Thermalization of the Wigner function

— a real time, non-perturbative quantum simulation based on the Schwinger Model

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In collaboration with Shuzhe Shi and Li Yan

2024/05/16 Spicy Gluons in Hefei

Thermalization of Quark Gluon Plasma

QGP in heavy-ion collisions:

How fast does it thermalize/isotropize

what are the most important processes contributing to this?

Schwinger model (1+1D QED)

$$H = \int \left(\bar{\psi}\left(\gamma^1(-i\partial_z - g A_1) + m\right)\psi + \frac{1}{2}\mathcal{E}^2 \right) \mathrm{d}z$$

Gauss law (Euler-Lagrange equation of gauge field) $\mathcal{E}(x) = g\int_0^x \bar{\psi}\gamma^0\psi$

Gauge fixing $A_1 = 0$

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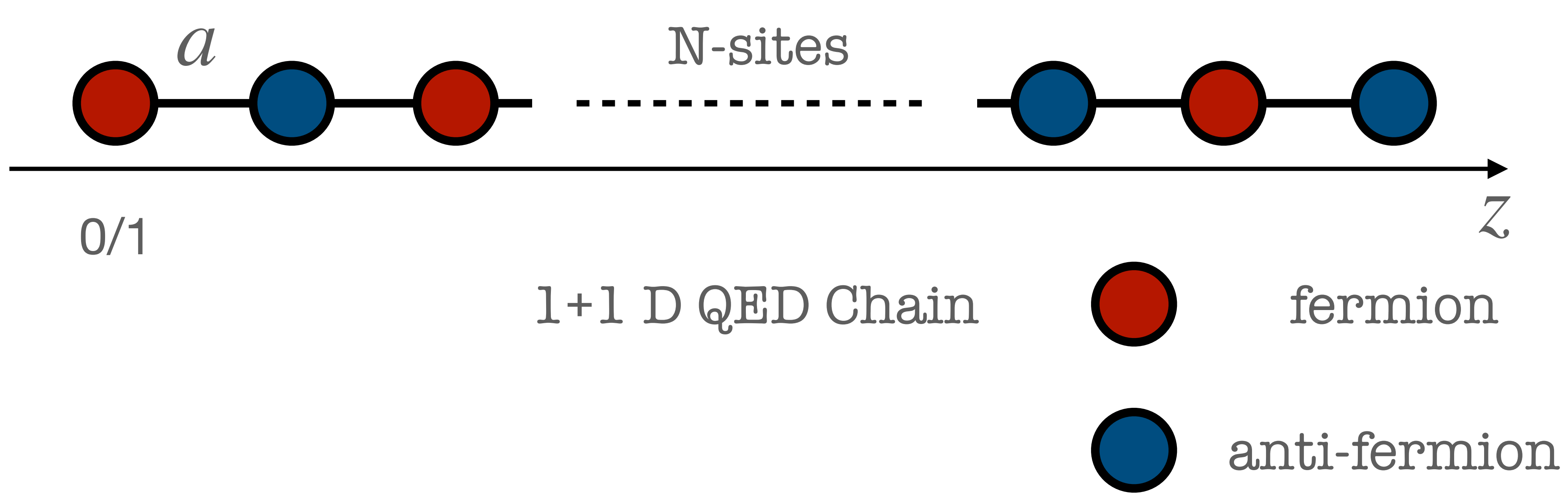
Discretization



$z_n = na$ $\quad \varepsilon_n = g^{-1}E(z_n)$ $\quad \phi_n = agA_0(z_n)$

$\chi_{2n} = a^{1/2}\psi_\uparrow(z_{2n})$ $\quad \chi_{2n+1} = a^{1/2}\psi_\downarrow(z_{2n+1})$

Hamiltonian with periodic boundary condition

$$H_{PBC} = \sum_{n=1}^{N} \left(-\frac{i}{2}\frac{1}{a}(\chi_n^\dagger e^{i\phi_n}\chi_{n+1} - \chi_{n+1}^\dagger e^{-i\phi_n}\chi_n) + (-1)^n m_0 \chi_n^\dagger \chi_n + \frac{ag^2}{2}\varepsilon_n^2 \right)$$

Dimension of fermion sector 2^N

Dimension of electric field sector M

Dimension of total Hilbert space $2^N \times M$

Discretized Gauss law $\varepsilon_{n+1} - \varepsilon_n = \chi_n^\dagger \chi_n$

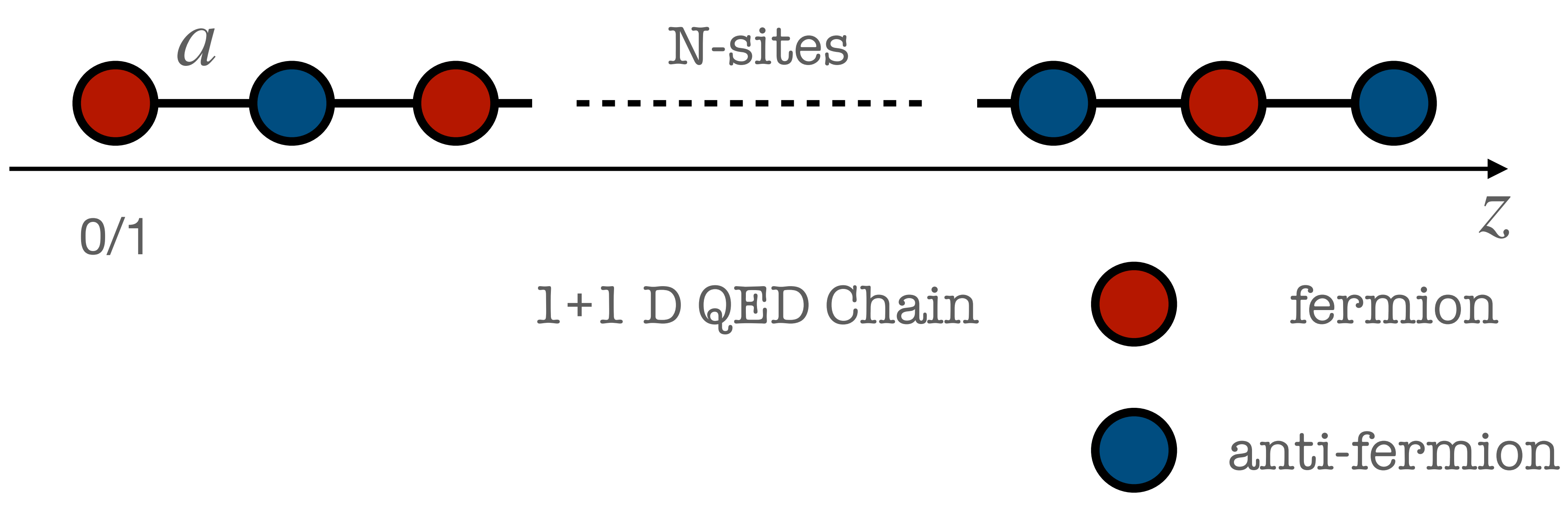
Schwinger model (1+1D QED)

