李政道研究所
TSUNG-DAO LEE INSTITUTE

Resolving Phase Transition Properties of Dense Matter through Tidal-excited g-mode from inspiralling neutron stars

Zhiqiang Miao (缪志强)
zq_miao@sjtu.edu.cn
Tsung-Dao Lee Institute, SJTU

In collaboration with：
Enping Zhou（周恩平） @HUST and Ang Li（李昂）@XMU

Based on MZQ+2024 ApJ 964, 31 (2305.08501)

SpicyGluons 胶麻2024

2024.5.16 @ Hefei

Outline

- Background
- G-mode induced by 1st PT interface
- Tidal seismology and GW
- Summary

❖Background

Fujimoto+ PRL 2022

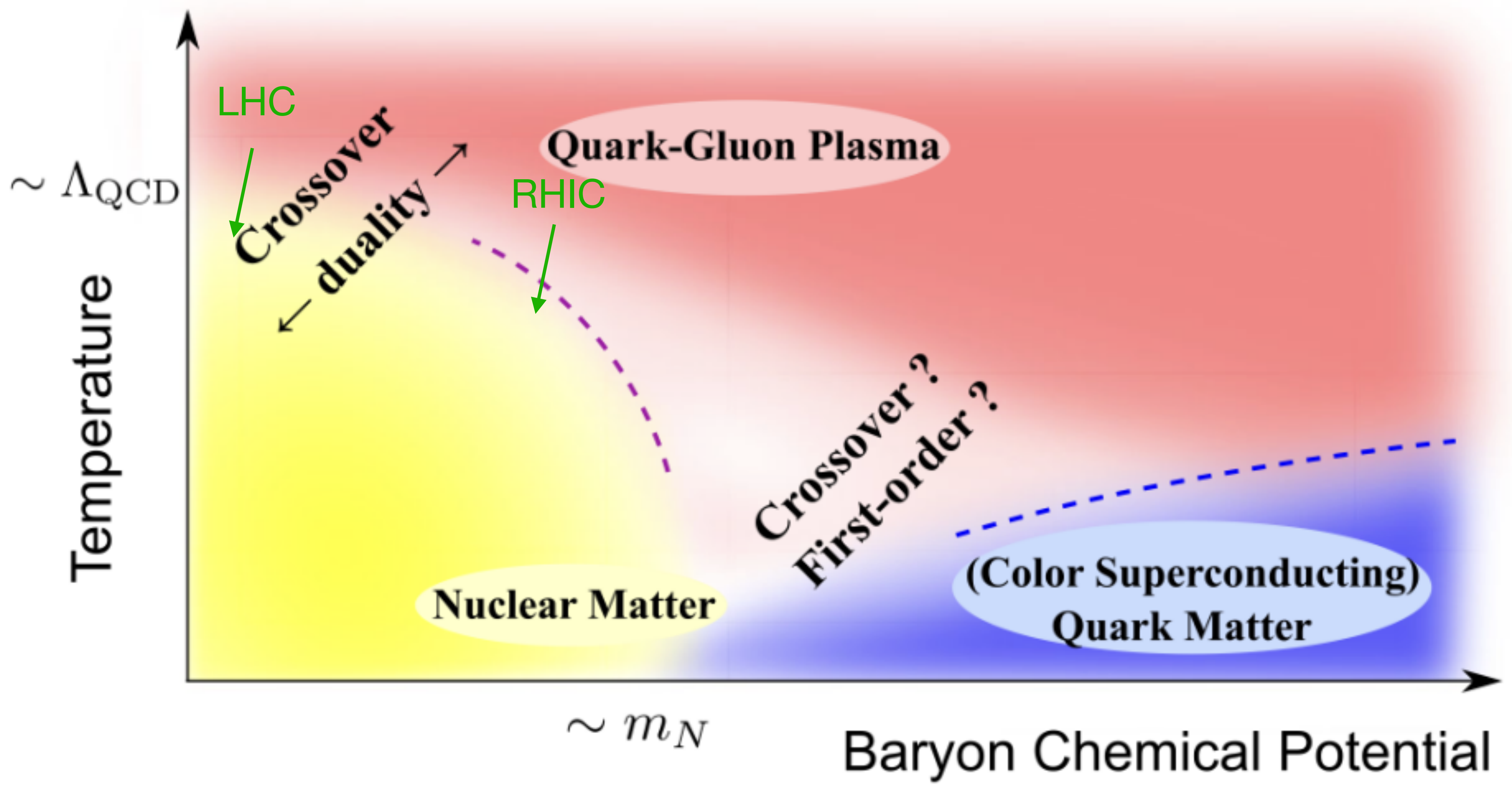
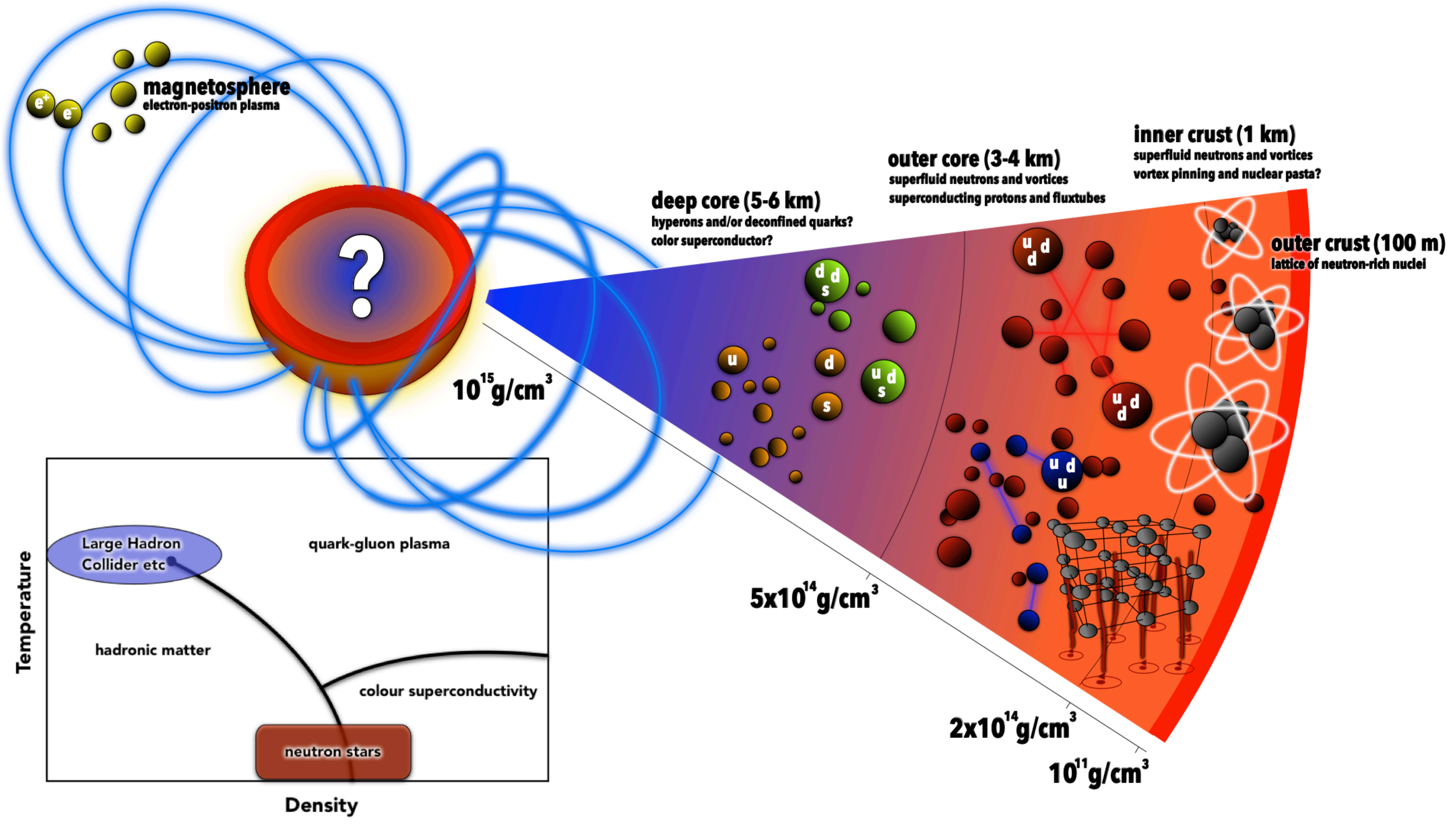


FIG. 1. Schematic QCD phase diagram. Deconfinement at high temperature and low density has been established to be a smooth crossover. A change to QM at low temperature is yet unresolved.

- If there is a 1st PT, then can we detect it by using neutron star observations?

