Estimates on the convergence of expansions at finite baryon chemical potentials

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Based on: Rui Wen, Shi Yin, Wei-jie Fu arXiv:2403.06770

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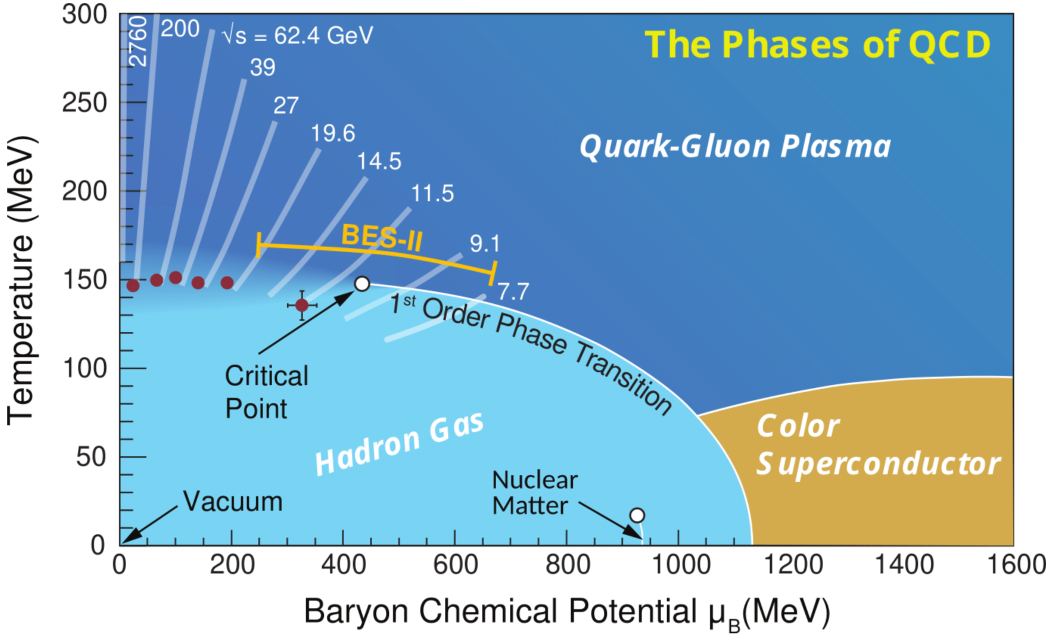
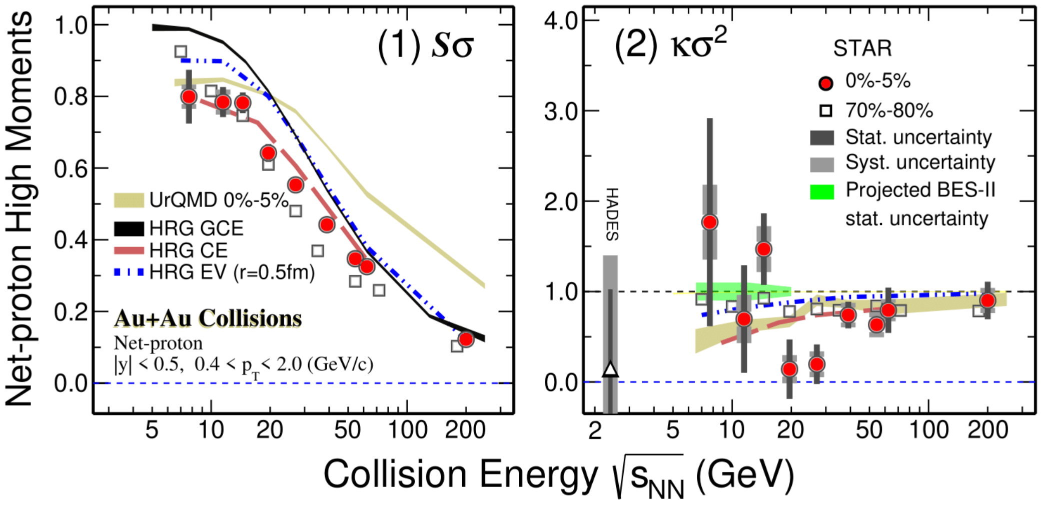
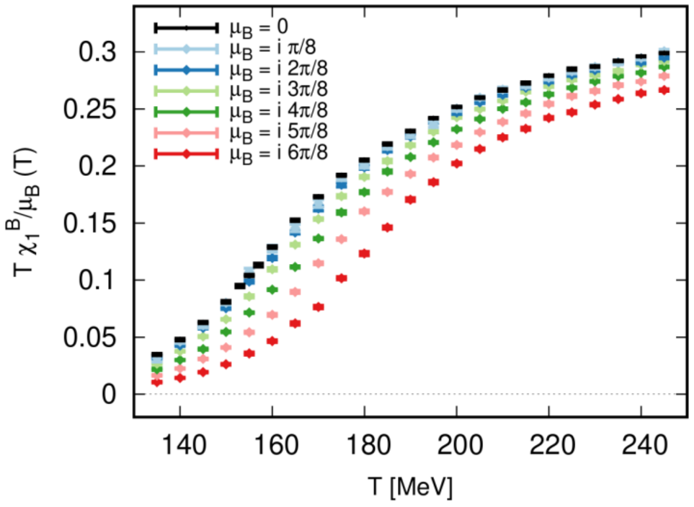
Introduction



