Nuclear Science Computing Center at CCNU

Baryon electric charge correlation as a magnetometer of QCD

Jin-Biao Gu (顾锦彪)

Central China Normal University

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In collaboration with H.-T. Ding, A. Kumar, S.-T. Li and J.-H. Liu

The 1st edition of Spicy Gluons Workshop for Young Scientists: The quark-gluon matter in extreme conditions
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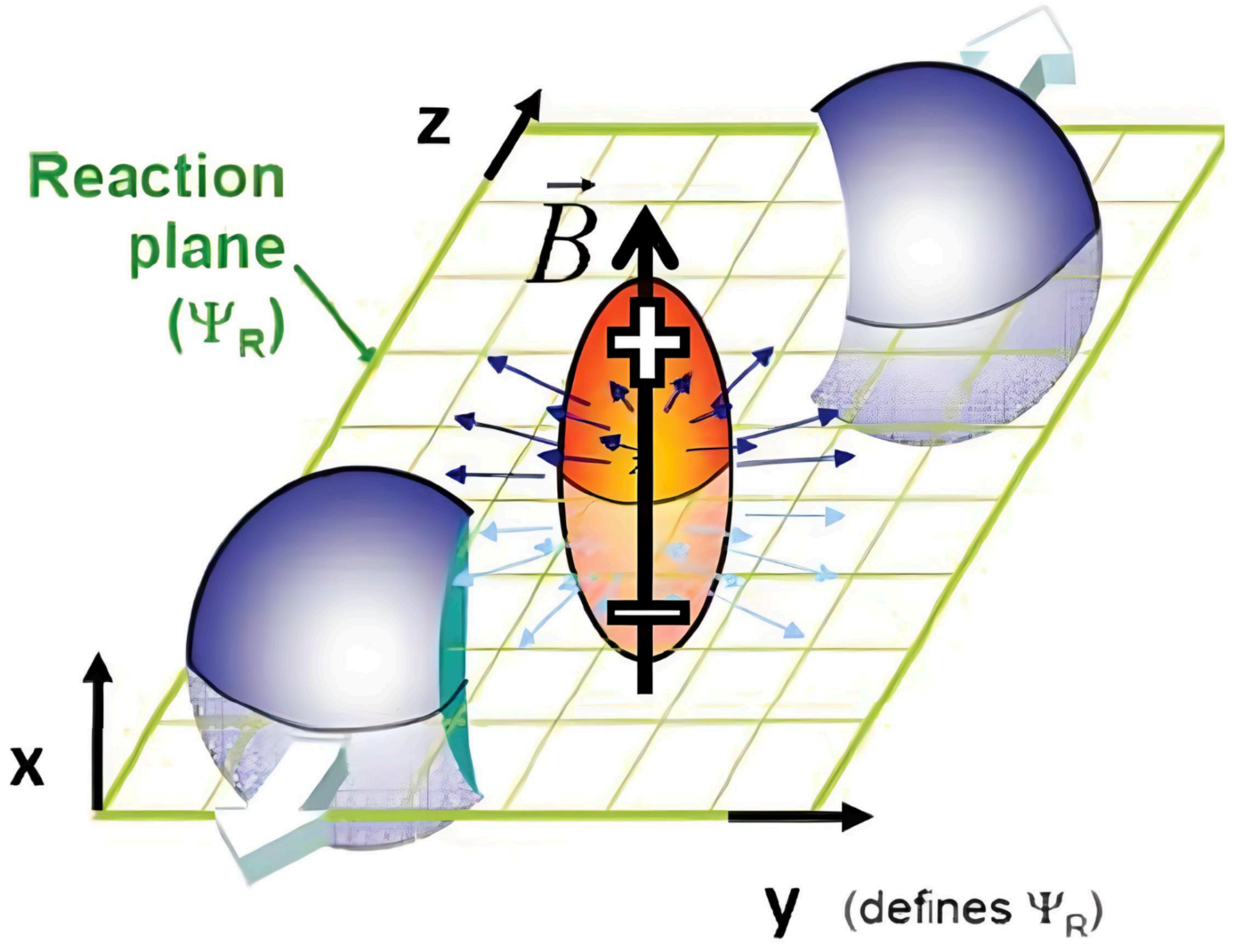
Outline

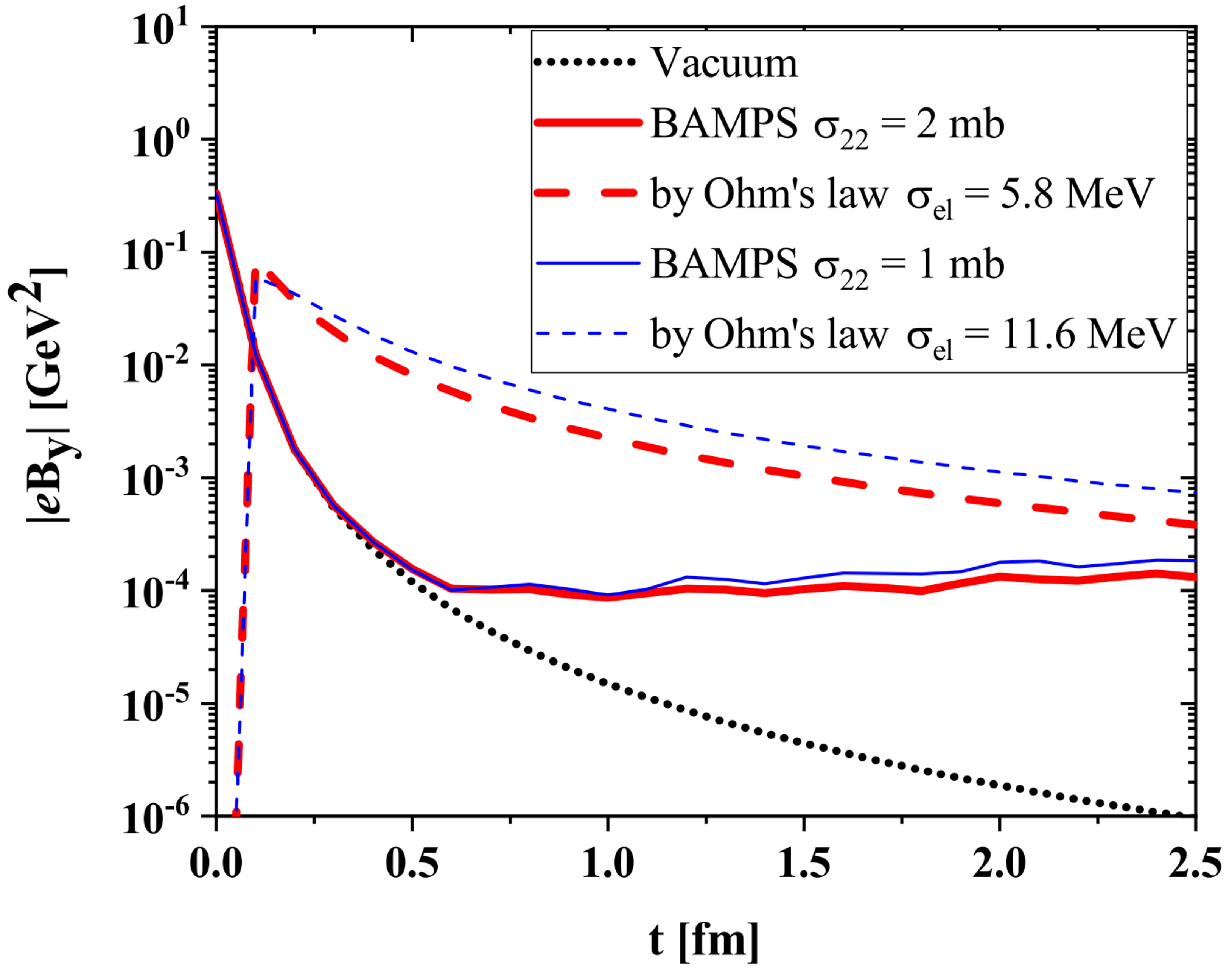
- Introduction and motivation
  - QCD in strong magnetic field
- Lattice Setup
- Lattice results
  - 2nd order fluctuations of conserved charges
  - Proxy for fluctuations in heavy-ion experiment
- Summary

Strong magnetic fields in heavy-ion collisions



$eB_{\tau=0}$ ~ 5 M_π^2 in RHIC $eB_{\tau=0}$ ~ 70 M_π^2 in LHC

W.-T. Deng et al. Phys.Rev.C 85 (2012) 044907

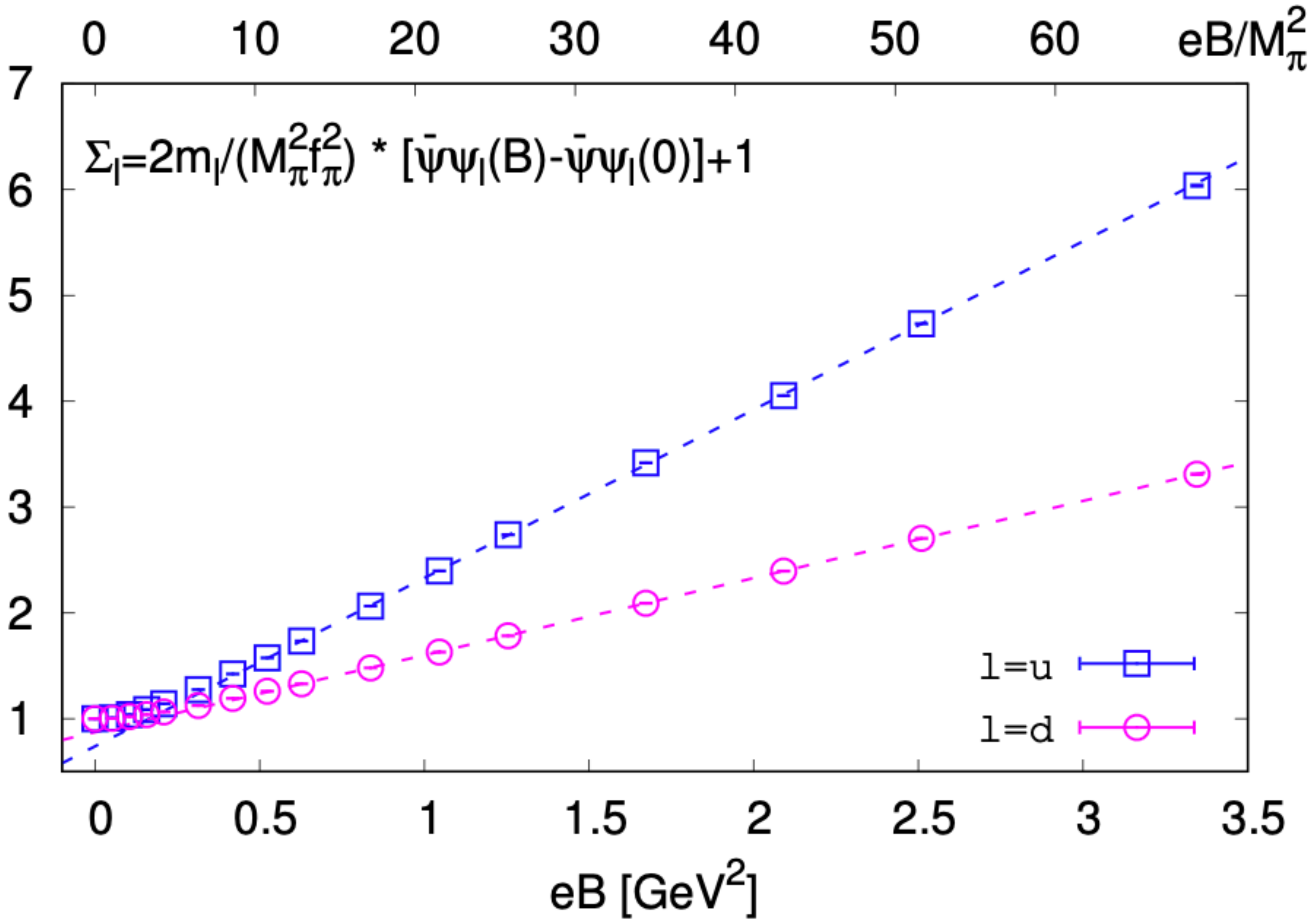


Z. Wang et al. Phys.Rev.C 105 (2022) L041901

The magnetic field is the key ingredient for the chiral magnetic effect

Questions: What observables are suitable as probes for magnetic fields?

