Dynamics of conserved net-baryon near QCD critical point

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In collaboration with **Huichao Song**

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QCD phase diagram

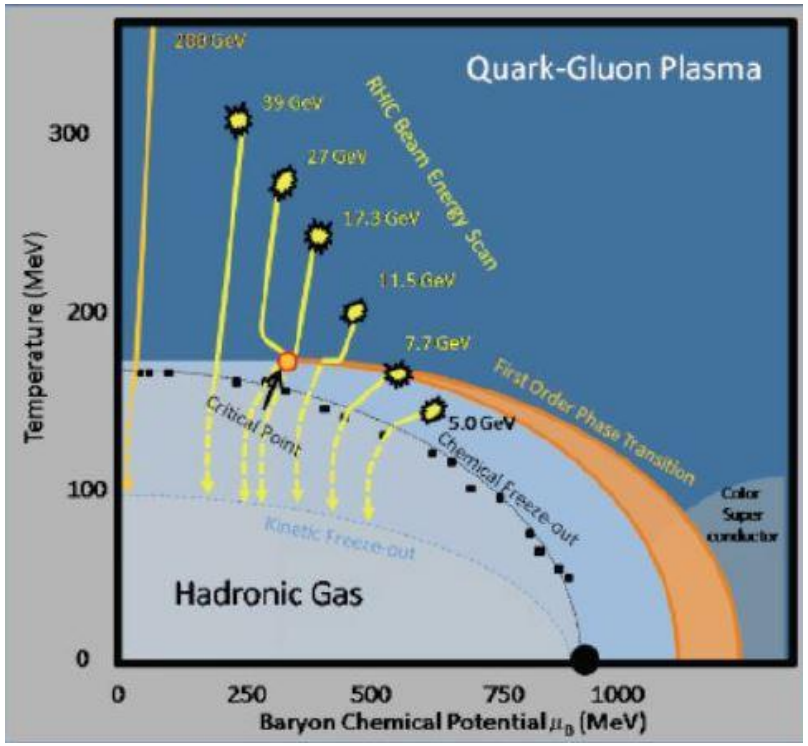
- **Lattice QCD** (small μ_B finite T):
  - Crossover
- **Effective models** (large μ_B)
  - 1st order phase trans.

→ Critical point

- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

→ **Heavy-ion collisions :**

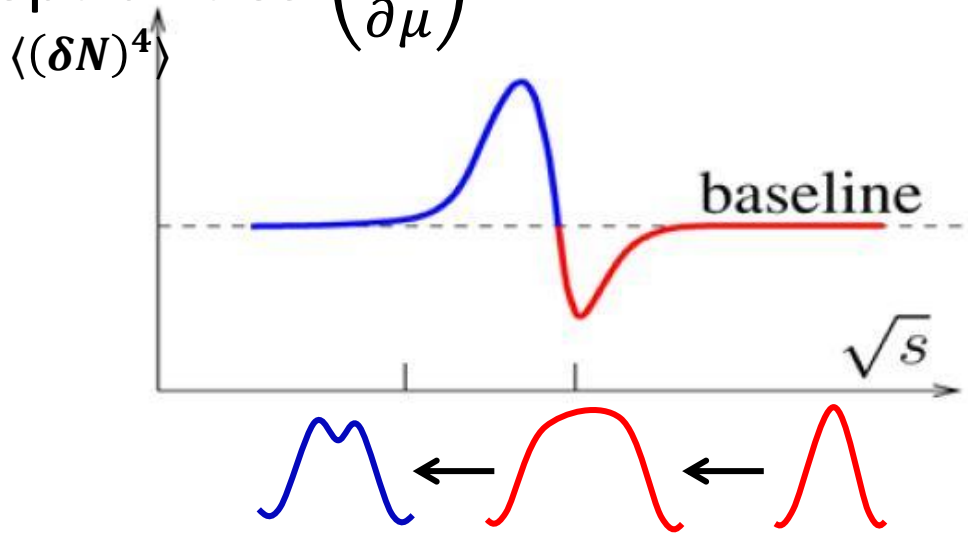
  - tuning $\sqrt{s_{NN}}$,mapping $T-\mu$ phase diagram: RHIC(BES),NICA,FAIR,J_PARC,HIAF….



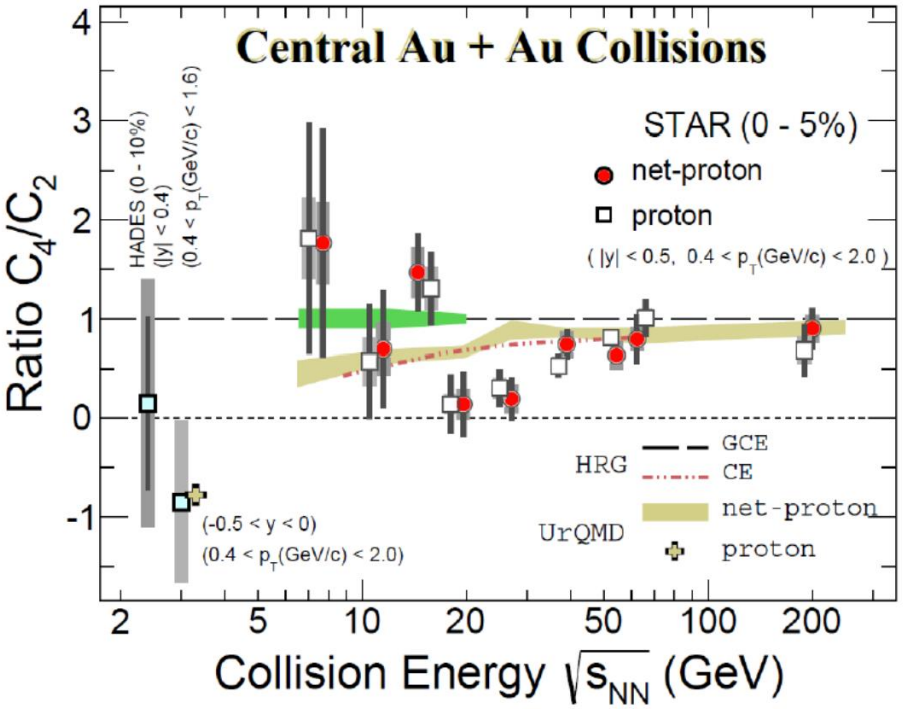
Net-proton fluctuations near critical point

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** of Net-Proton Cumulant

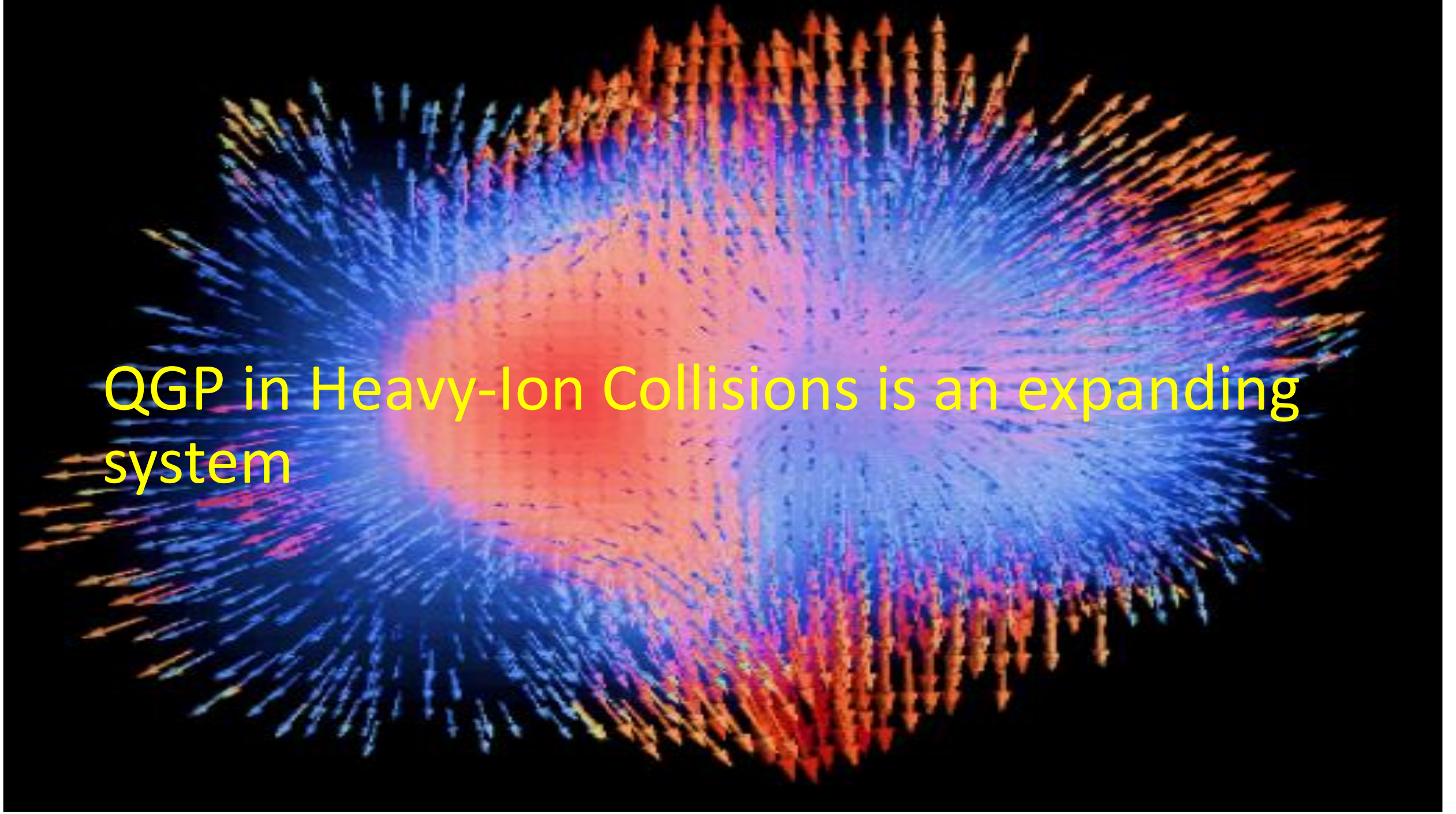
$\sim$ susceptibilities $\left(\frac{\partial}{\partial \mu}\right)^n \ln Z$