Adam Bzdak et.al. Phys.Rept. 853 (2020) 1-87



J. Adam et al. (STAR), Phys. Rev. Lett.126.092301



Taylor expansions and Padé approximants for cumulants of conserved charge fluctuations at nonvanishing chemical potentials HotQCD Collaboration D. Bollweg et.al. Phys.Rev.D 105 (2022) 7, 074511

S. Bors ´any et.al. Phys. Rev. Lett. 126, 232001 (2021)

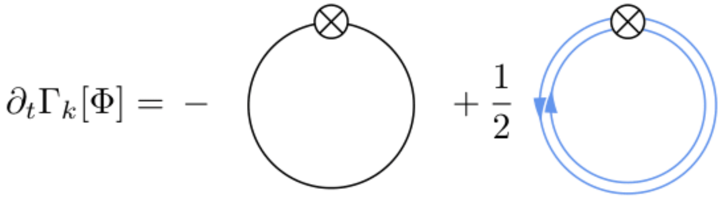
Polyakov-loop extended quark-meson model

The effective action of 2 flavor PQM model

$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}\Big[\gamma_\mu\partial_\mu - \gamma_0(\mu + igA_0)\Big]q + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2$$
$$+h_k\bar{q}(T^0\sigma + i\gamma_5\vec{T}\cdot\vec{\pi})q + V_k(\rho) - c\sigma + V_{\text{glue}}(L,\bar{L})\Big\}$$

$$\phi = (\sigma, \vec{\pi})$$
$$\rho = \phi^2/2$$

LPA: $Z_{q/\phi,k} = 1, \partial_k h_k = 0.$



The flow equation

$$\partial_k V_k(\rho) = \frac{k^3}{4\pi^2}\Big[3l_0^{(B)}(m_\pi) + l_0^{(B)}(m_\sigma) - 4N_cN_f l_0^{(F)}(m_f)\Big]$$

At imaginary chemical potential

$$l_0^{(F)}(m_f) = \frac{k}{3E}(1 - n_f(L,\bar{L}) - \bar{n}_f(\bar{L},L))$$
$$= \frac{k}{3E}[1 - 2\mathrm{Re}(n_f)]\,,$$

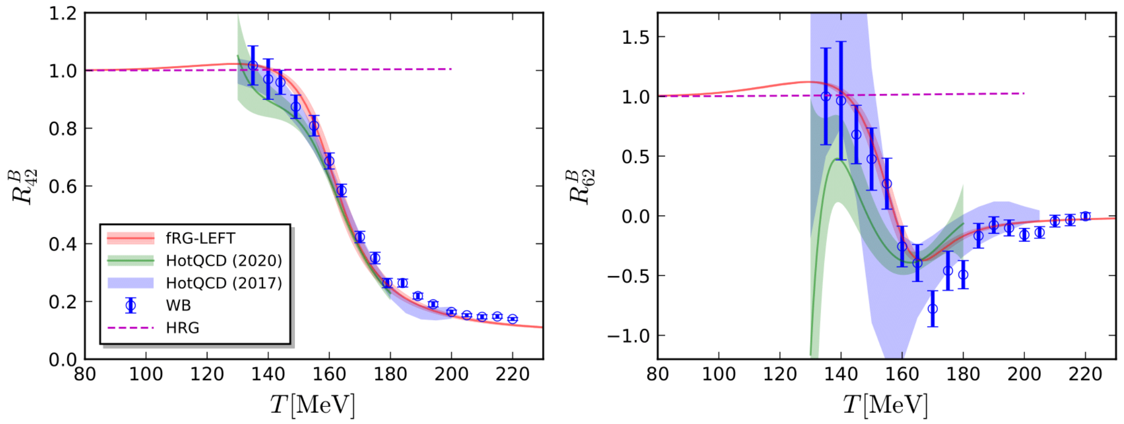
scale-matching

$$T_{\mathrm{QCD}}^{(N_f=2+1)} = c\; T_{\mathrm{PQM}}^{(N_f=2)}$$
$$\mu_{B\,\mathrm{QCD}}^{(N_f=2+1)} = c\; \mu_{B\,\mathrm{PQM}}^{(N_f=2)}$$

$$c = \frac{T_{c\,\mathrm{QCD}}^{(N_f=2+1)}}{T_{c\,\mathrm{PQM}}^{(N_f=2)}} = \frac{156\,\mathrm{MeV}}{215\,\mathrm{MeV}} = 0.726$$