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

A clear effect but Not accessible in HIC experiments!

Fluctuations of net baryon number, electric charge and strangeness

Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}\left(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s\right) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{\mathrm{BQS}}}{i!j!k!} \left(\frac{\mu_{\mathrm{B}}}{T}\right)^i \left(\frac{\mu_{\mathrm{Q}}}{T}\right)^j \left(\frac{\mu_{\mathrm{S}}}{T}\right)^k$$

C. Allton et al., Phys.Rev. D66 (2002) 074507

Taylor expansion coefficients at $\mu = 0$ are computable in LQCD

$$\chi_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\right|_{\mu_{u,d,s}=0}$$

$$\chi_{ijk}^{\mathrm{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_{\mathrm{B}}/T\right)^i \partial\left(\mu_{\mathrm{Q}}/T\right)^j \partial\left(\mu_{\mathrm{S}}/T\right)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}$$

$$\mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}$$

$$\mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}$$

LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112

μ_{Q} can expanded as $\hat{\mu}_{\mathrm{Q}} = q_1\hat{\mu}_{\mathrm{B}} + q_3\hat{\mu}_{\mathrm{B}}^3 + \mathcal{O}(\hat{\mu}_{\mathrm{B}}^5)$, and the leading order coefficient:

$$q_1 = \frac{r\left(\chi_2^{\mathrm{B}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{BS}}\right) - \left(\chi_{11}^{\mathrm{BQ}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{QS}}\right)}{\left(\chi_2^{\mathrm{Q}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{QS}}\chi_{11}^{\mathrm{QS}}\right) - r\left(\chi_{11}^{\mathrm{BQ}}\chi_2^{\mathrm{S}} - \chi_{11}^{\mathrm{BS}}\chi_{11}^{\mathrm{QS}}\right)}$$

A. Bazavov et al. Phys. Rev. Lett. 109 (2012) 192302

with $r = n_{\mathrm{Q}}/n_{\mathrm{B}}$ is the ratio of net electric charge to net baryon number density in the colliding nuclei

Lattice Setup

- Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- $N_f = 2 + 1$
- Lattice sizes : $32^3 \times 8,\ 48^3 \times 12$
- $m_s^{\text{phy}}/m_l = 27,\ M_\pi \approx 135\ \text{MeV}$
- T window : $(144\ \text{MeV},\ 165\ \text{MeV})$, i.e. $(0.9T_{pc},\ 1.1T_{pc})$
- eB window: $0 \leq eB < 9M_\pi^2$

$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}, \quad N_b = 0,\ 1,\ 2,\ 3,\ 4,\ 6$$

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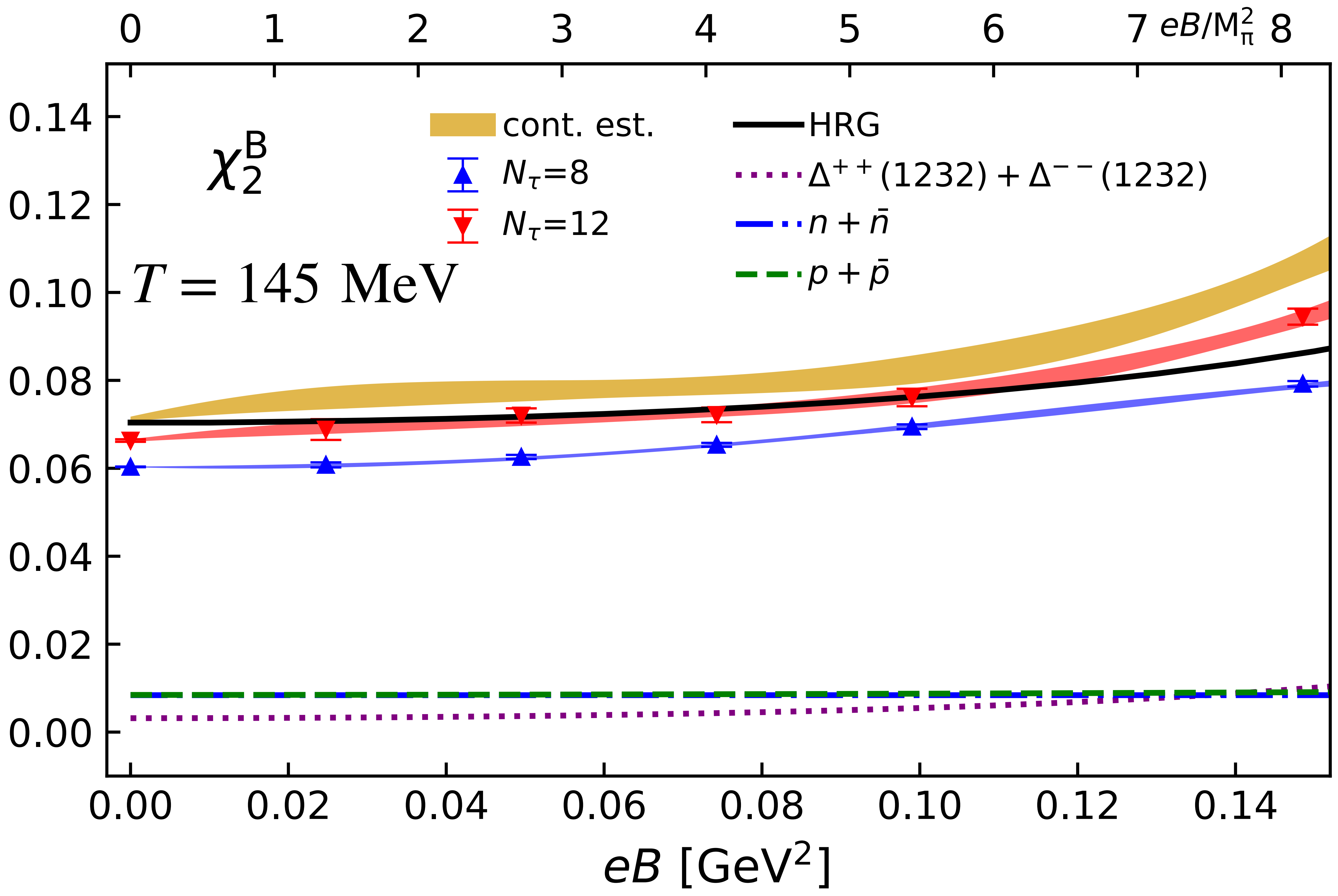


- χ_2^B increases $\sim 45\%$ at $eB \sim 8M_\pi^2$
- Hadron Resonance Gas model (HRG):

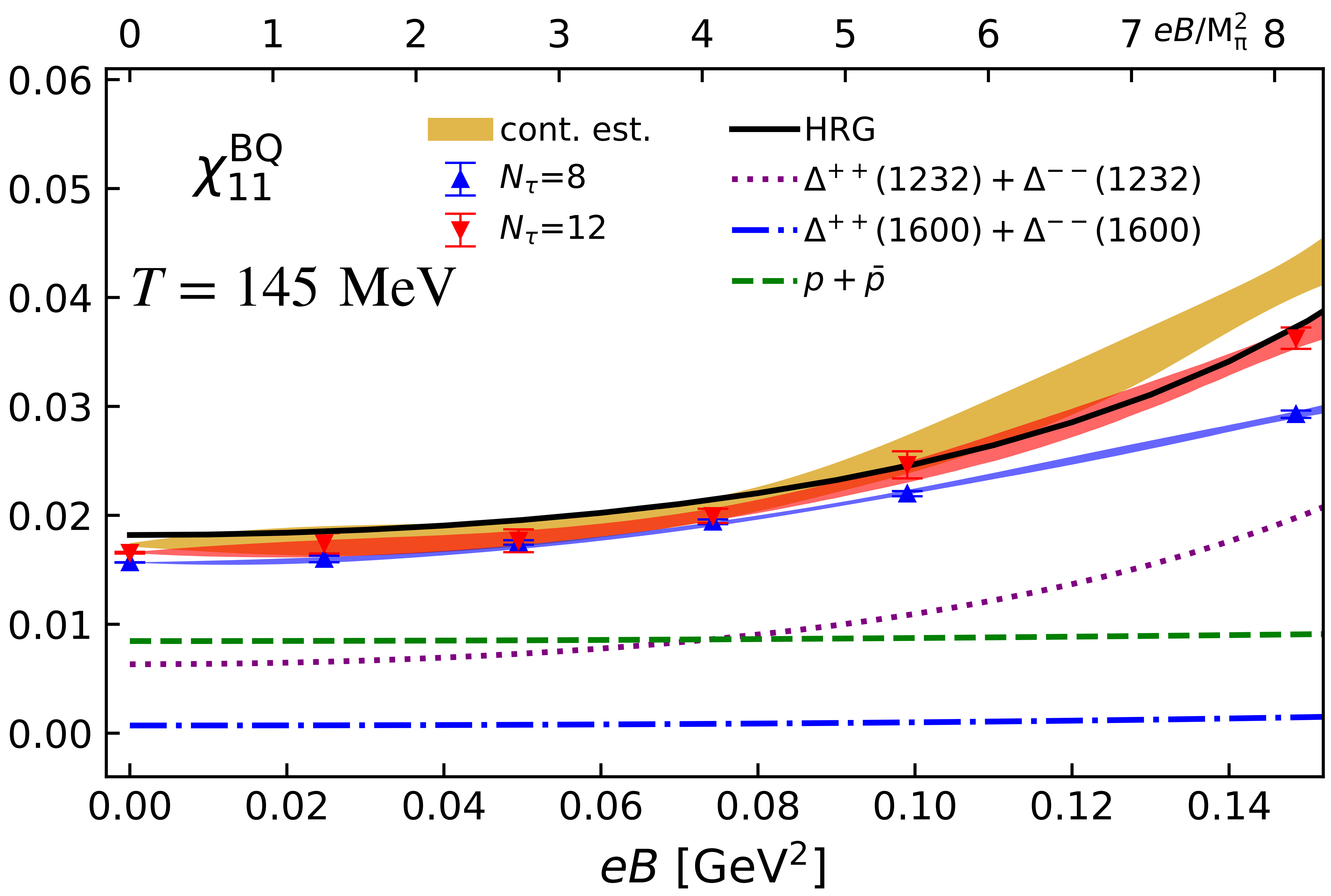
  Pressure arising from charged hadrons ($eB \neq 0$):

$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i|\, B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1\left(\frac{n\varepsilon_0}{T}\right)$$

where $\varepsilon_0 = \sqrt{m_i^2 + 2\,|q_i|\, B\left(l + 1/2 - s_z\right)}$,

K_1 is the first-order modified Bessel function

Contributions from Individual hadrons in HRG model



- ✦ χ_{11}^{BQ} increases $\sim$ **140%** at $eB \sim 8M_\pi^2$
- ✦ The results of HRG model are consistent with LQCD up to $eB \sim 6M_\pi^2$
- ✦ p contributions are almost independent on magnetic field
- ✦ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ give most of the contributions of magnetic field dependence of χ_{11}^{BQ}
- ✦ $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ are not measurable in HIC experiments

Proxy construction based on the HRG

$\Delta^{++}(1232) \rightarrow p + \pi^{+}$: branching ratio almost **100%** !