Gate Representation (Jorden-Wigner Representation)

$$\chi_n = \frac{X_n - iY_n}{2}\Pi_{m=1}^{n-1}(-iZ_m)$$
$$\chi_n^\dagger = \frac{X_n + iY_n}{2}\Pi_{m=1}^{n-1}(iZ_m)$$

$$X_n = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y_n = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z_n = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Discretization



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Real time evolution and Thermal average

Energy-eigenstates $\{|n\rangle\}$

Operator $\mathcal{O}$

Initial pure state $|\Psi\rangle_0 = \sum_n c_n |n\rangle$

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Real-time

Time evolving state $|\Psi\rangle_t = \sum_n c_n e^{-iE_n t} |n\rangle$

Time evolving expectation value

$$\langle \Psi_t | \mathcal{O} | \Psi \rangle_t = \sum_{n,n'} c_n \, c^*_{n'} e^{i(E_{n'} - E_n)t} \langle n' | \mathcal{O} | n \rangle$$

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Thermal

Temperature $\beta := \{\sum_n |c_n|^2 E_n = \frac{\sum_n e^{-\beta E_n} E_n}{\sum_n e^{-\beta E_n}}\}$

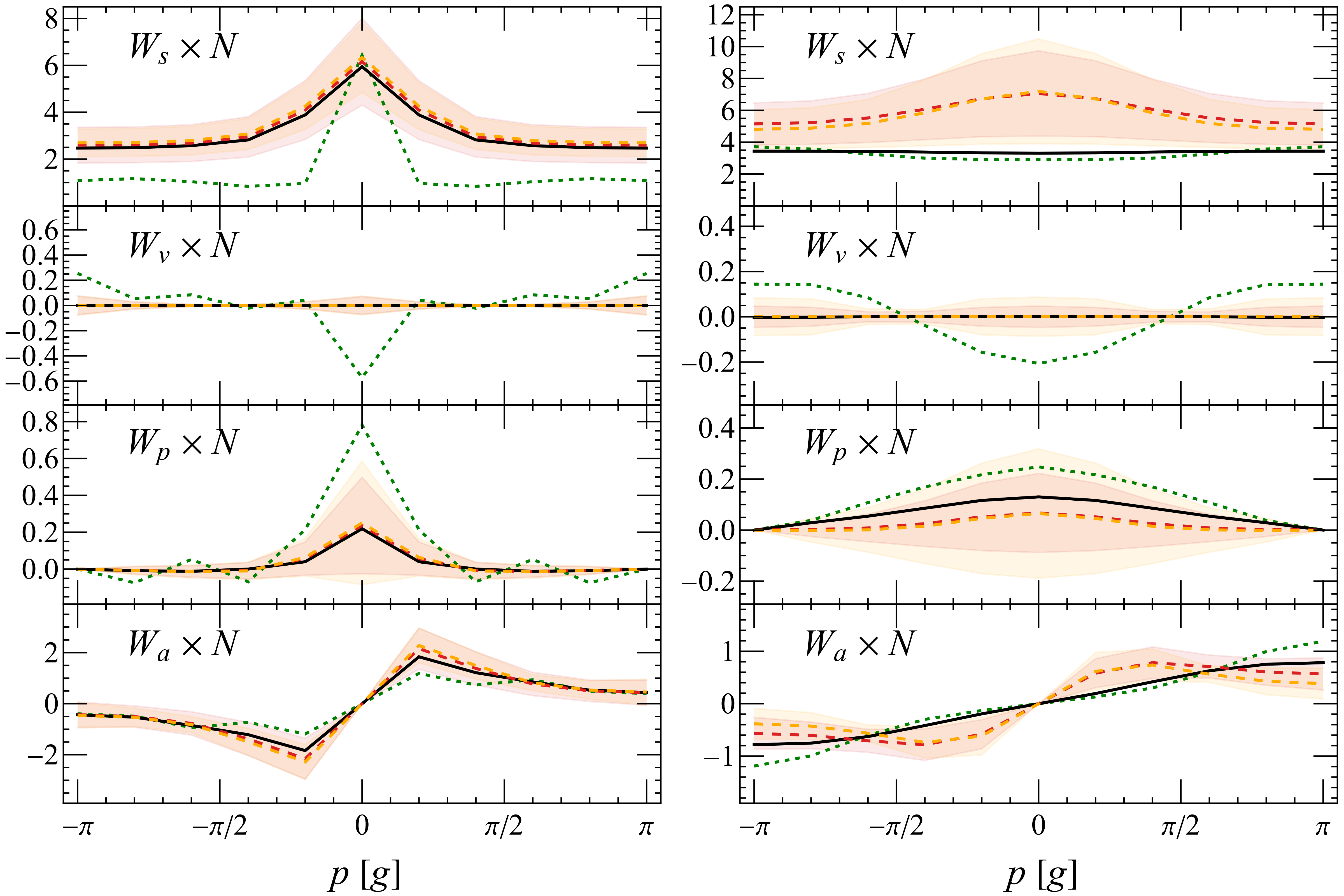
Canonical average $\langle\mathcal{O}\rangle_\beta = tr(\rho_T \mathcal{O}) = \frac{\sum_n e^{-\beta E_n}\langle n|\mathcal{O}|n\rangle}{\sum_n e^{-\beta E_n}}$