M.Stephanov, PRL 107,052301



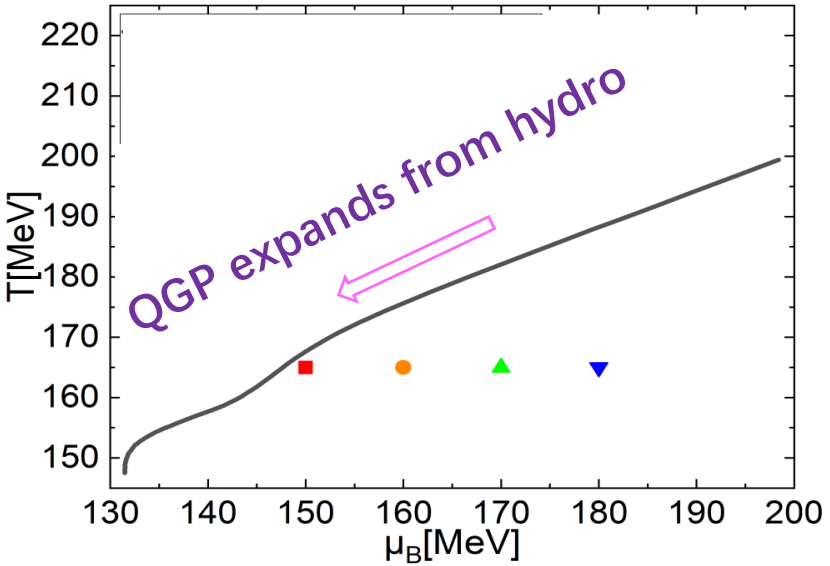
STAR, PRL 126,092301
STAR,PRL 128,202303

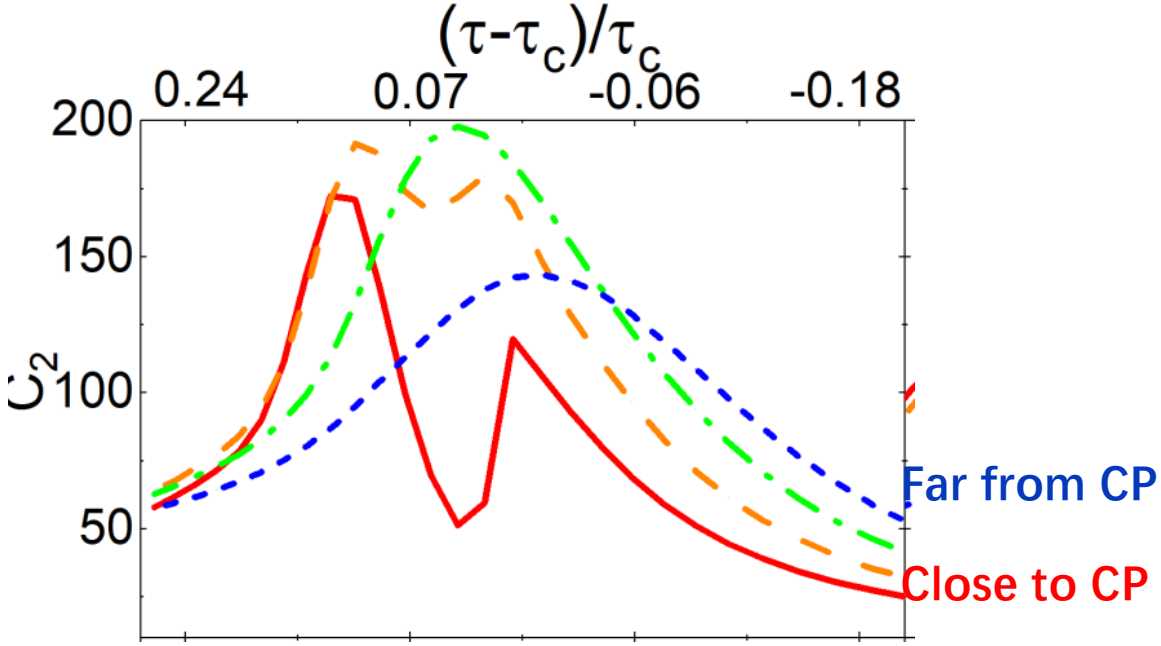

QGP in Heavy-Ion Collisions is an expanding system

Fluctuations is non-trivial in expanding QGP

S.Tang, SW, H.Song, PRC(2023)

- Fluctuations is largest at critical point in equilibrium case. $C_2 \sim \xi^2$
- Expanding QGP => Critical Slowing Down. $\tau_{relax} \sim \xi^z, z = 3$
- Critical slowing down effects suppress the fluctuations
- Largest fluctuations **not necessary** closest to critical point





Dynamical models near QCD critical point

Modeling in expanding QGP: Hydrodynamics + Critical fluctuations

- **Model A** (order parameter field)
  S.Mukherjee et al15’ 16’, L.Jiang et al17’, S.Wu et al 19’, S.Tang et al 23’,
- **Model B** (conserved field)
  M.Sakaida et al 17’, S.Wu et al 19’, M.Nahrgang et al 19’, G.Pihan et al 22’…
- **Model H** (conserved order parameter field + momentum+…)
  it is hard and in progress
- **Non-equilibrium chiral hydrodynamics** (hydro + order parameter)
  M. Nahrgang et al 11’12’14’16’19’
- **Hydro+, hydro++…** (hydro + slow modes)
  M. Stephanov et al 18’19’20’, N. Abbasi et al 22’, L. Du et al 20’,……
- **Fluctuating hydrodynamics** (hydro + noise)
  J.Kapusta et al 12’,12’, K.Murase et al 13’, X.An et al 19’,21’…
- **Hydro-kinetics** (deterministic fluctuating hydro)
  D.Teaney et al 17’18’19’22’…

See reviews: e.g. Lipei Du et al. 2402.10183; Xin An et al., 2108.13867; Shanjiu Wu, et al.,2104.13250; Marcus Bluhm et al., 2001.08831; Adam Bzdak et al.,1906.00936;M.Asakawa et al.,1512.05308

