Spicy Gluons (胶麻) 2024: Workshop for Young Scientists on the quark-gluon matter in extreme conditions

Elastic energy loss of heavy quarks in a soft-hard factorized approach

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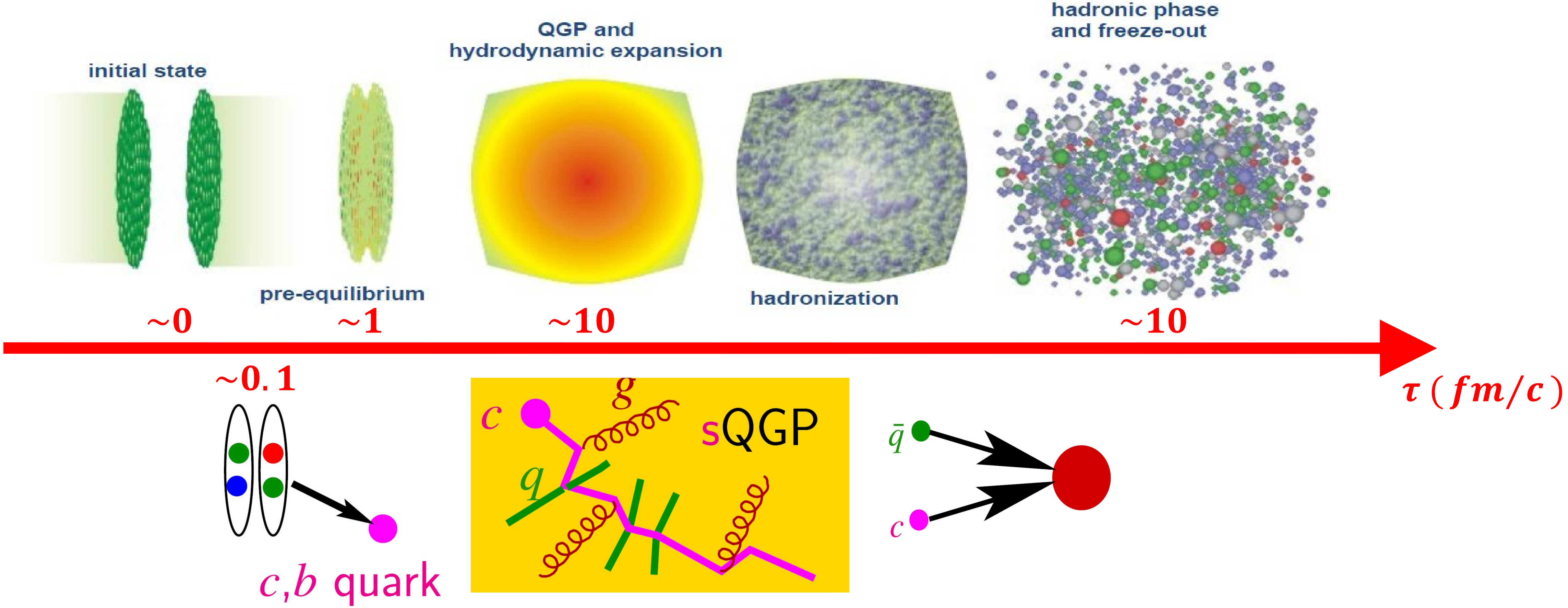
Outline

- Heavy quarks as probes of QGP
- Soft-hard factorization model
- Collision energy loss for heavy quarks
- High-energy approximation
- Transport coefficient
- Summary
- Appendix