❖Background



Graber&Andersson 2017

- The densest observable object in the universe. For M = 1.4$M_{\odot}$, R≈10km, average density ~few times nuclear density ($\sim 10^{14}$ g/cm^3)

- The ideal laboratory for exploring physics under extreme conditions.
  - Gravity
  - Electromagnetism
  - Strong interaction
  - Weak interaction

Outline

- Background
- G-mode induced by 1st PT interface
- Tidal seismology and GW
- Summary

❖Neutron star seismology and normal modes

李政道研究所
TSUNG-DAO LEE INSTITUTE

- Basic equations (Newtonian, normal modes)
  - Non-rotating star in equilibrium (the background star)

$$\frac{dp}{dr} = -\frac{\rho M}{r^2}$$

  - perturbation equations

$$\xi(r,t) = (\xi^r \hat{\boldsymbol{r}} + \xi^h r\nabla) Y_{lm}(\theta, \phi) e^{i\omega t}$$

$$\partial_t^2 \boldsymbol{\xi} = \frac{\delta\rho}{\rho^2}\nabla p - \frac{1}{\rho}\nabla\delta p - \nabla\delta\Phi \qquad \text{(Euler equation)}$$

$$\delta\rho = -\nabla\cdot(\rho\boldsymbol{\xi}) \qquad \text{(Continuity)}$$

⬇

$$\mathscr{L}\boldsymbol{\xi} - \rho\omega^2\xi = 0$$

$$\mathscr{L}\boldsymbol{\xi} = \rho[-\nabla(\frac{\Gamma p}{\rho}\nabla\cdot\boldsymbol{\xi}) - \nabla(\frac{1}{\rho}\boldsymbol{\xi}\cdot\nabla p) + \nabla\delta\Phi]$$

❖Neutron star seismology and normal modes

TSUNG-DAO LEE INSTITUTE

- Basic equations (Newtonian, normal modes)
  - Non-rotating star in equilibrium (the background star)

$$\frac{dp}{dr} = -\frac{\rho M}{r^2}$$

  - perturbation equations

$$\xi(r,t) = (\xi^r \hat{\boldsymbol{r}} + \xi^h r\nabla) Y_{lm}(\theta,\phi) e^{i\omega t}$$

$$\partial_t^2 \boldsymbol{\xi} = \frac{\delta\rho}{\rho^2}\nabla p - \frac{1}{\rho}\nabla\delta p - \nabla\delta\Phi \qquad \text{(Euler equation)}$$

$$\delta\rho = -\nabla\cdot(\rho\boldsymbol{\xi}) \qquad \text{(Continuity)}$$

⬇

$$\mathscr{L}\boldsymbol{\xi} - \rho\omega^2\xi = 0$$

$$\mathscr{L}\boldsymbol{\xi} = \rho[-\nabla(\frac{\Gamma p}{\rho}\nabla\cdot\boldsymbol{\xi}) - \nabla(\frac{1}{\rho}\boldsymbol{\xi}\cdot\nabla p) + \nabla\delta\Phi]$$

- Some difference in GR (quasi-normal modes)

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2 d\Omega$$

$$\frac{dp}{dr} = -\frac{(\rho+p)(M+4\pi r^3 p)}{r(r-2M)} \qquad \text{(TOV equation)}$$

$$\xi^r = r^{l-1}e^{-\lambda}WY_{lm}e^{i\omega t}, \quad \xi^\theta = -r^{l-2}V\partial_\theta Y_{lm}e^{i\omega t}, \quad \xi^\phi = -r^{l-2}\sin^{-2}\theta V\partial_\phi Y_{lm}e^{i\omega t}$$

$$h_{\mu\nu} = -r^l H_0 e^{i\omega t}Y_{lm}dt^2 - r^l H_0 e^{i\omega t}Y_{lm}dr^2 - r^l K e^{i\omega t}Y_{lm}r^2(d\theta^2 + \sin^2\theta d\phi^2) - 2i\omega r^{l+1}H_1 e^{i\omega t}Y_{lm}dtdr$$

$$(p+\varepsilon)u^\nu\nabla_\nu u^\mu + \perp^{\mu\nu}\nabla_\nu p = 0$$

$$\nabla_\mu(nu^\mu) = 0 \qquad \text{(Continuity)}$$

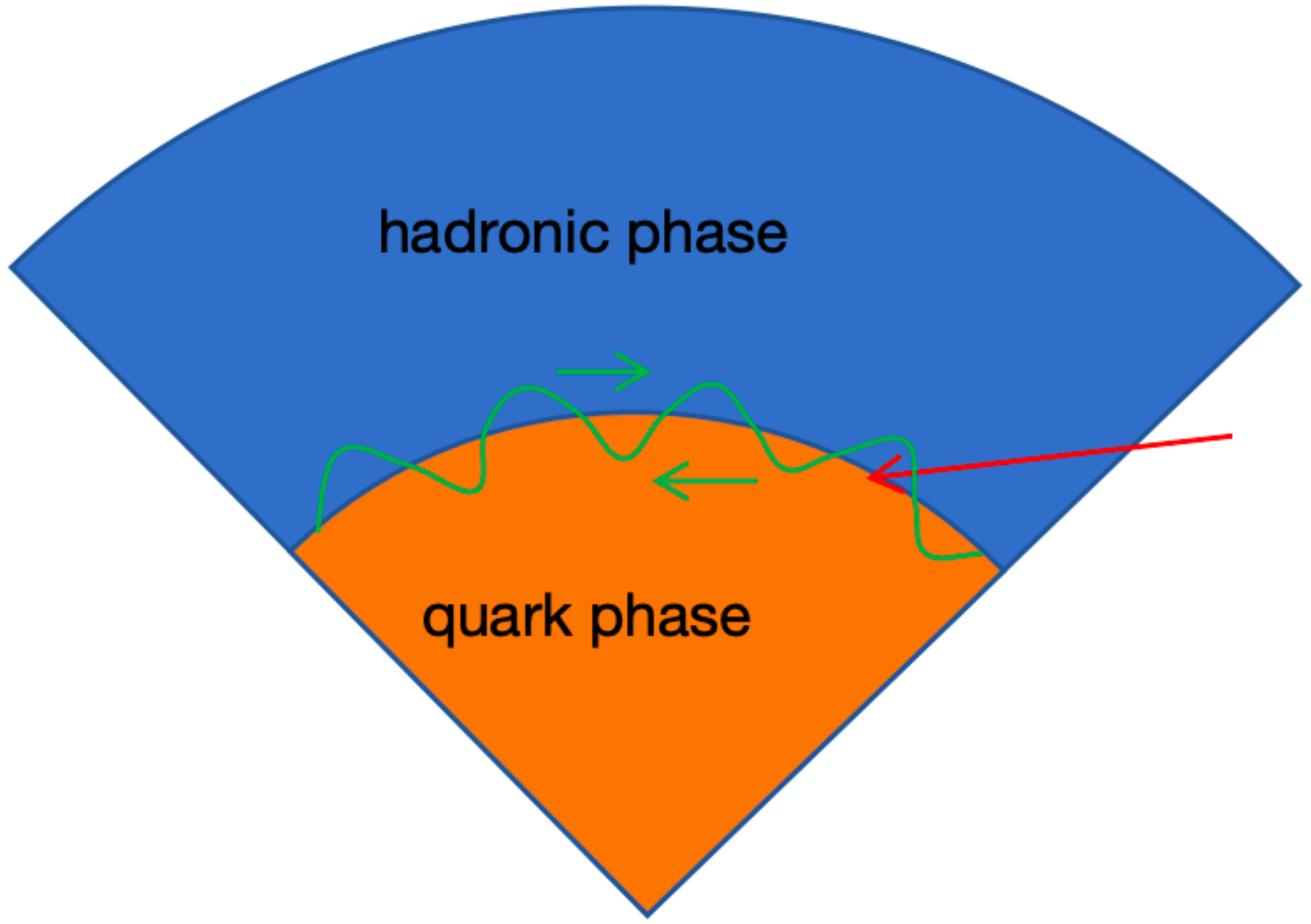
⬇

$$[\mathcal{L} - (p+\varepsilon)e^{-2\Phi}\omega_\alpha^2]\xi_\alpha^\mu = 0$$

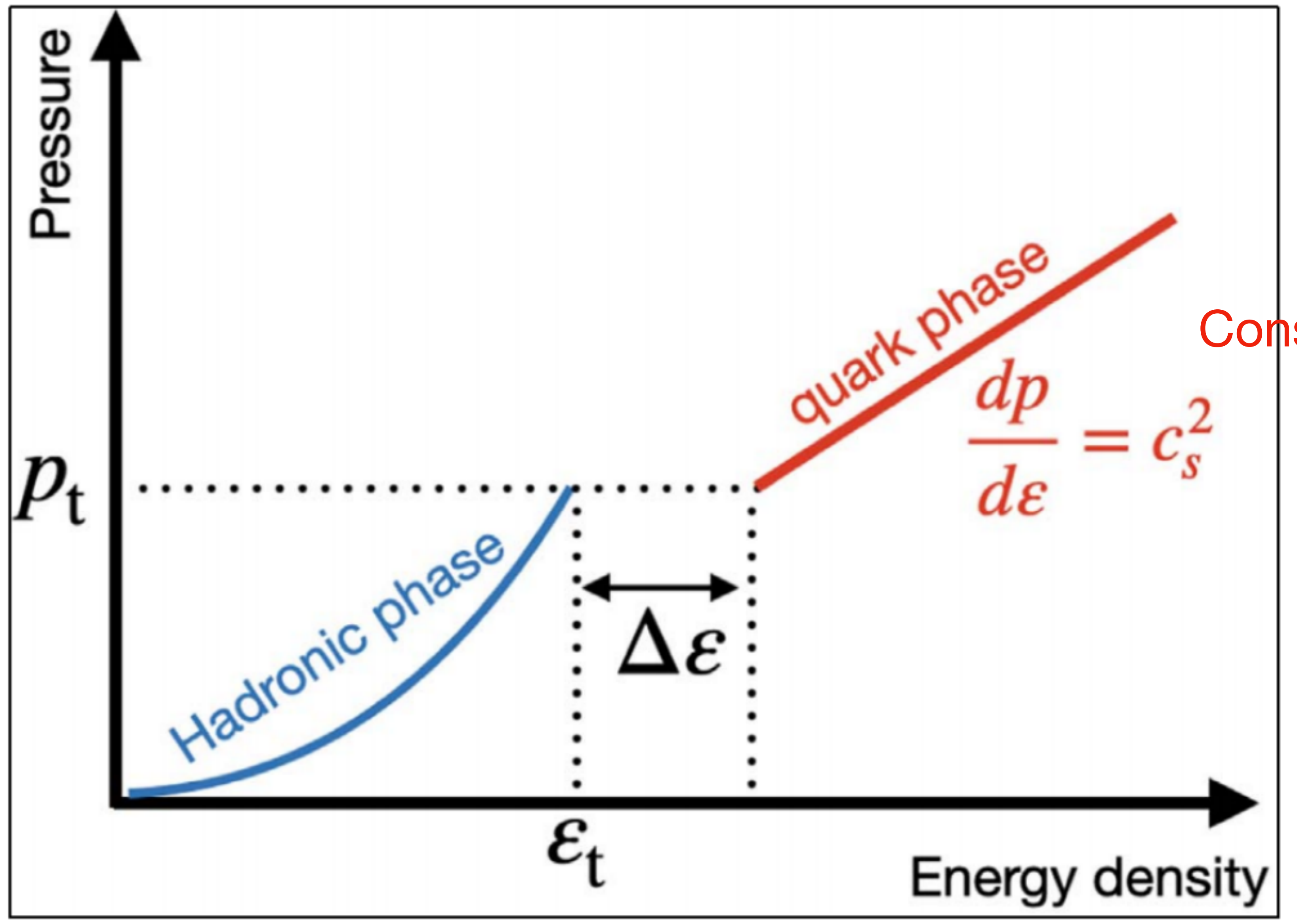
In GR, the frequency is a complex, the imaginary part represent the damping due to gravitational wave radiation.