HRG: Fluctuations expressed in terms of stable hadronic states:

$$\chi_{ijk}^{\mathrm{BQS}}\left(T, \hat{\mu}_{\mathrm{B}}, \hat{\mu}_{\mathrm{Q}}, \hat{\mu}_{\mathrm{S}}\right) = \sum_{R} \mathrm{B}_{R}^{i} \mathrm{Q}_{R}^{j} \mathrm{S}_{R}^{k} \frac{\partial^{l} p_{R}/T^{4}}{\partial \hat{\mu}_{R}^{l}}$$

net- B : $\tilde{p} + \tilde{n} + \tilde{\Lambda} + \tilde{\Sigma}^{+} + \tilde{\Sigma}^{-} + \tilde{\Xi}^{0} + \tilde{\Xi}^{-} + \tilde{\Omega}^{-}$

net- Q : $\tilde{\pi}^{+} + \tilde{K}^{+} + \tilde{p} + \tilde{\Sigma}^{+} - \tilde{\Sigma}^{-} - \tilde{\Xi}^{-} - \tilde{\Omega}^{-}$

net- S : $\tilde{K}^{+} + \tilde{K}^{0} - \tilde{\Lambda} - \tilde{\Sigma}^{+} - \tilde{\Sigma}^{-} - 2\tilde{\Xi}^{0} - 2\tilde{\Xi}^{-} - 3\tilde{\Omega}^{-}$

B_R, Q_R, S_R are the baryon number, electric charge and strangeness of the species R

R. Bellwied et al. ,Phys. Rev. D 101, 034506 (2020)

In HIC, fluctuations are related to the variance or covariance of net-multiplicity for Identified π, K, p

e.g. the proxy for χ_{11}^{BQ} is $\sigma_{Q^{PID},p}^{1,1} = \sigma_{p}^{2} + \sigma_{p,\pi}^{1,1} + \sigma_{p,K}^{1,1}$

STAR, Phys.Rev.C 100 (2019) 1, 014902 ; STAR, Phys.Rev.C 105(2019) 2, 029901

In HRG:

$$\sigma_{p}^{2} = \sum_{R} \left(P_{R\rightarrow\tilde{p}}\right)\left(P_{R\rightarrow\tilde{p}}\right) \frac{\partial^{2} p_{R}/T^{4}}{\partial \hat{\mu}_{R}^{2}}$$

$$\sigma_{p,\pi}^{1,1} = \sum_{R} \left(P_{R\rightarrow\tilde{p}}\right)\left(P_{R\rightarrow\tilde{\pi}^{+}}\right) \frac{\partial^{2} p_{R}/T^{4}}{\partial \hat{\mu}_{R}^{2}}$$

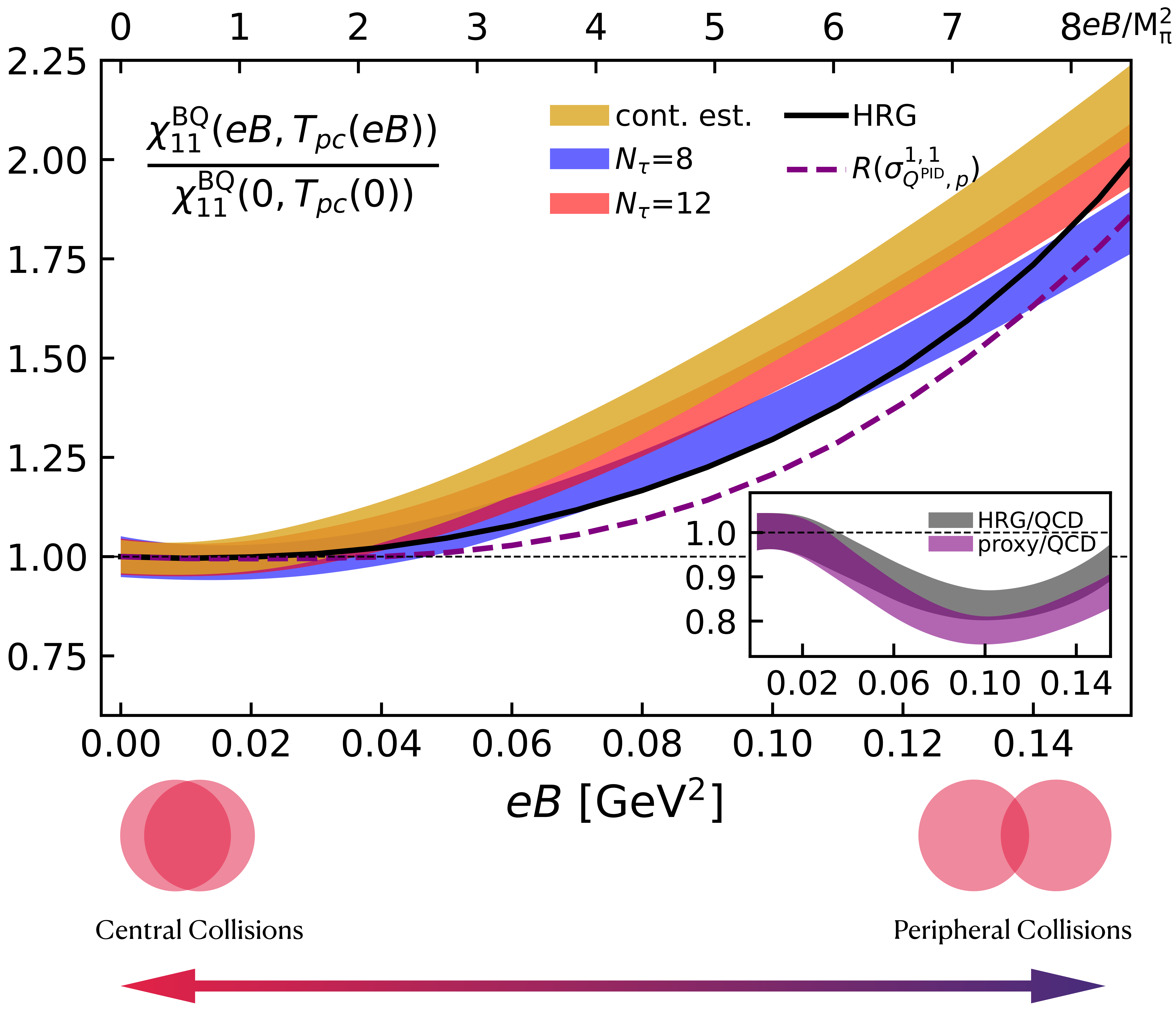
$$\sigma_{p,K}^{1,1} = \sum_{R} \left(P_{R\rightarrow\tilde{p}}\right)\left(P_{R\rightarrow\tilde{K}^{+}}\right) \frac{\partial^{2} p_{R}/T^{4}}{\partial \hat{\mu}_{R}^{2}}$$

where $P_{R\rightarrow i} = \sum_{\alpha} \mathrm{N}_{R\rightarrow i}^{\alpha} n_{i,\alpha}^{R}$

$n_{i,\alpha}^{R}$: numbers of i produced by R in decay channel α

$\mathrm{N}_{R\rightarrow i}^{\alpha}$: Branching ratio of channel α

Proxy for χ_{11}^{BQ} along the transition line



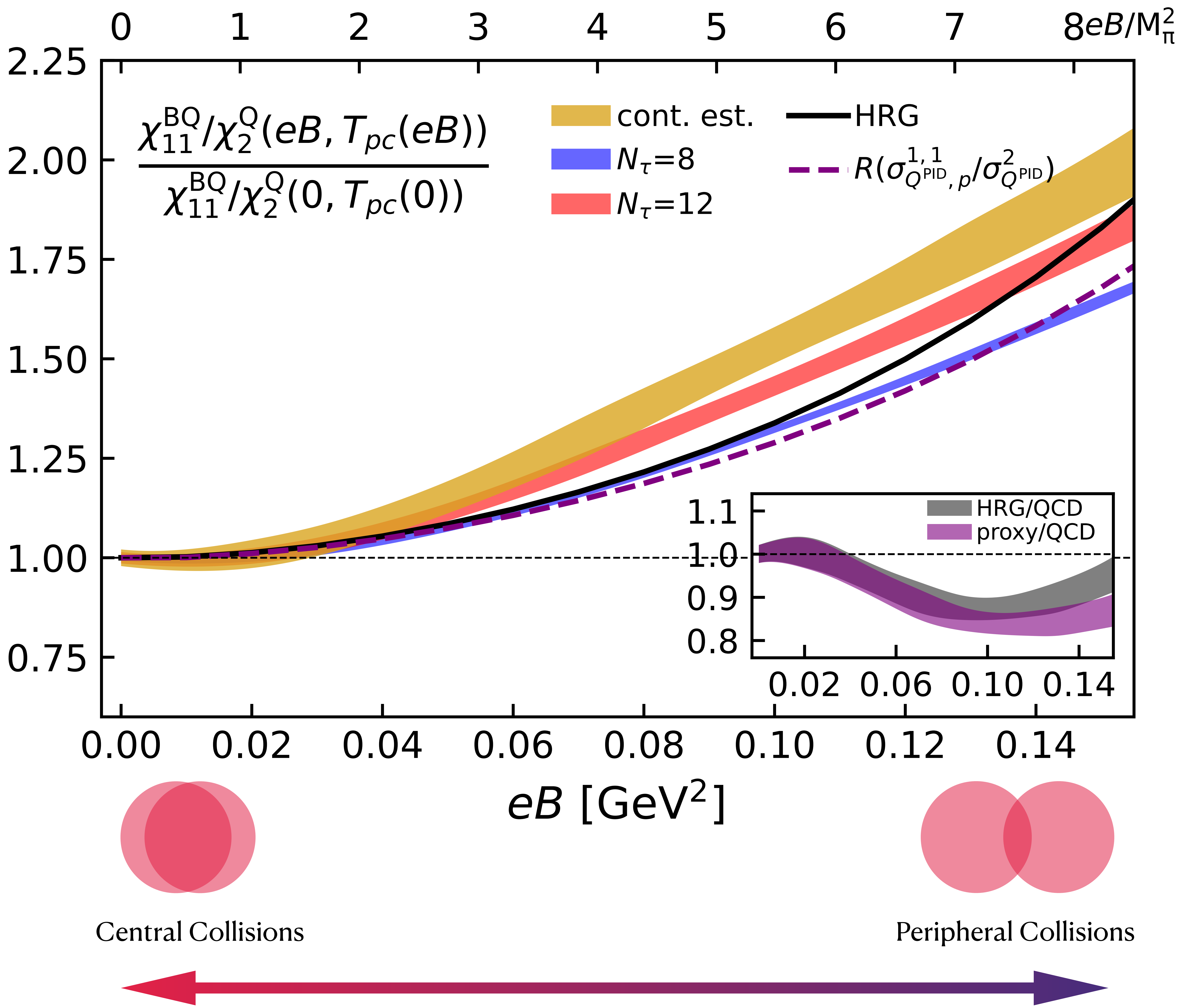
✦At $eB \simeq 8M_\pi^2$, ratio of $\chi_{11}^{BQ} \sim$ **2.1**