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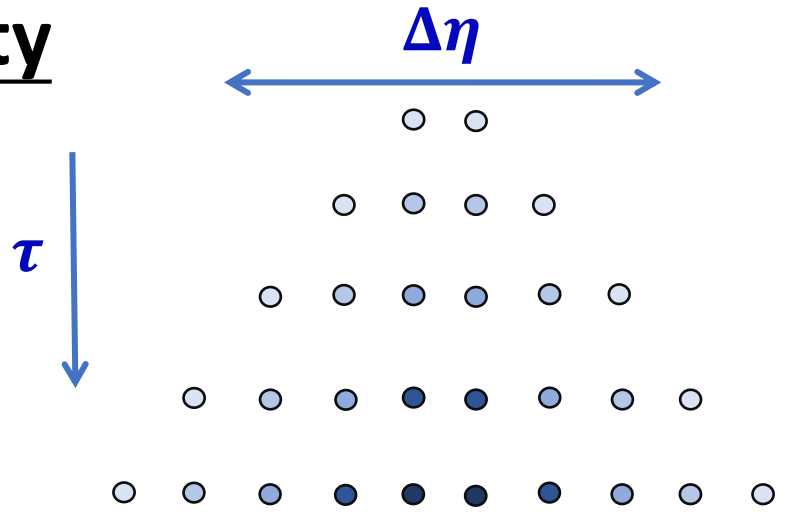
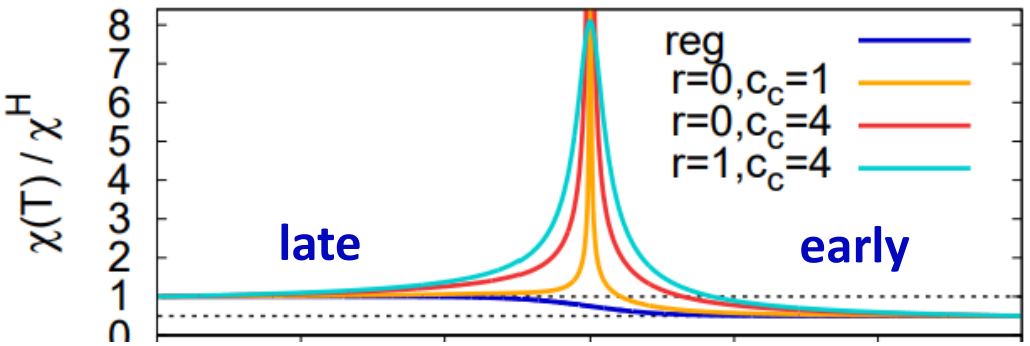
Dynamics of conserved net-baryon density

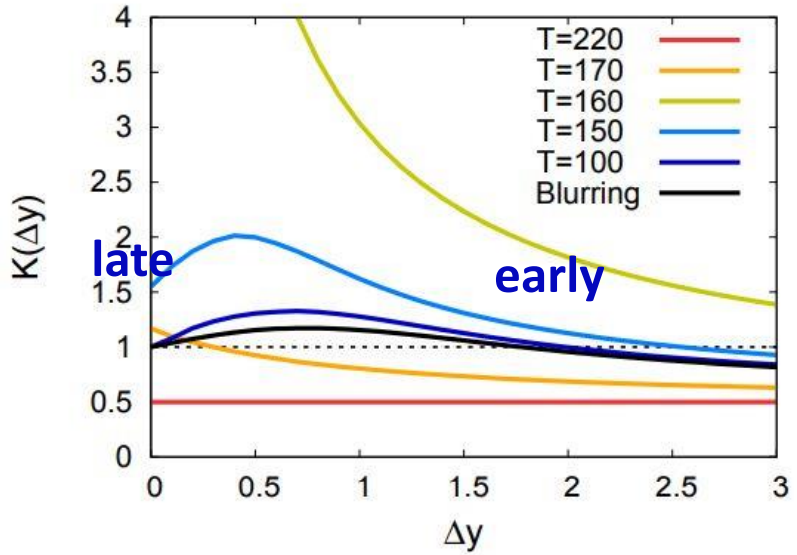


- Diffusion of conserved baryon near critical point:

$$\partial_\mu N^\mu = 0 \Rightarrow \partial_\tau n = \nabla^2 n + \text{noise}$$

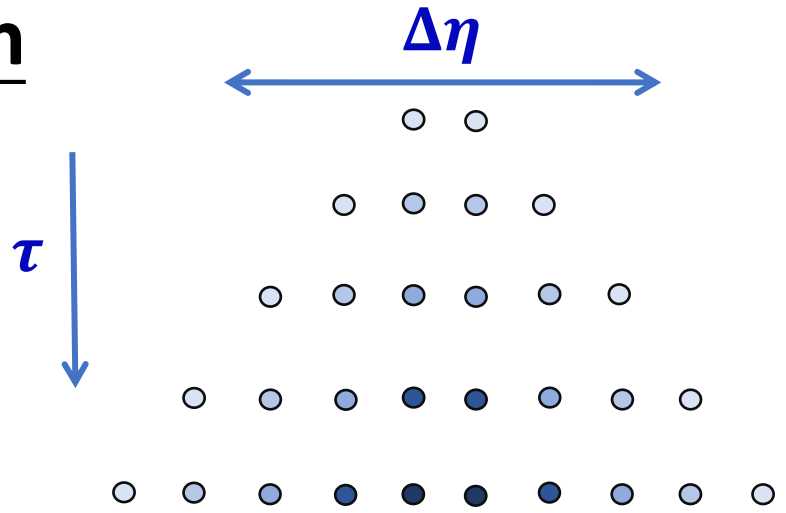
- The process of diffusion consumes time.
- Correlation of larger $\Delta\eta$ preserves the information of the early evolution.





Sakaida et al, PRC.95.064905(2017)

Conserved net-baryon with non-Gaussian

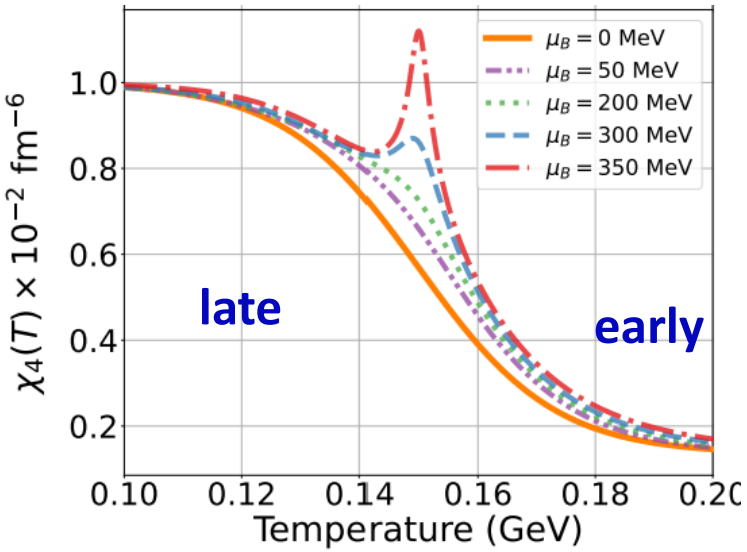


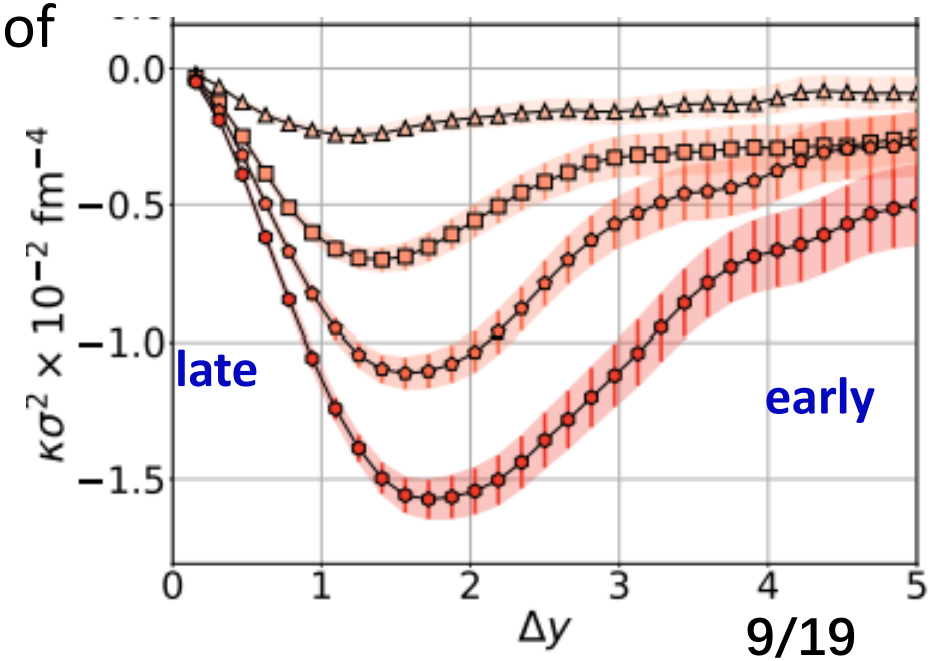
- Diffusion of conserved baryon near critical point:

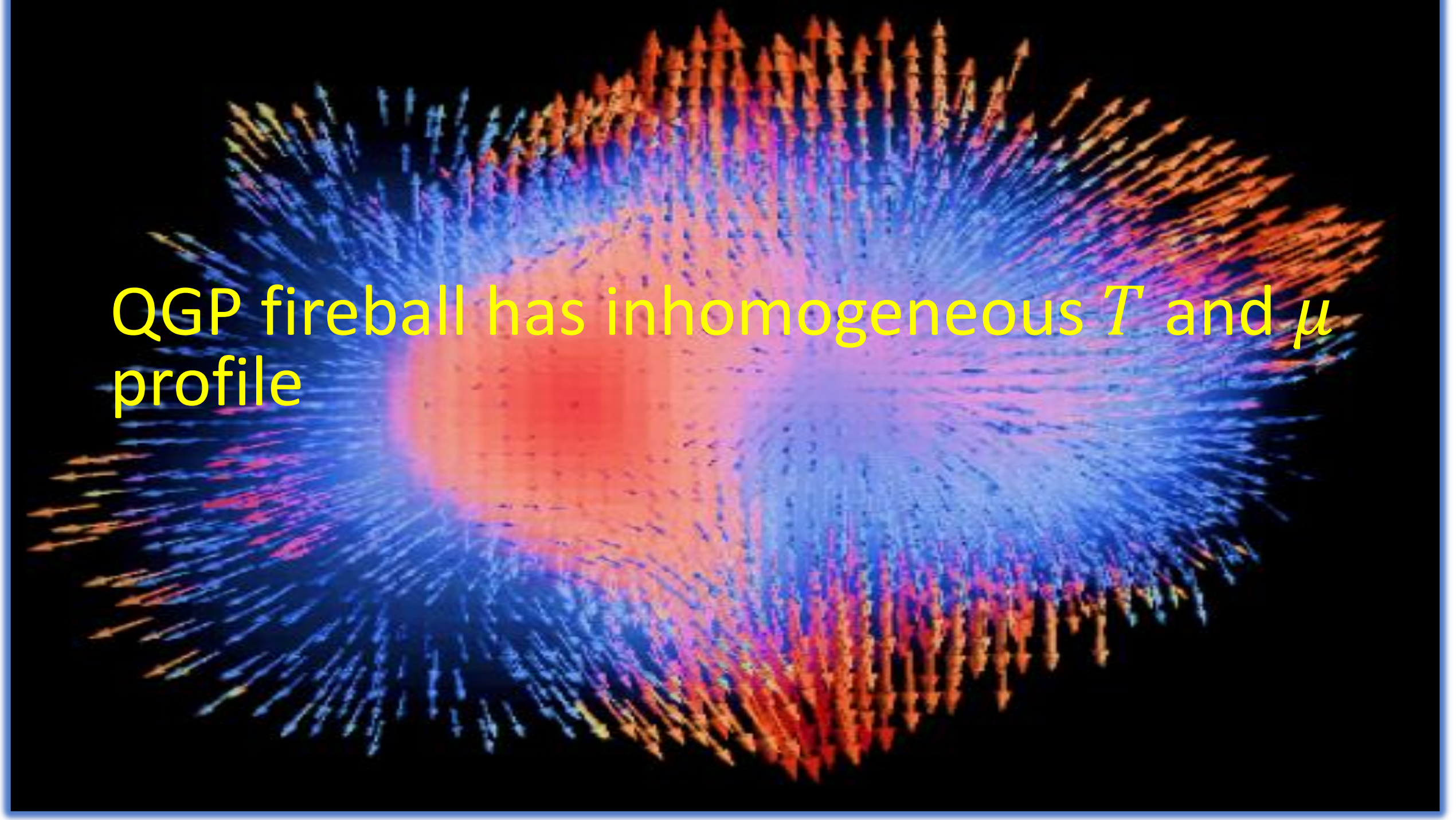
$$\partial_\mu N^\mu = 0 \Rightarrow \partial_\tau n = \nabla^2(n + n^2 + n^3) + \text{noise}$$

- The process of diffusion consumes time.
- Correlation of larger $\Delta\eta$ preserves the information of the early evolution.

G.Pihan et al., PRC.107.014908(2022)



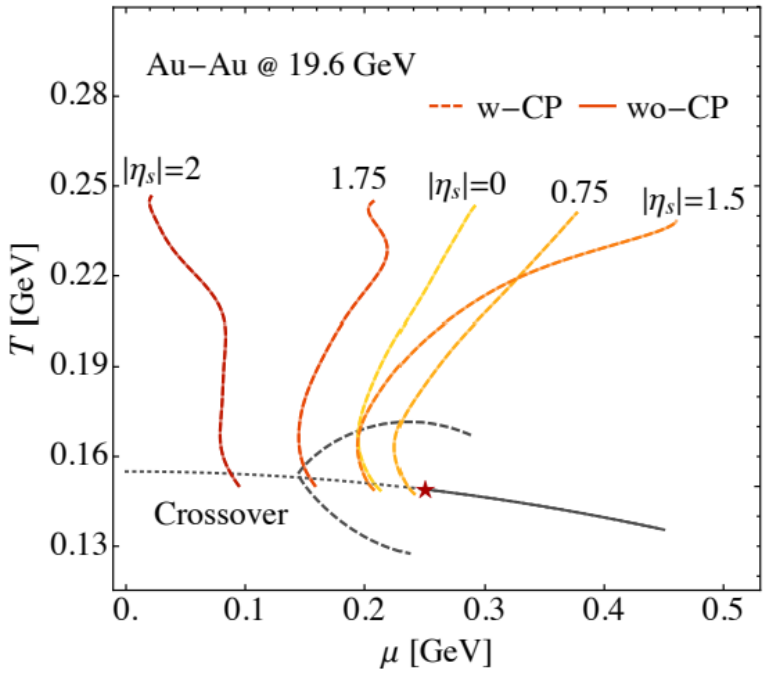



QGP fireball has inhomogeneous T and μ profile

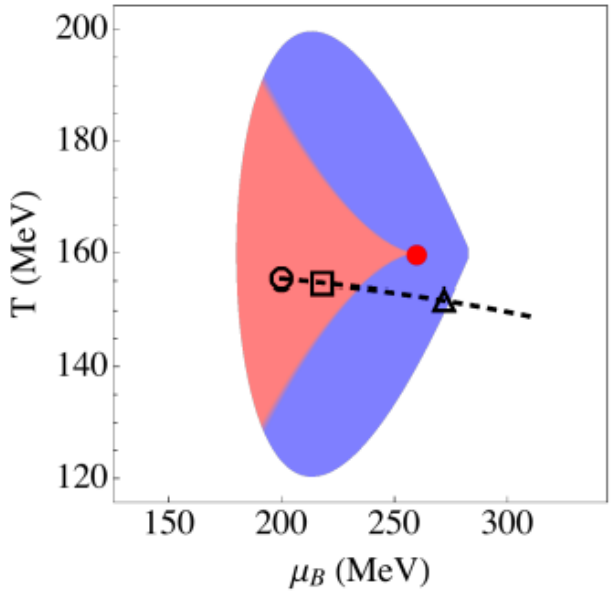
Inhomogeneous QGP profile

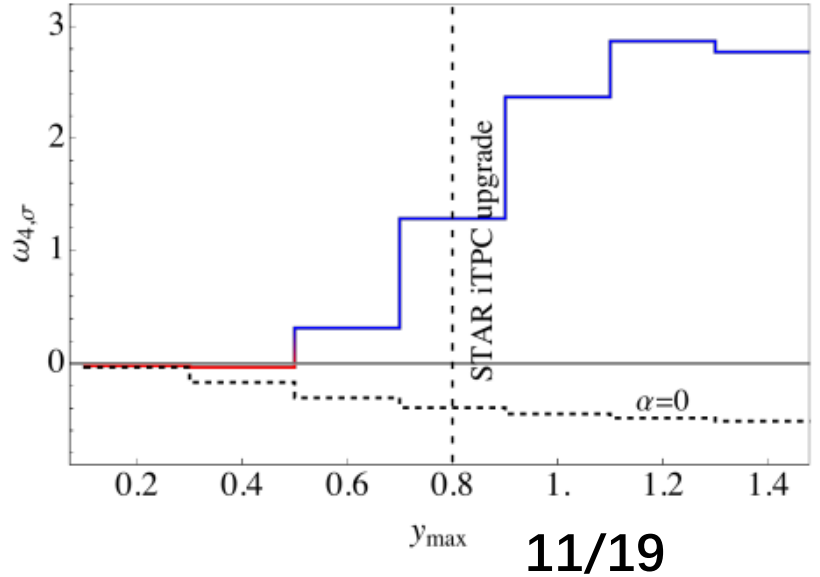
- Different rapidity of the QGP profile evolves along different trajectories on the QCD phase diagram
- Different rapidity detect different region of critical region