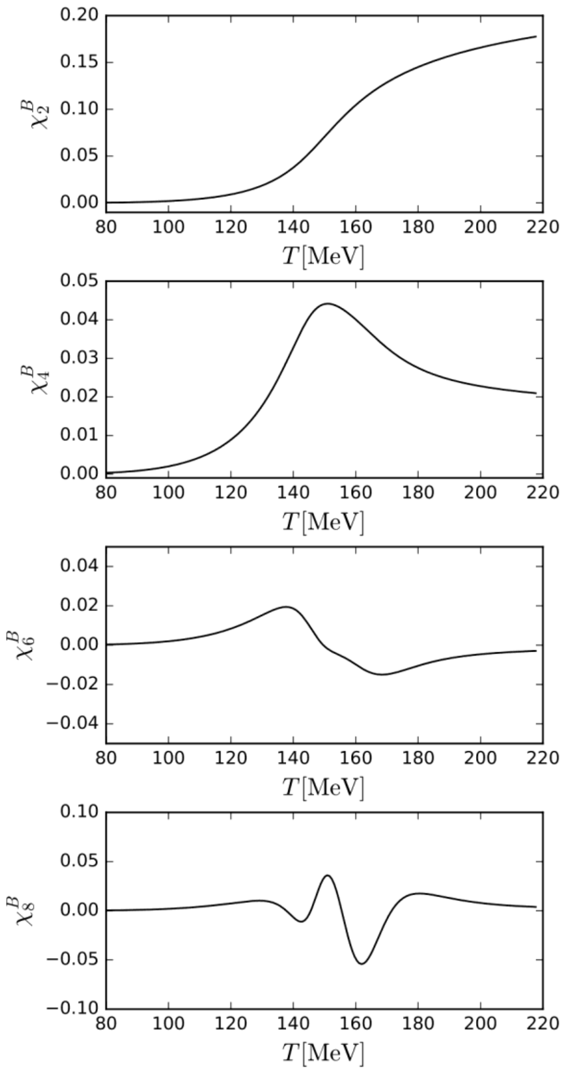
The generalized susceptibilities

$$\chi_i^B = \frac{\partial^i}{\partial \hat{\mu}_B{}^i} \frac{p}{T^4}$$



$R_{42}^B = \chi_4^B / \chi_2^B$ $\qquad$ $R_{62}^B = \chi_6^B / \chi_2^B$

Wei-jie Fu, Xiaofeng Luo, Jan M. Pawlowski, Fabian Rennecke, Rui Wen and Shi Yin arXiv:2101.06035



This work

Expansion scheme

The generalized susceptibilities

$$\chi_i^B = \frac{\partial^i}{\partial \hat{\mu}_B{}^i} \frac{p}{T^4}$$

◆ Taylor expansion

$$\frac{p(T,\hat{\mu}_B) - p(T,0)}{T^4} = \sum_{n=1} \frac{1}{(2n)!} \chi_{2n}^B(T,0) \hat{\mu}_B^{2n}$$

◆ Padé approximants

$$P[m,n] \equiv \frac{p(T,\hat{\mu}_B) - p(T,0)}{T^4} = \frac{\sum_{i=1}^{n/2} a_i \cdot \hat{\mu}_B^{2i}}{1 + \sum_{j=1}^{m/2} b_j \cdot \hat{\mu}_B^{2j}} .$$

$$\frac{\partial^i P[m,n]}{\partial \hat{\mu}_B{}^i} = \chi_i^B$$

For example

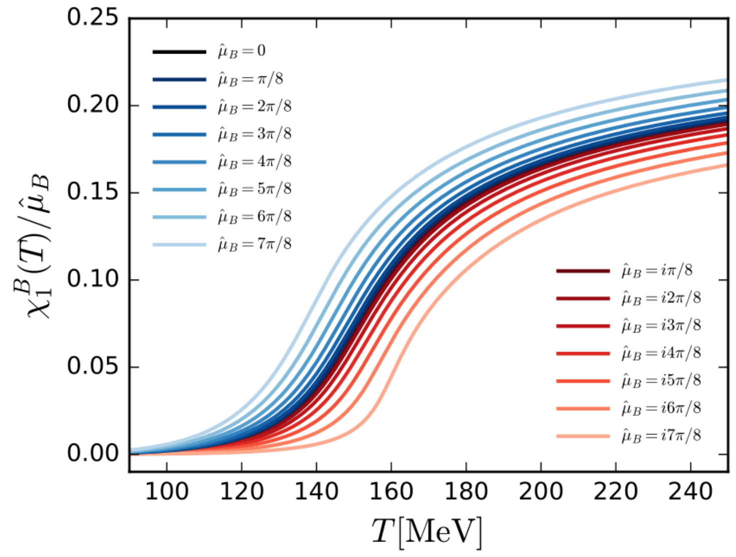
$$P[4,2] = \frac{60\chi_2^B \chi_4^B \hat{\mu}_B^2 + \left(5(\chi_4^B)^2 - 2\chi_2^B \chi_6^B\right)\mu_B^4}{120\chi_4^B - 4\chi_6^B \hat{\mu}_B^2}$$