$$R(\sigma^{1,1}_{Q^{PID},p}) = \sigma^{1,1}_{Q^{PID},p}(eB)/\sigma^{1,1}_{Q^{PID},p}(eB=0)$$

✦The proxy $R(\sigma^{1,1}_{Q^{PID},p})$ can represent **80∼85%** of the LQCD results

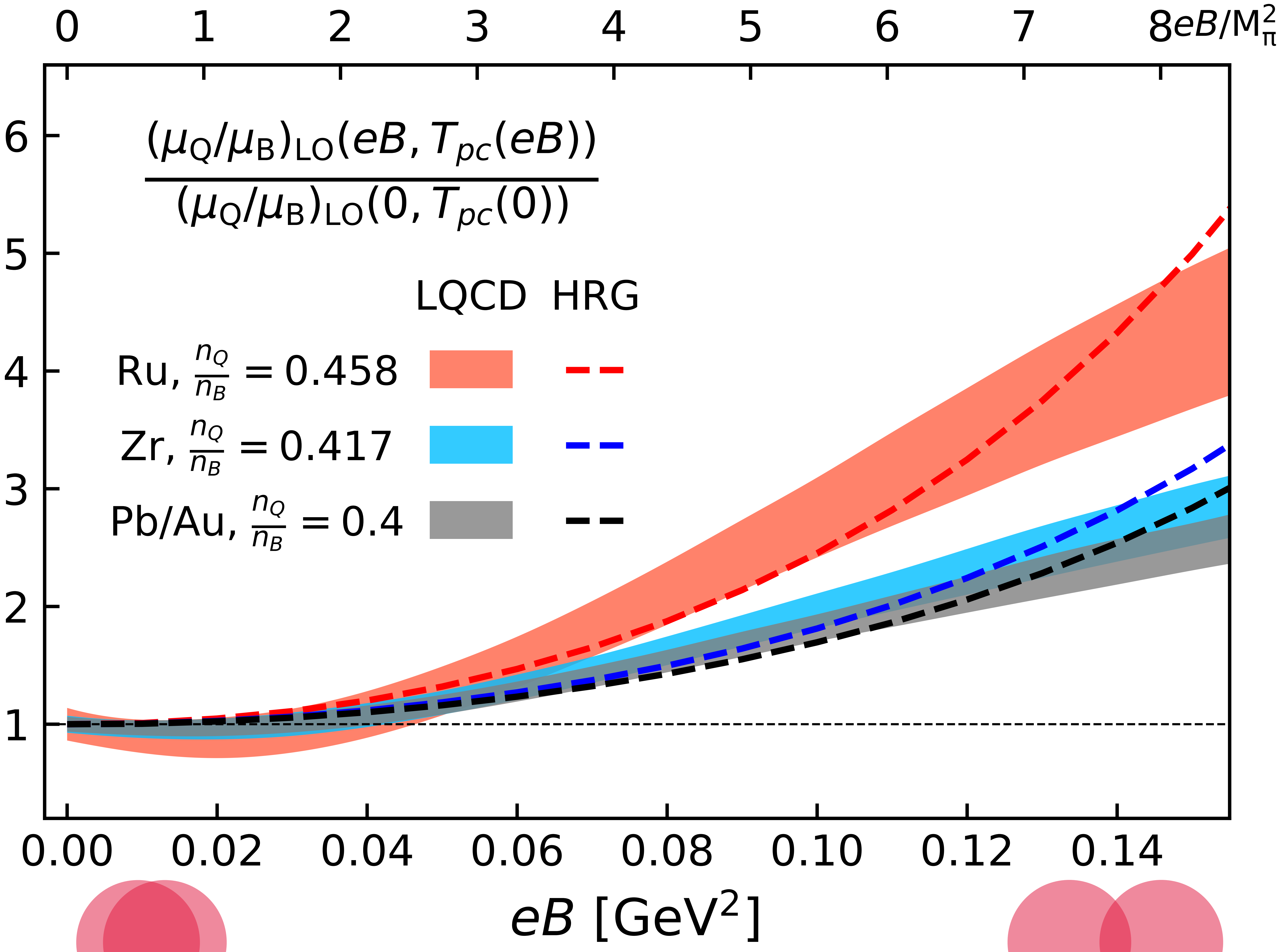
✦$R(\sigma^{1,1}_{Q^{PID},p})$ is a reasonable proxy for χ_{11}^{BQ}

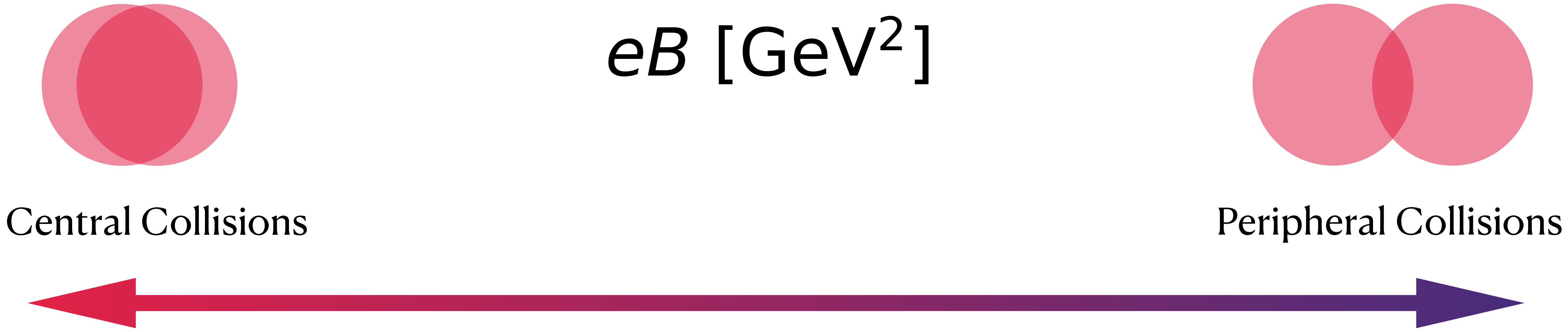
Proxy for $\chi_{11}^{BQ}/\chi_2^{Q}$ along the transition line



- At $eB \simeq 8M_\pi^2$, ratio of $\chi_{11}^{BQ}/\chi_2^{Q} \sim$ **1.9**
- The proxy $R(\sigma^{1,1}_{Q^{PID},p}/\sigma^2_{Q^{PID}})$ can represent ~**85%** of the LQCD results
- $R(\sigma^{1,1}_{Q^{PID},p}/\sigma^2_{p})$ is a reasonable proxy for $\chi_{11}^{BQ}/\chi_2^{Q}$

Dependence of $(\mu_Q/\mu_B)_{LO}$ on the magnetic field





$$\hat{\mu}_Q/\hat{\mu}_B = q_1 + q_3\ \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4)$$

