



安徽大學
Anhui University

形变奇特核的研究

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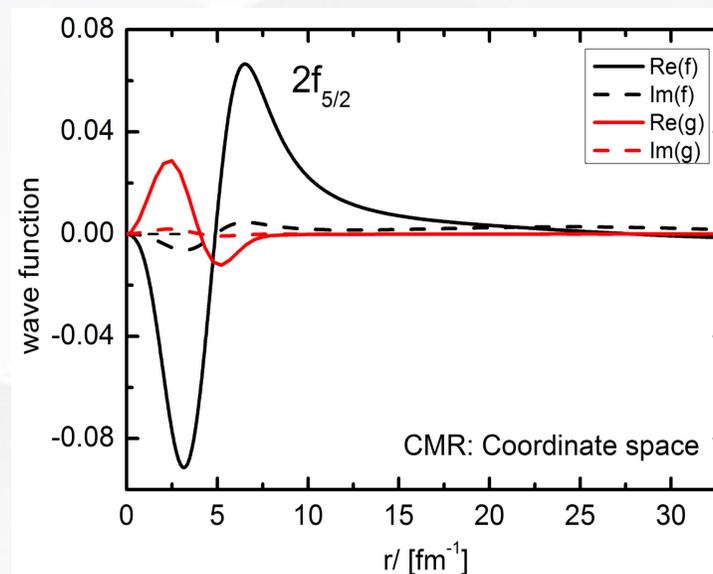
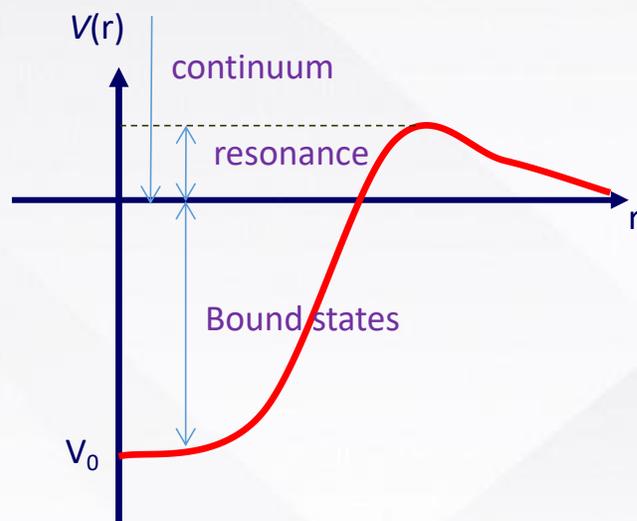
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总结