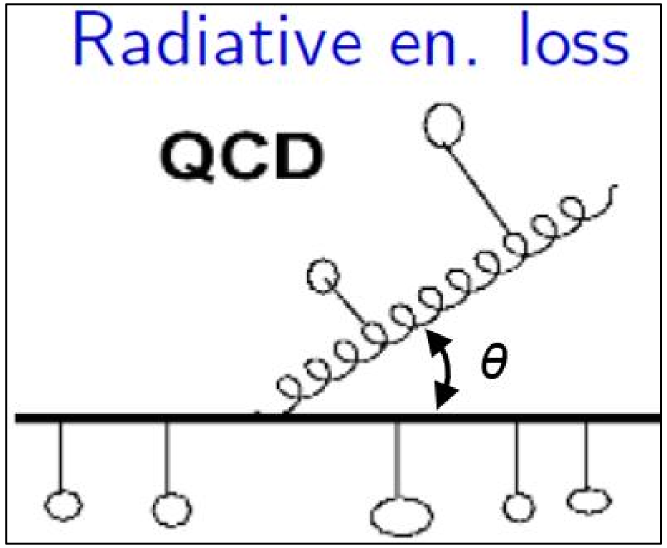
Heavy quarks as probes of QGP

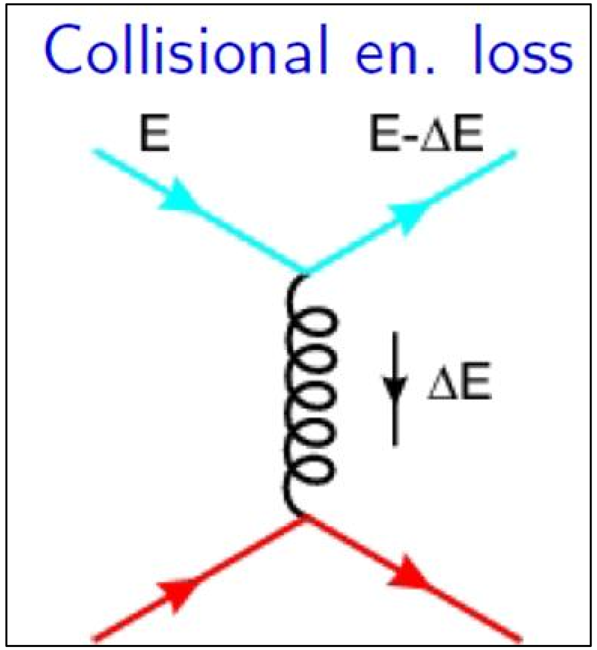


- $m_Q \gg \Lambda_{QCD}$: their initial production can be well described by pQCD
- $m_Q \gg T$: thermal abundance in QGP is negligible ~ final multiplicity set by the initial hard production
- $m_Q \gg gT$: many soft scatterings necessary to change significantly the momentum of HQ~Brownian motion

Heavy quarks as probes of QGP

[for illustration]

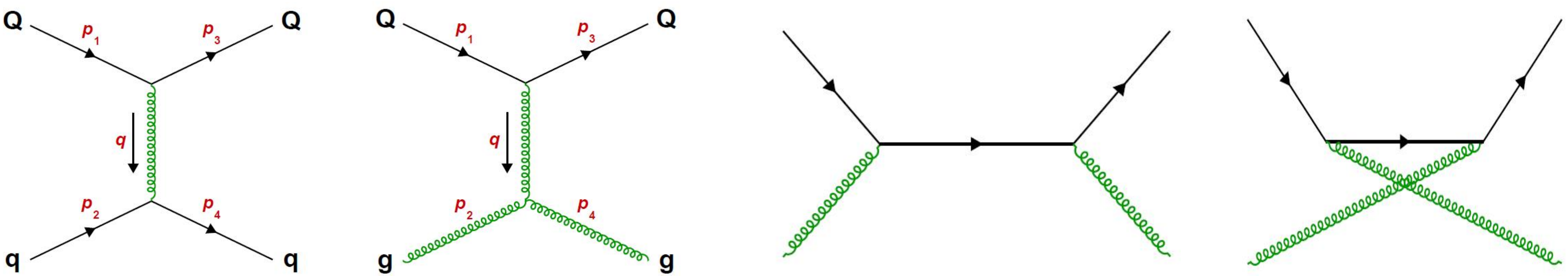




- Energy loss via (in)elastic interaction with the medium constituents
  - ✓ gluon radiation (inelastic) and collisional (elastic)
  - ✓ interplay between them ~ the collisional energy loss is nontrivial at low and moderate energy

Phys. Rev. C 74 (2006) 064907

Soft-hard factorization model



- Divergence from *t*-channel contribution: $\frac{d\sigma}{dt} \propto \overline{|\mathcal{M}^2|} \propto \frac{1}{t^2}$ ~ infrared divergence when $t \to 0$

- Eliminate this divergence via a soft-hard approach
  - ✓ hard collisions: $|t| > |t^*|$, where the pQCD Born approximation is valid
  - ✓ soft collisions: $|t| < |t^*|$, where the *t*-channel long wavelength gluons are screened by the mediums ~ they feel the presence of the medium and require the resummation ~ Hard Thermal Loop (HTL) approximation

$$\int_{-\infty}^{0} dt = \underbrace{\int_{t^*}^{0} dt}_{\textbf{soft}} + \underbrace{\int_{-\infty}^{t^*} dt}_{\textbf{hard}}$$

