Polyakov loop potential and QCD thermodynamics

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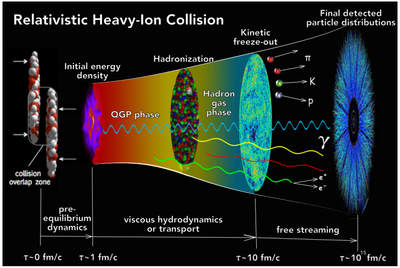
Introduction

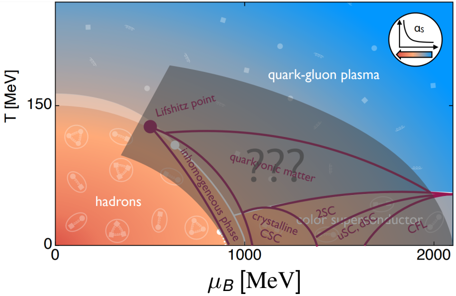
The QCD phase structure and thermodynamics:

- **Evolution** of the early Universe, and the origin of visible mass; heavy-ion collision experiment.
- **Phase transitions** at finite temperature, baryon density, etc.
- **Phenomena:** nuclear liquid-gas transition, critical end point (CEP), spatial modulations, color superconductivity $\cdots$
- **Observables** to connect theory and experiment; e.g. particle yields, (collective) flows, jets, $\cdots$

The Present and Future of QCD, QCD Town Meeting White Paper, 2023.

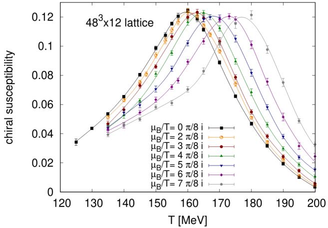
Correlations in a moat regime, F. Rennecke, ECT*Trento, 2023.

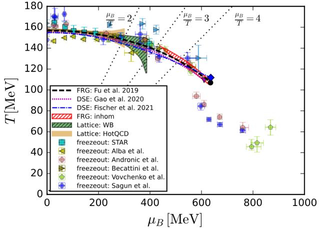


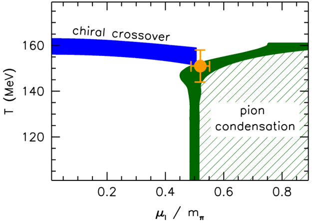


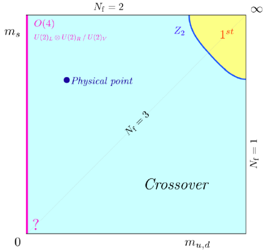
Chiral condensate $\langle\bar{q}q\rangle$, varying T, μ_B, μ_I (isospin), quark mass, N_f, magnetic field, $\cdots$

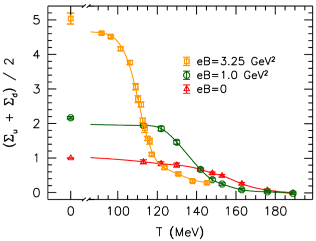
[1] Borsanyi et al., Phys. Rev. Lett. 125: 052001 (2020). [2] Fu, Commun. Theor. Phys. 74: 097304 (2022).
[3] Brandt et al., Phys. Rev. D 97: 054514 (2018). [4] Cuteri et al., JHEP 11 (2021) 141. [5] D'Elia et al., Phys. Rev. D 104: 114512 (2021).











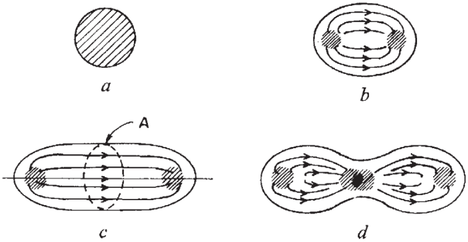
Color/quark confinement: less understood...

Phenomenology:

(i) gluons carry color charge – non-Abelian;

(ii) strong colour-electric flux connecting the quark and (anti-)quark prevents their separation;

(iii) eventually, the energy density is sufficient for another $q\bar{q}$ pair to materialise [1].



Polyakov loop:
the thermal analog of the Wilson loop:

$$L(\boldsymbol{r}) = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} \exp\left[\mathrm{i} \int_0^\beta d\tau\, A_0(\boldsymbol{r}, \tau) \right].$$

with $\beta = 1/T$. In the heavy quark picture [2]:

$$\exp(-\beta F_{q\bar{q}}(\boldsymbol{r})) \xrightarrow{\text{large } |\boldsymbol{r}|} \langle L(\mathbf{0})\rangle \langle L^\dagger(\boldsymbol{r})\rangle.$$

$L = 0$ reflects $F_{q\bar{q}}(\boldsymbol{r}) \to \infty$ ("confinement").