李政道研究所 TSUNG-DAO LEE INSTITUTE

❖The (discontinuity) g-mode



interface



Constant-speed-of-sound (CSS) model

$$\varepsilon_t, \Delta\varepsilon, c_s^2$$

Hadronic EOS:
NL3ωρ (stiff) and QMF(soft)

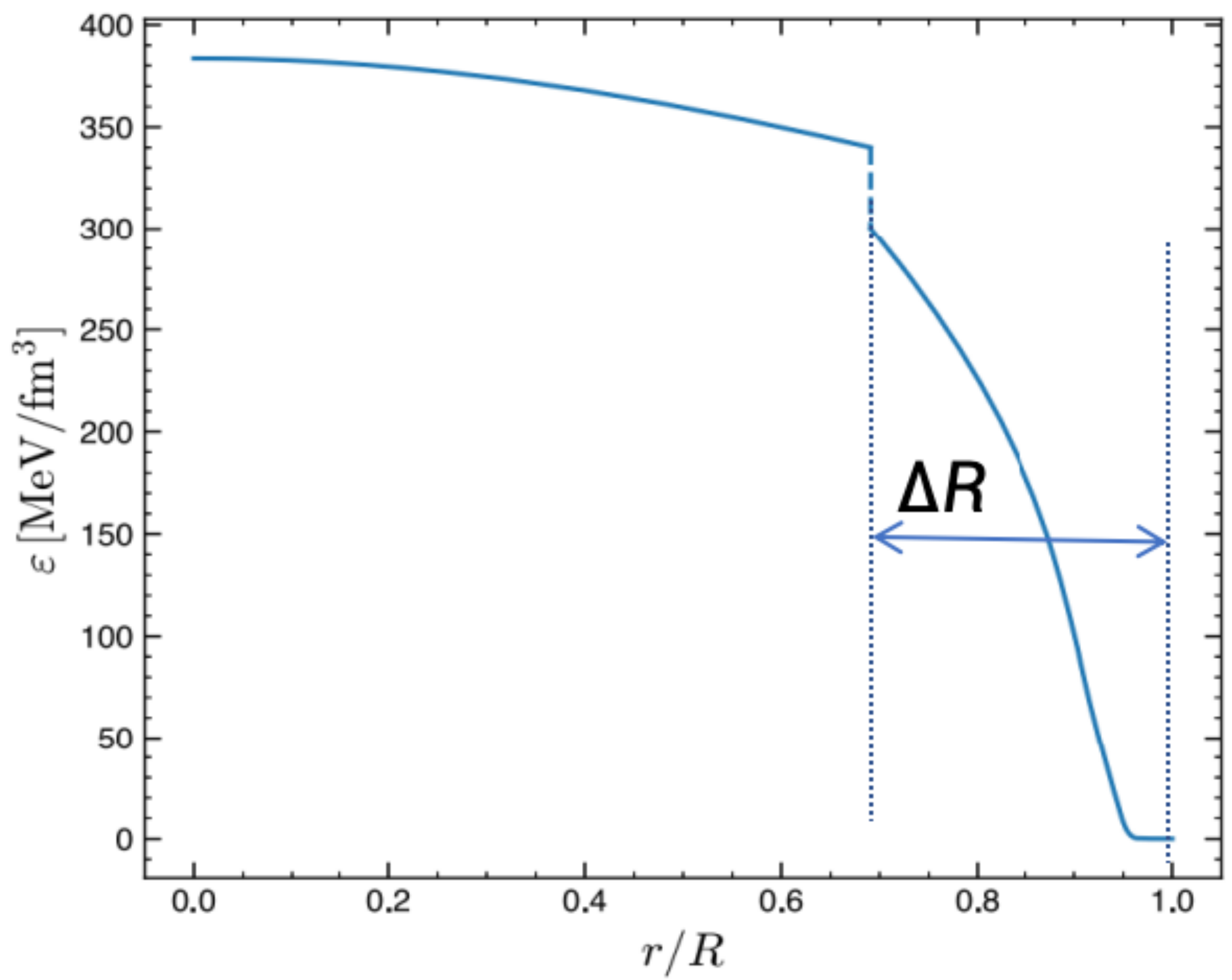
$$\omega^2 \propto \frac{\Delta\varepsilon}{\varepsilon_t}\frac{\Delta R}{R}\frac{GM}{R^3}$$

$$\omega^2 \approx K\left[\frac{6}{5}\frac{\Delta\varepsilon}{\varepsilon_{\rm trans}}(1-x^5) + 1 + \eta - \sqrt{(1+\eta)^2 - \frac{12}{5}\frac{\Delta\varepsilon}{\varepsilon_{\rm trans}}(\kappa-1)(1-x^5)}\right] \times \left[\frac{\varepsilon_{\rm trans}+\Delta\varepsilon}{\varepsilon_{\rm trans}} - \frac{2}{5}\frac{\Delta\varepsilon}{\varepsilon_{\rm trans}}(1-x^5)\right]^{-1}\frac{M}{R^3},$$

$\eta = (1/5)(\Delta\varepsilon/\varepsilon_{\rm trans})[3(\kappa-1)+(3+2\kappa)x^5]$

$\kappa = (M_c/M)(R/R_{\rm trans})^3$

$x = R_{\rm trans}/R$



MZQ+2024

❖How can we detect the modes?

李政道研究所
TSUNG-DAO LEE INSTITUTE

- Different from geoseismology on Earth and helioseismology on Sun, we can't directly detect the seismic wave of NS oscillation and can hardly to resolve the surface emission of NSs.

- GW signals are very faint, only possible for galactic events (like supernova or pulsar glitch) and f-mode.

$$h \approx 4 \times 10^{-23} \left(\frac{E}{10^{-9} M_\odot c^2} \right)^{1/2} \left(\frac{\tau}{0.1\ \mathrm{s}} \right)^{-1/2} \left(\frac{f}{2\ \mathrm{kHz}} \right)^{-1} \left(\frac{d}{10\ \mathrm{kpc}} \right)^{-1}$$

- We may detect the orbital phase change induced by mode excitation in BNS inspiral.

李政道研究所
TSUNG-DAO LEE INSTITUTE

Outline

- Background
- G-mode induced by 1st PT interface
- Tidal seismology and GW
- Summary

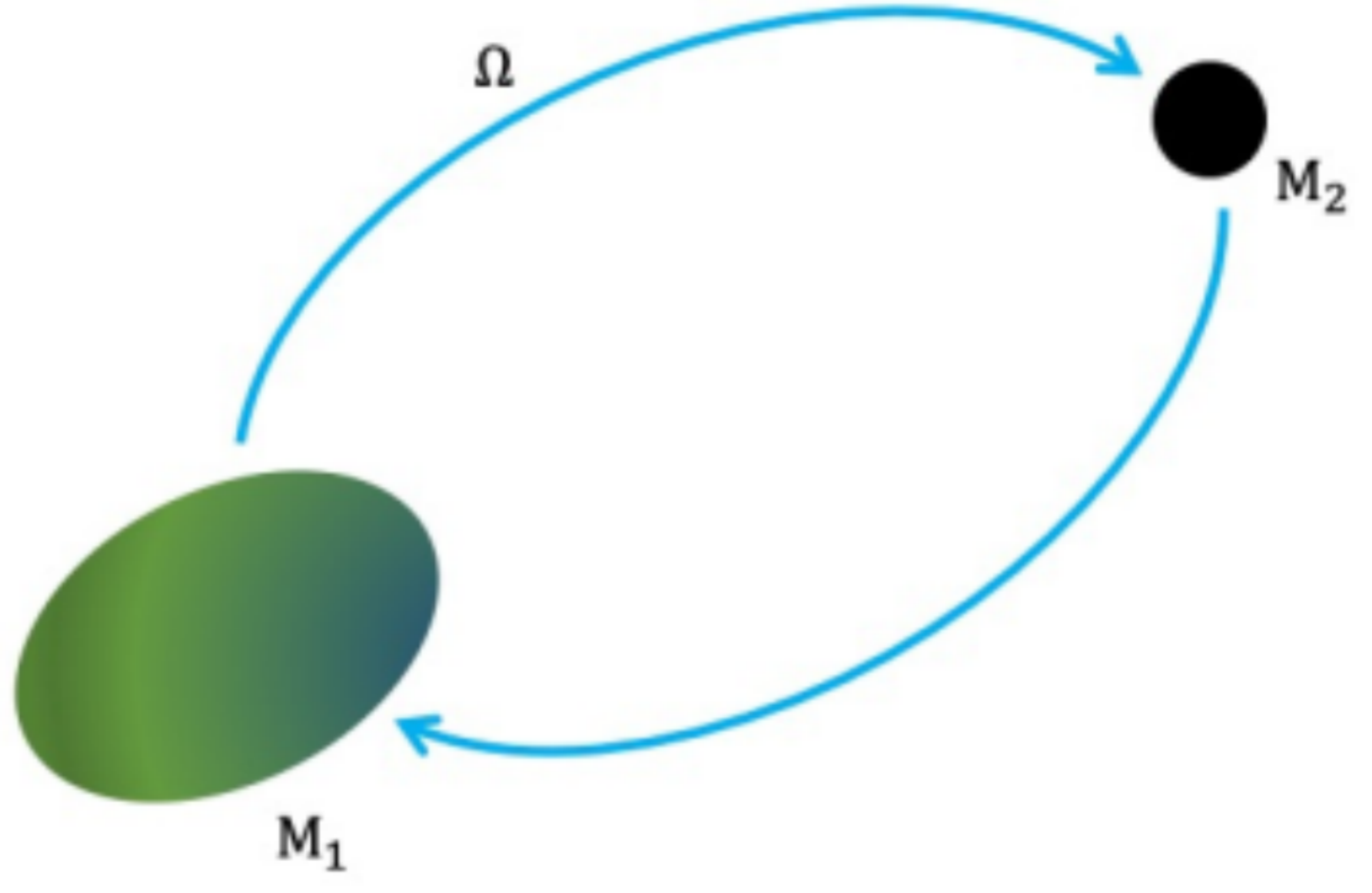
❖Stellar response to tidal field

- Oscillation under tidal force

$$\left(\rho\frac{\partial^2}{\partial t^2}+\mathcal{L}\right)\vec{\xi}=-\rho\nabla U$$