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}$$

$$r = n_Q/n_B$$

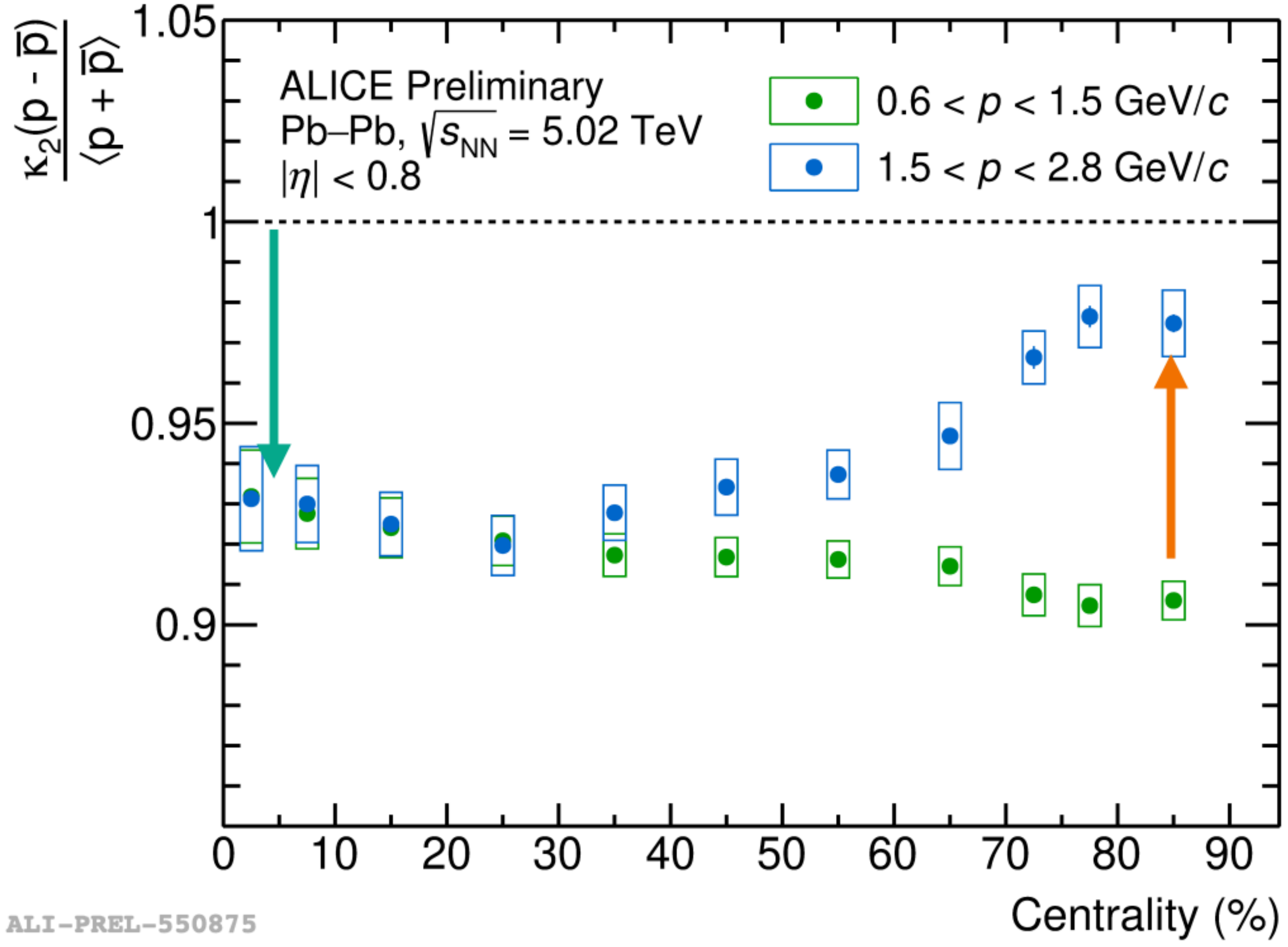
✦At $eB \simeq 8M_\pi^2$,

Ratio of $(\mu_Q/\mu_B)_{LO}$ for Pb, Au, Zr ~2.4

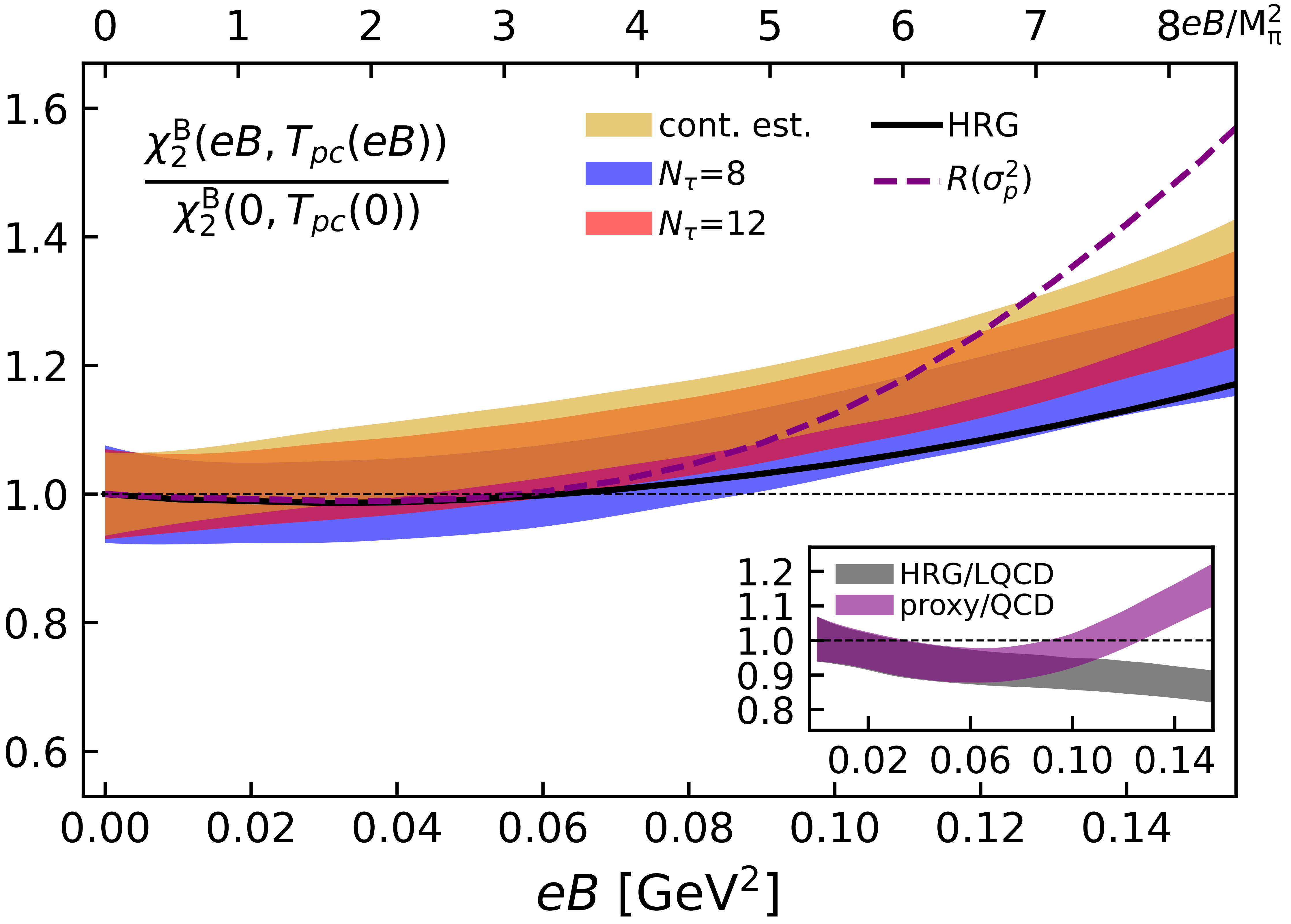
Ratio of $(\mu_Q/\mu_B)_{LO}$ for **Ru ~ 4**