Moreover, Polyakov loop is the order parameter of $Z(N_c)$ center symmetry:

$$L \to zL, \quad z \in Z(N_c).$$

[1] Jaffe, Nature 268: 21 (1977).

[2] McLerran and Svetitsky, Phys. Rev. D, 24: 450 (1981).

Polyakov loop effective potential and the glue dynamics

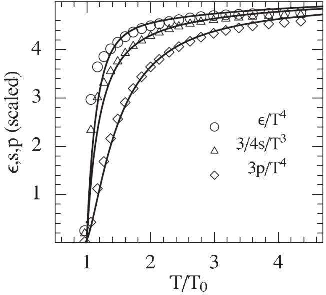
Polyakov loop potential $V[A_0]$ is the effective glue potential in terms of a (spatially constant) temporal background gauge field A_0;

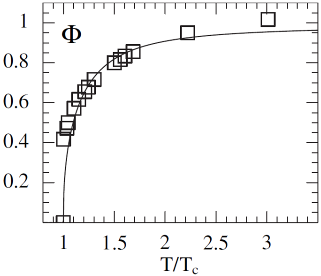
i.e. the glue pressure / free energy.

Fit of $V[A_0]$ by the Yang-Mills lattice QCD pressure, to assist the low energy effective models, e.g. the Nambu–Jona-Lasinio (NJL) model [1,2]:

$$V[A_0] = V[L[A_0], L^\dagger[A_0]; T],$$
$$L_{\text{pNJL}} = L_{\text{NJL}} + \bar{q}\gamma_\mu \delta_{\mu 0} A_0 q - V[A_0].$$

More recent investigations go beyond this, for a direct evaluation of the glue dynamics via the background field A_0 [3,4,5].





[1] Ratti, Thaler and Weise, Phys. Rev. D 73: 014019 (2006). [2] Schaefer, Pawlowski and Wambach, Phys. Rev. D 76: 074023 (2007). [3] Braun, Haas, Marhauser, Pawlowski, Phys. Rev. Lett. 106: 022002 (2011). [4] Fister and Pawlowski, Phys. Rev. D 88: 045010 (2013). [5] Fischer, Fister, Luecker and Pawlowski, Phys. Lett. B 732: 273 (2014).

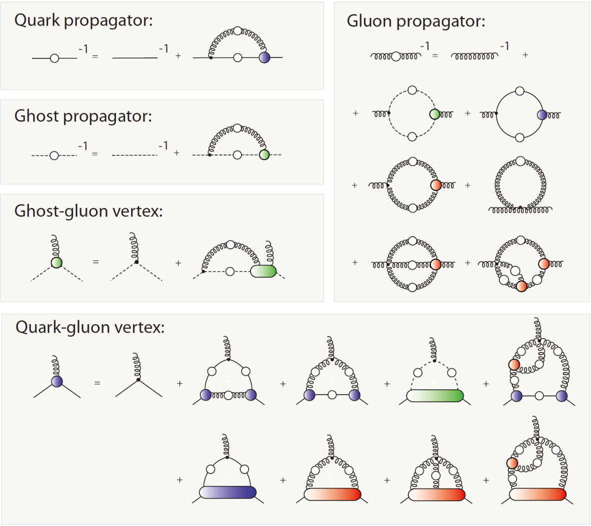
I. A_0 and glue dynamics via Dyson-Schwinger equations

YL, F. Gao, Y. X. Liu and J. Pawlowski, Phys. Rev. D submitted, 2310,18383.
YL, F. Gao, Y. X. Liu and J. Pawlowski, in preparation.

Dyson-Schwinger equations (DSEs) for QCD

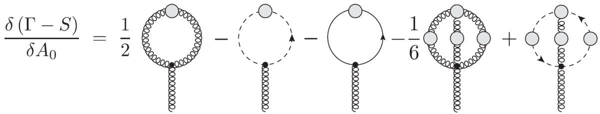
- "Equation of motion" of the quantum field theory - "least action principle".
- Continuum field theory approach for the **full** nonperturbative QCD Green functions.
- Each equation is "nonperturbatively" exact; price to pay: an infinite tower of coupled equations.
- Truncation is required:
  from modeling to gauge symmetry - Slavnov-Taylor identities;
- Recent review on DSE at finite (T, μ_B):
  Fischer, Prog. Part. Nucl. Phys. 105: 1-60 (2019).