Micro-canonical average $\langle\mathcal{O}\rangle_{MC} = \frac{\sum_{n:|E_n-E|\le\Delta E}\mathcal{O}_{n,n}}{\sum_{n:|E_n-E|\le\Delta E}}$

Equal time Wigner function for 1+1D system

$$W_{ab}(t,z,p) = \int \langle \Psi_t | \bar{\psi}_a(z+\frac{y}{2})\psi_b(z-\frac{y}{2}) | \Psi_t \rangle e^{ipy} dy$$

Decomposition $\quad W = W_s + W_v\,\gamma^0 + W_a\,\gamma^1 - iW_p\,\gamma^5$



m=0 $\qquad$ m=2g

$W_s : \quad n_f + n_{\bar{f}}$

$W_v : \quad n_f - n_{\bar{f}}$

$W_a : \quad \chi_f + \chi_{\bar{f}}$

$W_p : \quad \chi_f - \chi_{\bar{f}}$

Equal time Wigner function for 1+1D system

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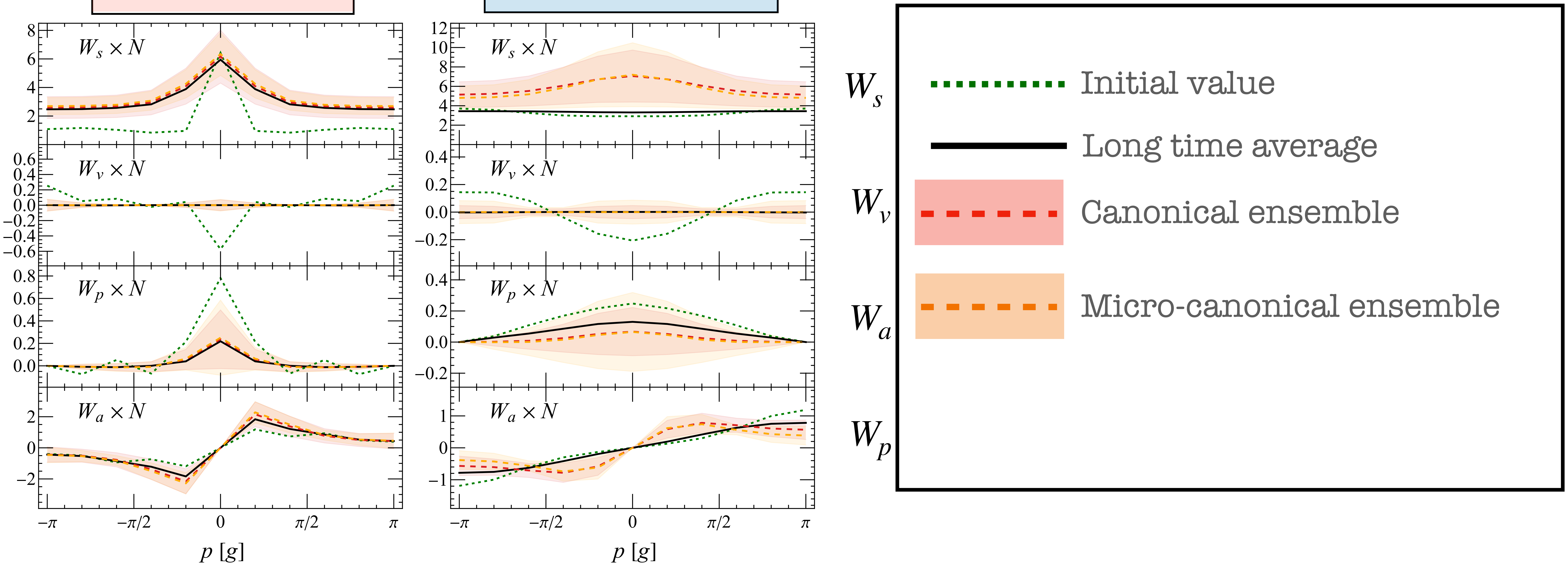
Thermalize