Collision energy loss for heavy quarks

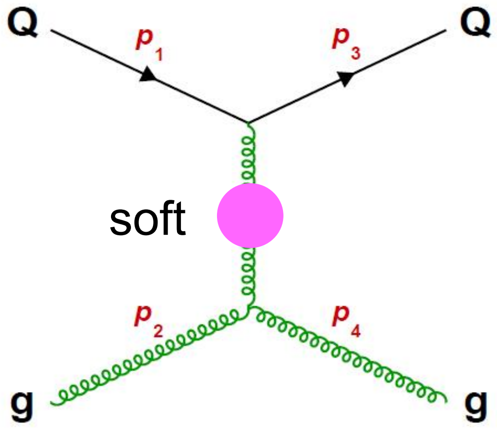
$$-\frac{dE}{dz} = \int d^3\vec{q}\,\frac{d\Gamma}{d^3\vec{q}}\,\frac{\omega}{v_1}$$

$$\Gamma = \Gamma_{(t)}^{soft} + \Gamma_{(t)}^{hard} + \Gamma_{(s+u)}$$

$$-\frac{dE}{dz} = \left[-\frac{dE}{dz}\right]_{(t)}^{soft} + \left[-\frac{dE}{dz}\right]_{(t)}^{hard} + \left[-\frac{dE}{dz}\right]_{(s+u)}$$

Collision energy loss for heavy quarks

- Soft components: $|t| < |t^*|$



✓ the *blob* indicates the dressed gluon propagator

✓ the medium effects are embedded in the HTL gluon self-energy

The relationship between the self-energy and the interaction rate

$$\Gamma(E) = -\frac{1}{2E}(1 - n_F(E))\,\mathrm{tr}[(\not{P} + M)\,\mathrm{Im}\Sigma(P)]$$

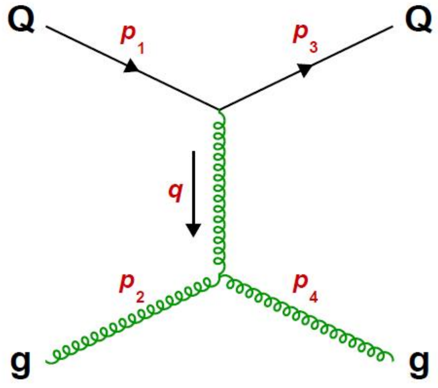
$$\Gamma^{soft}_{(t)}(E_1, T) = C_F g^2 \int_q \int d\omega\, \bar{n}_B(\omega)\delta(\omega - \vec{v}_1 \cdot \vec{q}\,) \left\{\rho_L(\omega, q) + \vec{v}_1^{\,2}\left[1 - (\hat{v}_1 \cdot \hat{q})^2\right]\rho_T(\omega, q)\right\}$$

$\rho_L(\omega,\ q)$ and $\rho_T(\omega,\ q)$ are the transverse and longitudinal parts of the HTL gluon spectral function, respectively

H. A. Weldon, Phys. Rev. D **28**, 2007 (1983)

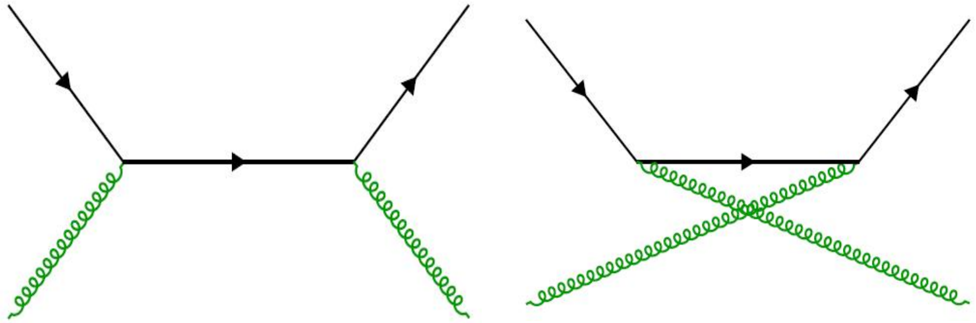
Collision energy loss for heavy quarks

- Hard components: $|t| > |t^*|$



$$\Gamma_{Qi}^{hard}(E_1,T) = \frac{1}{2E_1}\int_{p_2}\frac{n(E_2)}{2E_2}\int_{p_3}\frac{1}{2E_3}\int_{p_4}\frac{\bar{n}(E_4)}{2E_4} \times \overline{|\mathcal{M}^2|}^{Qi}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