$$U=-GM'\sum_{lm}\frac{4\pi}{2l+1}\frac{r^l}{D^{l+1}}Y^*_{lm}\left(\frac{\pi}{2},\Phi\right)Y_{lm}(\theta,\phi)$$

$$=-GM'\sum_{lm}W_{lm}\frac{r^l}{D(t)^{l+1}}e^{-im\Phi(t)}Y_{lm}(\theta,\phi)$$



- Decompose into normal modes

$$\vec{\xi}(\mathbf{r},t)=\sum_\alpha a_\alpha(t)\vec{\xi}_\alpha(\mathbf{r}), \qquad \left(\mathcal{L}-\rho\omega_\alpha^2\right)\vec{\xi}_\alpha(\mathbf{r})=0,$$

$$\ddot{a}_\alpha+\omega_\alpha^2 a_\alpha=\frac{GM_2W_{lm}Q_\alpha}{D^{l+1}}e^{-im\Omega_{\rm orb}t}$$

- Tidal overlap integral

$$Q_{nl}=\int d^3x\rho\boldsymbol{\xi}^*_{nlm}\cdot\nabla[r^lY_{lm}(\theta,\phi)]$$

$$=\int_0^R\rho l r^{l+1}dr[\xi^r_{nl}(r)+(l+1)\xi^\perp_{nl}(r)]$$

- Quasi-equilibrium (static) tide

$$\omega_\alpha\gg m\Omega_{\rm orb} \qquad a_\alpha\sim\frac{e^{i\Omega_{\rm orb}t}}{\omega_\alpha^2D^{l+1}}$$

- Resonant tide

$$\omega_\alpha\simeq m\Omega_{\rm orb} \qquad a_\alpha\sim\frac{e^{i\Omega_{\rm orb}t}}{(\omega_\alpha^2-m^2\Omega_{\rm orb}^2)D^{l+1}}$$

❖Stellar response to tidal field

- Oscillation under tidal force

$$\left(\rho\frac{\partial^2}{\partial t^2}+\mathcal{L}\right)\vec{\xi}=-\rho\nabla U$$

$$U=-GM'\sum_{lm}\frac{4\pi}{2l+1}\frac{r^l}{D^{l+1}}Y$$

$$=-GM'\sum_{lm}W_{lm}\frac{r^l}{D(t)^{l+1}}e$$

- Decompose into normal modes

$$\vec{\xi}(\mathbf{r},t)=\sum_{\alpha}a_\alpha(t)\vec{\xi}_\alpha(\mathbf{r}), \qquad \left(\mathcal{L}-\rho\omega_\alpha^2\right)\vec{\xi}_\alpha(\mathbf{r})=0,$$

$$\ddot{a}_\alpha+\omega_\alpha^2 a_\alpha=\frac{GM_2W_{lm}Q_\alpha}{D^{l+1}}e^{-im\Omega_{\rm orb}t} \qquad Q_{nl}=$$

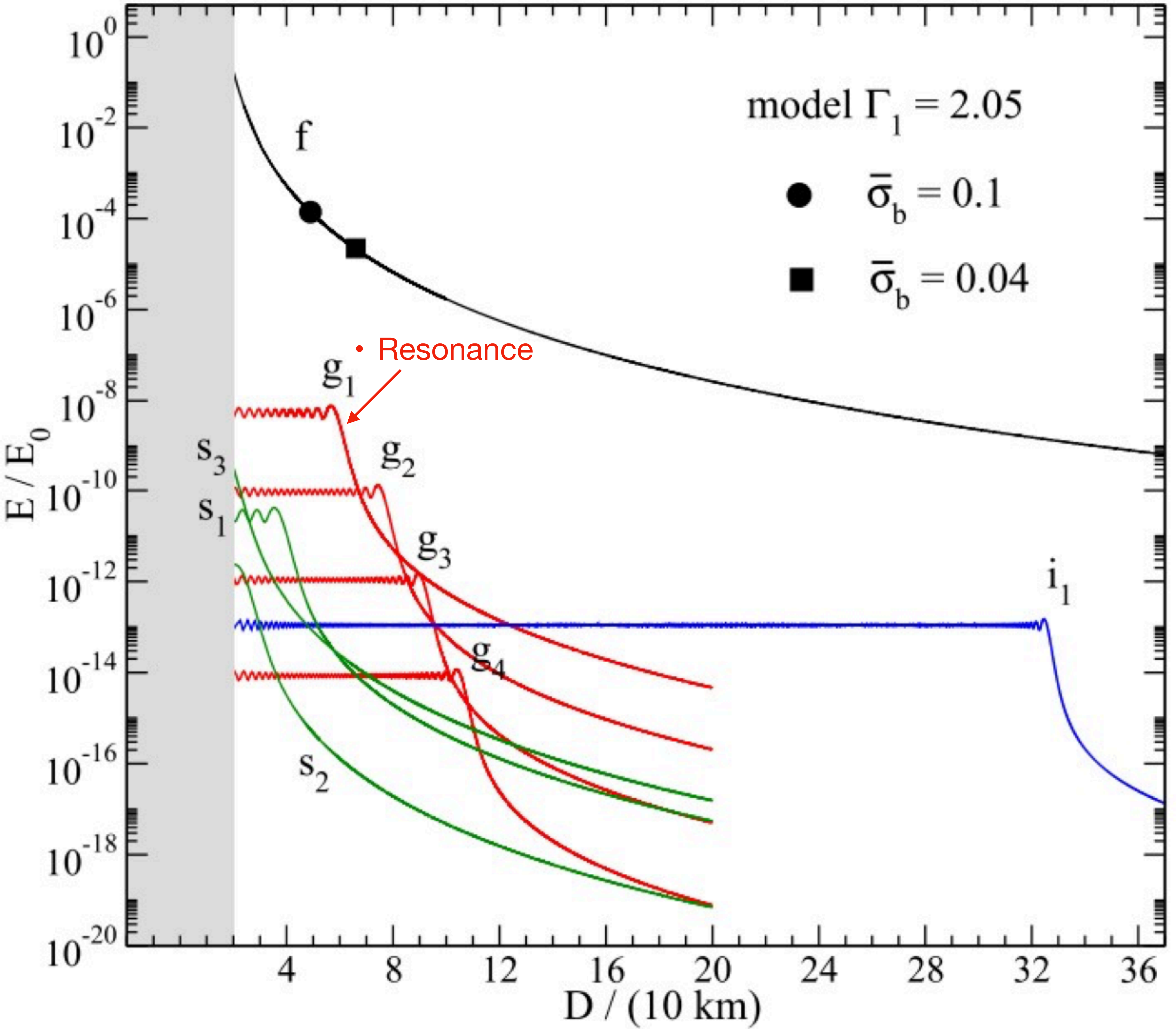
$$=$$

- Quasi-equilibrium (static) tide

$$\omega_\alpha \gg m\Omega_{\rm orb} \qquad a_\alpha \sim \frac{e^{i\Omega_{\rm orb}t}}{\omega_\alpha^2 D^{l+1}}$$

- Resonant tide

$$\omega_\alpha \simeq m\Omega_{\rm orb} \qquad a_\alpha \sim \frac{e^{i\Omega_{\rm orb}t}}{(\omega_\alpha^2-m^2\Omega_{\rm orb}^2)D^{l+1}}$$



Passamonti+21

❖Resonant tides

- The resonance is almost instaneous at lower frequency

$$t_{res} \simeq 0.01s\mathcal{M}_{1.2}^{-5/6} f_{600}^{-11/6} \ll t_D \simeq 0.1s\mathcal{M}_{1.2}^{-5/3} f_{600}^{-8/3}$$

Resonance Orbit decay

- The energy transfer from orbit to stellar oscillation is

$$\Delta E \simeq 5 \times 10^{49} \mathrm{erg} f_{600}^{1/3} Q_{0.01}^2 M_{1.4}^{-2/3} R_{12}^2 q (\frac{2}{1+q})^{5/3}$$

- Which implies a sudden GW phase change at resonance frequency

$$\delta\Phi = \frac{\omega_{mode} \Delta E}{P_{GW}} \simeq -0.12 f_{600}^{-2} Q_{0.01}^2 M_{1.4}^{-4} R_{12}^2 \frac{2q}{1+q}$$

Lai+1994

❖Signature in gravitational waveform

$$h(f) = \mathcal{A}e^{i\Psi(f)}$$

$$\Psi(f, \phi_c, t_c) = \begin{cases} 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4}\left(\frac{8\pi G\mathcal{M}f}{c^3}\right)^{-5/3} \\ 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4}\left(\frac{8\pi G\mathcal{M}f}{c^3}\right)^{-5/3} - \left(1 - \frac{f}{f_a}\right)\delta\phi_a \end{cases}$$

- Before resonance, i.e., $f < f_a$
- After the resonance, i.e., $f > f_a$

(Flanagan+2007, Yu+2017)