(Schematic plots from: S.-X. Qin, Mini-workshop at Nankai University, Tianjin, Jan. 2024)



Polyakov loop potential via DSE

A_0 is the gluon one-point function; the corresponding DSE evaluates $\delta V[A_0]/\delta A_0$ [1]:



in the background field formalism [3]. We start from SU($N_c = 2$) case for example:

$$A_0 = \frac{2\pi T}{g}\varphi\lambda_3, \qquad \frac{\delta V[A_0]}{\delta A_0} \to V'(\varphi) \equiv \partial_\varphi V(\varphi);$$

one-loop truncation [1,2]: $V(\varphi) = V_{\rm glue}(\varphi) + V_q(\varphi) = \frac{1}{2}V_{\rm Weiss}(\varphi) + V_a(\varphi) + V_c(\varphi) + V_q(\varphi)$,

$$V_a'(\varphi) = \sum\!\!\!\!\!\!\!\int_k (\omega_k + 2\pi T\varphi)[G_a^E(\boldsymbol{k}, \omega_k + 2\pi T\varphi) + 2G_a^M(\boldsymbol{k}, \omega_k + 2\pi T\varphi)],$$

$$V_c'(\varphi) = -2\sum\!\!\!\!\!\!\!\int_k (\omega_k + 2\pi T\varphi)G_c(\boldsymbol{k}, \omega_k + 2\pi T\varphi),$$

$$V_q'(\varphi) = -\frac{1}{2}\sum\!\!\!\!\!\!\!\int_p \mathrm{tr}_{C,D}\left[\lambda^3\gamma_4\, G_q(\boldsymbol{p}, \omega_p + \pi T\varphi\lambda^3)\right].$$

[1] Fischer et al., Phys. Lett. B 732: 273 (2014). [2] Fister and Pawlowski, Phys. Rev. D 88: 045010 (2013).
[3] Abbott, Nucl. Phys. B 185 (1981) 189-203.

Polyakov loop potential via DSE

The full QCD propagators $G_{a,c,q}$ (q-quark, a-gluon, c-ghost) at finite (T, μ_B) take input from a recently proposed DSE computation with an optimised truncation scheme [1].

For SU(3), the general structure of A_0:

$$A_0 = \frac{2\pi T}{g} (\varphi_3 \lambda_3 + \varphi_8 \lambda_8) .$$

The eigenvalues $\varphi_{3,8}$ correspond to the representations of gluons / quarks, e.g.:

$$\varphi_3 = \{\pm\varphi, \pm\varphi/2, \pm\varphi/2, 0, 0\} , \qquad \text{adjoint rep.,}$$
$$\varphi_3 = \{\pm\varphi/2, 0\} , \qquad \text{fundamental rep.}$$

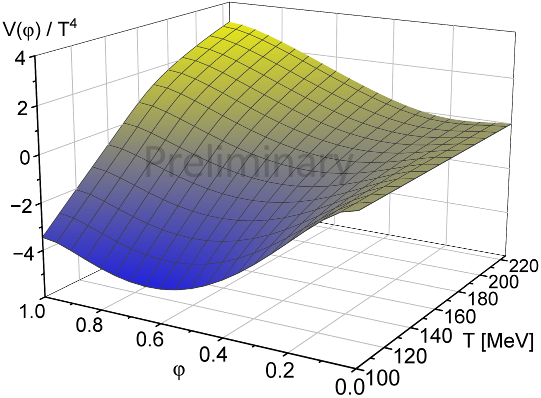
φ_8 reflects the difference between the conjugated Polyakov loops $\mathcal{L}$ and $\mathcal{L}^\dagger$; estimated to be small, see e.g. [3], up to $\mu_B \lesssim 500\,\text{MeV}$.
At present, we just focus on the “center average” gauge [2]: $\varphi_8 = 0$.

[1] **YL**, Gao, Liu and Pawlowski, Phys. Rev. D submitted, 2310,18383.

[2] Fischer, Fister, Luecker and Pawlowski, Phys. Lett. B 732: 273 (2014).

[3] Fu, Pawlowski and Rennecke, Phys. Rev. D 101: 054032 (2020).