- Other contributions： s+u channel



$$\Gamma_{Qg(s+u)}^{hard}(E_1,T) = \frac{1}{2E_1}\int_{p_2}\frac{n(E_2)}{2E_2}\int_{p_3}\frac{1}{2E_3}\int_{p_4}\frac{\bar{n}(E_4)}{2E_4} \overline{|\mathcal{M}^2|}_{Qg(s+u)}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

Collision energy loss for heavy quarks

$$-\frac{dE}{dz}=\int d^3\vec{q}\,\frac{d\Gamma}{d^3\vec{q}}\,\frac{\omega}{v_1}$$

$$\Gamma=\Gamma^{soft}_{(t)}+\Gamma^{hard}_{(t)}+\Gamma_{(s+u)}$$

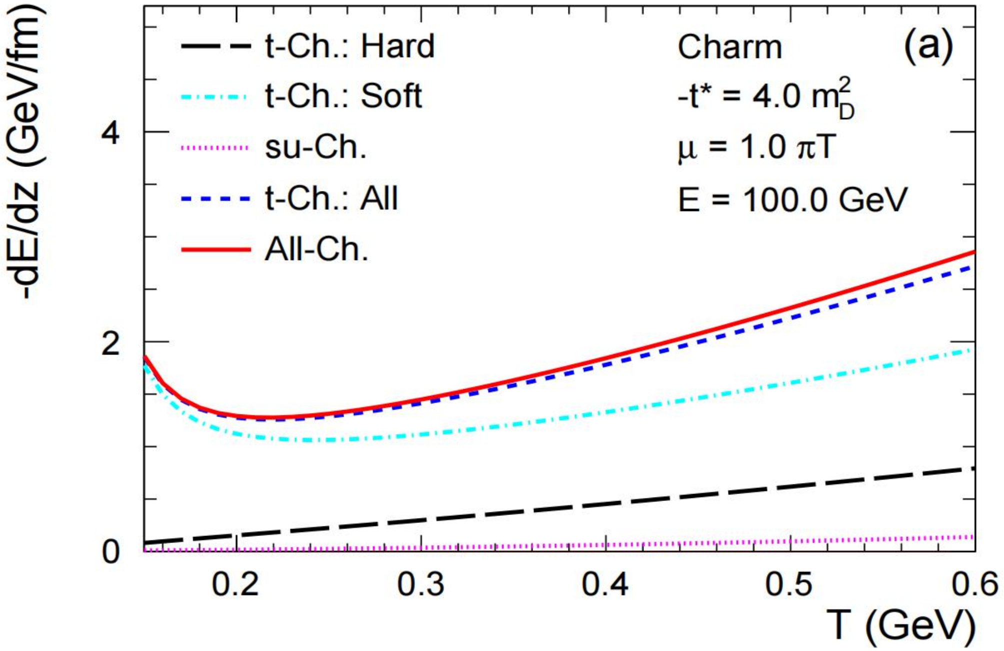
$$-\frac{dE}{dz}=\left[-\frac{dE}{dz}\right]^{soft}_{(t)}+\left[-\frac{dE}{dz}\right]^{hard}_{(t)}+\left[-\frac{dE}{dz}\right]_{(s+u)}$$

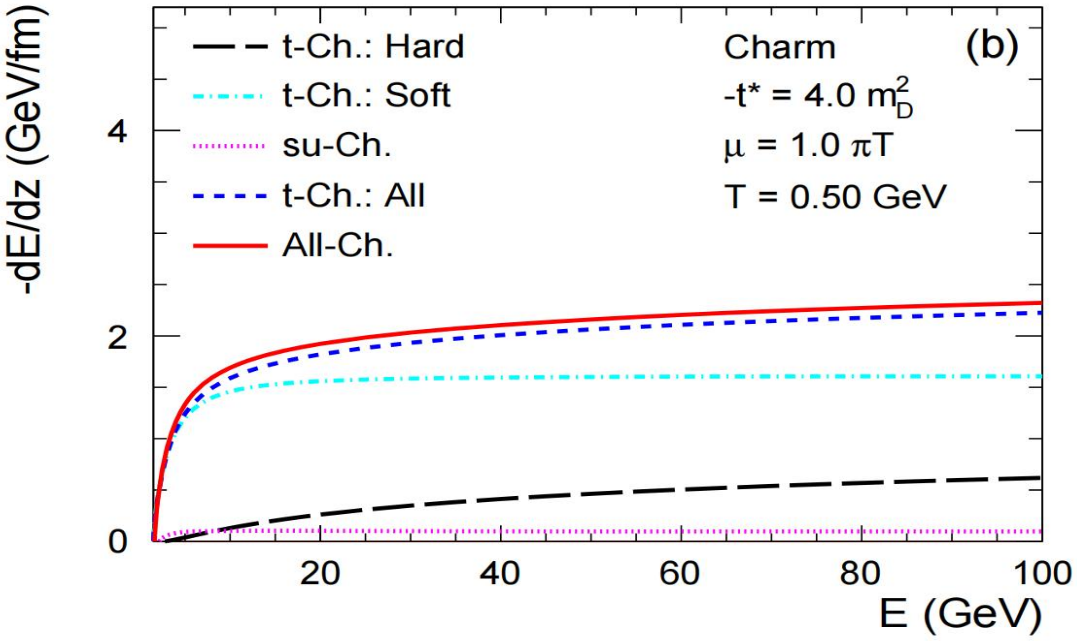
$$\left[-\frac{dE}{dz}\right]^{soft}_{(t)}=\frac{C_F g^2}{8\pi^2 v_1^2}\int_{t*}^{0}dt\,(-t)\int_0^{v_1}dx\,\frac{x}{(1-x^2)^2}\left[\rho_L(t,x)+(v_1^2-x^2)\rho_T(t,x)\right],$$