npositio

Not Thermalize

$_v\,\gamma^0 + W_a\,\gamma^1 - iW_p\,\gamma^5$



m=0

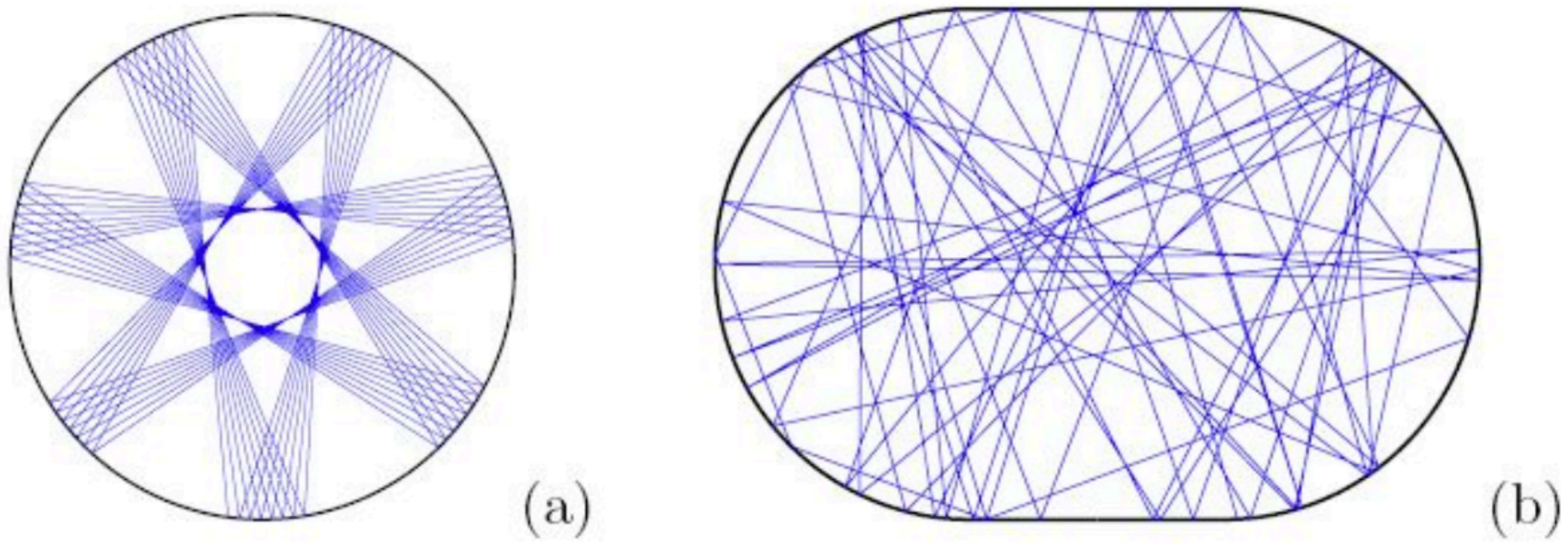
m=2g

Eigenstate thermalization Hypothesis [Mark Srednicki Phys. Rev. E **50**, 888]

ETH: A chaotic quantum system in a finitely excited energy eigenstate behaves thermally when probed by few-body operators

$$\langle E_a | \mathcal{O} | E_b \rangle = f_{\mathcal{O}}(E)\delta_{ab} + \Omega^{-1/2}(E) r_{ab} \qquad E = \frac{E_a + E_b}{2}$$