1

研究背景

- Taylor 在《散射理论》中提到 (Taylor, 1987), 共振态是散射实验中最引人注目的现象, 广泛存在于原子、分子、原子核和化学反应中。

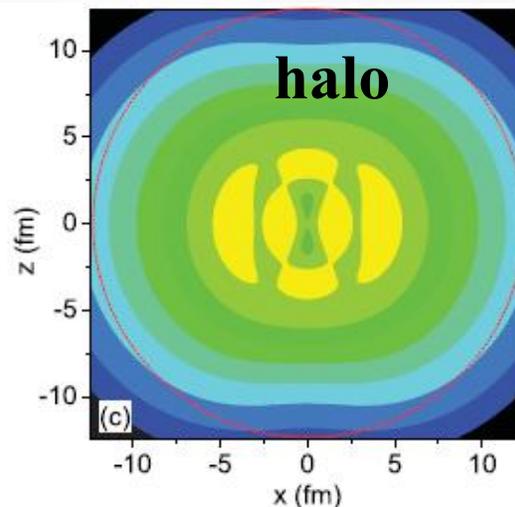
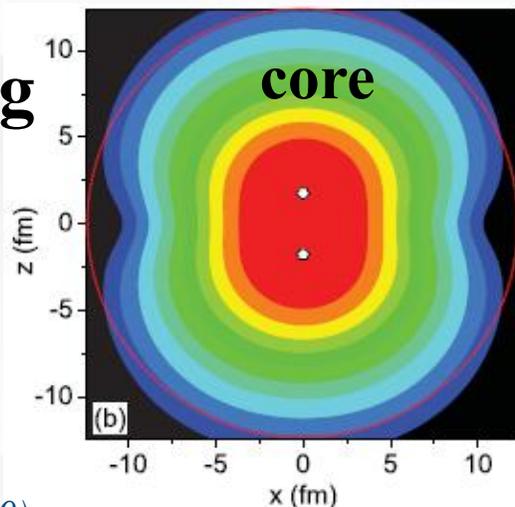
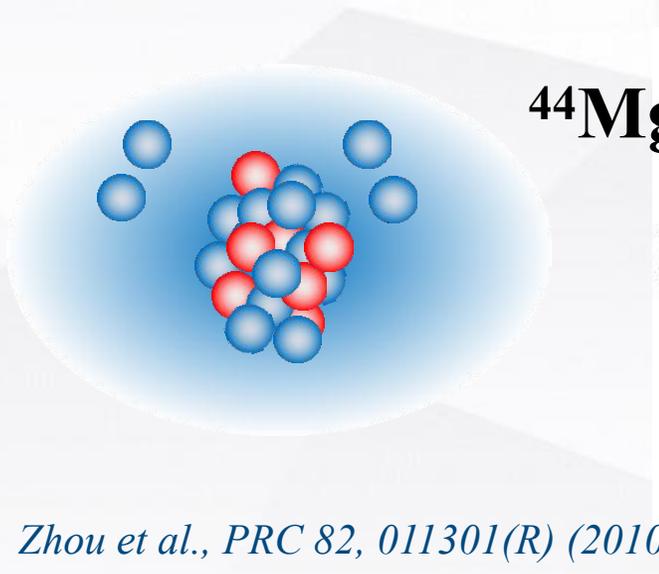
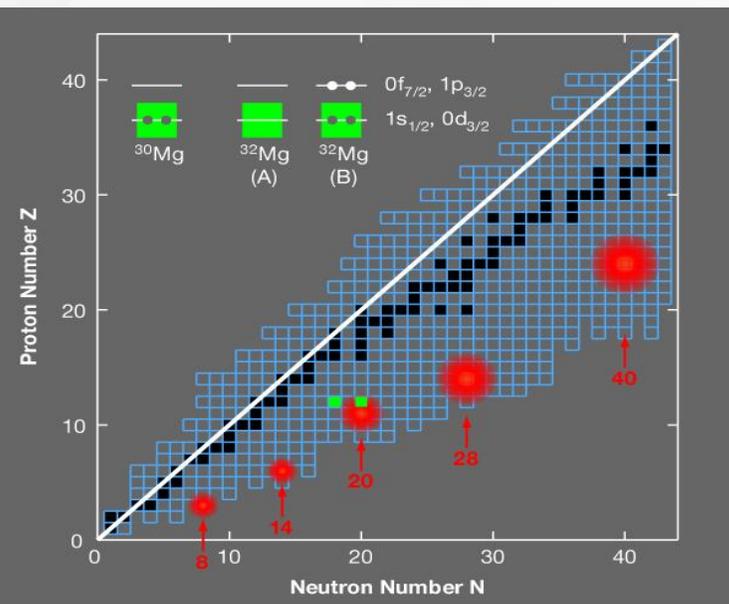
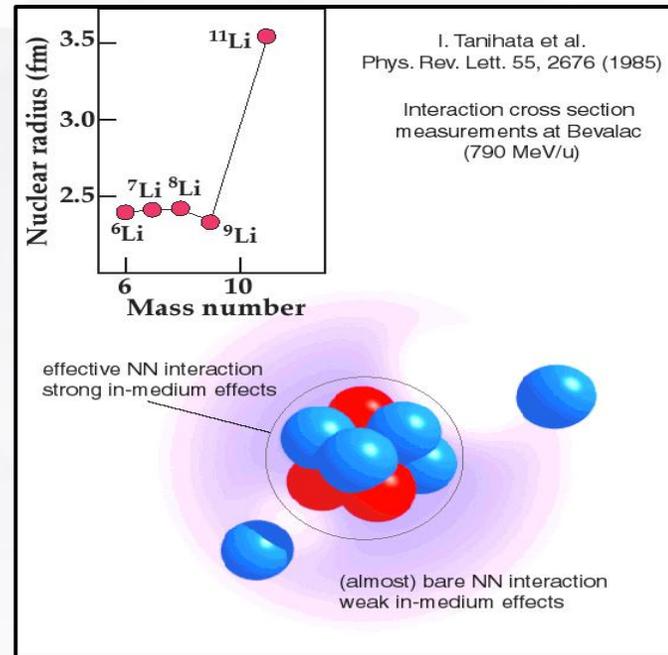