$$\left[-\frac{dE}{dz}\right]_{(s+u)}=\frac{1}{256\pi^3\vec{p}_1^{\,2}}\int_0^{\infty}d|\vec{p}_2|\;E_2 n(E_2)\int_{-1}^{1}d(cos\psi)\int_{t_{min}}^{0}dt\,\frac{b}{a^3}\,\overline{|\mathcal{M}^2|}_{Qg(s+u)}$$

$$\left[-\frac{dE}{dz}\right]^{hard}_{(t)}=\sum_{i=q,g}\left[-\frac{dE}{dz}\right]^{hard}_{Qi(t)}=\frac{1}{256\pi^3\vec{p}_1^{\,2}}\sum_{i=q,g}\int_{|\vec{p}_2|_{min}}^{\infty}d|\vec{p}_2|\;E_2 n(E_2)\int_{-1}^{cos\psi|_{max}}d(cos\psi)\int_{t_{min}}^{t^*}dt\,\frac{b}{a^3}\,\overline{|\mathcal{M}^2|}_{Qi(t)}$$

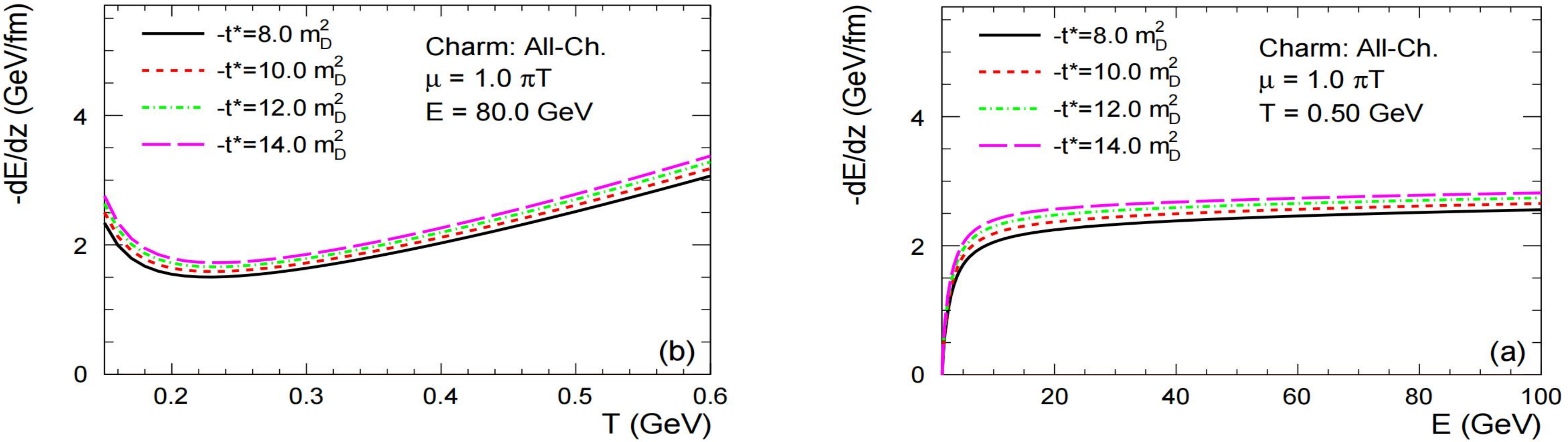
Collision energy loss for heavy quarks





- s + u channel contributions are negligible
- the soft contribution is close to the combined result , reflecting its dominance in the whole temperature range.