Classical trajectories of a bouncing particle in a cavity



Integral system

Non-ergodic

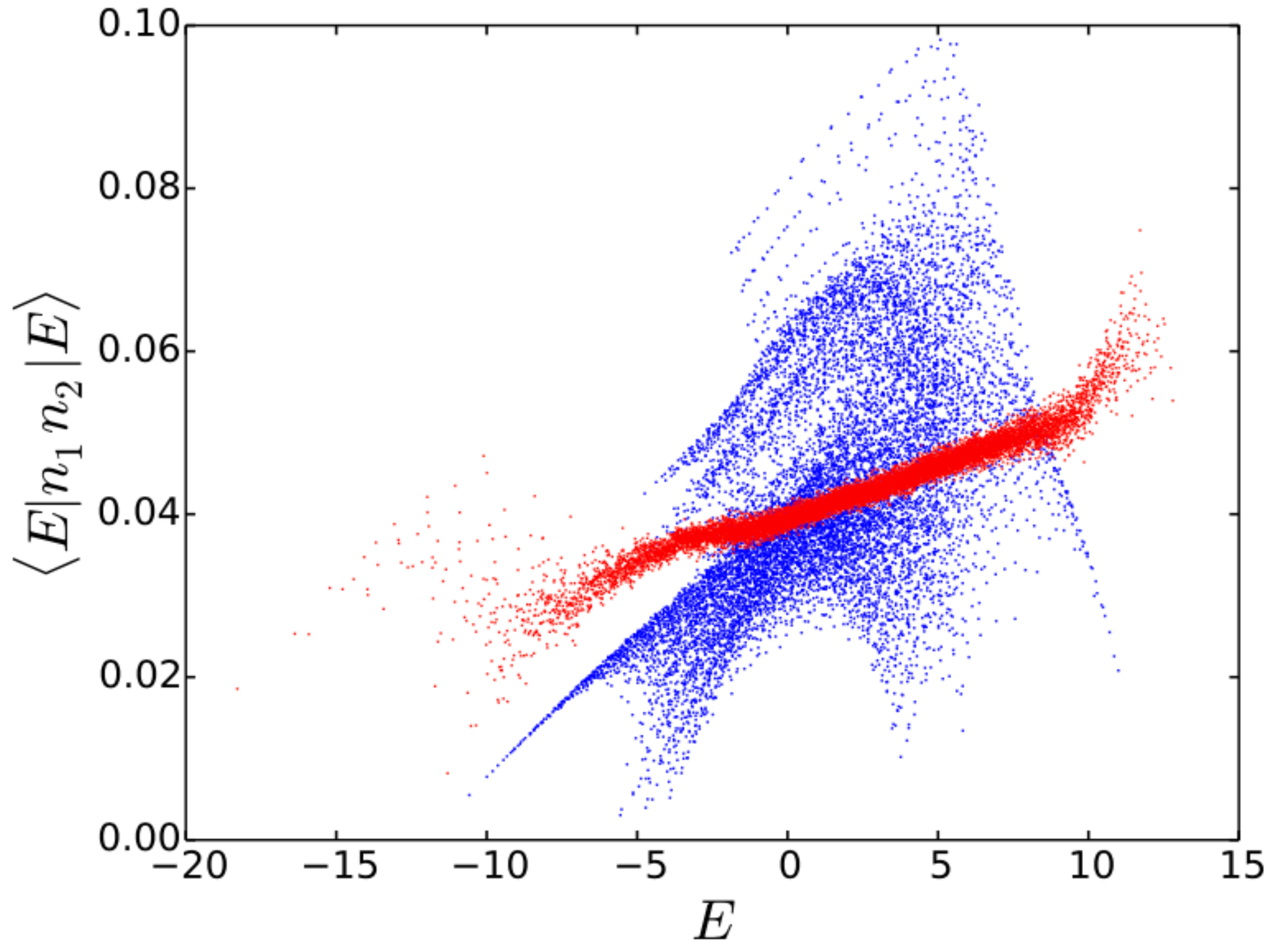
Non-chaotic

Non-integral system

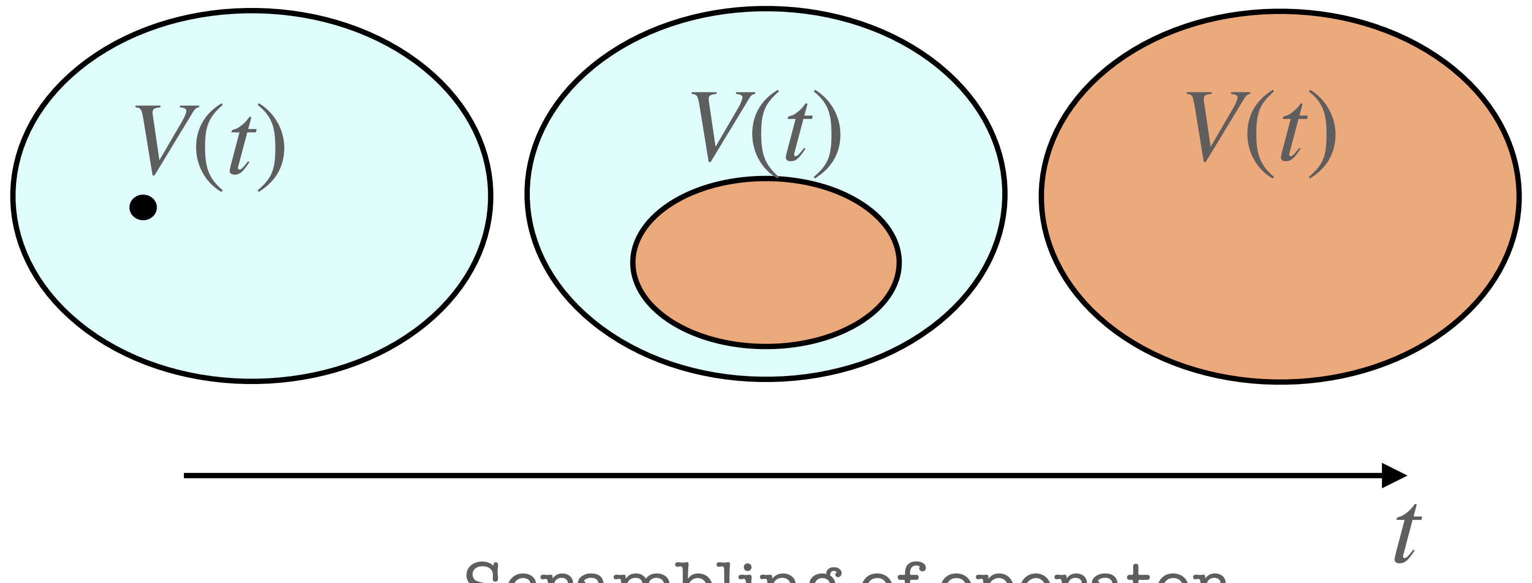
Ergodic

Chaotic Bunimovich stadium

Quantum explanation