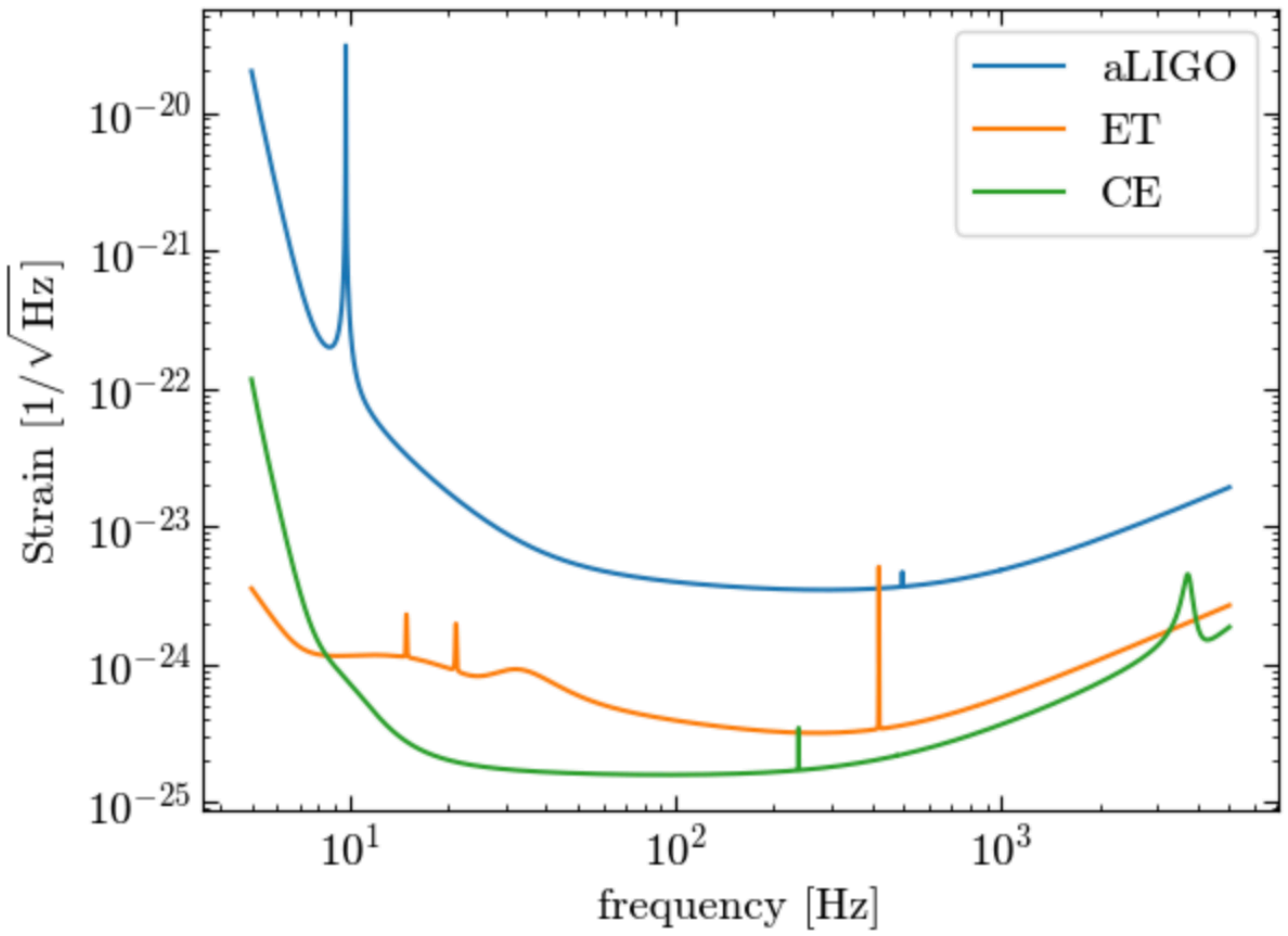
❖Sensitivity curve with future ground-base detectors

李政道研究所
TSUNG-DAO LEE INSTITUTE

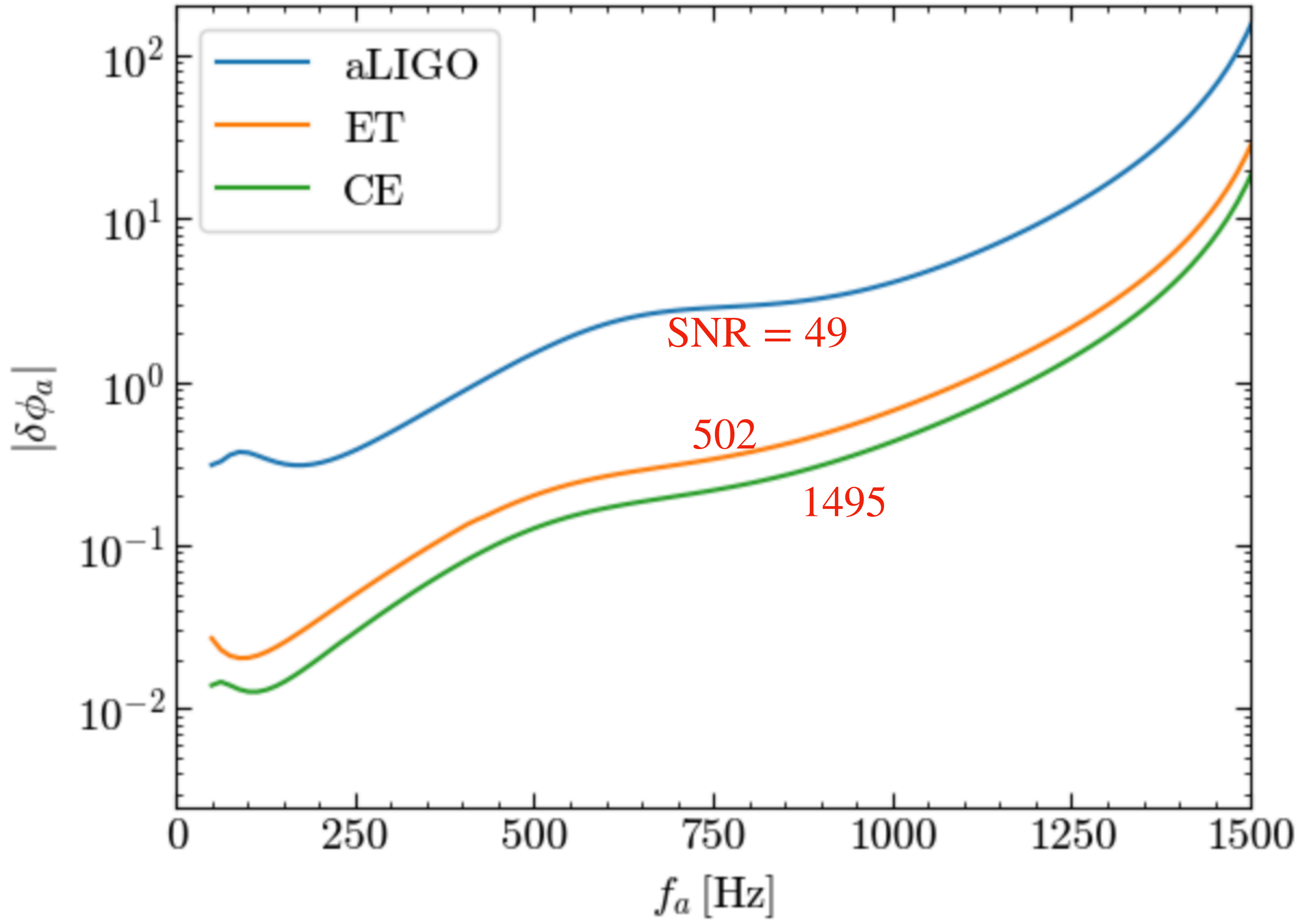
Fisher Information Matrix

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta_i} \Big| \frac{\partial h}{\partial \theta_j} \right), \qquad (h_1|h_2) = 2\int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df, \qquad \Delta\theta_i = \sqrt{(\Gamma^{-1})_{ii}},$$

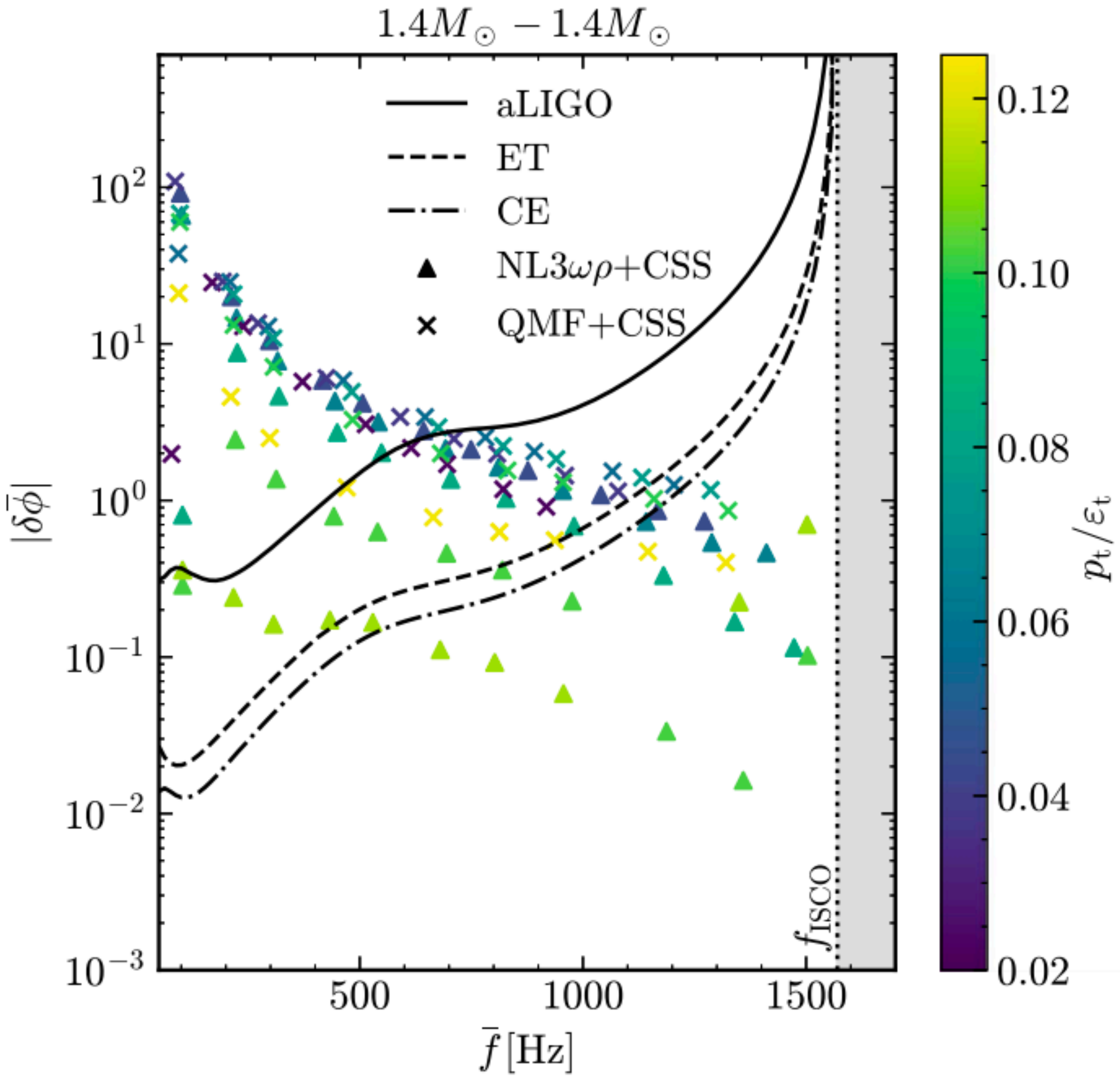
$$\{\theta_i\} = \{\mathcal{M}, q, \Lambda_1, \Lambda_2, d_L, t_c, \phi_c, |\delta\phi_a|, f_a\}$$

