '24	224(16)(7)	277(7)(8)	$a^2 \sim 0.006\ \mathrm{fm}^2$
两格距下结果中心值相差	4%	1%	离散误差估计：3%

衰变常数比值

f_V/f_P	D_s^*/D_s	$J/\psi/\eta_c$
chiQCD '20	1.10(3)(2)	-
chiQCD '24	1.096(35)(4)	1.059(15)

Heavy quark symmetry breaking (~10%)

f_V^T/f_V	D^*	D_s^*	J/ψ
chiQCD '20	0.91(3)(2)	0.92(3)(2)	-
chiQCD '24	0.88(5)(3)	0.907(15)(13)	0.96(1)

D_s^*衰变常数的误差相对较小

$$Z_T^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})/Z_V = 1.072(10)$$

Y. Bi, ..., ZL et al., PRD108.054506 (2023)

D_s^*总宽度及纯轻衰变分支比

HPQCD给出D_s^*总宽度：0.070(28) keV

以及 $\mathrm{Br}(D_s^* \to l\nu) = 3.4(1.4) \times 10^{-5}$

(PRL112, 212002 (2014))

$\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)\ \mathrm{keV}$

$D_s^* \to D_s\gamma$分支比的实验值93.5(7)% PDG

D_s^*总宽度：0.0587(54) keV

Y. Meng, ..., ZL et al., arXiv:2401.13475 (PRD109.074511)

$D_s^* \to l\nu\ (l = e, \mu)$分支比实验测量，结合总宽度，可给出$f_{D_s^*}|V_{cs}|$

BESIII对$D_s^{*+} \to e^+\nu_e$分支比的首个测量结果：

[2304.12159, PRL131.141802(2023)]

$$(2.1^{+1.2}_{-0.9\,\mathrm{stat}} \pm 0.2_{\mathrm{syst}}) \times 10^{-5}$$

- BESIII + HPQCD

$$f_{D_s^*}|V_{cs}| = (207.9^{+59.4}_{-44.6\,\mathrm{stat.}} \pm 9.9_{\mathrm{syst.\,exp}} \pm 41.5_{\mathrm{syst.\,latt}})\ \mathrm{MeV}$$

- BESIII + arXiv:2401.13475

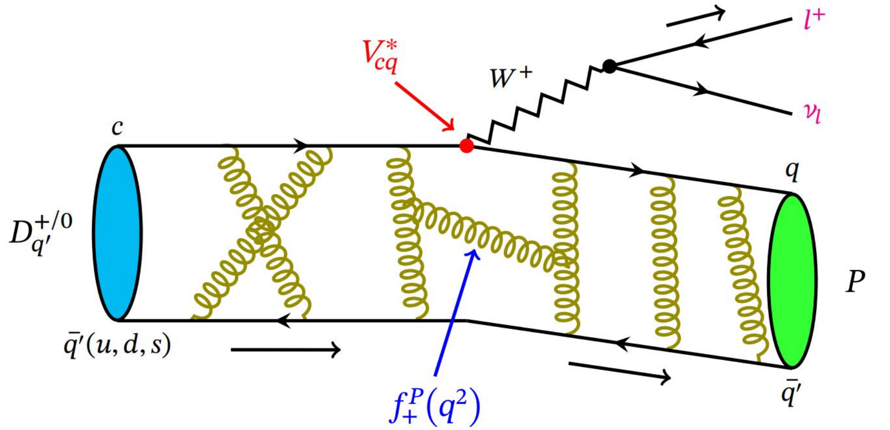
$$f_{D_s^*}|V_{cs}| = (190.5^{+55.1}_{-41.7\,\mathrm{stat.}} \pm 9.1_{\mathrm{syst.\,exp}} \pm 8.7_{\mathrm{syst.\,latt}})\ \mathrm{MeV}$$

STCF提高实验精度

(李东浩 et al., arXiv:2407.03697) + (arXiv:2401.13475)

$f_{D_s^*} = 277(11)$ MeV → $\Gamma_{D_s^* \to l\nu} = 2.5(2) \times 10^{-6}$ keV → $\mathrm{Br} = 4.26(52) \times 10^{-5}$