Diagonal term of operator matrix element on energy basis

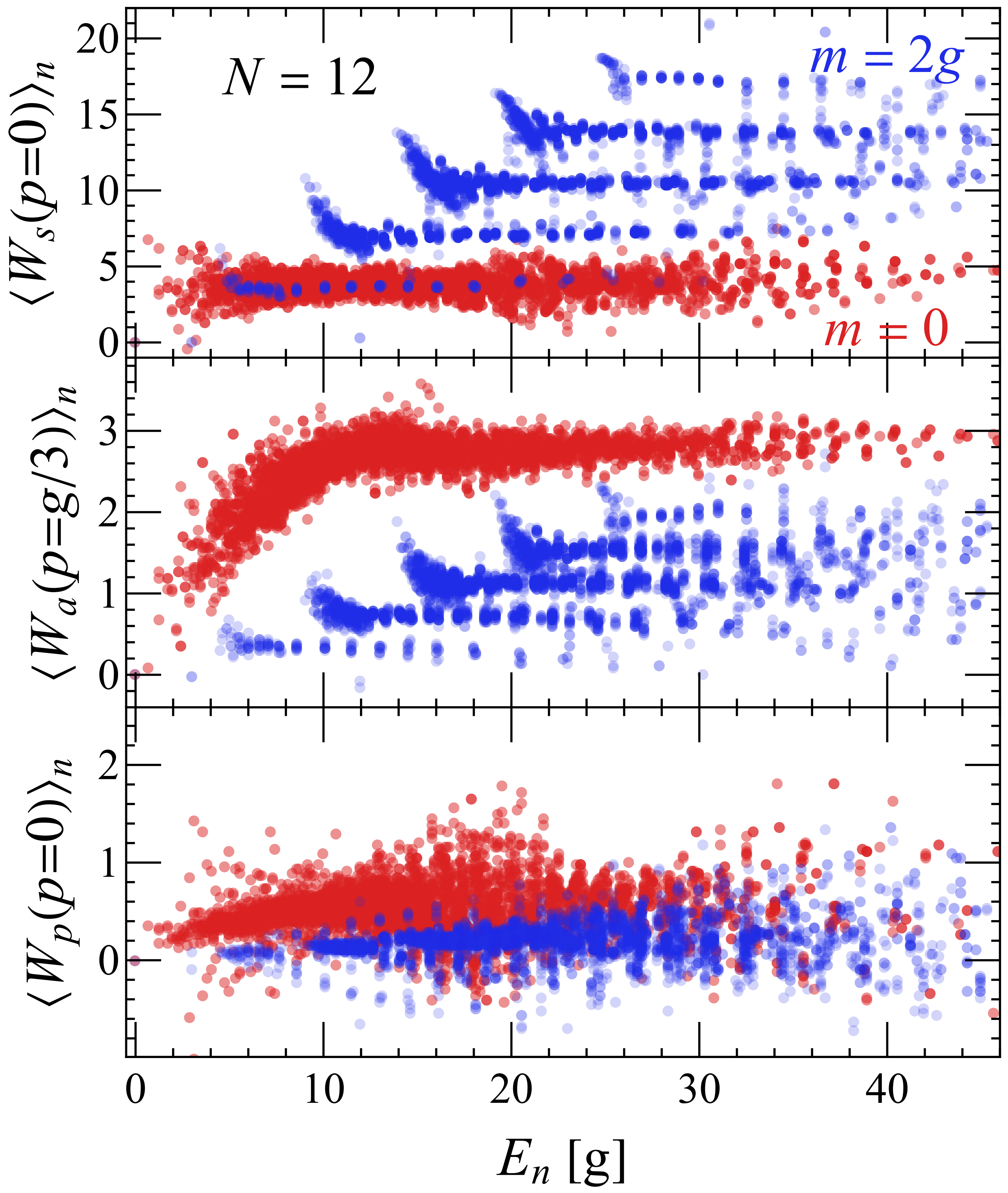


Scrambling of operator

Many-body localization

Eigenstate thermalization Hypothesis

Very large fermion mass -> Almost conserved quantity -> particle number \ Chirality