L.Du et al., PhysRevC.104.064904



J.Brewer et al., PhysRevC.98.061901

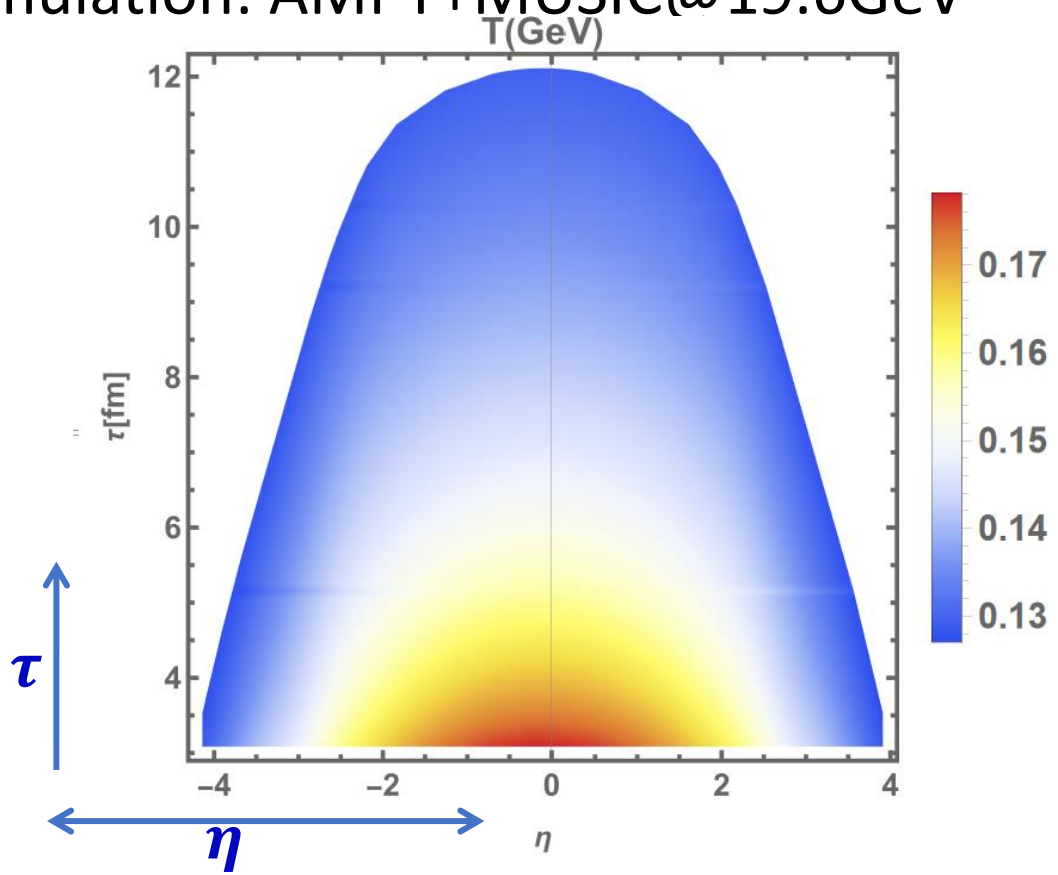


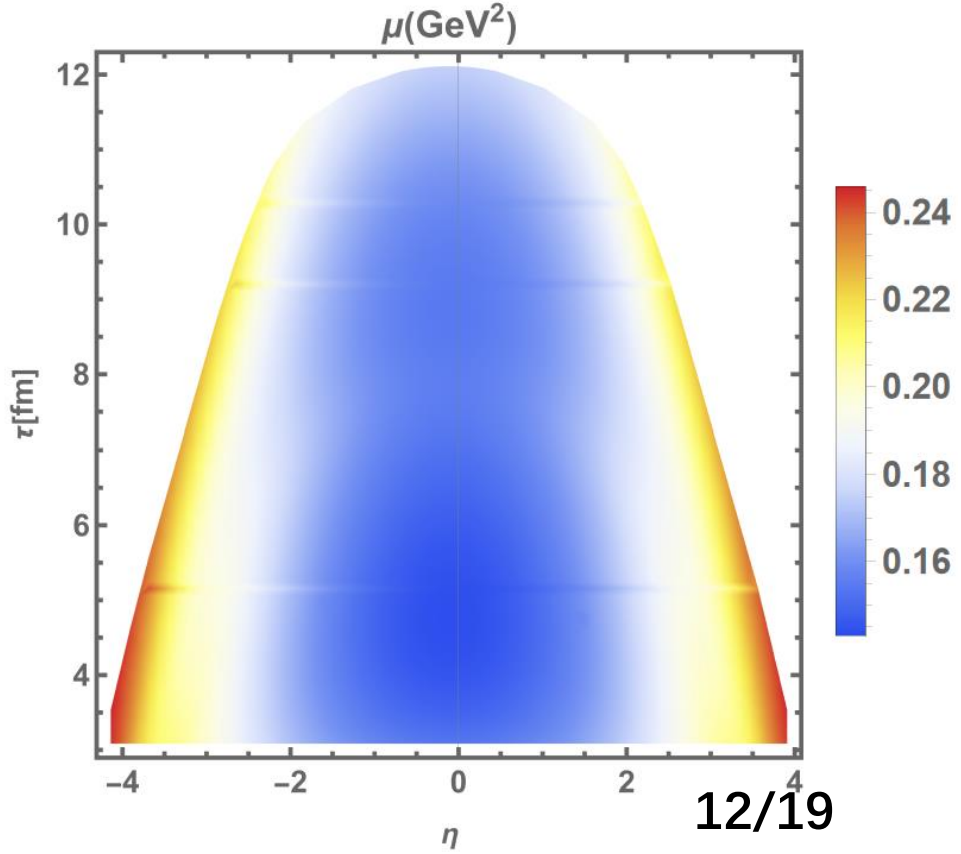


Inhomogeneous T and μ profile from hydro simulation

- This talk aims to study the inhomogeneous QGP profile effects on the diffusion of net-baryon
- Hydro simulation: AMPT+MUSIC@19.6GeV