Collision energy loss for heavy quarks



- Energy loss behave with a mild sensitivity to the intermediate cutoff scale t^*, supporting the validity of the soft-hard approach when the coupling is not terribly small.

High-energy approximation (HEA)

1.High energy approach $E_1 \to \infty$

2.Weak coupling approximation $m_D^2 \ll -t^* \ll T^2$

3.Large momentum transfer $-t \approx \tilde{s} \approx s \gg m_1^2 \iff -\tilde{u} \ll \tilde{s} \approx s$

$$\left[-\frac{dE}{dz}\right]^{soft}_{(t)} \Rightarrow \left[-\frac{dE}{dz}\right]^{soft-HEA}_{(t)} \approx \frac{C_F}{16\pi}\left(\frac{N_c}{3}+\frac{N_f}{6}\right)g^4T^2 ln\frac{-2t^*}{m_D^2}$$

$$\left[-\frac{dE}{dz}\right]^{hard}_{(t)} \Rightarrow \left[-\frac{dE}{dz}\right]^{hard-HEA}_{Qq(t)} \approx \frac{N_f N_c}{216\pi}g^4T^2\left(ln\frac{8E_1T}{-t^*}-\frac{3}{4}+c\right)$$

$$\left[-\frac{dE}{dz}\right]^{hard-HEA}_{Qg(t)} = \frac{N_c^2-1}{96\pi}g^4T^2\left(ln\frac{4E_1T}{-t^*}-\frac{3}{4}+c\right)$$

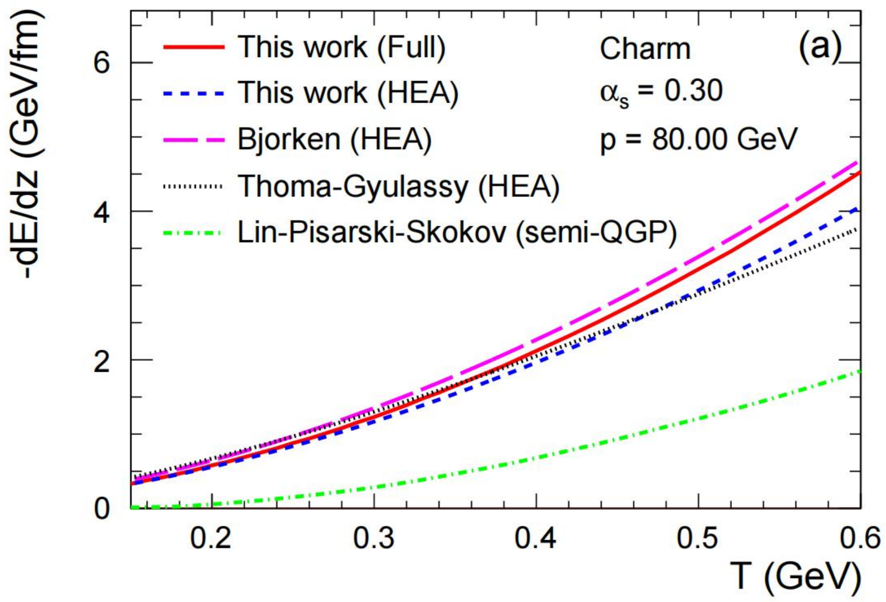
$$\left[-\frac{dE}{dz}\right]_{(s+u)} \Rightarrow \left[-\frac{dE}{dz}\right]^{hard-HEA}_{Qg(s+u)} = \frac{N_c^2-1}{432\pi}g^4T^2\left(ln\frac{4E_1T}{m_1^2}-\frac{5}{6}+c\right)$$