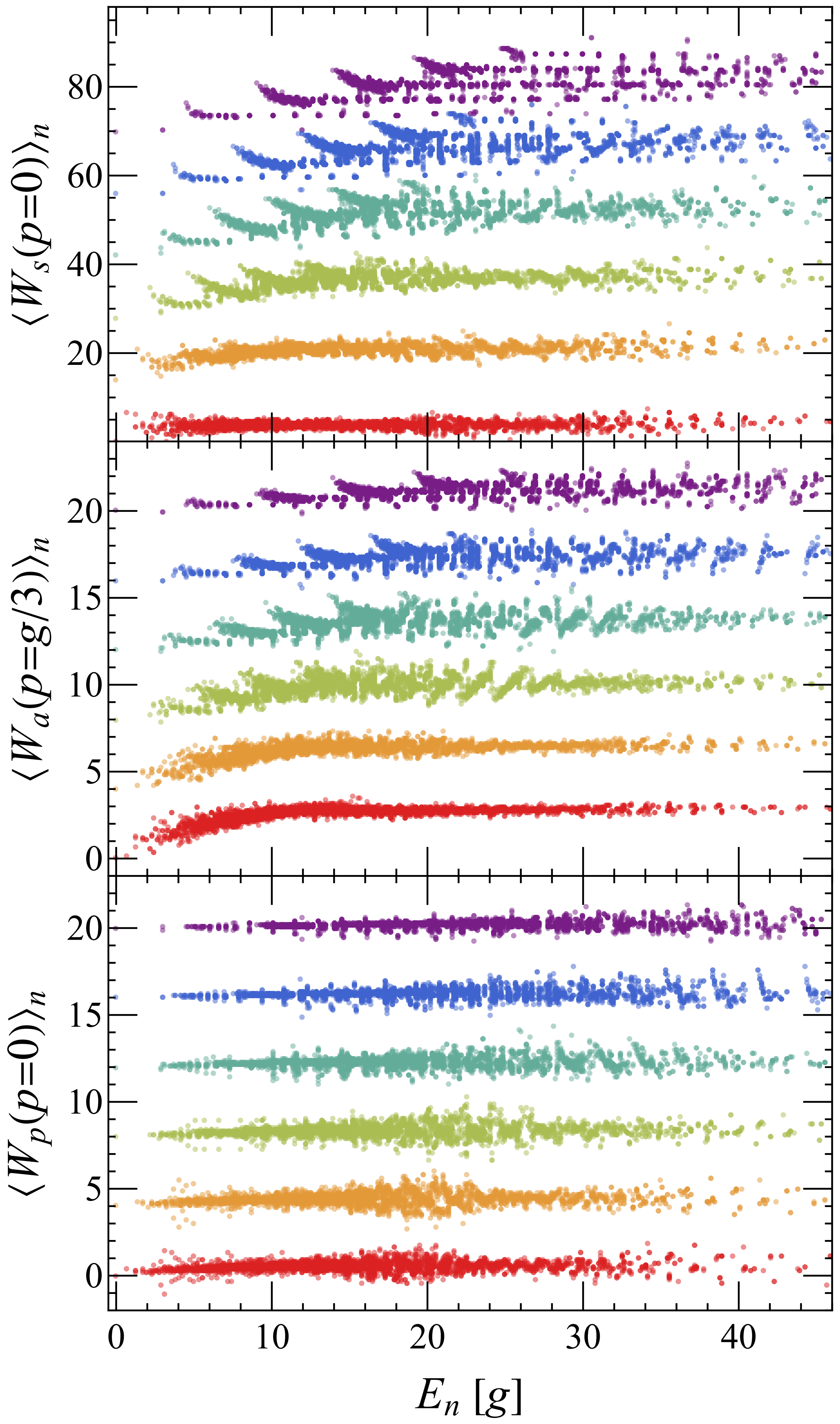


Energy Degeneracy

Not change with Unitary Time Evolution

Localized in Focker space

Eigenstate thermalization Hypothesis



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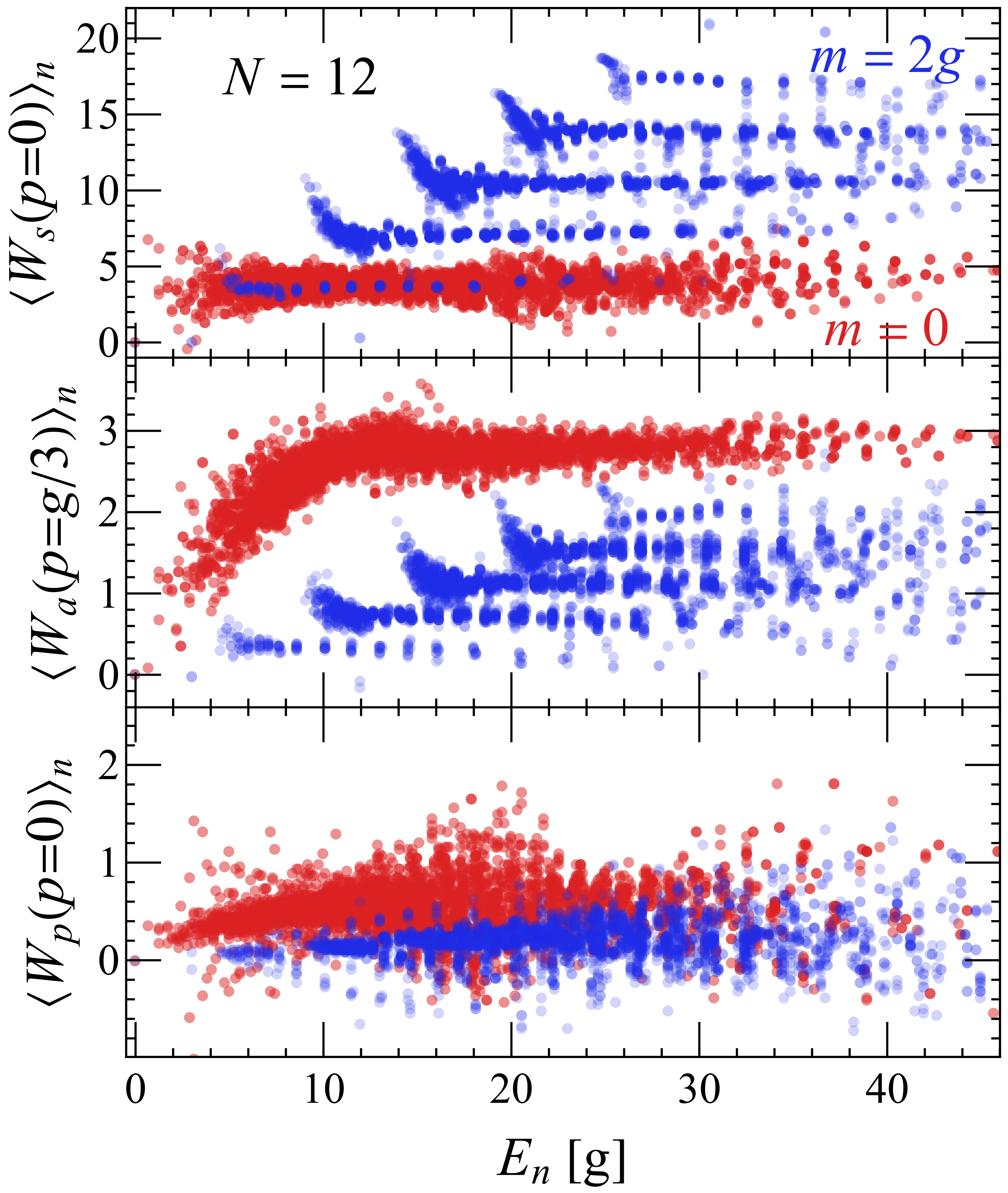