- 上面左图展示共振态在散射理论中的定义, 当一个入射粒子的能量大于零, 且小于此时势垒的最大值时, 可认为该粒子处于共振态。右图是共振态的波函数, 可看出有些单粒子共振态的波函数在半径很大的地方不为零。

1 研究背景

共振态的研究意义

- 在核物理领域，许多新奇现象的出现，共振扮演着重要角色。
- ^{11}Li 晕的解释、Zr和Ca巨晕的预言、 ^{31}Ne 和 $^{42,44}\text{Mg}$ 形变晕的认识都需要考虑连续谱的贡献，尤其是连续谱中共振态的贡献。在异常中质比核中，共振能级反转是传统幻数改变的原因之一。
- 单粒子共振态的能量和寿命也是天体演化过程中化学元素核合成研究的关键输入量。



➤ 研究核单粒子共振的方法:

- ◆ R-Matrix method, *E. Wigner et al., PR 72, 29 (1947).*
- ◆ K-Matrix method, *J. Humblet et al., PRC 44, 2530 (1991).*
- ◆ The phase-shift method, *J. R. Taylor, Scattering Theory (Wiley, New York, 1972).*
- ◆ J-Matrix method, *H. A. Yamani et al., J. Math. Phys. 16, 410 (1975).*
- ◆ Coupled-Channel Approach, *I. Hamamoto, PRC 72, 024301 (2005).*
- ◆ Jost function method, *B. N. Lu, E. G. Zhao, and S. G. Zhou, PRL 109, 072501 (2012).*
- ◆ Real stabilization method (RSM), *A. U. Hazi and H. S. Taylor, PRA 1, 1109 (1970);
Y.K.Ho, Phys. Rept. 99, 1 (1983).*
- ◆ \implies RMF-RSM, *L. Zhang, S.G. Zhou, J. Meng, and E.G. Zhao, PRC 77, 014312(2008).*

- ◆ **Analytic continuation in the coupling constant (ACCC)**, *V. I. Kukulin et al., Theory of Resonances: Principles and Applications (KluwerAcademic, Dordrecht, 1989).*
- ◆ \implies **RMF-ACCC**, *S. C. Yang, J. Meng, S. G. Zhou, CPL 18, 196 (2001); S. S. Zhang, J. Meng, S. G. Zhou et al., PRC 70, 034308 (2004);*
J. Y. Guo, et al., PRC 72, 054319 (2005); PRC 74, 02432 (2006).
- ◆ **Green's Function method**, *E.N. Economou, Green's Function in Quantum Physics (Springer-Verlag, Berlin, 2006); Y. Zhang et al., Phys. Rev. C83, 054301.*
- ◆ \implies **RMF-GF**, *T.T. Sun, S.Q. Zhang, Y. Zhang, J.N. Hu, and J. Meng, Phys.Rev.C 90, 054321 (2014)*
- ◆ **Complex scaling method (CSM)**, *Kenichi Yoshida, PRC72, 064311 (2005).*
- ◆ \implies **RMF-CSM**, *J.Y. Guo, S.G. Zhou, J. Meng, and E.G. Zhao, PRC 82, 034318(2010).*

- 复动量表象 (CMR) 方法是类束缚态的方法, 因此计算上更加简单方便。 [T. Berggren 1968 NPA]
- 2016年, 本课题组建立了RMF-CMR方法。它不仅能够统一描述束缚态、共振态和连续谱, 而且能够很好地描述窄共振态和宽共振态。 [N. Li 2016 PRL]

PRL 117, 062502 (2016)

PHYSICAL REVIEW LETTERS

week ending
5 AUGUST 2016

Probing Resonances of the Dirac Equation with Complex Momentum Representation

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Resonance plays critical roles in the formation of many physical phenomena, and several methods have been developed for the exploration of resonance. In this work, we propose a new scheme for resonance by solving the Dirac equation in the complex momentum representation, in which the resonant states are exposed clearly in the complex momentum plane and the resonance parameters can be determined precisely without imposing unphysical parameters. Combined with the relativistic mean-field theory, this method is applied to probe the resonances in ^{120}Sn with the energies, widths, and wave functions being obtained. Compared to other methods, this method is not only very effective for narrow resonances, but also can be reliably applied to broad resonances.