粲介子半轻衰变



courtesy arXiv:2103.00908

- $D\to\pi l\nu$， $D\to K l\nu$可用于确定 $|V_{cd}|$和$|V_{cs}|$

$$\frac{d\Gamma(D\to K\ell\nu)}{dq^2}=(\text{known})\,|\mathbf{p}_K|^3\,|V_{cs}|^2\,|f_+^{D\to K}(q^2)|^2$$

- 需要形状因子$f_+(q^2)$

$$\langle K|V^\mu|D\rangle=f_+(q^2)\left(p_D^\mu+p_K^\mu-\frac{m_D^2-m_K^2}{q^2}q^\mu\right)+f_0(q^2)\frac{m_D^2-m_K^2}{q^2}q^\mu$$

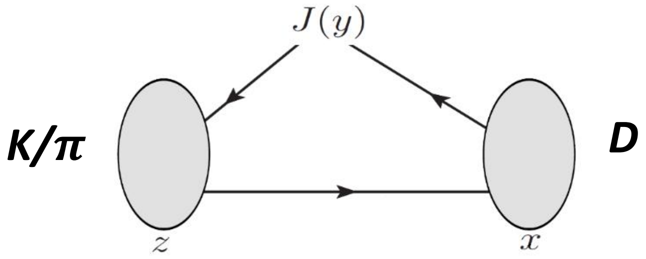
$$f_+(0)=f_0(0)$$

对于$l=e,\mu$, 形状因子f_0对衰变宽度的贡献较小（正比于m_l^2）

- 标量流形状因子 $\langle K|S|D\rangle=f_0^{D\to K}(q^2)\frac{M_D^2-M_K^2}{m_c-m_s}$ (手征格点费米子有$Z_SZ_m=1$, 无需计算重整化常数)

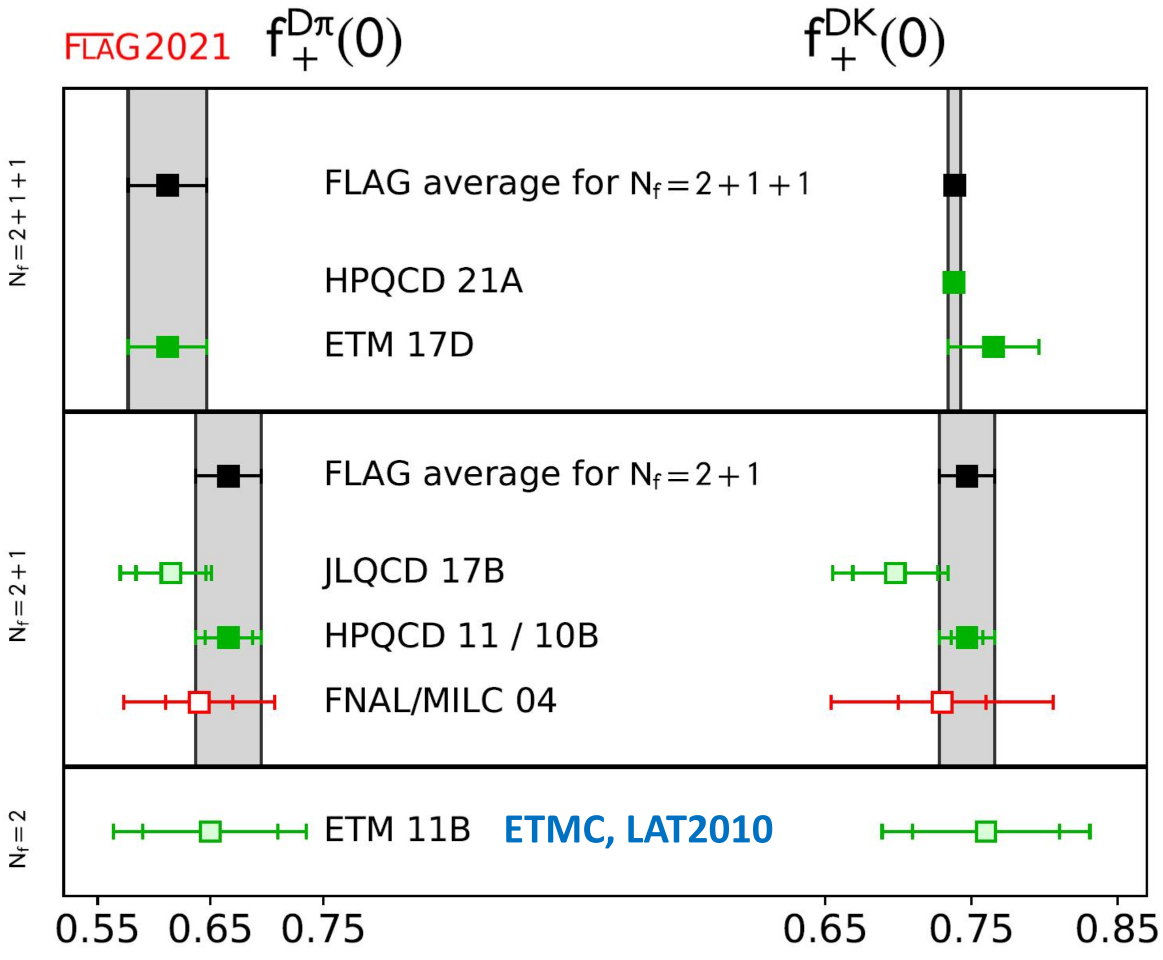
- 初末态强子四动量：p、p'