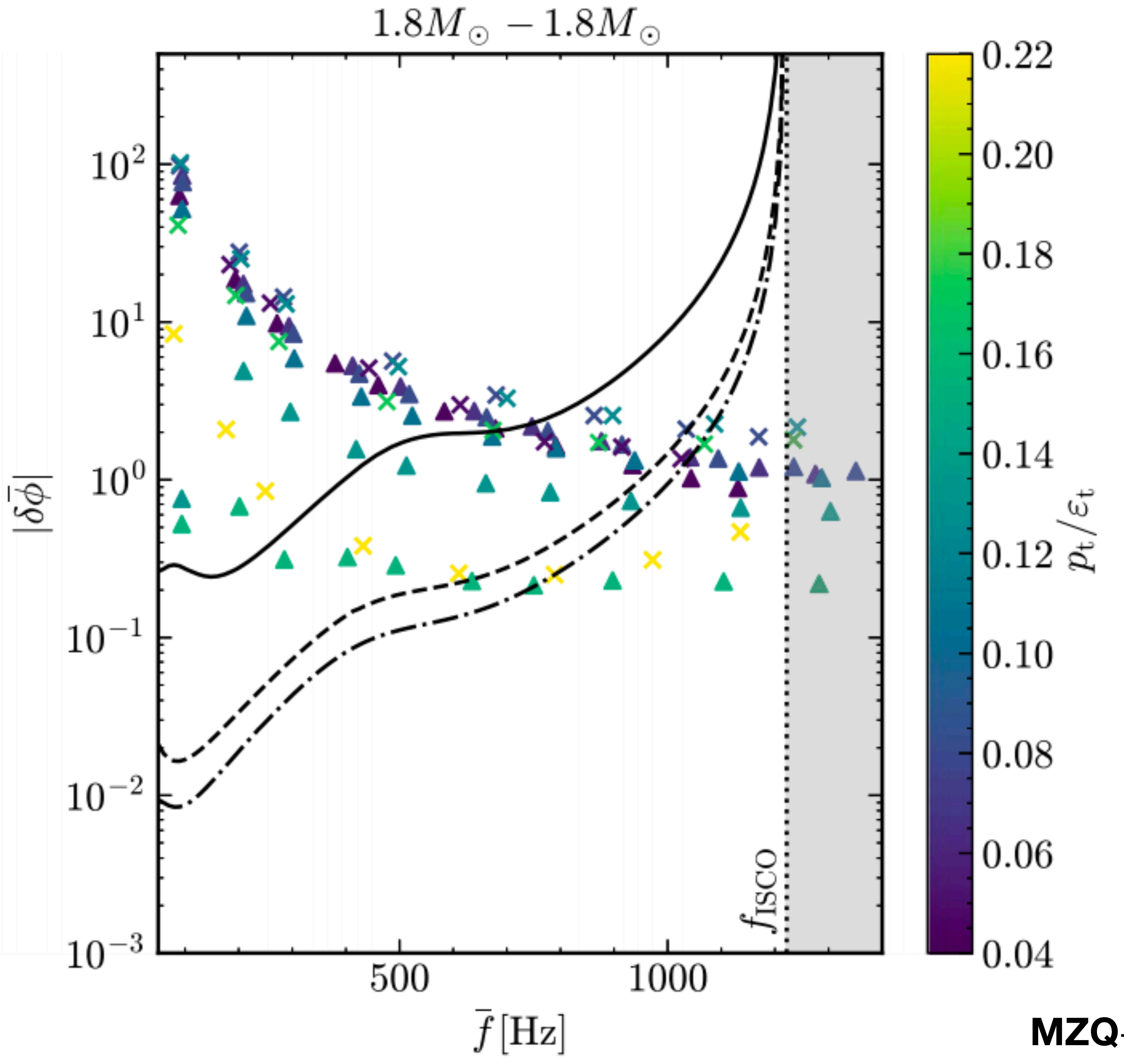


$1.4M_\odot - 1.4M_\odot$ @ 100Mpc



❖Detectability of the g-mode





MZQ+2024

❖Data analysis of GW170817

- Waveform model: IMRPhenomD_NTidal + resonance

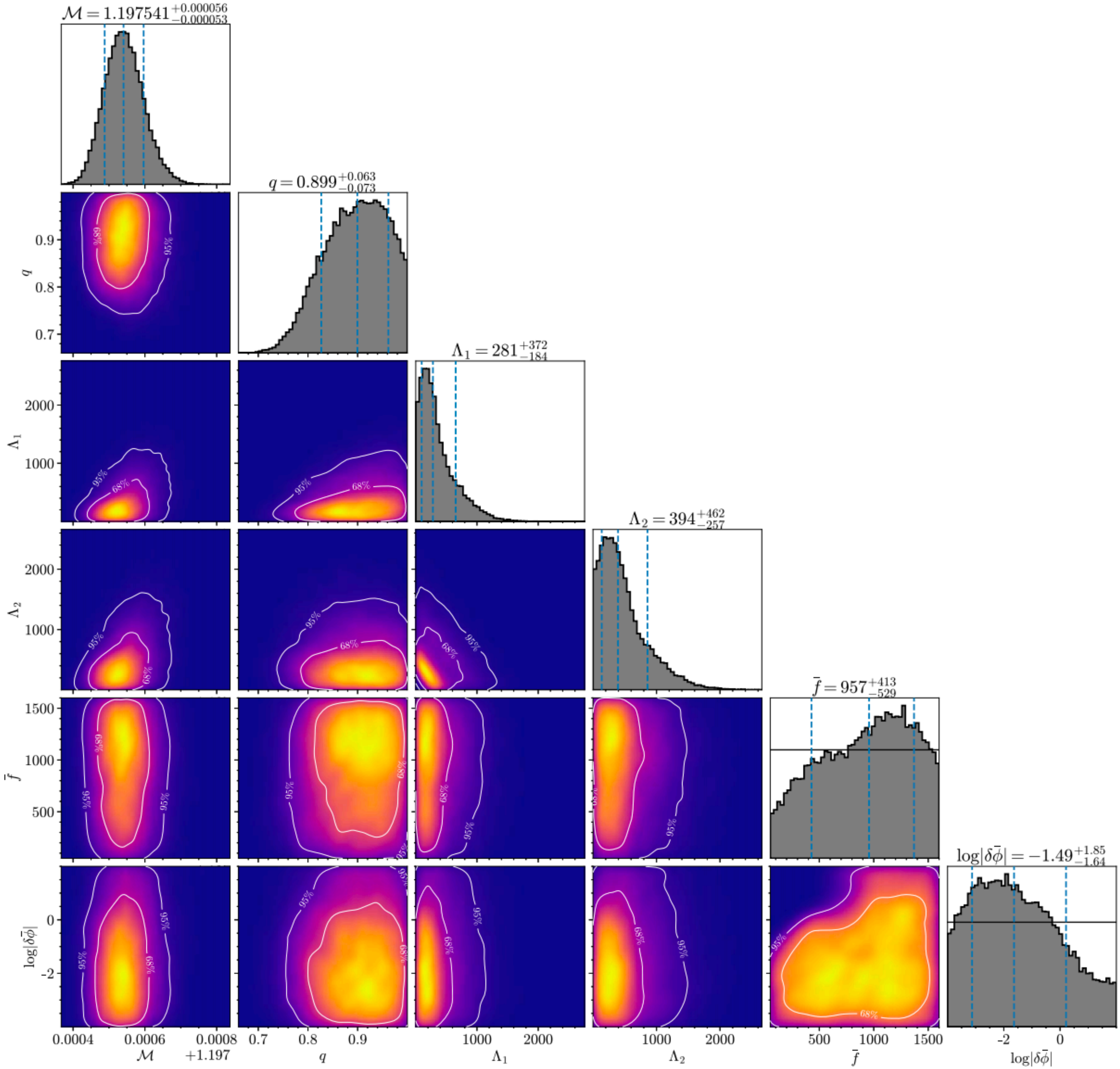
MZQ+2024

$$\{\theta_i\} = \{\mathcal{M},\, q,\, \Lambda_1,\, \Lambda_2,\, \chi_{1z},\, \chi_{2z},\, \theta_{\rm jn},\, t_c,\, \phi_c,\, \Psi,\, |\delta\bar{\phi}|,\, \bar{f}\}. \quad \text{(A3}$$