- 2021年，我们在建立了 RMF-CMR-GF 方法 [*Luo Y.X. et. al. 2021 PRC*]，能够准确清晰地获得了共振参数，而且还能提高 $E = 0$ 附近共振态的准确性。

PHYSICAL REVIEW C **104**, 014307 (2021)

Role of quadrupole deformation and continuum effects in the “island of inversion” nuclei $^{28,29,31}\text{F}$

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Background: The peculiar properties of nuclei in the so-called “island of inversion” around $Z = 10$ and $N = 20$ are the focus of current nuclear physics research. Recent studies showed that ^{28}F has a negative-parity ground state and thus lies within the southern shore of the island of inversion, and ^{29}F presents a halo structure in its ground state, but it is unclear which effects, such as deformation, shell evolution due to tensor forces, or couplings to the continuum, lead to this situation.

Purpose: We investigate the role of quadrupole deformation and continuum effects on the single-particle structure of $^{28,29,31}\text{F}$ from a relativistic mean-field approach and show how both phenomena can lead to a negative-parity ground state in ^{28}F and halo structures in $^{29,31}\text{F}$.

Methods: We solve the Dirac equation in the complex-momentum (Berggren) representation for a potential with quadrupole deformation at the first order obtained from relativistic mean-field calculations using the NL3 interaction and calculate the continuum level densities using the Green’s function method.

- 考虑到许多核是形变的，为了描述奇特核，2023年，我们发展了适用于形变系统的 DRMF-CMR-BCS理论 [Luo Y.X. et. al. 2023 PRC]。该理论提供了真实的物理共振态，是研究晕核的有效方法。

PHYSICAL REVIEW C **108**, 024320 (2023)

Research on exotic nuclei in deformed relativistic mean-field theory plus BCS in complex momentum representation

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Study of exotic nuclei is one of the important frontiers in nuclear physics. The coupling of weakly bound states and resonant states, deformation, and pairings play important roles at the formation of exotic phenomena. To deal with these uniformly, we develop the deformed relativistic mean field theory in complex momentum representations with BCS pairings. ^{44}Mg is chosen as an illustration example. The calculated binding energy indicates that ^{44}Mg is a weakly bound nucleus. There are several broad resonant states with low orbital angular momentum near the Fermi surface, and the occupation of these levels is responsible for the halo structure in ^{44}Mg . The available density distributions suggest that ^{44}Mg is a deformed halo nucleus with prolate core and oblate halo, which agree with the deformed relativistic Hartree-Bogoliubov in continuum calculations. In particular, the role of resonances is clearly demonstrated in the halo formation, which is helpful to understand the physical mechanism of deformed exotic nuclei.

2

RMF-CMR-GF方法及其结果

用RMF-CMR-GF方法来研究形变核的结构和性质。这个系统的哈密顿量可写成：

$$H = \boldsymbol{\alpha} \cdot \mathbf{p} + V(\mathbf{r}) + \beta(M + S(\mathbf{r})) \quad (1)$$

粒子在具有排斥的矢量势 $V(\mathbf{r})$ 和具有吸引的标量势 $S(\mathbf{r})$ 中运动的狄拉克方程为：

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + V(\mathbf{r}) + \beta(M + S(\mathbf{r}))]\psi(\mathbf{r}) = \varepsilon\psi(\mathbf{r}), \quad (2)$$

其中 $\boldsymbol{\alpha}$ 和 β 为狄拉克矩阵， M 和 \mathbf{p} 分别为核子的质量和动量。 $E = \varepsilon - M$ 表示单粒子能量， ψ 为波函数。为了在Dirac方程中引入四极形变，我们将矢量势和标量势 $V(\mathbf{r})$ 和 $S(\mathbf{r})$ 写为以下形式：

$$\begin{aligned} V(\vec{r}) &= V_0 f(r) - \beta_2 V_0 k(r) Y_{20}(\mathcal{Q}, \varphi) \\ S(\vec{r}) &= S_0 f(r) - \beta_2 S_0 k(r) Y_{20}(\mathcal{Q}, \varphi) \end{aligned} \quad (3)$$

这里的 β_2 为四极形变参数，径向函数 $f(r)$ 和 $k(r)$ 为WS形式： $f(r) = \frac{1}{1 + e^{-\frac{r-R}{a}}}$ 和 $k(r) = \frac{r df(r)}{dr}$ 。公式(4)的形式是类似于变形的WS势在 β_2 中的一阶展开。为了获得共振态，我们将在复动量表象下去求解Dirac方程。在复动量表象下Dirac方程可表示为：

$$\int d\mathbf{k}' \langle \mathbf{k} | H | \mathbf{k}' \rangle \psi(\mathbf{k}') = \varepsilon \psi(\mathbf{k}), \quad (4)$$