Polyakov loop potential via DSE



Full SU(3) potential $V(\varphi)$ at $\mu_B = 0$
as a function of φ and T,

where the SU(3) glue potential (gluon+ghost) can be derived by the decomposition:

$$V_{\text{glue}}^{\text{SU(3)}}(\varphi) = V_{\text{glue}}^{\text{SU(2)}}(\varphi) + 2\,V_{\text{glue}}^{\text{SU(2)}}\left(\frac{\varphi}{2}\right).$$

The minimum of the potential:

$$\partial_\varphi V(\varphi) = 0$$

yields the equation of motion of $A_0(\varphi)$ and the corresponding Polyakov loop:

$$\mathcal{L}(\varphi) = \frac{1}{3}(1 + 2\cos\pi\varphi).$$

Similar shape of $V(\varphi)$ found in:

Fischer et al., Phys. Lett. B 732: 273 (2014).

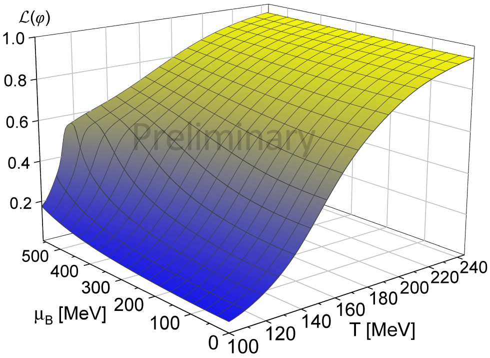
Deconfinement phase transition via DSE

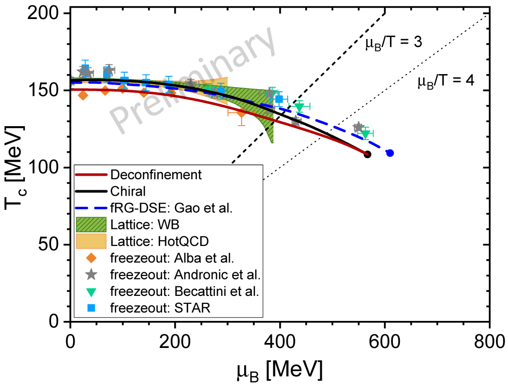
Polyakov loop at finite (T, μ_B), and the deconfinement phase transition:

critical temperature defined by the susceptibility: $\chi_{\mathcal{L}} = \partial\mathcal{L}/\partial T$.

Close connection between the deconfinement and chiral phase transitions:

similar transition temperature; possible coincidence of CEP.





II. Relevance of A_0 / Polyakov loop in QCD thermodynamics

YL, F. Gao, Y. X. Liu and J. Pawlowski, in preparation.

Feed back of A_0 on the full quark propagator:

$$G_q^{-1}(\tilde{p}) = \mathrm{i}(\omega_p + \mathrm{i}\mu_q + gA_0)\gamma_4 \, Z_q^E(\tilde{p}) + \mathrm{i}\gamma \cdot \boldsymbol{p}\, Z_q^M(\tilde{p}) + Z_q^E M_q(\tilde{p}),$$

Impacts on the baryon number density:

$$n_q(A_0(\varphi); T, \mu_q) = -T \sum_{\omega_p} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \mathrm{tr}_{C,D} \left\{\gamma_4 \left[G_q(\boldsymbol{p})\right]\right\},$$

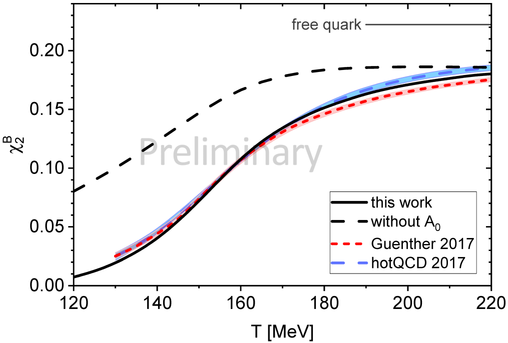
and the baryon number susceptibilities: $\chi_k^B = \dfrac{\partial^k (P/T^4)}{\partial(\mu_B/T)^k} = \dfrac{\partial^{k-1}(n_B/T^3)}{\partial(\mu_B/T)^{k-1}}$.