S.Wu et al., in preparation



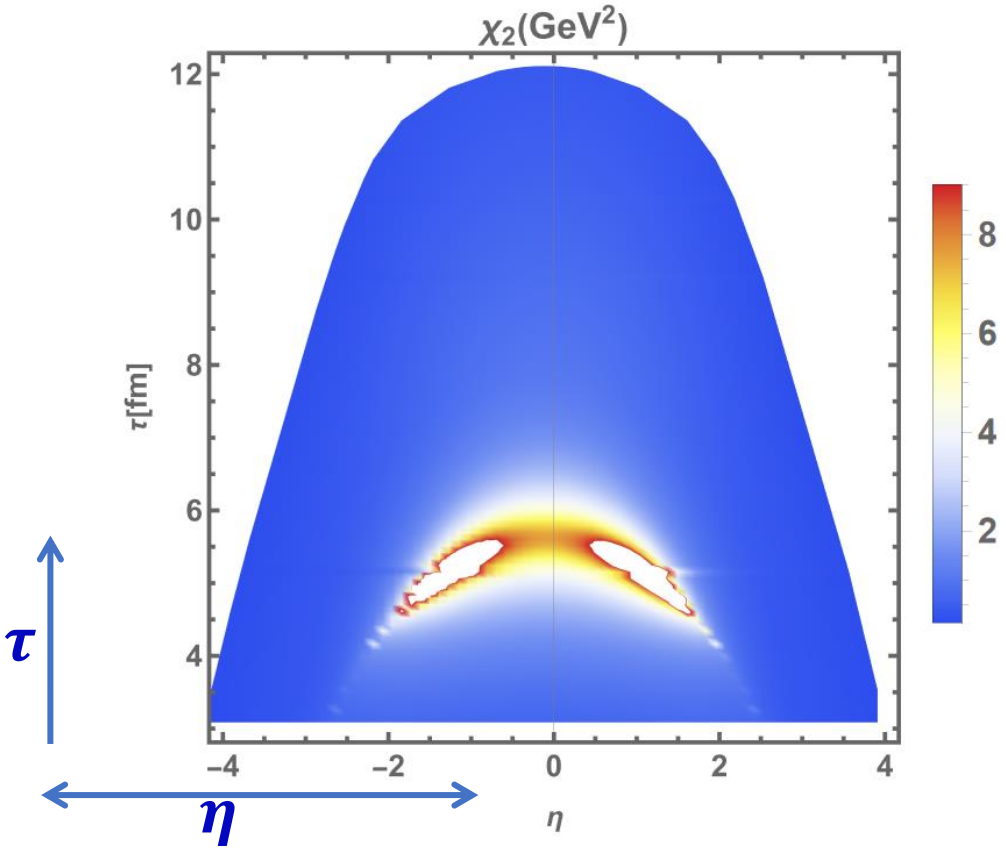


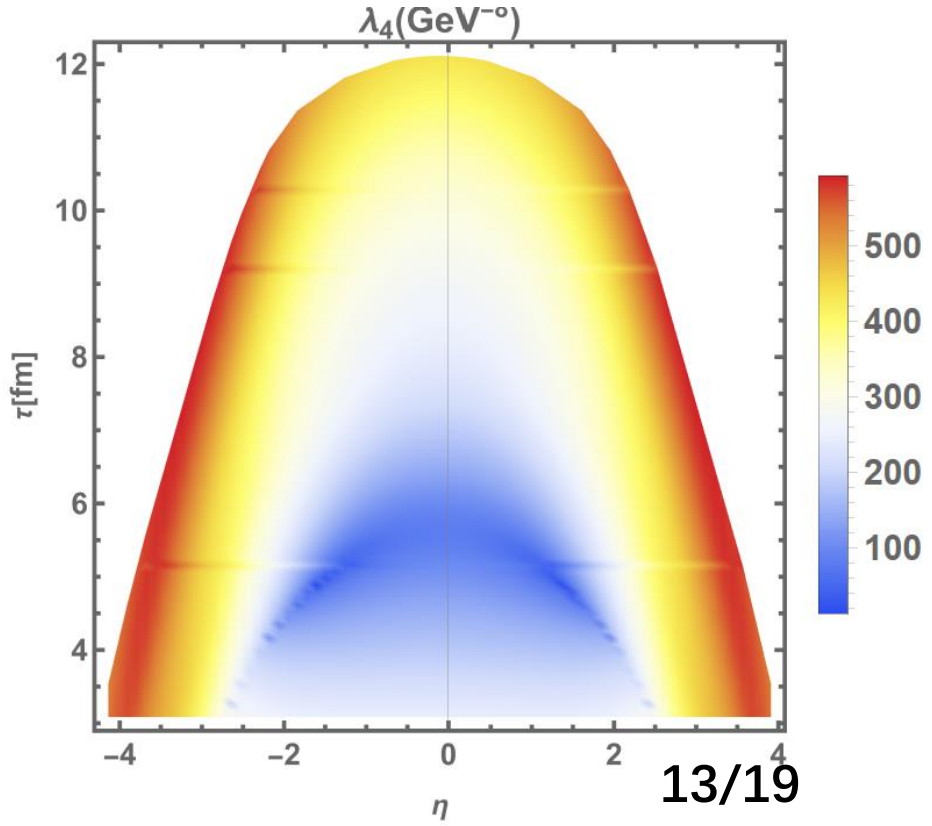
Conserved net-baryon with inhomogeneous T and μ profile

- Diffusion of conserved baryon near critical point: S.Wu et al., in preparation

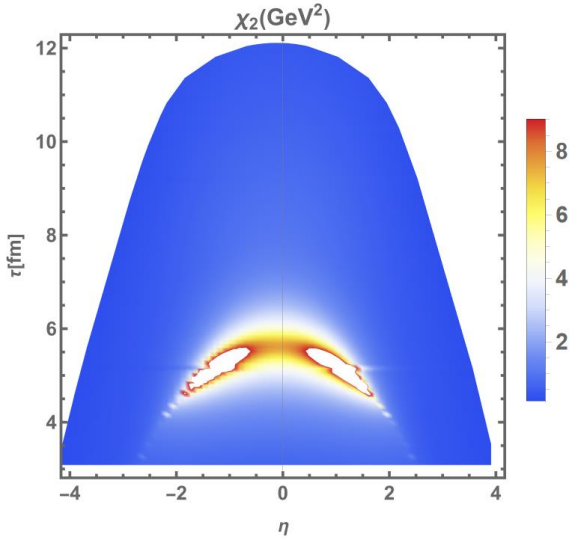
$$\partial_\mu N^\mu = 0 \Rightarrow \partial_\tau n = \nabla^2\left(\frac{n}{\chi_2} + \lambda_3 n^2 + \lambda_4\, n^3\right) + \text{noise}$$

χ_2, λ_4 from Ising model





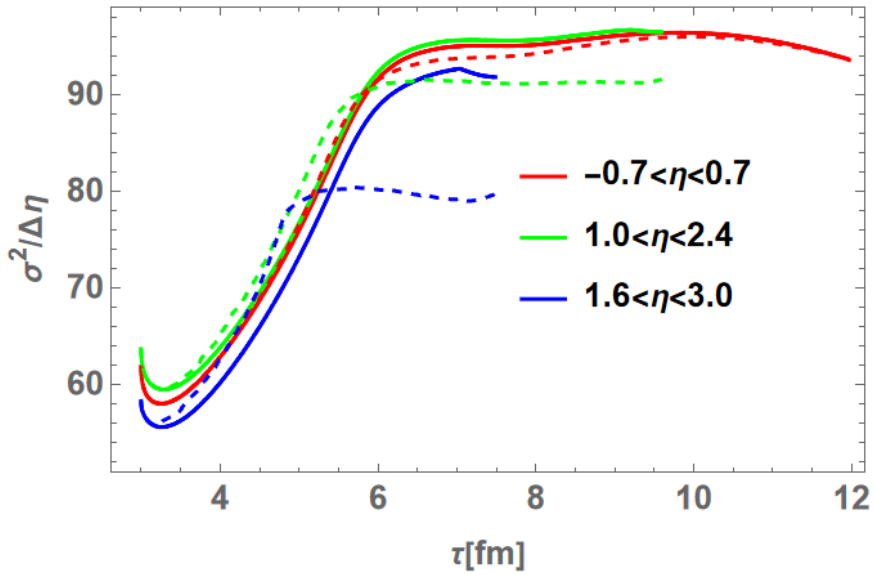
Time evolution of conserved net-baryon fluctuations