◆ T ′ expansion

S. Bors ´any et.al. Phys. Rev. Lett. 126, 232001 (2021)

$$\frac{\chi_1^B(T,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0)$$

$$T' = T\left(1 + \kappa_2^B(T)\hat{\mu}_B^2 + \kappa_4^B(T)\hat{\mu}_B^4 + \kappa_6^B(T)\hat{\mu}_B^6 + \mathcal{O}(\hat{\mu}_B^8)\right)$$



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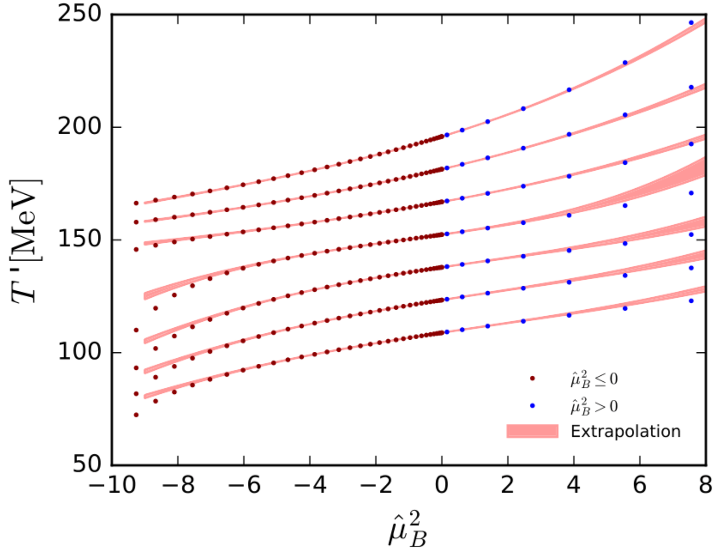
Compare with Taylor expansion

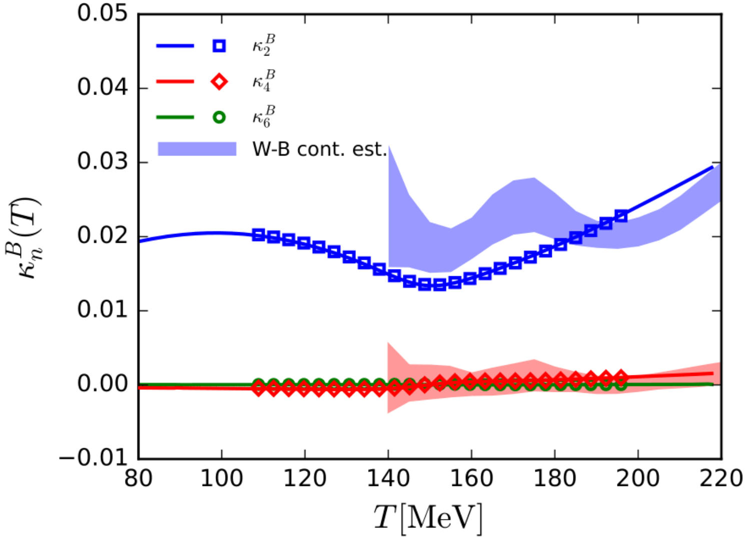
$$\frac{\chi_1^B(T,\hat{\mu}_B)}{\hat{\mu}_B} = \sum_{n=1} \frac{1}{(2n-1)!}\chi_{2n}^B(T,0)\cdot\hat{\mu}_B^{2n-2}\,.$$

$$\frac{\chi_4^B(T)}{3!} = \frac{\partial\chi_2^B}{\partial T}T\kappa_2^B(T)\,,$$

$$\frac{\chi_6^B(T)}{5!} = \frac{\partial\chi_2^B}{\partial T}T\kappa_4^B(T) + \frac{1}{2!}\frac{\partial^2\chi_2^B}{(\partial T)^2}T^2(\kappa_2^B(T))^2\,,$$

$$\frac{\chi_8^B(T)}{7!} = \frac{\partial\chi_2^B}{\partial T}T\kappa_6^B(T) + \frac{1}{2!}\frac{\partial^2\chi_2^B}{(\partial T)^2}T^2(2\kappa_2^B(T)\kappa_4^B(T)) + \frac{1}{3!}\frac{\partial^3\chi_2^B}{(\partial T)^3}T^3(\kappa_2^B(T))^3\,.$$





The markers denote the results obtained from the fitting of imaginary chemical potentials, and the solid lines stand for those calculated Taylor expansion coefficients.