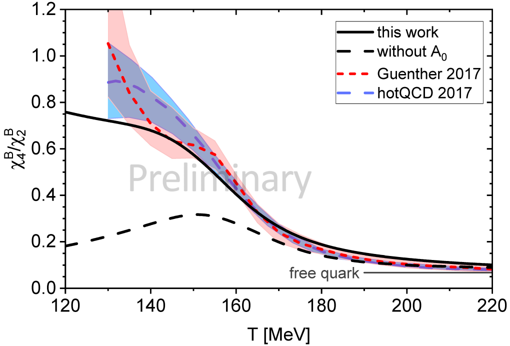
Baryon number density method [1,2] to calculate the QCD thermodynamic functions, i.e. equation of state (EoS):

$$P(T, \mu_B) = P_{Latt.}(T, \mathbf{0}) + \int_0^{\mu_B} n_B(T, \mu)\, d\mu.$$

[1] Gao and Oldengott, Phys. Rev. Lett. 128: 131301 (2022).
[2] **YL**, Gao, Liu and Pawlowski, Phys. Rev. D submitted, 2310,18383.

"Observables" of the deconfinement: baryon number fluctuations.





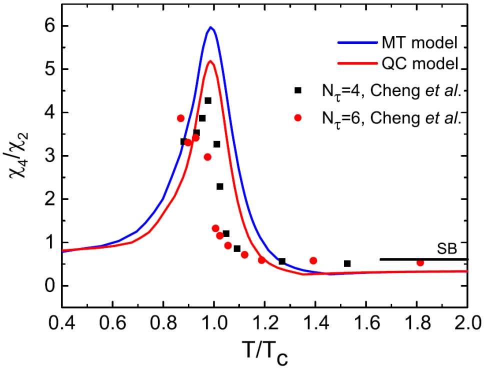
$\mu_B = 0$ results up to $k = 4$ are consistent with the lattice QCD benchmarks [1,2];
Kurtosis χ_4^B/χ_2^B approaches to 1 at low T: the hadron resonance gas (HRG) limit.

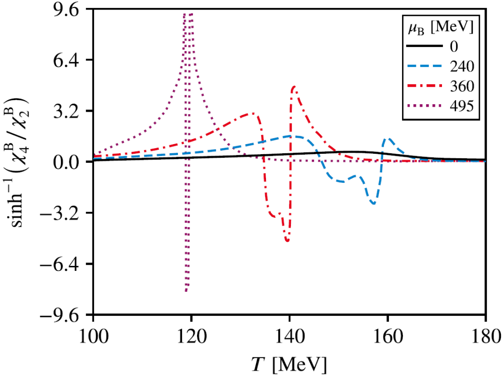
[1] HotQCD Collab. Phys. Rev. D 95: 054504 (2017).
[2] Guenther et al (WB Collab.) Nucl. Phys. A 967: 720-723 (2017).

Kurtosis without A_0: other examples.

Left: Xin, Qin and Liu, Phys. Rev. D 90: 076006 (2014) $(\chi_4^q/\chi_2^q \simeq 9\,\chi_4^B/\chi_2^B$ is plotted);

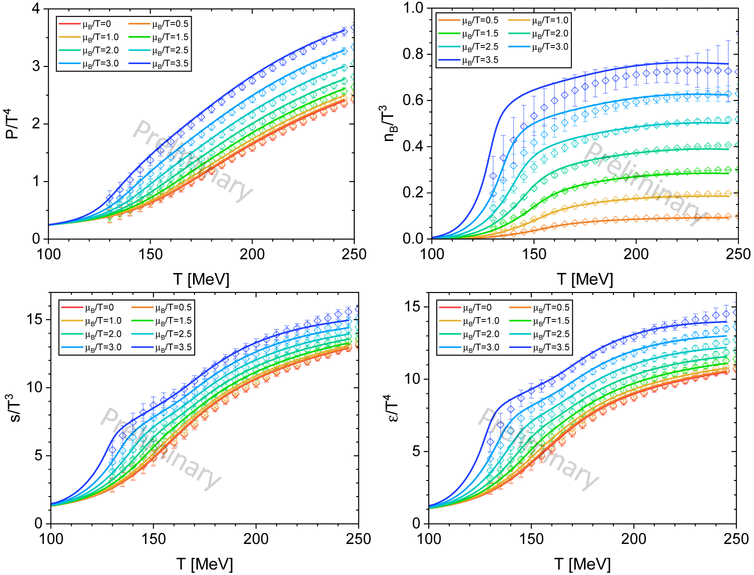
Right: Isserstedt, Buballa, Fischer and Gunkel, Phys. Rev. D 100: 074011 (2019).





Gluon background field and QCD thermodynamics

Equation of state: DSE (functional) vs. lattice QCD (extrapolation) [Borsanyi et al. PRD (2022)].