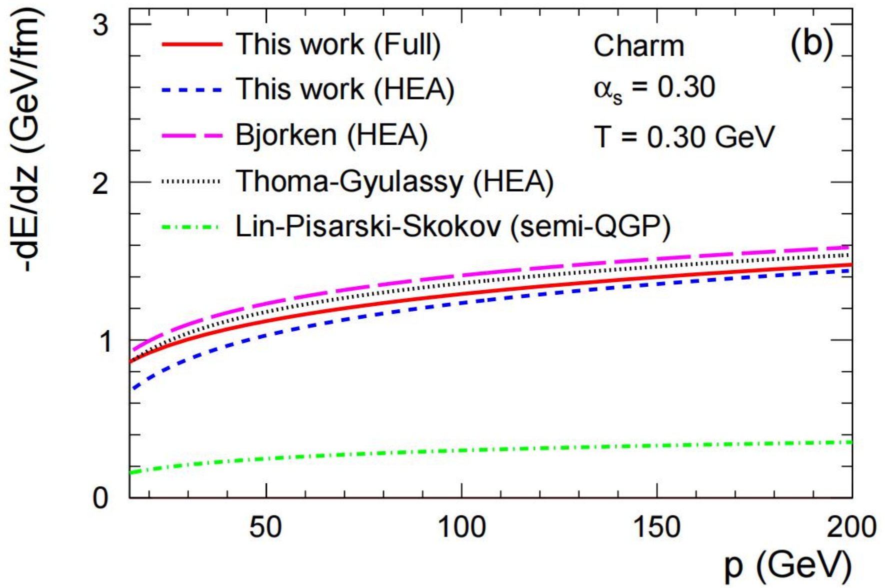
$$\left[-\frac{dE}{dz}\right]^{HEA}_{Qq+Qg} = \frac{4}{3}\pi\alpha_s^2T^2\left[\left(1+\frac{N_f}{6}\right)ln\frac{E_1T}{m_D^2} + \frac{2}{9}ln\frac{E_1T}{m_1^2}+d(N_f)\right],$$

Phys.Rev.D 77,114017(2008)

- The sum of these contributions cancels the t^*-dependence

High-energy approximation (HEA)





Bjorken Fermilab Report No. PUB-82/59-THY (1982)

Limit high energy (heavy quark mass is zero)

Thoma-Gyulassy Nuclear Physics B 351, 491 (1991)

Consider the velocity of the heavy quark

$$v = \sqrt{E^2 - m_Q^2}/E$$

Kinematic reduction of maximum momentum transfer q_{MAX}

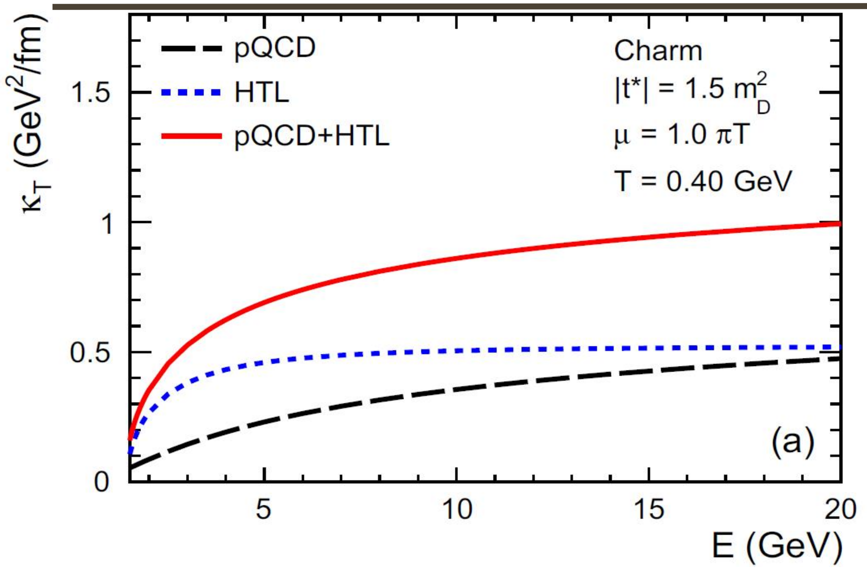
Lin-Pisarski-Skokov

S. Lin, R. D. Pisarski, and V. V. Skokov, Physics Letters B **730**, 236 (2014).

Work in the semi Quark–Gluon Plasma, which assumes that this region is dominated by the non-trivial holonomy of the thermal Wilson line. Collisional energy loss is suppressed by powers of the Polyakov loop

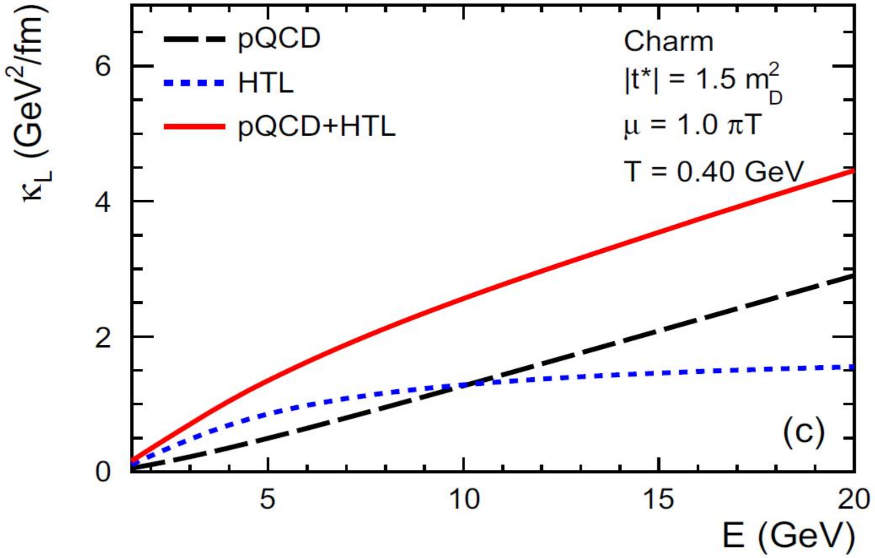
- Qualitatively, all the models give similar results.