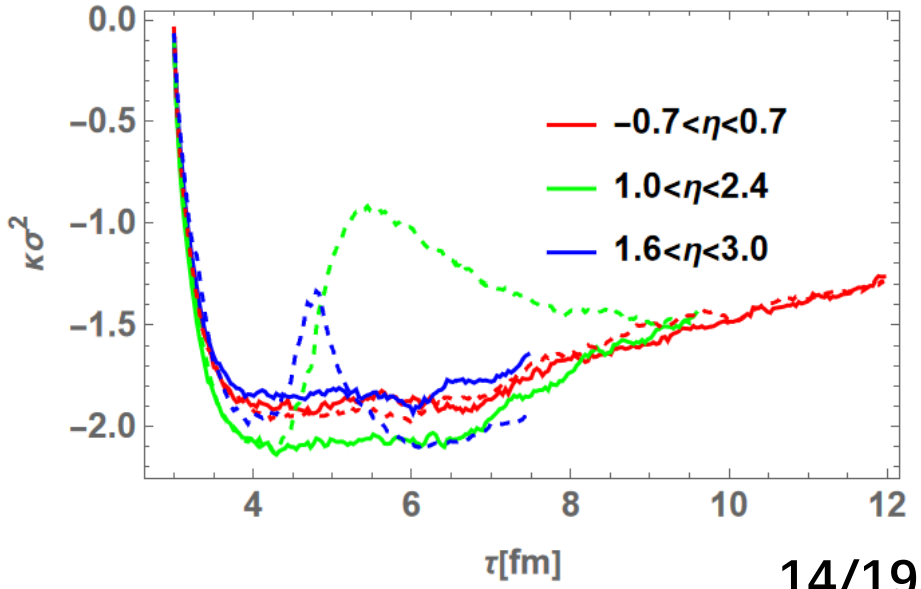


S.Wu et al., in preparation

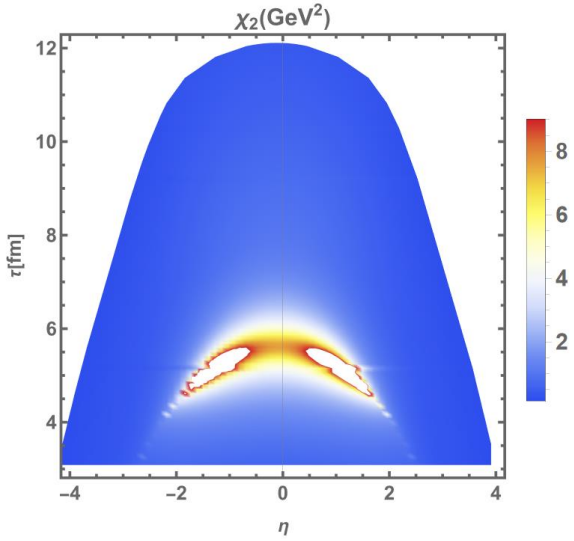
Inhomogeneous profile effects is significant at large rapidity

Solid: uniform profile; Dashed: inhomogeneous profile





Rapidity dependence of conserved net-baryon fluctuations



S.Wu et al., in preparation

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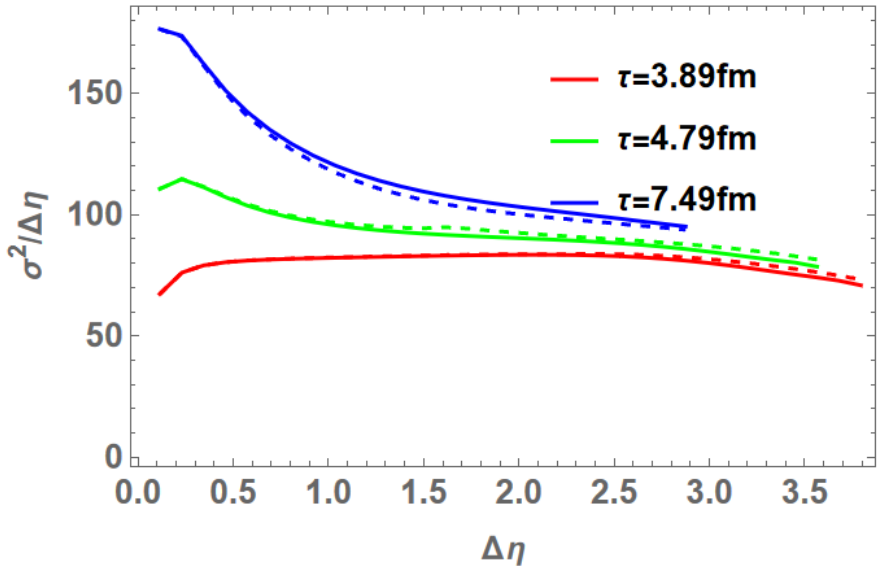
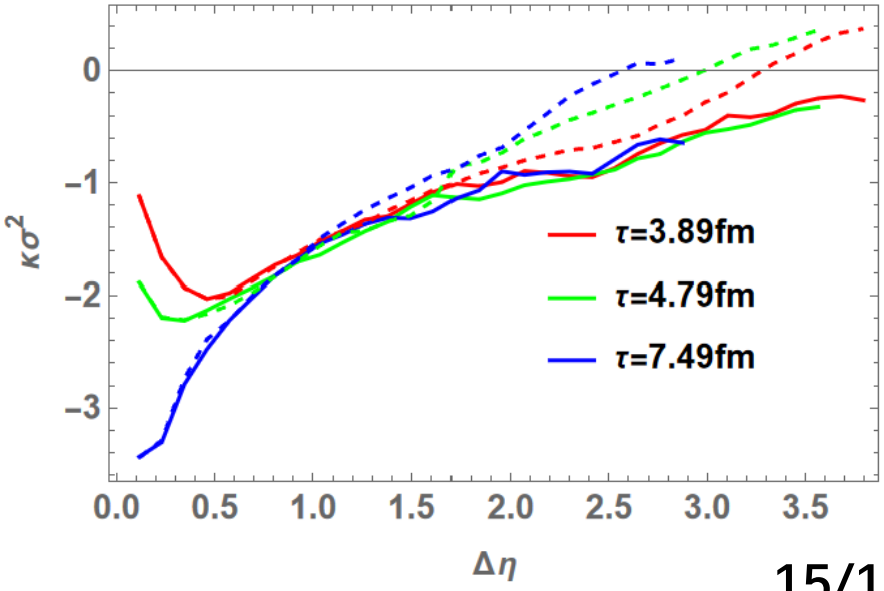
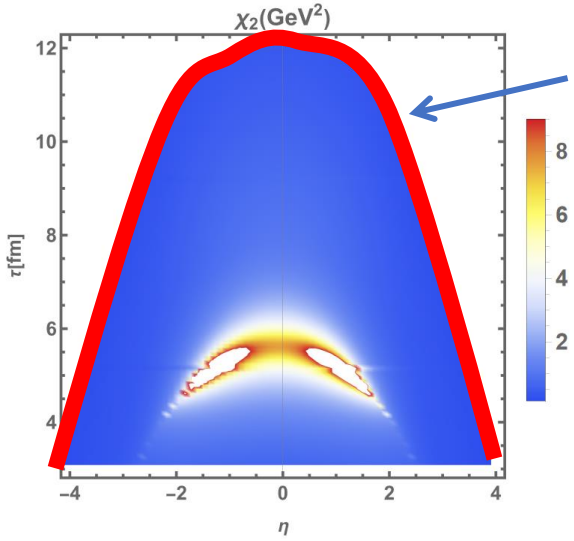




Conserved net-baryon fluctuations at freeze-out surface



Freeze-out surface

S.Wu et al., in preparation

Net-baryon number at freeze-out surface

$$N_B = g \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^0} \int d\sigma_\mu p^\mu f(\boldsymbol{x}, \boldsymbol{p})$$

In Bjorken limit

$$= \frac{2gA}{(2\pi)^2} \int_{-\Delta\eta}^{\Delta\eta} d\eta \tau_f \exp\left(\frac{\mu}{T}\right) T^3 \left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right)$$

Net-baryon fluctuations at freeze-out surface

$$\delta N_B = \frac{2gAm^2}{(2\pi)^2} \int_{-\Delta\eta}^{\Delta\eta} d\eta \exp\left(\frac{\mu}{T}\right) \frac{\delta n_B(\tau_f, \eta)}{\chi} K_2\left(\frac{m}{T}\right)$$