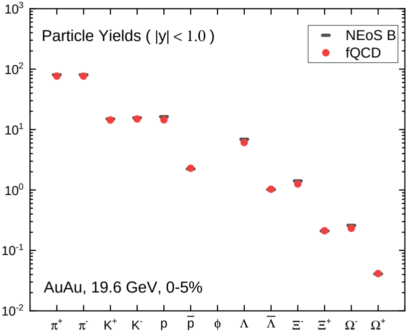
III. Impacts of Polyakov loop in the “Little Bang” and “Big Bang”

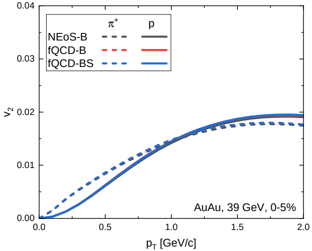
YL, F. Gao, B. C. Fu, H. C. Song, Y. X. Liu, Phys. Rev. D accepted, 2310.16345.

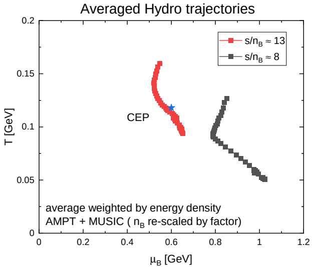
F. Gao, J. Harz, C. Hati, **YL**, I. Oldengott and G. White, Phys. Rev. Lett. submitted, 2309.00672.

Order parameter framework for EoS [1]: allows direct incorporation of the Polyakov loop data (fRG) [2]; Combined with the hydrodynamic simulation (MUSIC). (NEoS: HRG + lattice QCD)

(Left: viscous hydro.; Right: ideal)





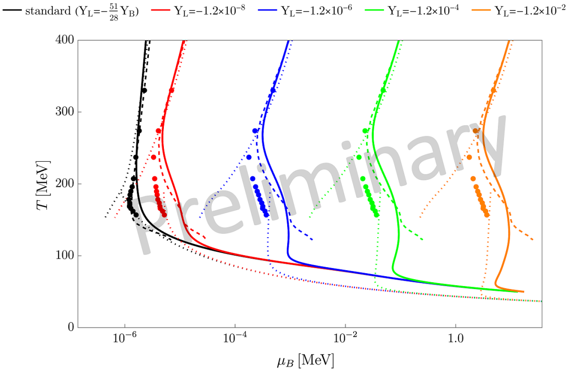


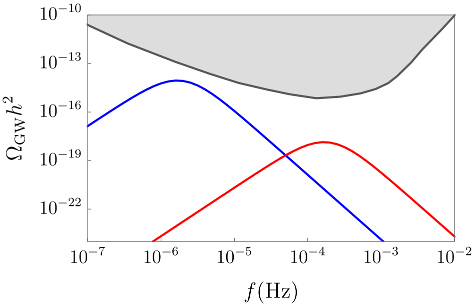
[1] **YL**, Gao, Fu, Song, Liu, 2310.16345.

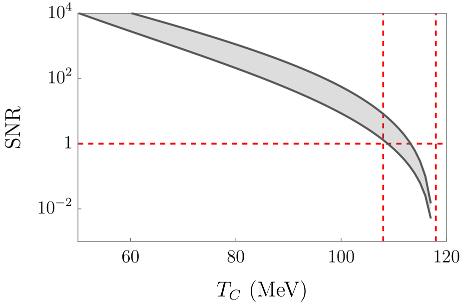
[2] Fu and Pawlowski, Phys. Rev. D 92: 116006 (2015).

"Big Bang": cosmological phase transition (the early Universe)

- Impacts on the cosmological trajectories:
  solid: with A_0; dashed: w/o. A_0; dotted: HRG [1];
- Gravitational wave signal in the sound shell model [2]
  for the cosmological 1st-order phase transition.







[1] Gao, Harz, Hati, **YL**, Oldengott and White, 2309.00672.
[2] Guo, Sinha, Vagie and White, JCAP 01 (2021) 001.

Summary and outlook

- Polyakov loop / A_0 is highly relevant for the QCD thermodynamics, particularly in the hadronic phase.
- Functional QCD approach confronted with the extrapolated lattice QCD EoS at finite μ_B.
- Impacts on the evolution of quark-gluon matter in HIC and the early Universe.