Transport coefficient



$$\kappa_T = \frac{1}{2}\int d\Gamma\, \mathbf{q}_T^2 = \frac{1}{2}\int d\Gamma \left[\omega^2 - t - \left(\frac{2\omega E_1 - t}{2|\mathbf{p}_1|}\right)^2\right]$$

$$\kappa_L = \int d\Gamma\, q_L^2 = \frac{1}{4\mathbf{p}_1^2}\int d\Gamma\, (2\omega E_1 - t)^2$$



- It is found that the soft components are significant at low energy, while they are compatible at larger values.

Summary

- We compute the energy loss per traveling distance of a heavy quark dE/dz from elastic scattering off thermal quarks and gluons at a temperature T, including the thermal perturbative description of soft scatterings ($-t <- t^*$) and a perturbative QCD-based calculation for hard collisions ($-t >- t^*$).

- It is found that the full dE/dz has a mild sensitivity to the intermediate scale t^*, supporting the validity of the soft-hard model when the coupling is not terribly small.

- We make a simplification of the complete results and obtain analytical results for the high-energy approximation. And are compared with the other models, qualitatively, all models give similar results

- Finally, we have used a soft-hard factorized model to investigate the heavy quark momentum diffusion coefficients $\kappa_{T/L}$ in the quark-gluon plasma

Thank you for your attention

Appendix

$$\rho_T(\omega, q) = \frac{\pi \omega m_D^2}{2q^3}(q^2-\omega^2)\Bigg\{\Big[q^2-\omega^2 + \frac{\omega^2 m_D^2}{2q^2}\left(1+\frac{q^2-\omega^2}{2\omega q} ln \frac{q+\omega}{q-\omega}\right)\Big]^2 + \left[\frac{\pi \omega m_D^2}{4q^3}(q^2-\omega^2)\right]^2\Bigg\}^{-1} \quad (A.15)$$

$$\rho_L(\omega, q) = \frac{\pi \omega m_D^2}{q}\Bigg\{\Big[q^2 + m_D^2\left(1-\frac{\omega}{2q} ln \frac{q+\omega}{q-\omega}\right)\Big]^2 + \left(\frac{\pi \omega m_D^2}{2q}\right)^2\Bigg\}^{-1}, \quad (A.16)$$

$$|\mathbf{p}_2|_{min} = \frac{|t^*| + \sqrt{(t^*)^2 + 4m_1^2|t^*|}}{4(E_1 + |\mathbf{p}_1|)} \quad (B.28)$$

$$cos\psi|_{max} = min\left\{1, \frac{E_1}{|\mathbf{p}_1|} - \frac{|t^*| + \sqrt{(t^*)^2 + 4m_1^2|t^*|}}{4|\mathbf{p}_1|\cdot|\mathbf{p}_2|}\right\} \quad (B.29)$$

$$t_{min} = -\frac{(s-m_1^2)^2}{s} \quad (B.30)$$

$$\omega_{max/min} = \frac{b \pm \sqrt{D}}{2a^2} \; with \quad (B.31)$$

$$a = \frac{s-m_1^2}{|\mathbf{p}_1|} \quad (B.32)$$

$$b = -\frac{2t}{\mathbf{p}_1^2}\left[E_1(s-m_1^2) - E_2(s+m_1^2)\right] \quad (B.33)$$

$$c = -\frac{t}{\mathbf{p}_1^2}\left\{t\left[(E_1+E_2)^2 - s\right] + 4\mathbf{p}_1^2\mathbf{p}_2^2 sin^2\psi\right\} \quad (B.34)$$

$$D = b^2 + 4a^2c = -t\left[ts + (s-m_1^2)^2\right]\cdot\left(\frac{4E_2 sin\psi}{|\mathbf{p}_1|}\right)^2 \quad (B.35)$$

$$G(\omega) = -a^2\omega + b\omega + c \quad (B.36)$$