B.Ling, T.Springer and M.Stephanov, PhysRevC.89.064901

Conserved net-baryon fluctuations at freeze-out surface

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S.Wu et al., in preparation

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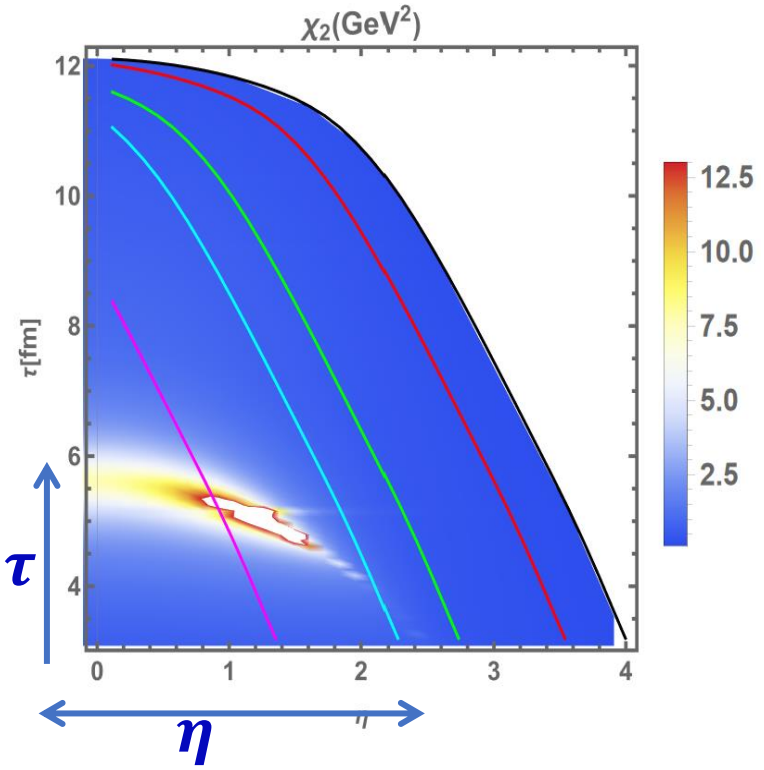
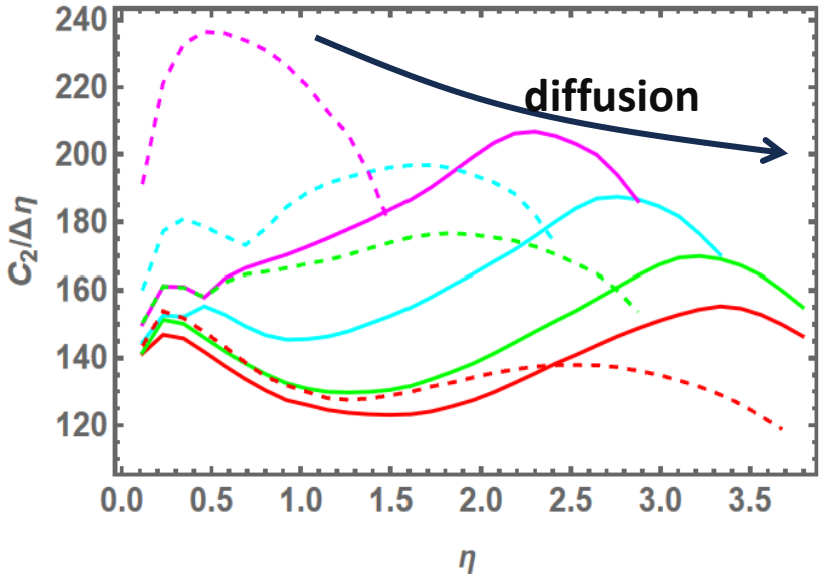
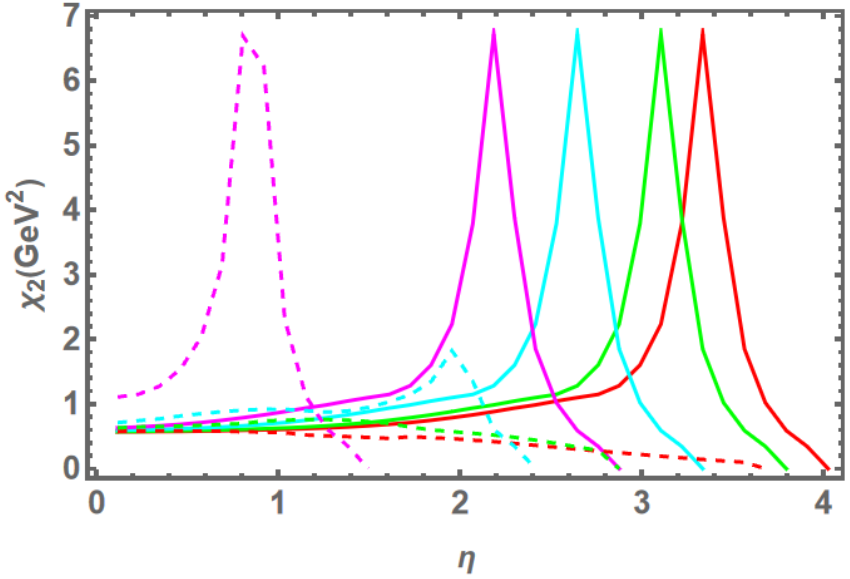


$C_2 = \langle(\delta n_B)^2\rangle$



χ_2



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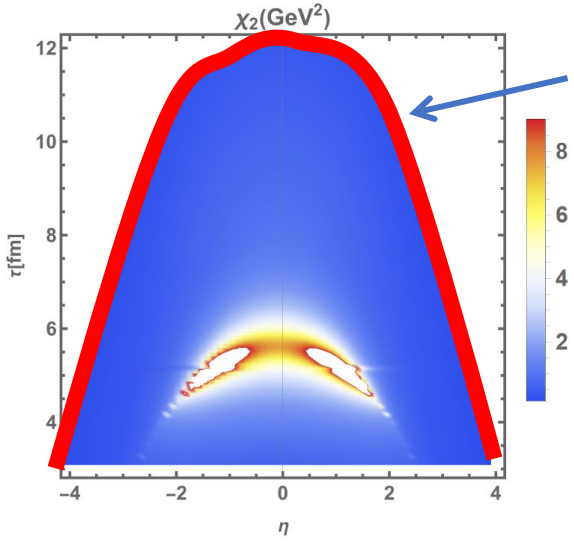

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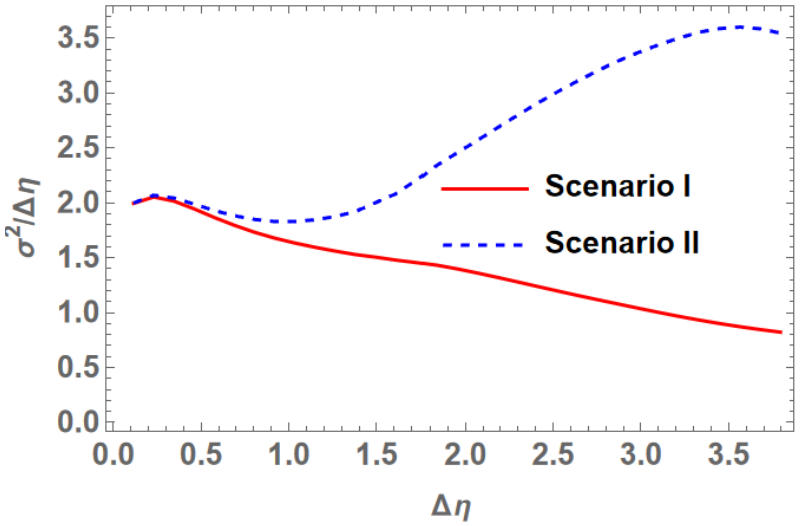

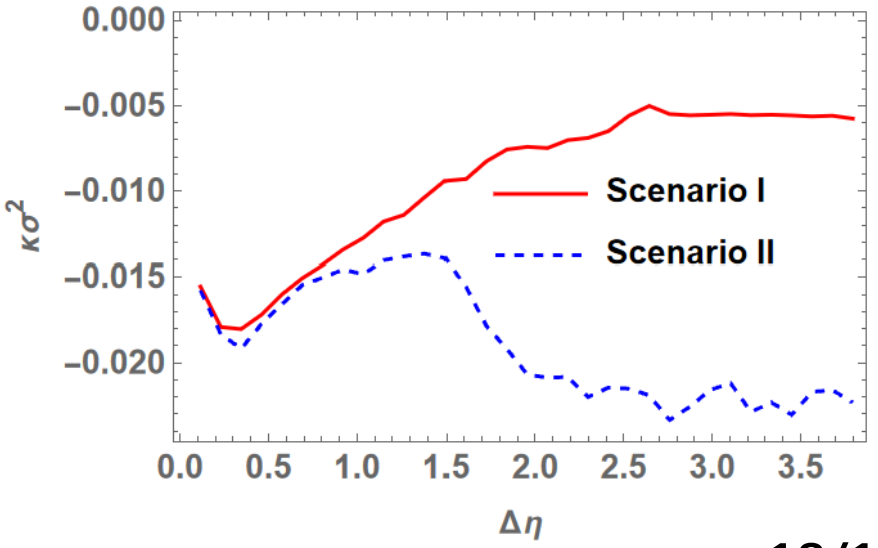

Summary

- Dynamical modeling the QGP evolution near the QCD critical point is essential for the study of fluctuations in heavy-ion experiments;
- The diffusion of conserved net-baryon density preserves the early evolution history and behaves non-monotonically with increasing rapidity;
- Considering the inhomogeneous T and μ profile has significant effects at large rapidity.

Thank you!