$q^2=(p-p')^2$，格点计算中3-动量取分立值



三点关联函数

$f_+(q^2=0)$ for $D\to\pi/K$



FLAG Review 2021, [arXiv:2111.09849]

2+1+1味结果

- $f_+^{D\pi}(0)=0.612(35)$ [ETM 17D: PRD96 (2017) 054514, 1706.03017]
- $f_+^{DK}(0)=0.7385(44)$ [ETM 17D: PRD96 (2017) 054514, 1706.03017. HPQCD 21A: PRD104 (2021) 034505, 2104.09883]

2+1味结果

- JLQCD 17: LAT2017 [1711.11235]
- FNAL/MILC/HPQCD 04: 一个格距，m_π大于500 MeV
- $f_+^{D\pi}(0)=0.666(29)$ [HPQCD 11, PRD84,114505, 1109.1501]
- $f_+^{DK}(0)=0.747(19)$ [HPQCD 10B, PRD82,114506, 1008.4562]

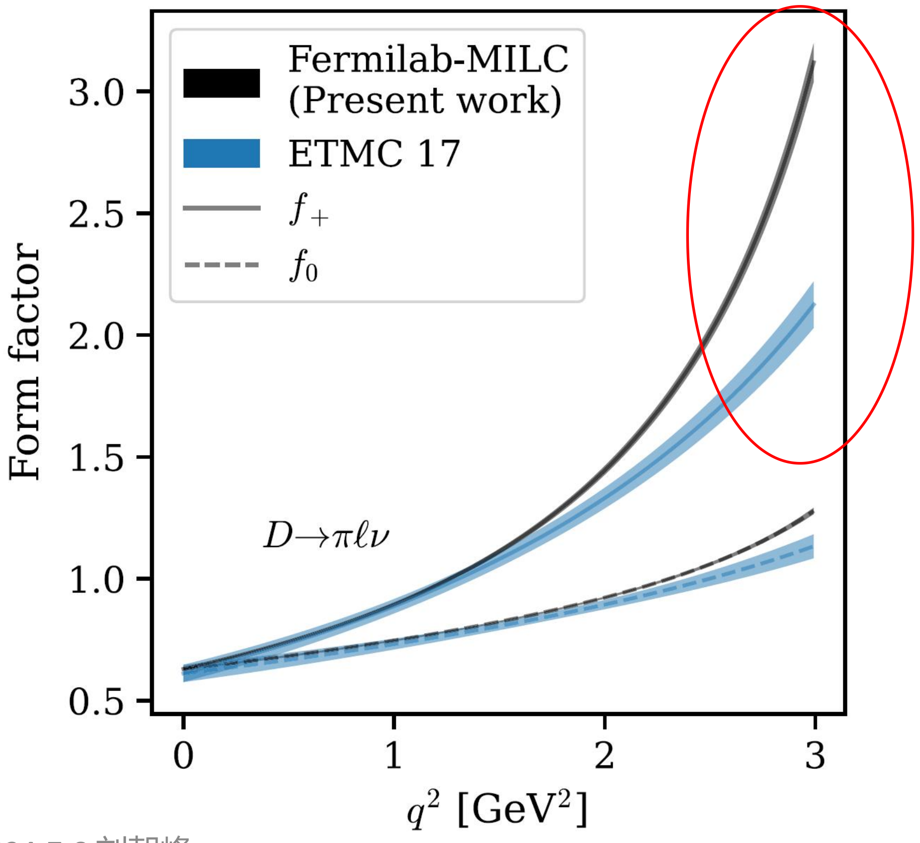
$D \to \pi/K,\ D_s \to K$形状因子

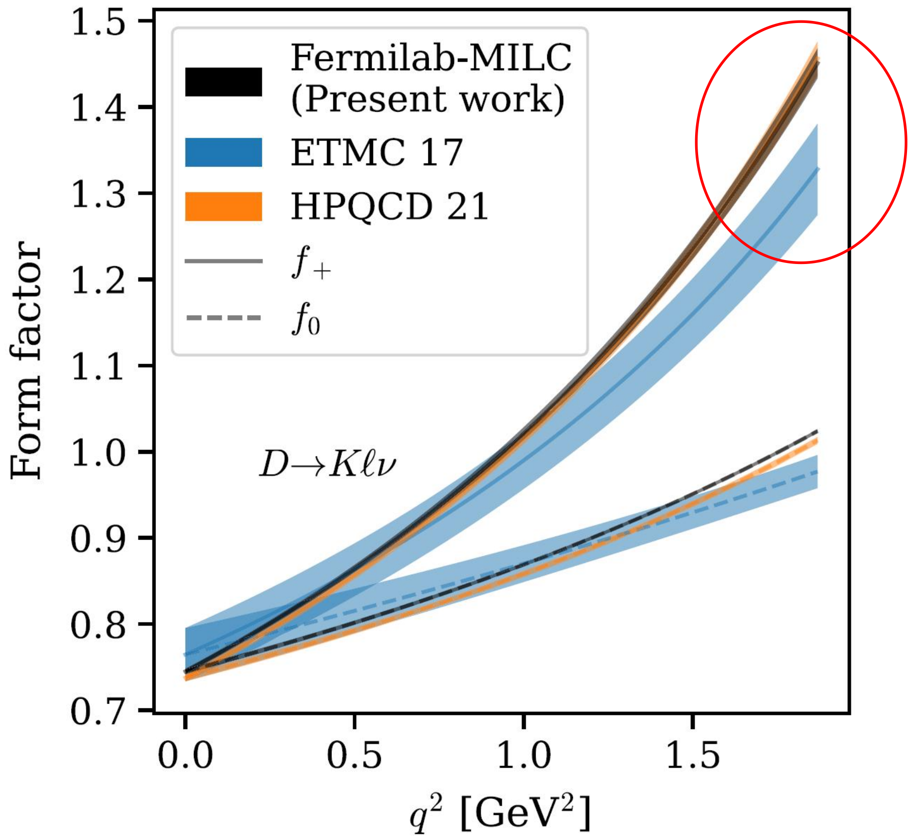
Fermilab/MILC, 2+1+1味HISQ

A. Bazavov et al., 2212.12648, PRD107.094516

- 四个格距：~0.12 fm - 0.04 fm
- $\mathcal{O}(1000)$组态数，多次测量/组态
- 两或三个体积@两个格距
- 物理轻夸克质量@三个格距

$$f_+^{D\pi}(0) = 0.6300(51)$$
$$f_+^{DK}(0) = 0.7452(31)$$
$$f_+^{D_sK}(0) = 0.6307(20)$$