The pressure

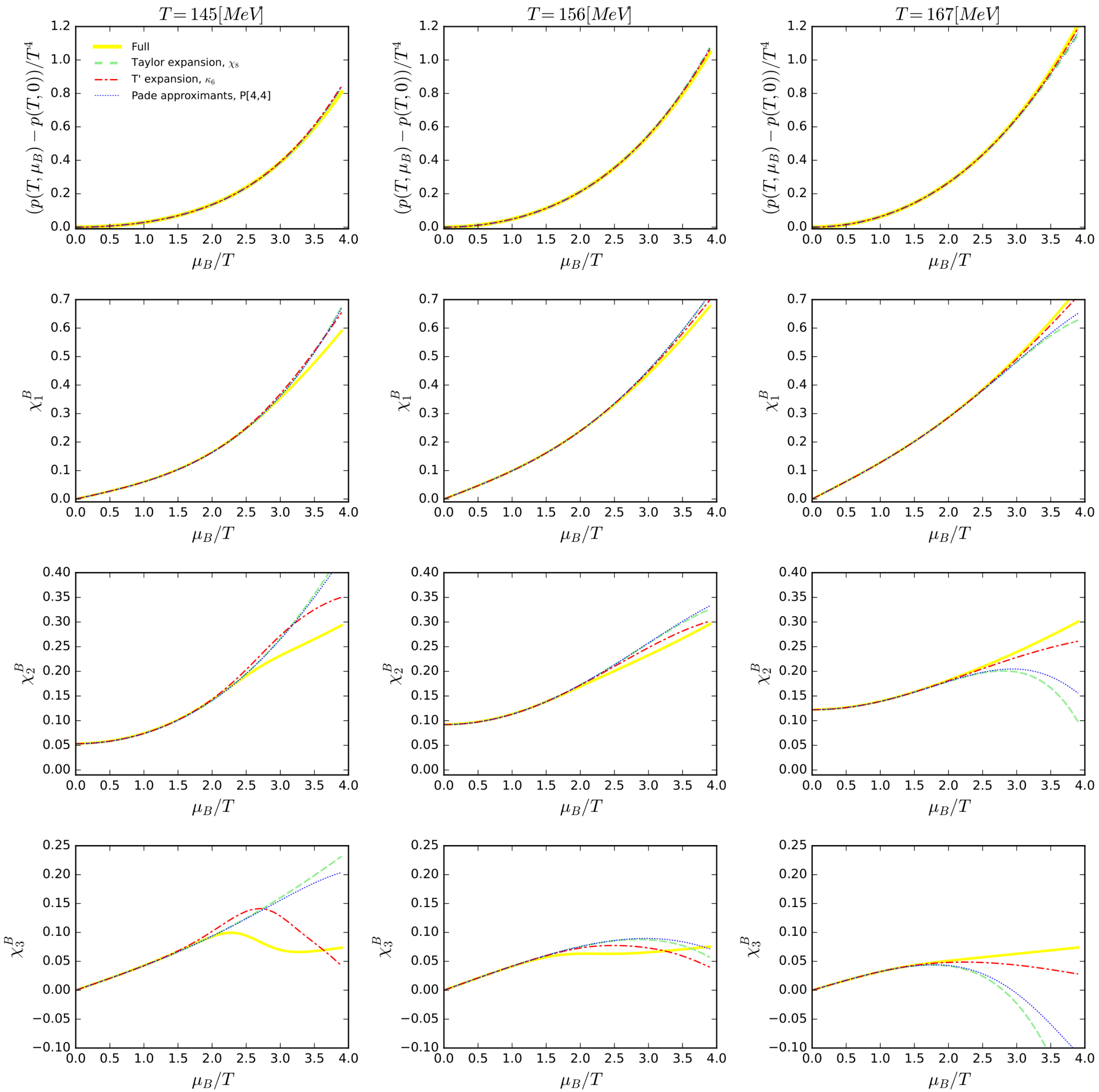
$$\frac{p(T,\hat{\mu}_B)-p(T,0)}{T^4}=\int_0^{\hat{\mu}_B} d\hat{\mu}'_B \chi_1^B(T,\mu'_B) \;=\int_0^{\hat{\mu}_B} d\hat{\mu}'_B \hat{\mu}'_B \chi_2^B(T',0)\,.$$

The first three order generalized susceptibilities

$$\chi_1^B(T,\mu_B) =\hat{\mu}_B \chi_2^B(T',0)\,,$$

$$\chi_2^B(T,\mu_B) =\chi_2^B(T',0)+\hat{\mu}_B \frac{\partial \chi_2^B(T',0)}{\partial T'}\frac{\partial T'}{\partial \hat{\mu}_B}\,,$$