In the future:

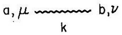
- Separation of $\mathcal{L}$ and $\mathcal{L}^\dagger$: improvements on φ_8 at finite μ_B.
- Higher order susceptibilities of χ_k^B, $k \geq 6$, and the critical phenomena.
- Charge and strangeness χ^Q and χ^S; also the cross correlators χ^{BQ} and χ^{BS}.
- More combined studies on the dynamical evolution.

Thanks for your attention!!

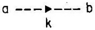
Back-up

Feynmann rules in the Landau–de-Witt gauge

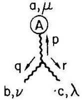
Abbott, Nucl. Phys. B 185 (1981) 189-203



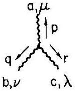
$$\frac{-i\delta_{ab}}{k^2+i\epsilon}\left[g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}(1-\alpha)\right]$$



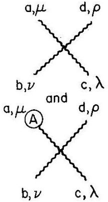
$$\frac{i\delta_{ab}}{k^2+i\epsilon}$$



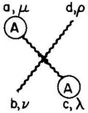
$$gf_{abc}\left[g_{\mu\lambda}\left(p-r-\frac{1}{\alpha}q\right)_\nu + g_{\nu\lambda}(r-q)_\mu + g_{\mu\nu}\left(q-p+\frac{1}{\alpha}r\right)_\lambda\right]$$



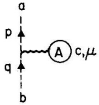
$$gf_{abc}\left[g_{\mu\lambda}(p-r)_\nu + g_{\nu\lambda}(r-q)_\mu + g_{\mu\nu}(q-p)_\lambda\right]$$



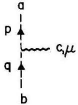
$$-ig^2\left[f_{abx}f_{xcd}(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}) + f_{adx}f_{xbc}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}) + f_{acx}f_{xbd}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\rho}g_{\nu\lambda})\right]$$



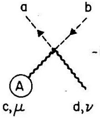
$$-ig^2\left[f_{abx}f_{xcd}\left(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}+\frac{1}{\alpha}g_{\mu\nu}g_{\lambda\rho}\right) + f_{adx}f_{xbc}\left(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-\frac{1}{\alpha}g_{\mu\rho}g_{\nu\lambda}\right) + f_{acx}f_{xbd}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\rho}g_{\nu\lambda})\right]$$



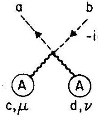
$$-gf_{acb}(p+q)_\mu$$



$$-gf_{acb}\,p_\mu$$



$$-ig^2 f_{acx}f_{xdb}g_{\mu\nu}$$



$$-ig^2 g_{\mu\nu}(f_{acx}f_{xdb}+f_{adx}f_{xcb})$$

Mind the "gap"

There are two alternative descriptions to the Polyakov potential:

(1) directly via background field A_0; more specifically, it is $\langle A_0 \rangle$.

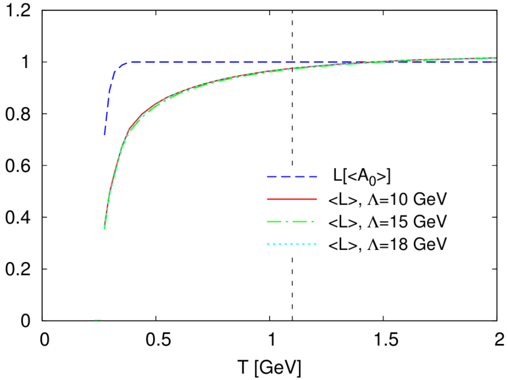
(2) via $\langle \mathcal{L}[A_0] \rangle$: expectation value of the traced loop.

(1) and (2) do not necessarily yield the same Polyakov loop, both for Yang-Mills theory and for full QCD. The Jensen inequality yields [1,2]:

$$\mathcal{L}[\langle A_0 \rangle] \geq \langle \mathcal{L}[A_0] \rangle,$$

the gap manifests the non-Gaussianity of the quark-gauge field / quark-Polyakov loop fluctuations: $\frac{1}{N_c^2}\langle \mathrm{Tr}\, P_1 P_2 \rangle \neq \langle L_1 \rangle \langle L_2 \rangle$ [2].

(Plot taken from Ref. [2])

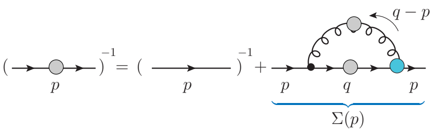


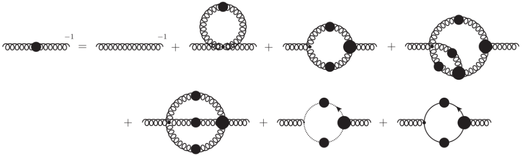
[1] Braun, Gies and Pawlowski, Phys. Lett. B 684 (2010) 262-267.