Lattice QCD meets experiments

Proxy for χ_2^B



I. Fokin et al. (ALICE Collaboration) @ Quark Matter 2023

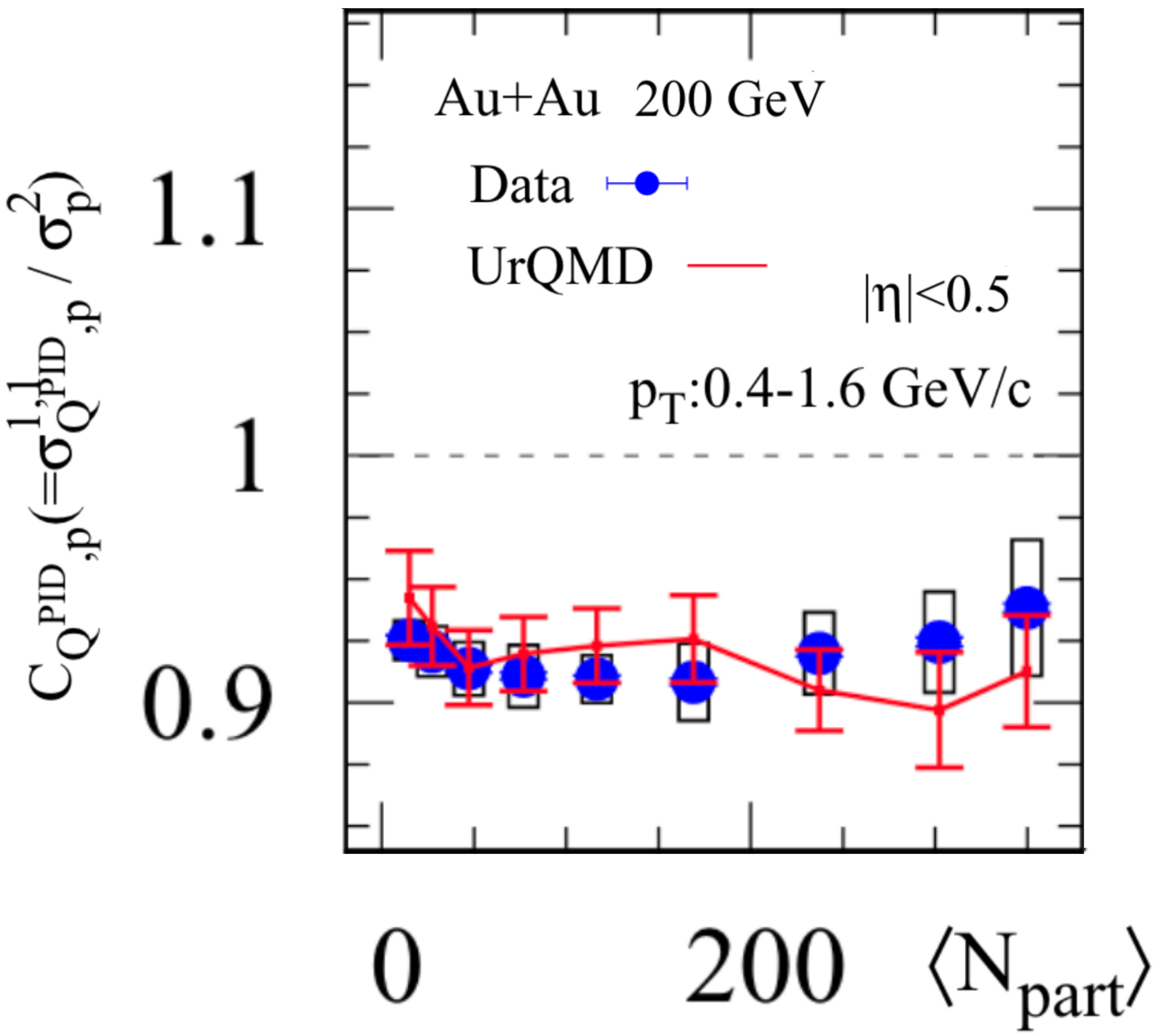


Central Collisions

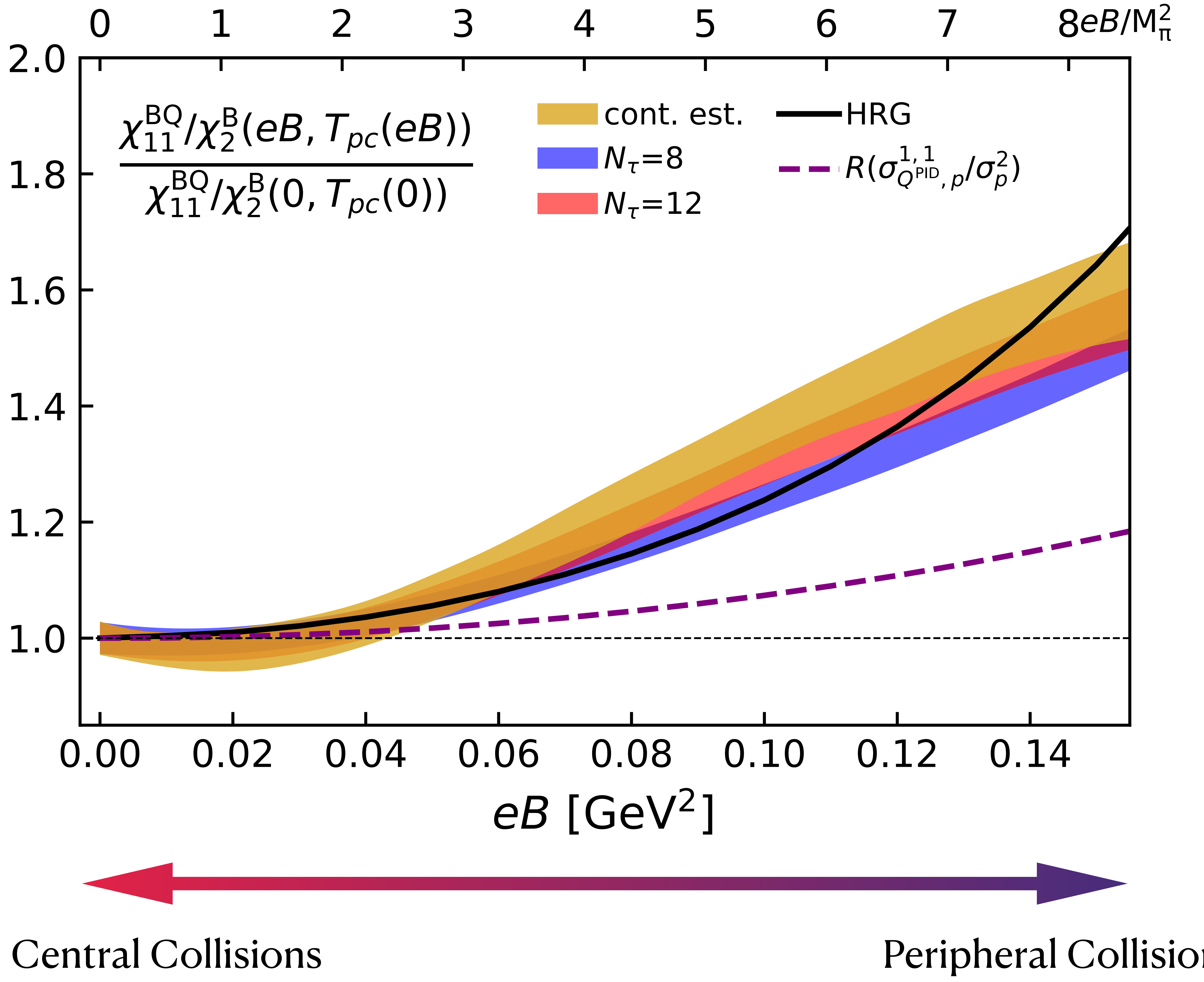
Peripheral Collisions

Lattice QCD meets experiments

Proxy for $\chi_{11}^{BQ}/\chi_2^{B}$

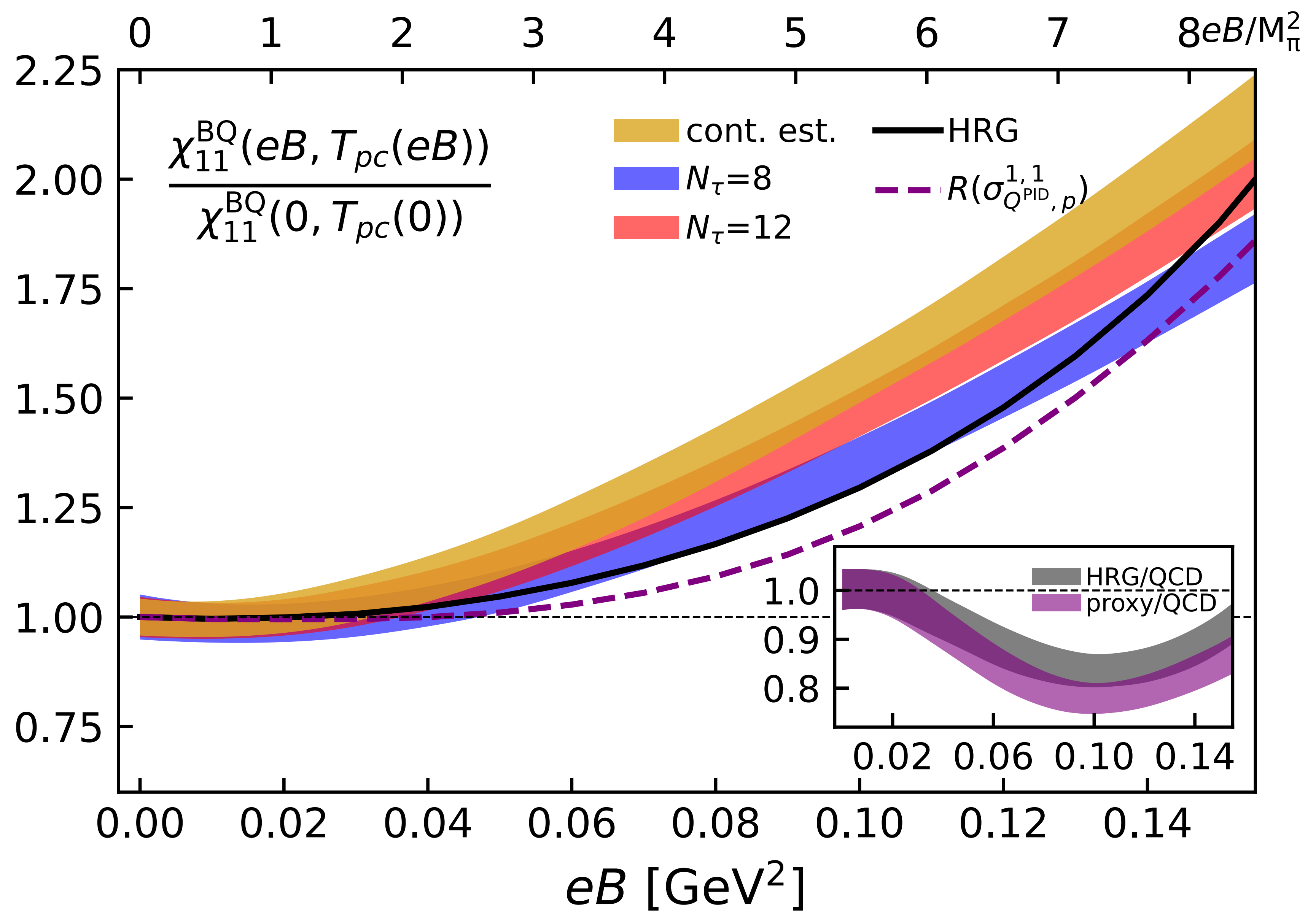


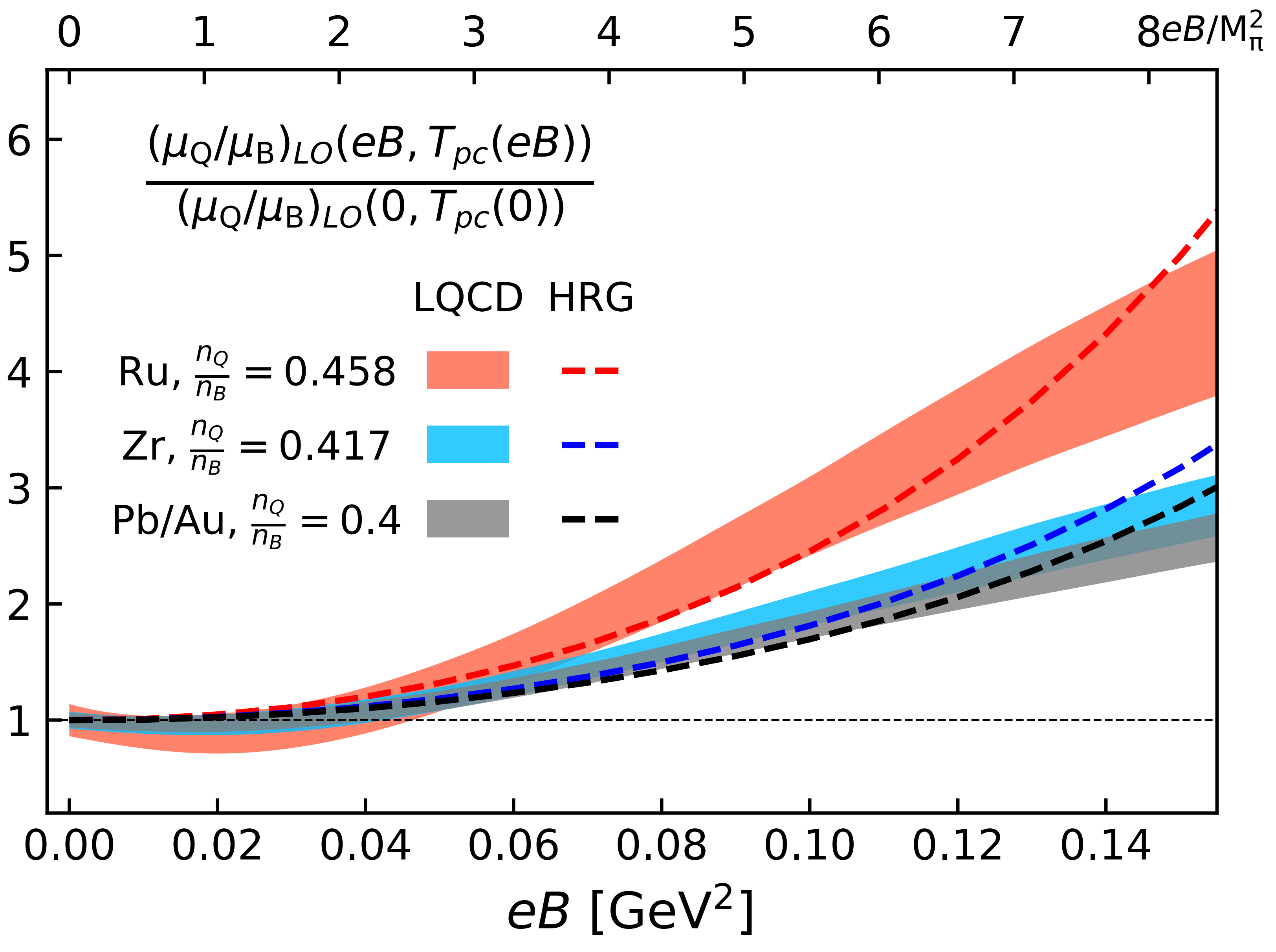
STAR, Phys. Rev. C 105 (2022) 029901(E)



Summary and outlook

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges based on LQCD computation on N_τ= 8 and 12 lattices
- $\chi_{11}^{\rm BQ}$ is strongly affected by eB, and a reasonable proxy is provided for measurement in HIC
- The $\mu_{\rm Q}/\mu_{\rm B}$ show a significant dependence on the magnetic field and is sensitive to the initial $n_{\rm Q}/n_{\rm B}$





- Computation of 4th order fluctuations is on the way

Backup

Lattice QCD in strong magnetic fields

B pointing to the z direction

$$u_x(n_x, n_y, n_z, n_\tau) = \begin{cases} \exp\left[-iqa^2 B N_x n_y\right] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$u_y(n_x, n_y, n_z, n_\tau) = \exp\left[iqa^2 B n_x\right]$$

$$u_z(n_x, n_y, n_z, n_\tau) = u_t(n_x, n_y, n_z, n_\tau) = 1$$

No sign problem !