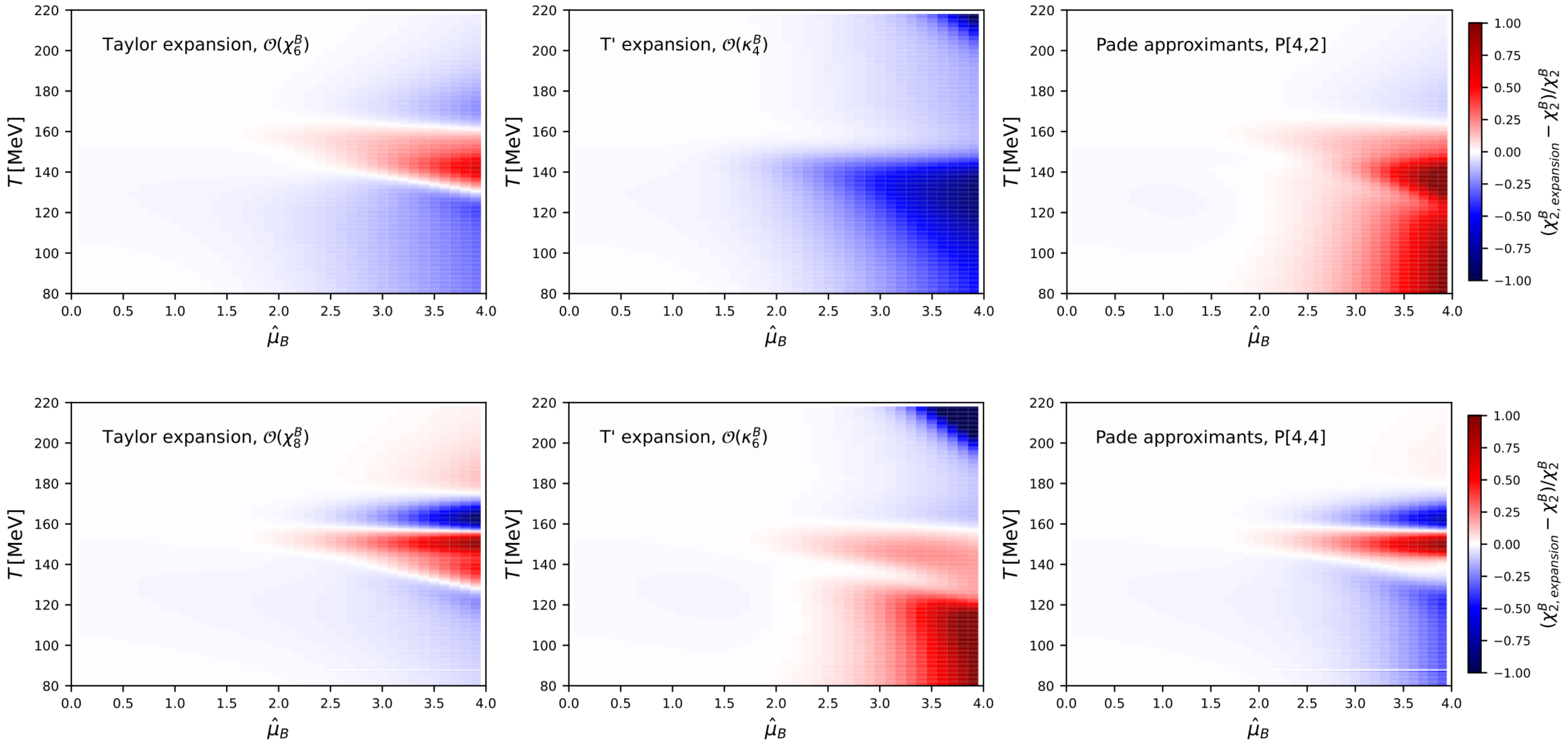
$$\chi_3^B(T,\mu_B) =2\frac{\partial \chi_2^B(T',0)}{\partial T'}\frac{\partial T'}{\partial \hat{\mu}_B}+\hat{\mu}_B\left(\frac{\partial^2 \chi_2^B}{\partial T'^2}\left(\frac{\partial T'}{\partial \hat{\mu}_B}\right)^2+\frac{\partial \chi_2^B}{\partial T'}\frac{\partial^2 T'}{\partial \hat{\mu}_B{}^2}\right).$$