[2] Herbst, Luecker and Pawlowski, arXiv: 1510.03830.

Optimised Dyson-Schwinger equations (DSEs) scheme

- Optimised tensor structures of the quark-gluon vertex [1,2,3];
- Self-consistent solutions for the quarks and gluons at finite (T, μ_B);
- Quantitatively reliable without model parameters.





$$\mathcal{T}_1^{\mu}(p,q) = \gamma^{\mu}, \quad \mathcal{T}_4^{\mu}(p,q) = \sigma_{\mu\nu} k^{\nu},$$

$$\lambda_1(p,q) = Z_c^{-1}(k^2)\frac{A(p^2)+A(q^2)}{2},$$

$$\lambda_4(p,q) = Z_A^{1/2}(k^2)\frac{B(p^2)-B(q^2)}{p^2-q^2}.$$

[1] Williams, Eur. Phys. J. A 51 (2015) 5, 57

[2] Cyrol, Mitter, Pawlowski and Strodthoff, Phys. Rev. D 97 (2018): 054006

[3] Gao, Papavassiliou and Pawlowski, Phys. Rev. D 103 (2021): 094013

* **YL**, Gao, Liu, Pawlowski, 2310.18383.

φ_8 and finite μ_B

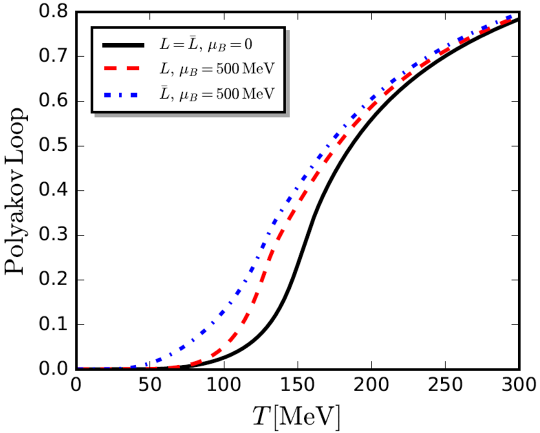
φ_8 manifests the difference between the conjugated Polyakov loops $\mathcal{L}$ and $\mathcal{L}^\dagger$:

$$\mathcal{L} = \frac{1}{3}\left[e^{-\mathrm{i}\frac{2\pi\varphi_8}{\sqrt{3}}} + 2e^{\mathrm{i}\pi\frac{\varphi_8}{\sqrt{3}}}\cos\pi\varphi_3\right].$$

Typically, $\varphi_8 = 0$ is the true minimum of the Polyakov potential at $\mu_B = 0$.

Current investigation shows that for 2+1-flavour QCD, such difference is small up to $\mu_B \approx 500\,\mathrm{MeV} \approx 0.8\,\mu_B^{\mathrm{CEP}}$.

Fu, Pawlowski and Rennecke, Phys. Rev. D 101: 054032 (2020).



Order parameter framework for QCD EoS

Partition function $Z = Z[G_X]$ in the quantum field theory is a functional of Green functions $\{G_X\}$; involves $X =$ quark, gluon, ghost etc.; G_X are in general complicated.

Alternative variables: $Z = Z[\mathcal{O}_X]$, where $\mathcal{O}$ are the **order parameters**: similar motivation as the density functional theory (DFT).

Connection between the order parameters and the quantum field:

$$\begin{aligned} &\mathcal{O}_q = \langle \bar{q}q \rangle \leftrightarrow M_q \text{ (mass function)}, && \text{for chiral phase transition,} \\ &\mathcal{O}_A = \langle A_0 \rangle \leftrightarrow \mathcal{L} \text{ (Polyakov loop)}, && \text{for deconfinement phase transition.} \end{aligned}$$

construction of an effective propagator $G_q[\mathcal{O}_X]$:

$$G_q^{-1}(\tilde{p}) \simeq \mathrm{i}(\omega_p + \mathrm{i}\mu_q + gA_0)\gamma_4 + \mathrm{i}\gamma \cdot \boldsymbol{p} + M_q.$$

The two order parameters have been widely studied in functional QCD; further proposed an Ising-type parametrisation [1] for the M_q data; see [2] for details.

[1] Parotto, Bluhm, Mroczek et.al. Phys. Rev. C 101: 034901 (2020).

[2] **YL**, Gao, Fu, Song, Liu, 2310.16345.