Quantization of the magnetic field

$$q_u = 2/3\ e$$
$$q_d = -1/3\ e$$
$$q_s = -1/3\ e$$

→

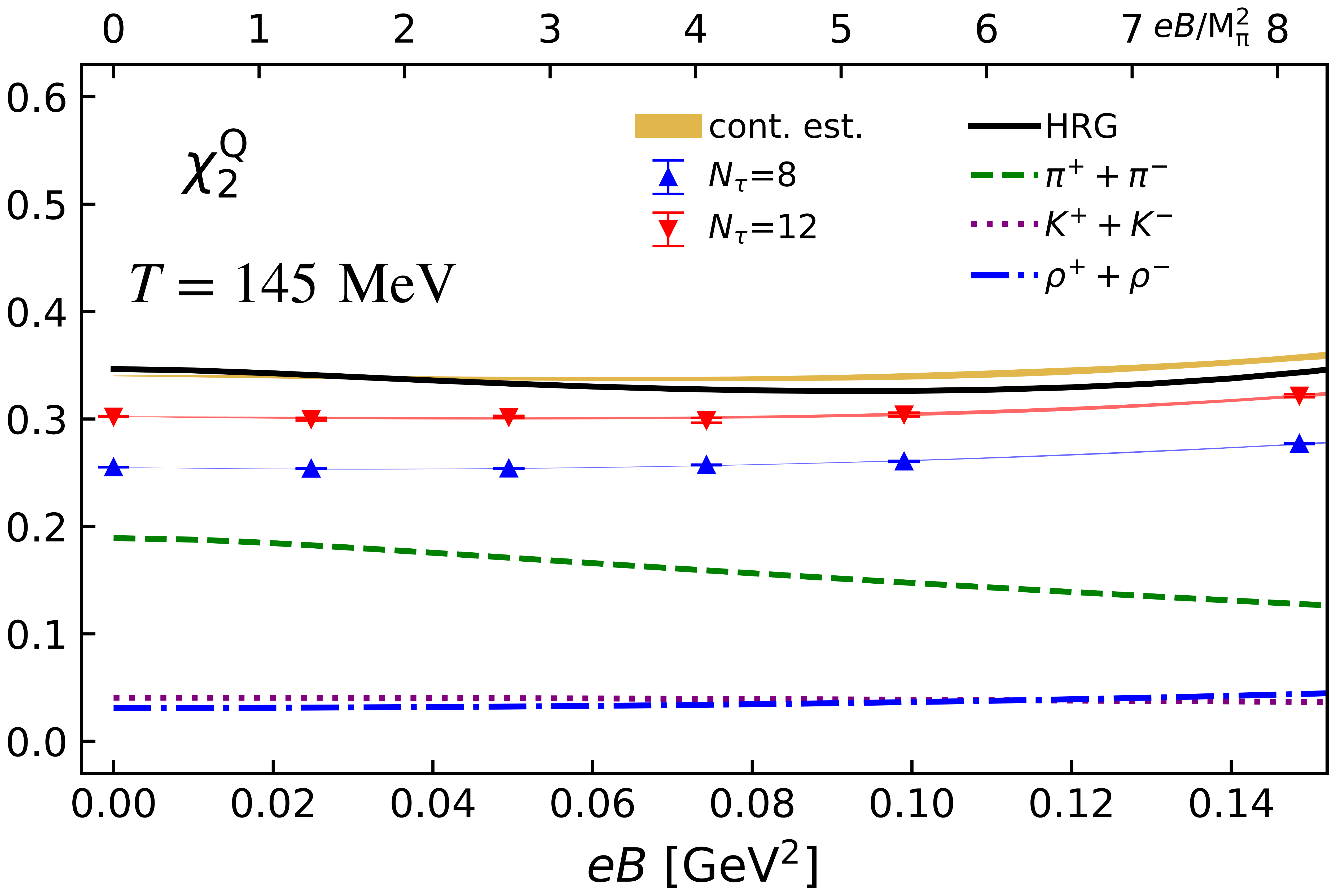
$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

a is changed to get the targeted T, $T = \dfrac{1}{aN_\tau}$

- Statistics($eB \neq 0$): N_τ=8: ~40000 (#N_{rv} : 603)
  N_τ=12: ~5000 (#N_{rv} : 102 ~ 705)

Landau gauge
G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

2nd order fluctuations below the transition temperature

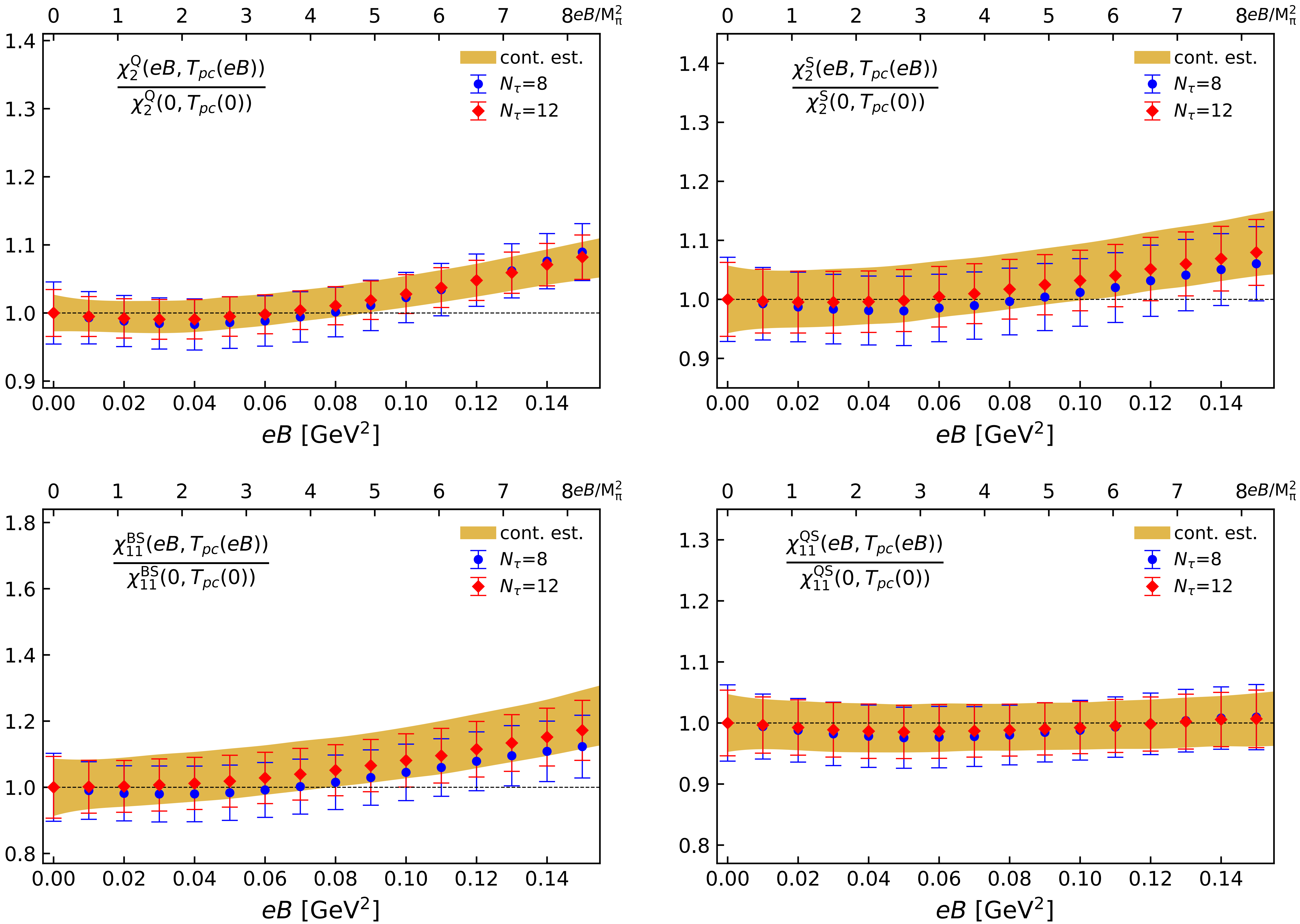


- χ_2^Q almost independent on eB
- HRG: Pressure arising from charged hadrons ($eB \neq 0$):

$$\frac{p_c^{M/B}}{T^4} = \frac{|q_i|\, B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} K_1\left(\frac{n\varepsilon_0}{T}\right)$$

where $\varepsilon_0 = \sqrt{m_i^2 + 2\,|q_i|\,B\,(l + 1/2 - s_z)}$, K_1 is the first-order modified Bessel function

Ratio for other 2nd order fluctuations and correlations



At $eB \simeq 8M_\pi^2$:

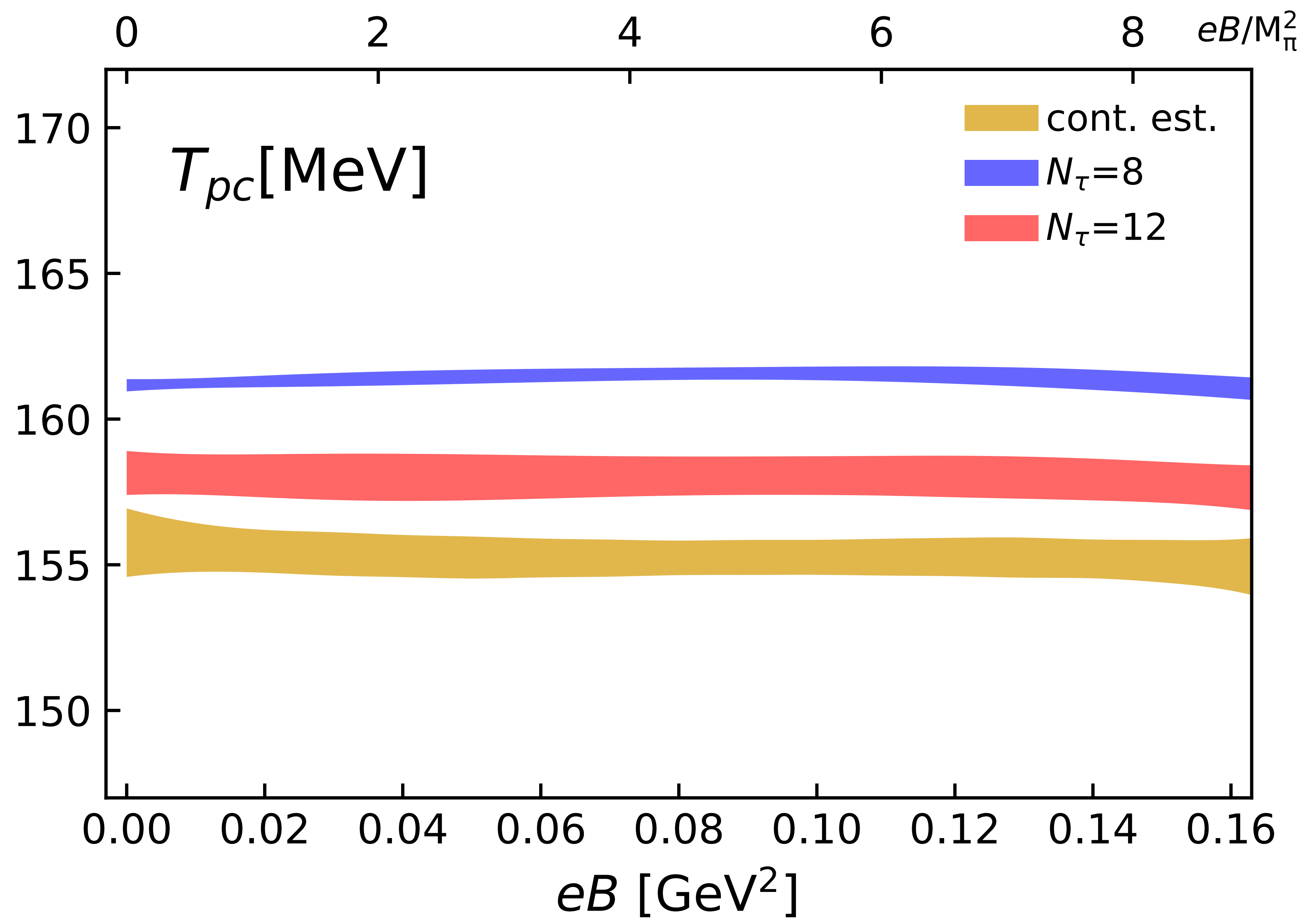
Ratio of χ_2^Q ~ 1.07

Ratio of χ_2^S ~ 1.1

Ratio of χ_{11}^{BS} ~ 1.2

Ratio of χ_{11}^{QS} ~ 1.03

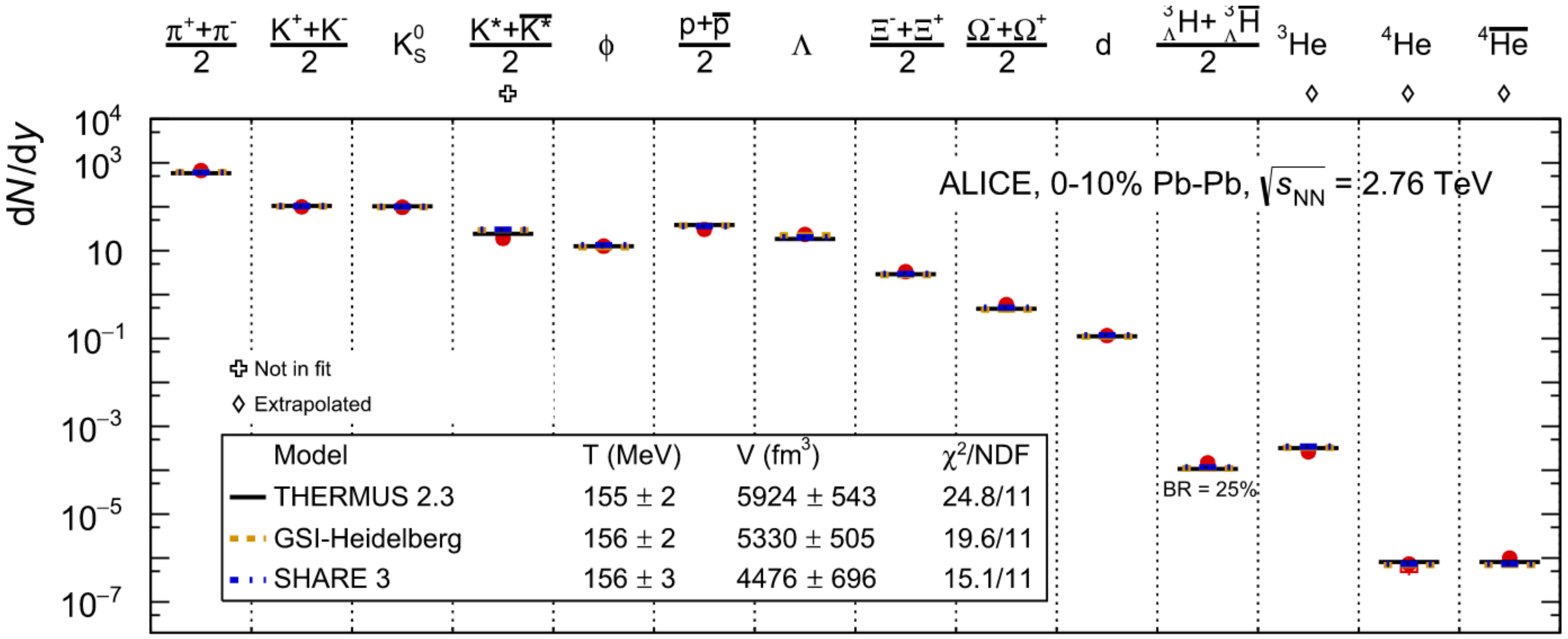
Transition line on $T - eB$ plane and $T_{\rm ch}$ in experiment



$$M = \frac{1}{f_K^4}\left[m_s\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) - \left(m_u + m_d\right)\langle\bar{\psi}\psi\rangle_s\right]$$

$$\chi_M(eB) = \frac{m_s}{f_K^4}\left[m_s\chi_l(eB) - 2\langle\bar{\psi}\psi\rangle_s(eB=0) - 4m_l\chi_{su}(eB=0)\right]$$

Finding the peak location of χ_M at each eB value



$T_{\rm ch} \simeq 156$ MeV

ALICE , Nucl.Phys.A 971 (2018) 1–20

Proxy in experiment

✦ Conserved charges susceptibilities in experiment:

$$\chi^2_\alpha = \frac{1}{VT^3}\kappa^2_\alpha, \quad \chi^{1,1}_{\alpha,\beta} = \frac{1}{VT^3}\kappa^{1,1}_{\alpha,\beta}$$

the second-order cumulants(κ) are the variance or covariance(σ) of the net-multiplicity N:

$$\kappa^2_\alpha = \sigma^2_\alpha = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)^2\rangle$$
$$\kappa^{1,1}_{\alpha,\beta} = \sigma^{1,1}_{\alpha,\beta} = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)(\delta N_\beta - \langle\delta N_\beta\rangle)\rangle$$

with $\delta N_\alpha = N_{\alpha^+} - N_{\alpha^-}$ and $\alpha, \beta = p, Q^{PID}, k$

- p : a proxy for the net-baryon
- k : a proxy for the net-strangeness
- Q^{PID}: identified π, k and p

STAR, Phys.Rev.C 100 (2019) 1, 014902

In experiment:

- p : a proxy for the net-baryon
- k : a proxy for the net-strangeness
- Q^{PID}: identified π, k and p

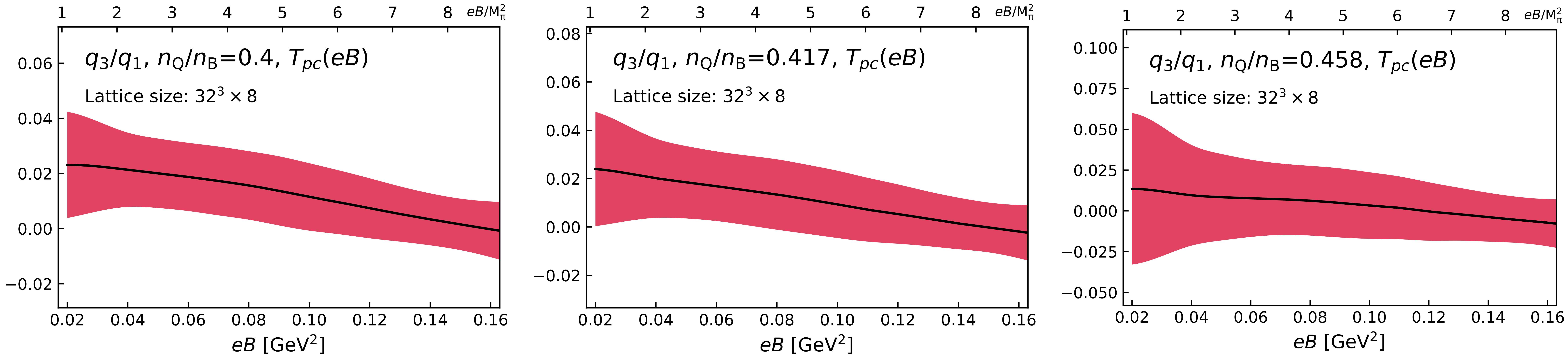
$$\sigma^{1,1}_{Q^{PID},p}(\chi^{BQ}_{11}) : \ \tilde{p}\tilde{p} + \tilde{p}\tilde{\pi}^+ + \tilde{p}\tilde{K}^+$$

$$\sigma^{1,1}_{Q^{PID},K}(\chi^{QS}_{11}) : \ \tilde{K}^+\tilde{p} + \tilde{K}^+\tilde{\pi}^+ + \tilde{K}^+\tilde{K}^+$$

The fluctuations are related to the variance or covariance of these net-multiplicities.

Dependence of $(\mu_Q/\mu_B)_{LO}$ on the magnetic field

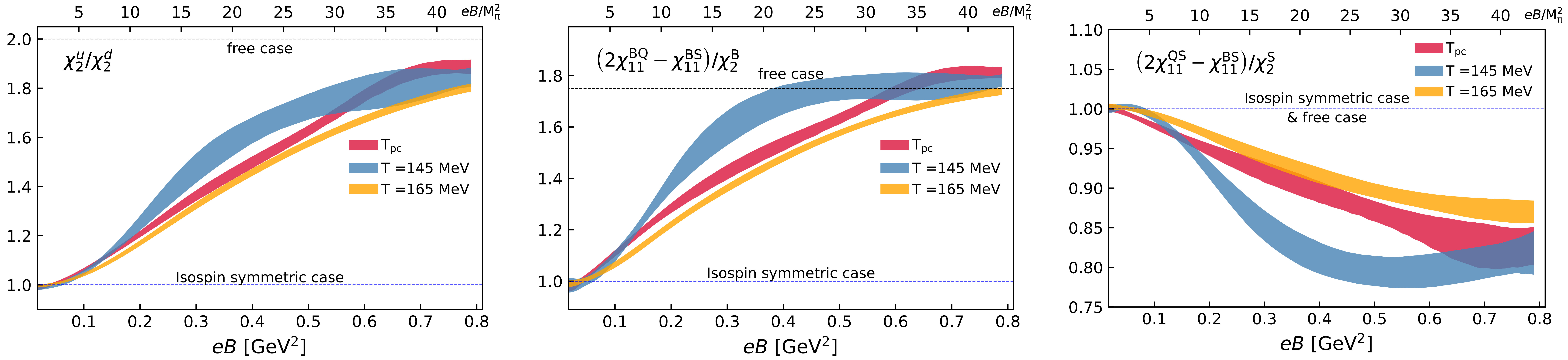
$$\hat{\mu}_Q/\hat{\mu}_B = q_1 + q_3\,\hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4) \qquad \hat{\mu}_B = \mu_B/T$$



q_3/q_1 in all cases remains within 2%

The next-to-leading order correction is negligible!

Isospin symmetry breaking in lattice



Due to $\chi_{11}^{us} = \chi_{11}^{ds}$ at $eB = 0$ case, we get:

$$2\chi_{11}^{QS} - \chi_{11}^{BS} = \chi_2^{S},$$

$$2\chi_{11}^{BQ} - \chi_{11}^{BS} = \chi_2^{B}$$