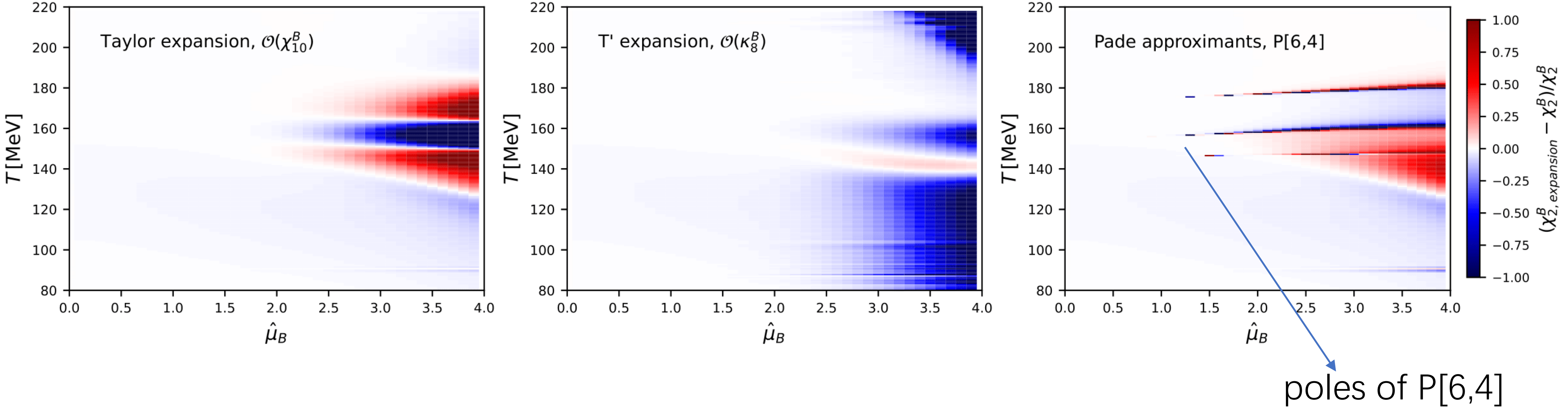
Comparison between the direct calculations of the pressure and the first three order generalized susceptibilities and those with the Taylor expansion, Padé approximants and the T ′ expansion.

$$p, \chi_1^B, \ \chi_2^B, \ \chi_3^B$$

$$\mu_B/T \lesssim 3.5, \ 3.0, \ 2.0, \ 1.5$$



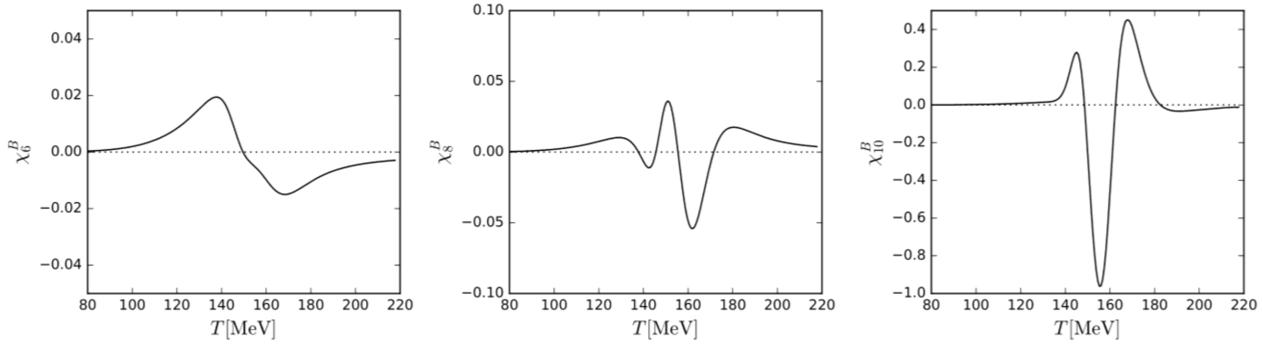
Relative errors of different expansion schemes for χ_2^B in the plane of T and $\hat{\mu}_B$.

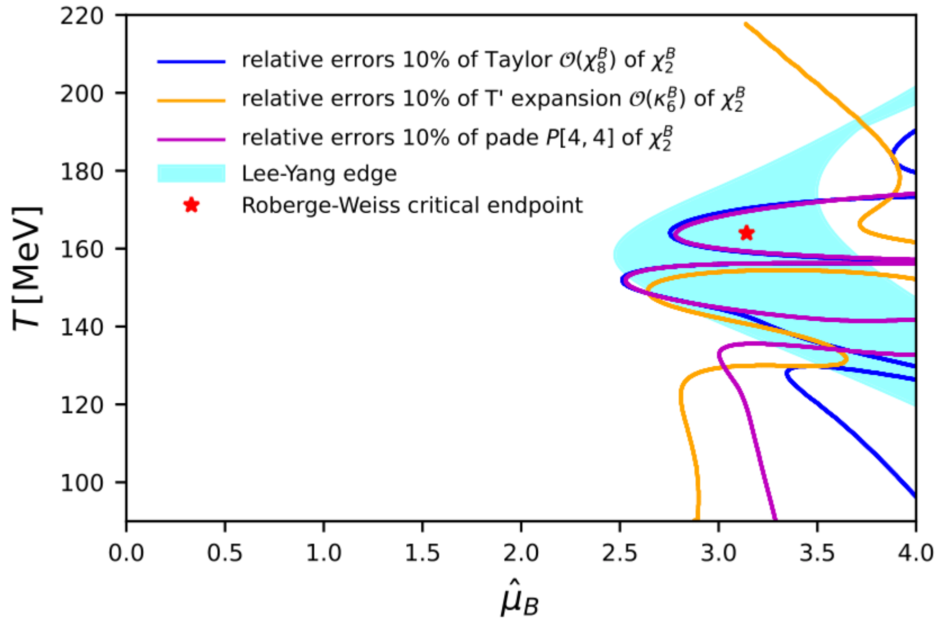


Relative errors of different expansion schemes for χ_2^B in the plane of T and $\hat{\mu}_B$.