We fix the location of the source to the position determined electromagnetic observations (Abbott et al. 2017c; Levan et 2017) with $\alpha(J2000) = 197^\circ\!.45$, $\delta(J2000) = -23^\circ\!.38$ a $z = 0.0099$. The priors of the parameters are chosen following those used in Abbott et al. (2019), with the excepti of the priors for $|\delta\bar{\phi}|$ and $\bar{f}$. For the mode resonance paramete

- H0: model without mode resonance
- H1: model with mode resonance

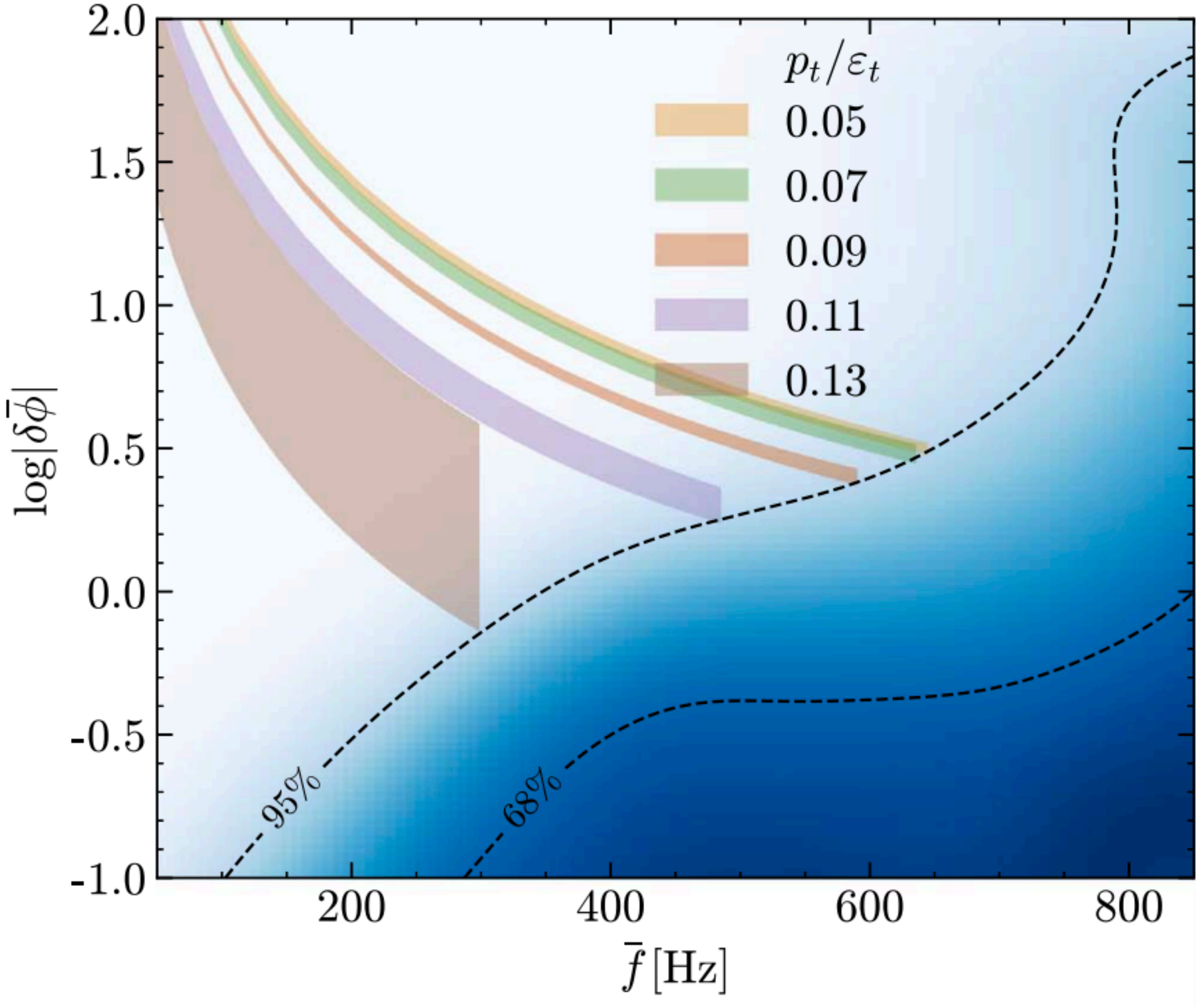
No detection of the signal, as $\mathrm{BF}^{H_1}_{H_0} \in [0.72, 1.11]$