Energy Degeneracy

Not change with Unitary Time Evolution

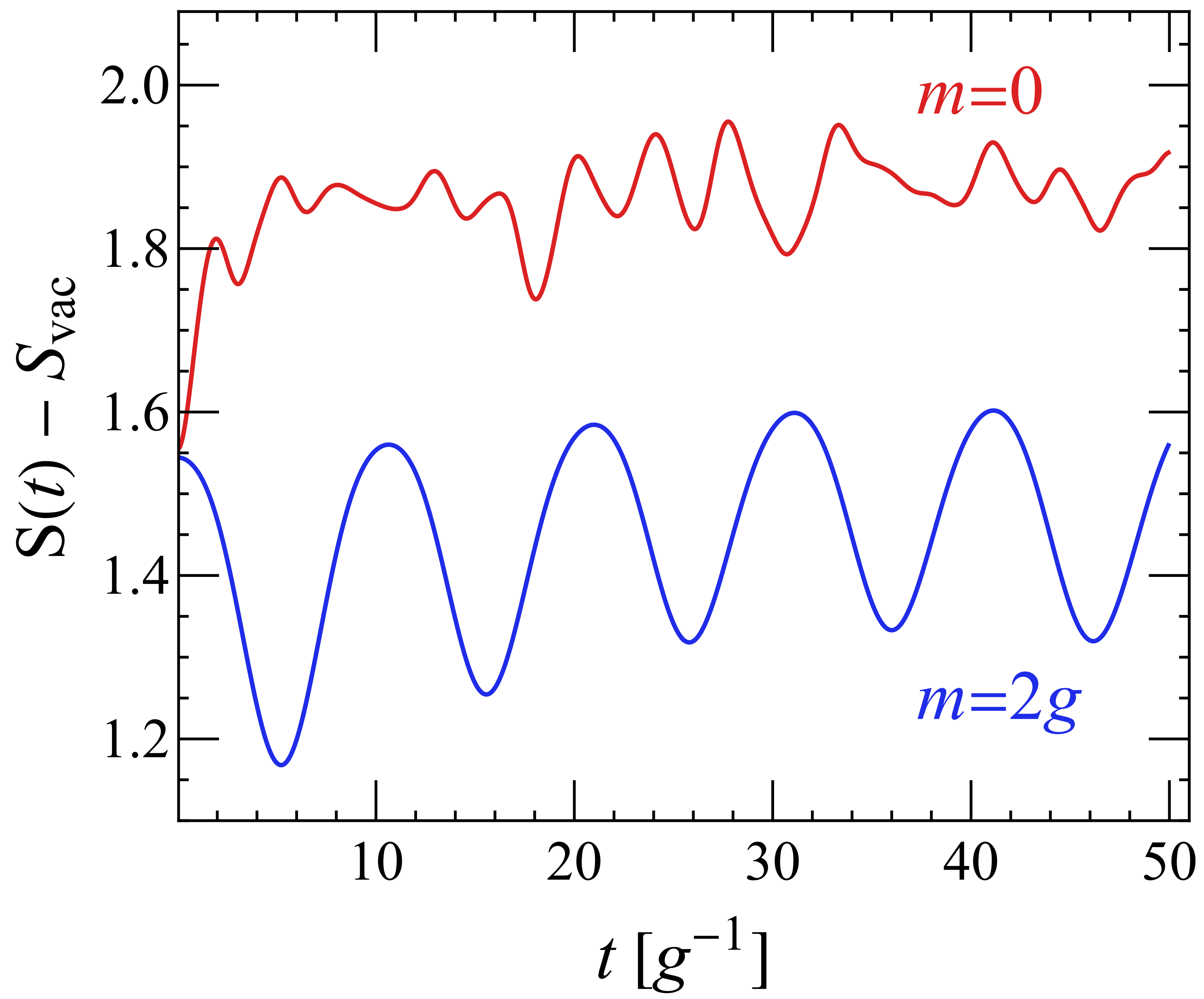
Localized in Focker space

Entanglement Entropy & Reduced density matrix

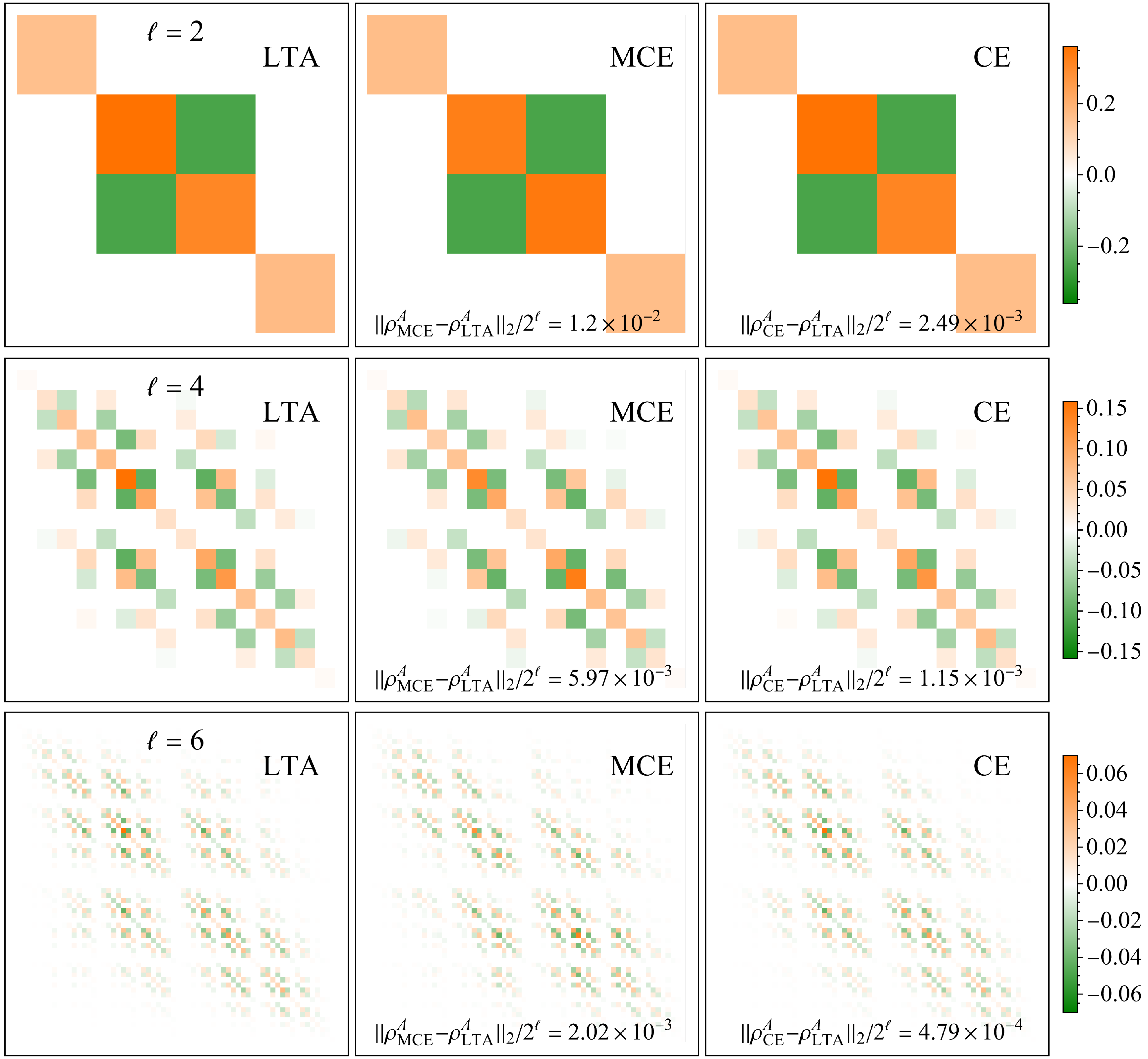
Subsystem eigenstate thermalization hypothesis

Diagonal $||\rho_a^A - \rho^A(E = E_a)|| \sim O[\Omega^{-1/2}(E_a)]$

Off-diagonal $||\rho_{ab}^A|| \sim O[\Omega^{-1/2}(E)], \quad E = \frac{1}{2}(E_a + E_b)$



Time evolution of entanglement entropy



Summary

We calculate the real time evolution of a closed system with quantum computing.

Find the momentum distribution function will thermalize when the system satisfies ETH.

After trace out part of the system, the subsystem ETH is verified.

Outlook

How does the system reach the thermal equilibrium:
hydrodynamics?
attractor?

Thanks for listening!🌶️🥟