Discussing the poles is not meaningful.

$$P[n,2] \xrightarrow{\text{poles}} \frac{(n+1)(n+2)\chi^B_{2n}}{\chi^B_{2n+2}}$$





Lee-Yang edge singularities

$$R_{\text{conv}} = \left| \frac{z_c}{z_0} \left(\frac{m_l^{\text{phys}}}{m_s^{\text{phys}}} \right)^{\frac{1}{\beta\delta}} - \frac{T - T_c^0}{T_c^0} \right|^{\frac{1}{2}} \frac{1}{\sqrt{\kappa_2}}.$$

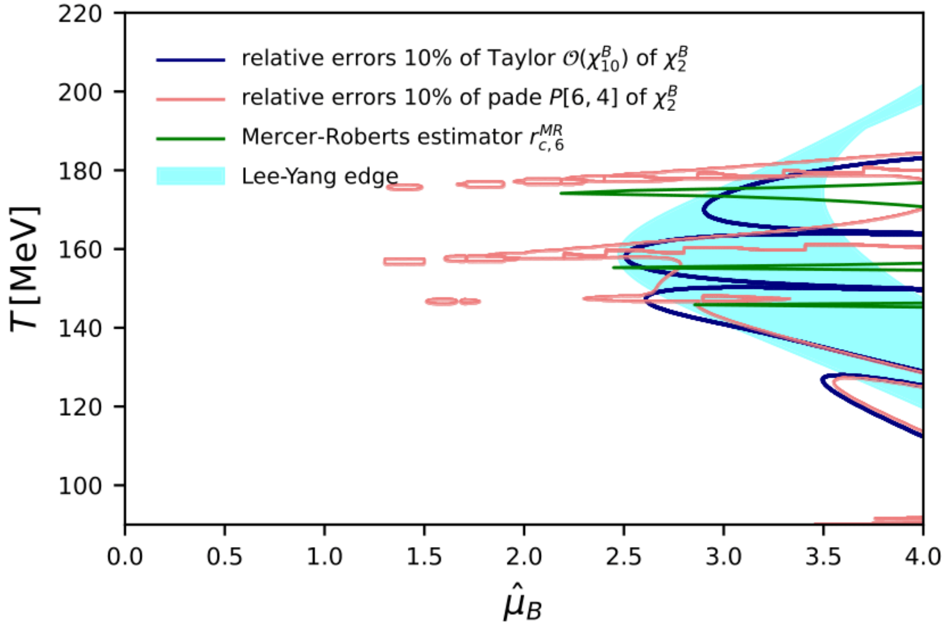
$$m_l^{\text{phys}} / m_s^{\text{phys}} = 1/27$$

$$\beta = 0.3989, \ \delta = 4.975$$

$$\text{the chiral limit } T_c^0 = 142.6 \text{ MeV}$$

$$|z_c| = 1.665 \quad z_0 \in [1, 2]$$

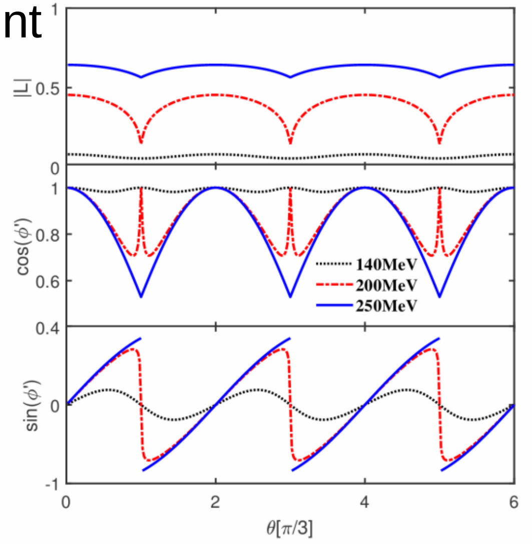
S. Mukherjee and V. Skokov Phys. Rev. D 103, L071501



Roberge-Weiss critical end point

$$\hat{\mu}_B^{RW} = i\pi$$

$$T^{RW} = 164 \text{ MeV}$$



Ke-xin Sun, RW, Wei-jie Fu
Phys.Rev.D 98 074028

Summary

- The consistent region near the critical temperature is smaller than those at high or low temperature.
- The T' expansion or the Padé approximants would hardly improve the consistent regions of expansion in comparison to the conventional Taylor expansion, within the expansion orders considered in this work.
- The consistent regions of the three different expansions are in agreement with the convergence radius of the Lee-Yang edge singularities.

Thanks for your attention