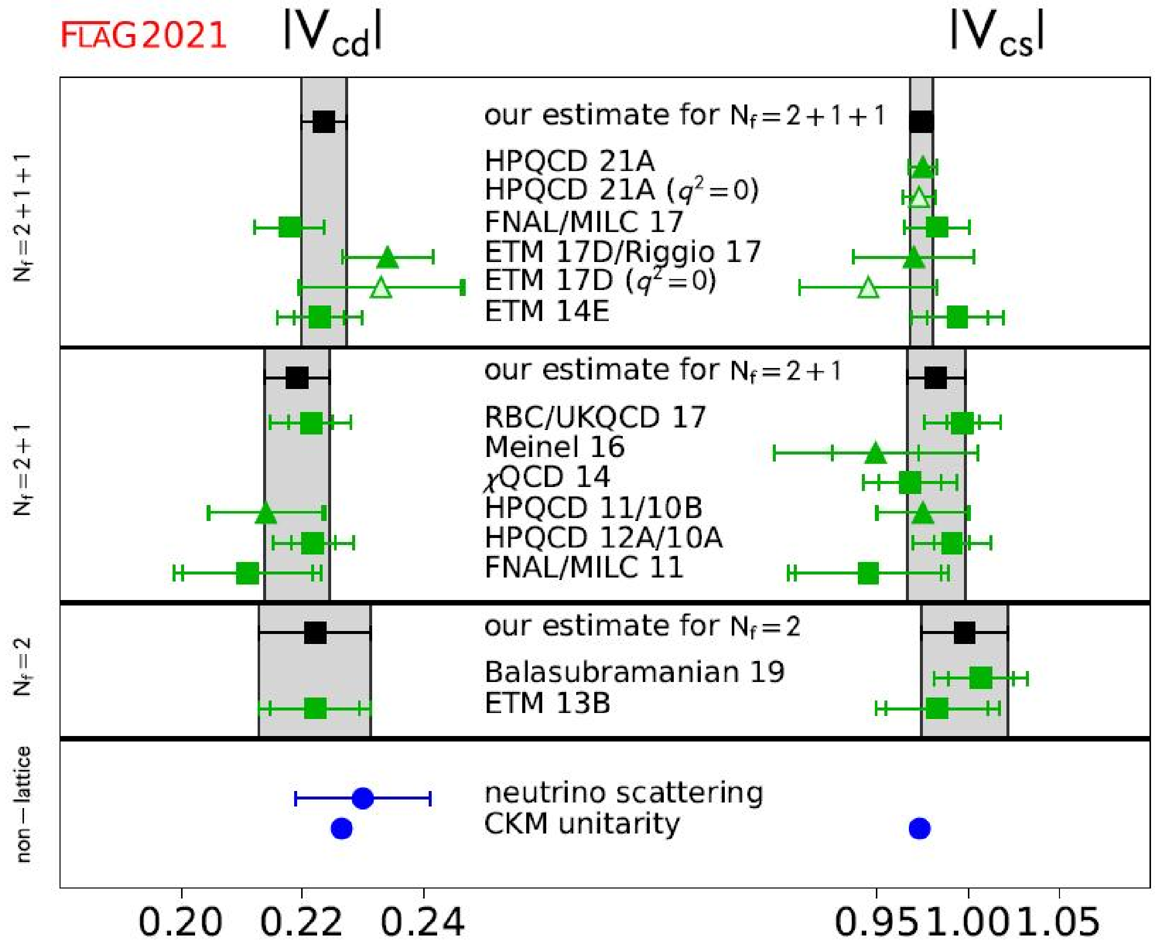


$|V_{cd}|$和$|V_{cs}|$

$f_D|V_{cd}| = 46.2(1.2)$ MeV, $f_{D_s}|V_{cs}| = 245.7(4.6)$ MeV   PDG2020, PTEP2020, 083C01

$f_+^{D\pi}(0)|V_{cd}| = 0.1426(18)$, $f_+^{DK}(0)|V_{cs}| = 0.7180(33)$ HFLAV, EPJC81(2021) 226, 1909.12524

- 纯轻过程，LQCD误差比实验误差小（2+1，2+1+1味）
- 介子半轻过程，LQCD误差比实验误差大
- 使用形状因子的q^2依赖显著降低$|V_{cx}|$的误差
- 重子半轻过程，实验误差主导



CKM幺正性（第二行）在10^{-2}精度上成立（2+1味）

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.01(3)$$

FLAG Review 2021, [arXiv:2111.09849]

总结

- 粲物理相关的格点QCD计算有助于精确检验标准模型
  - 衰变常数
  - 形状因子
- 计算精度向 1% 以内努力，考虑同位旋破缺效应

- 算符重整化常数
- 粲介子衰变常数
- 形状因子

ZL et al. (χQCD), 1312.7628, PRD90.034505
Y. Bi, ..., ZL et al., 1710.08678, PRD97.094501
F. He, ..., ZL et al., 2204.09246, PRD106.114506
Y. Bi, ..., ZL et al., 2302.01659, PRD108.054506
χQCD, PRD92.034517 (2015), arXiv:1410.3343
χQCD, CPC45.023109 (2021), arXiv:2008.05208
Y. Meng et al., 2401.13475, PRD109.074511
D. Li et al. (χQCD), arXiv:2407.03697
沈庭弘, 张振宇, ZL et al., in progress

谢谢！

(后有2024年格点QCD研讨会广告)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