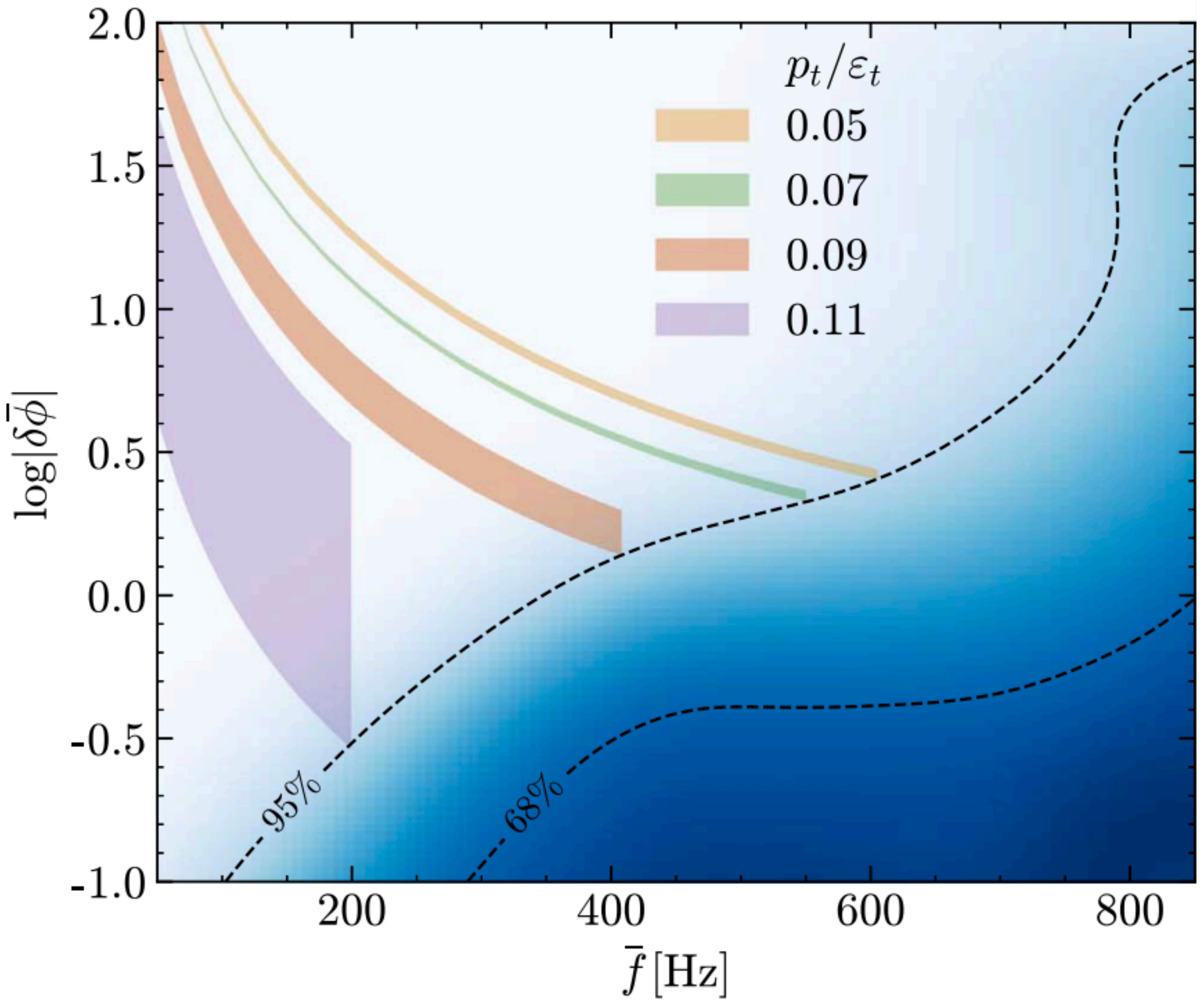
❖Data analysis of GW170817

李政道研究所 TSUNG-DAO LEE INSTITUTE

QMF+CSS

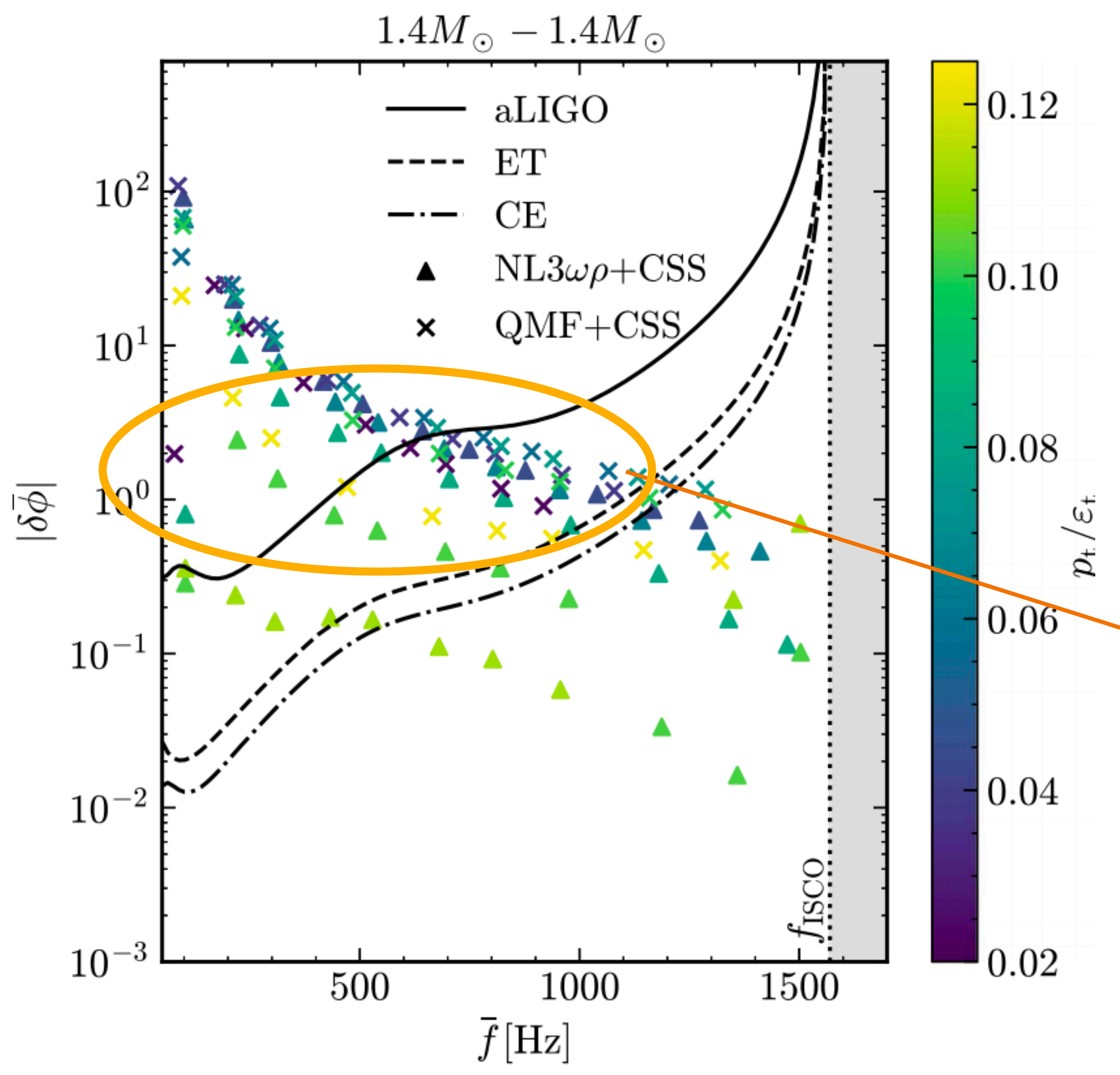


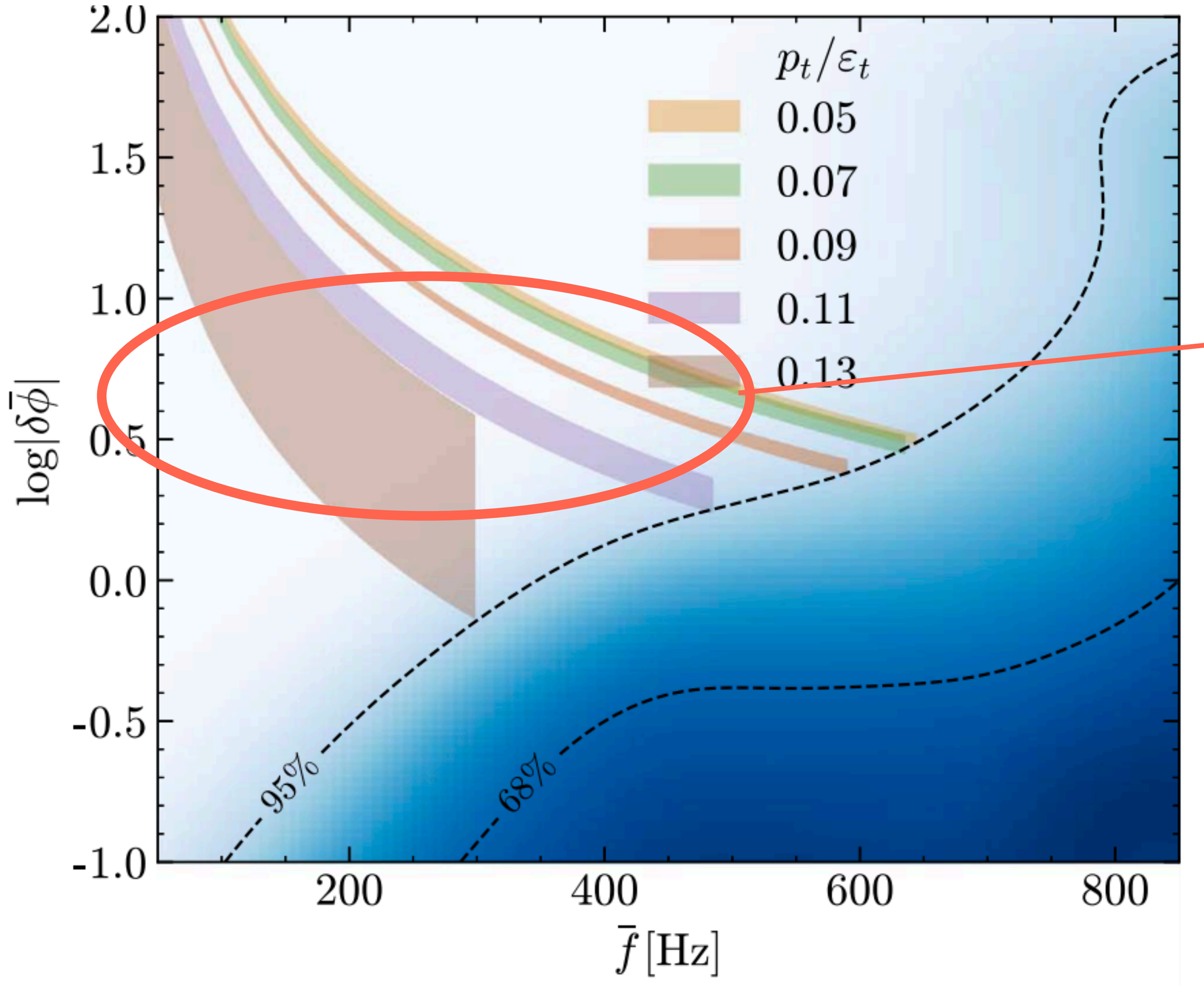
NL3ωρ+CSS

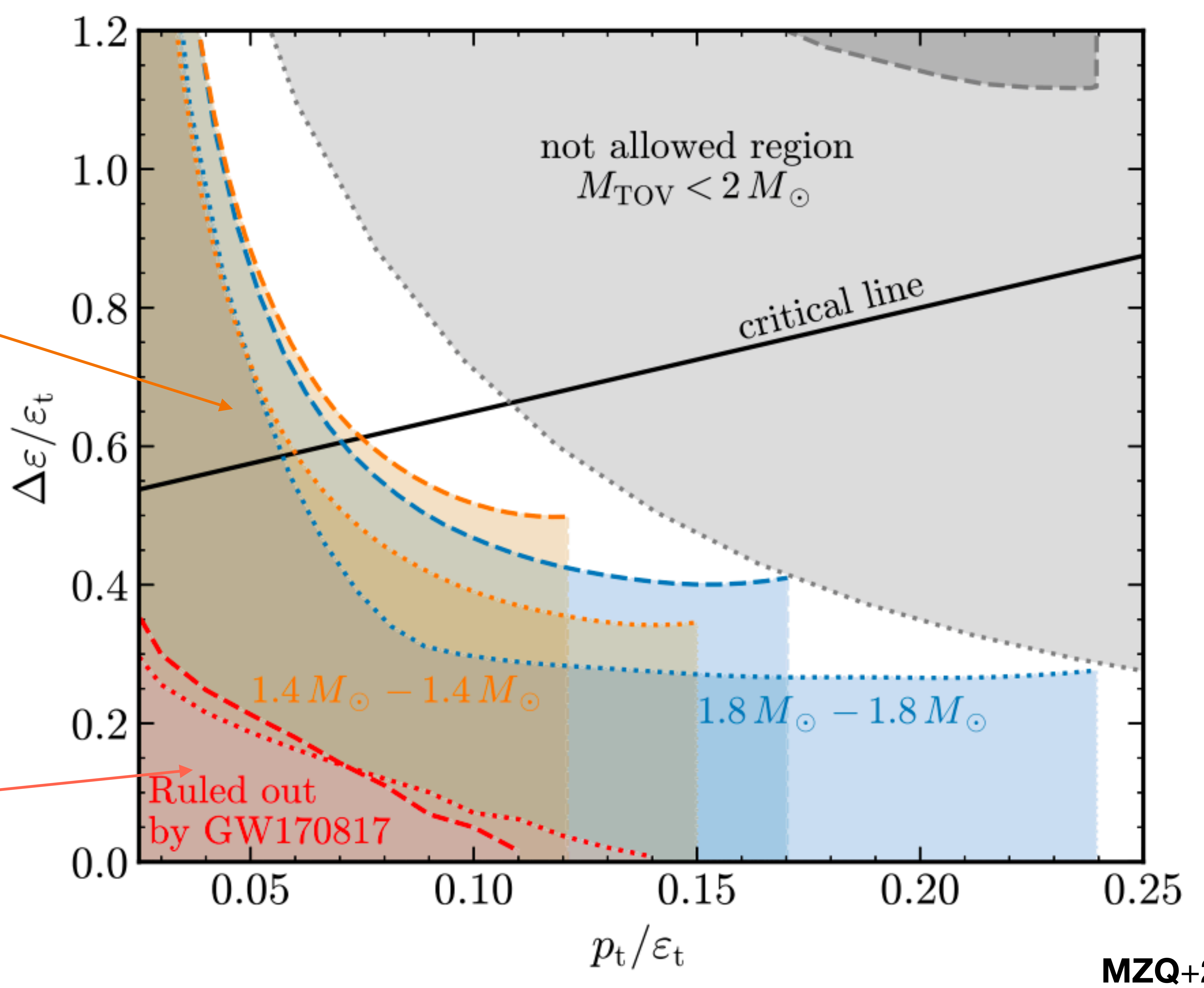


MZQ+2024

❖Constraints on phase transition properties







MZQ+2024

Summary

✓1st phase transition —> discontinuity g-mode —> resonance signature in GW waveform

✓A case study of the g-mode resonance in GW170817 data has ruled out the possibility of a weak phase transition taking place at low density.

Thank you for your attention!