其中 $\psi(\vec{k})$ 是动量空间的波函数。对于一个轴对称形变系统，宇称 π 和总角动量 m_j 的第三个分量都是好量子数。因此，我们可以将波函数分为角向部分和径向部分，如下：

$$\psi(\mathbf{k}) = \begin{pmatrix} f(k)\phi_{l j m_j}(\Omega_k) \\ g(k)\phi_{\tilde{l} j m_j}(\Omega_k) \end{pmatrix}, \quad (5)$$

将上述的波函数代入Dirac方程，并经过一系列变换，我们可以将前述的Dirac方程变为一个对称矩阵方程。求解Dirac方程变为求解对称矩阵的本征解。所有束缚态和共振态可通过对角化哈密顿量得到。单粒子态的密度分布可表示为：

$$\rho^N(E) = \sum_b^{N_b} \delta(E - E_b) + \frac{1}{\pi} \sum_r^{N_r} \frac{\Gamma_r / 2}{(E - \varepsilon_r)^2 + \Gamma_r^2 / 4} + \frac{1}{\pi} \sum_c^{N - N_b - N_r} \frac{E_c^I}{(E - E_c^R)^2 + E_c^I{}^2}, \quad (6)$$

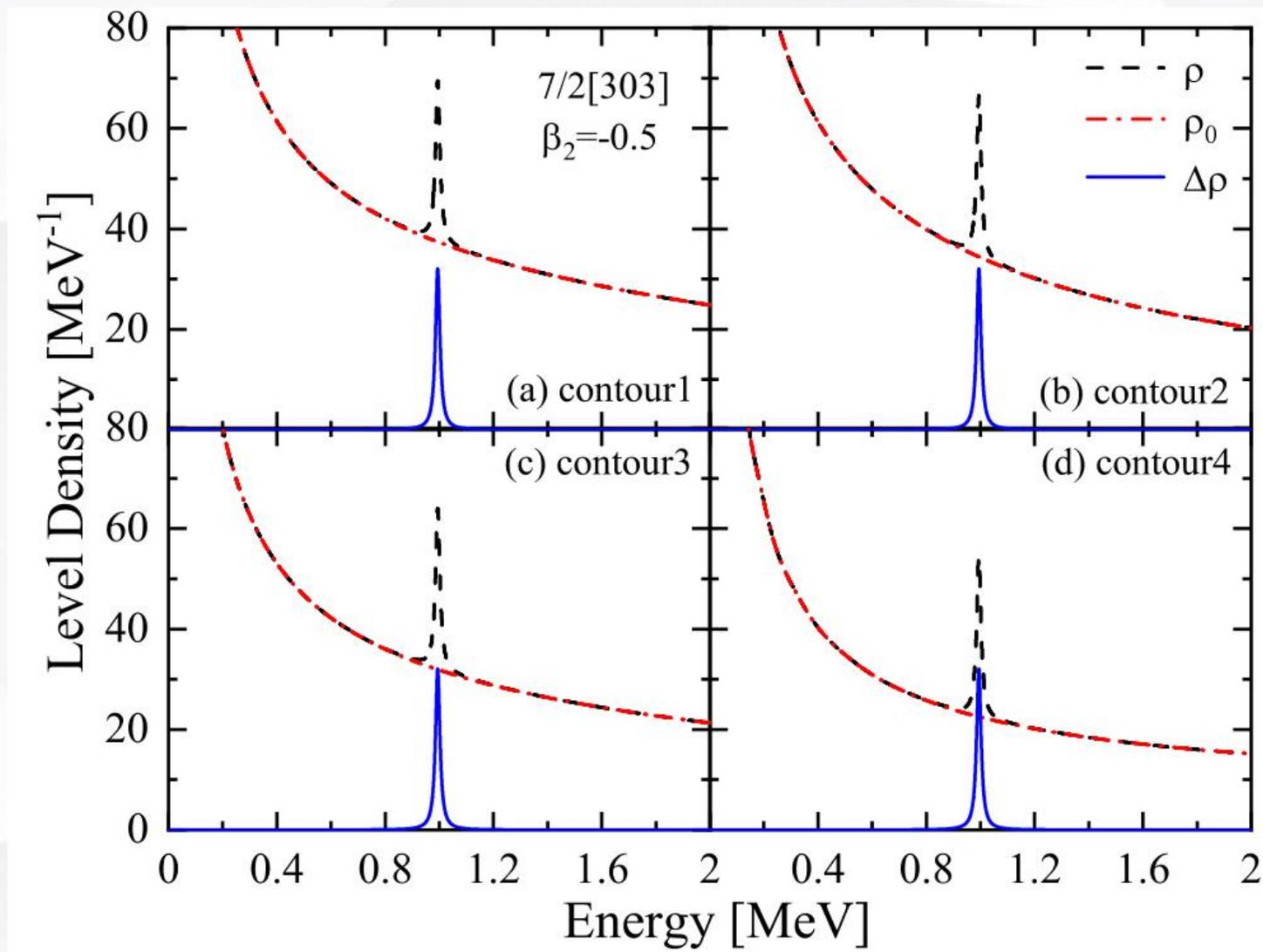
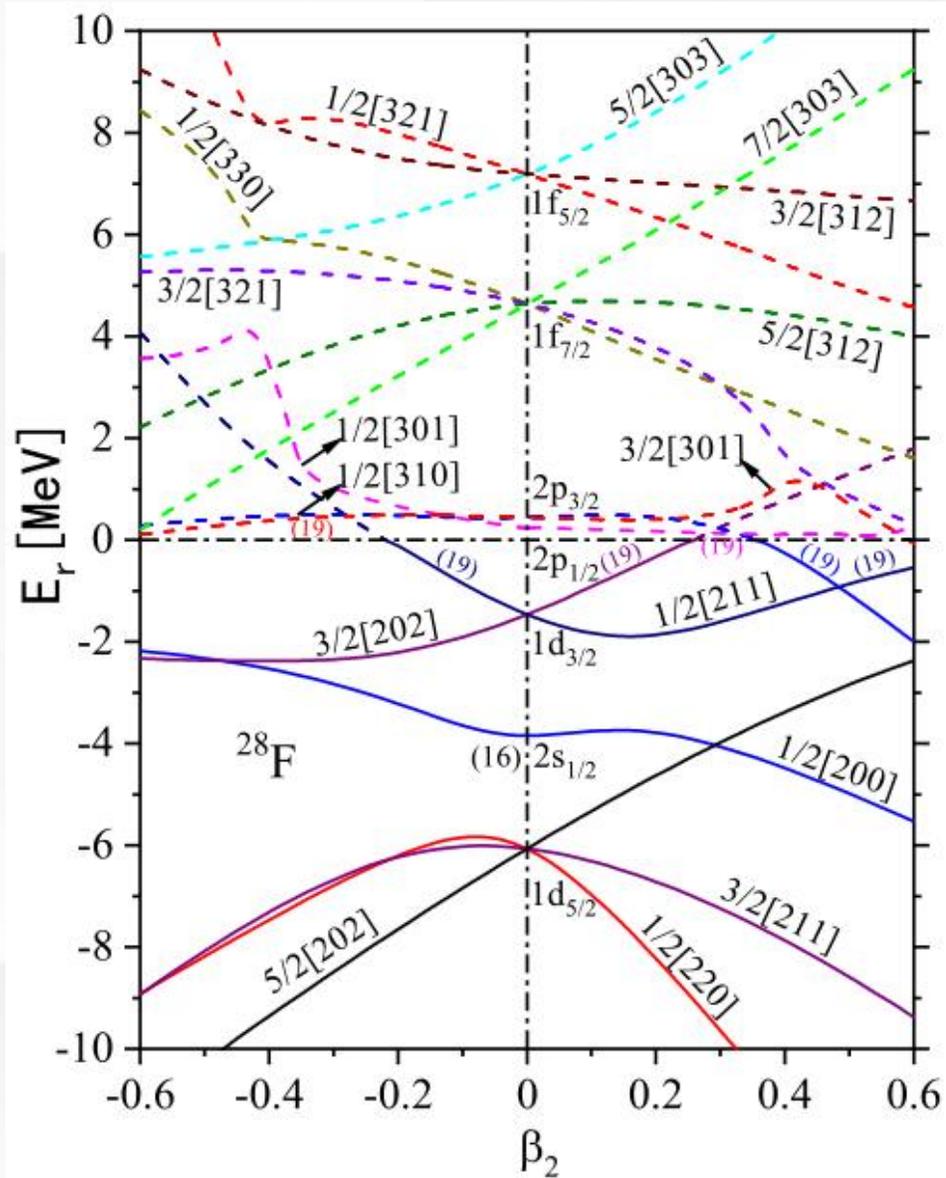
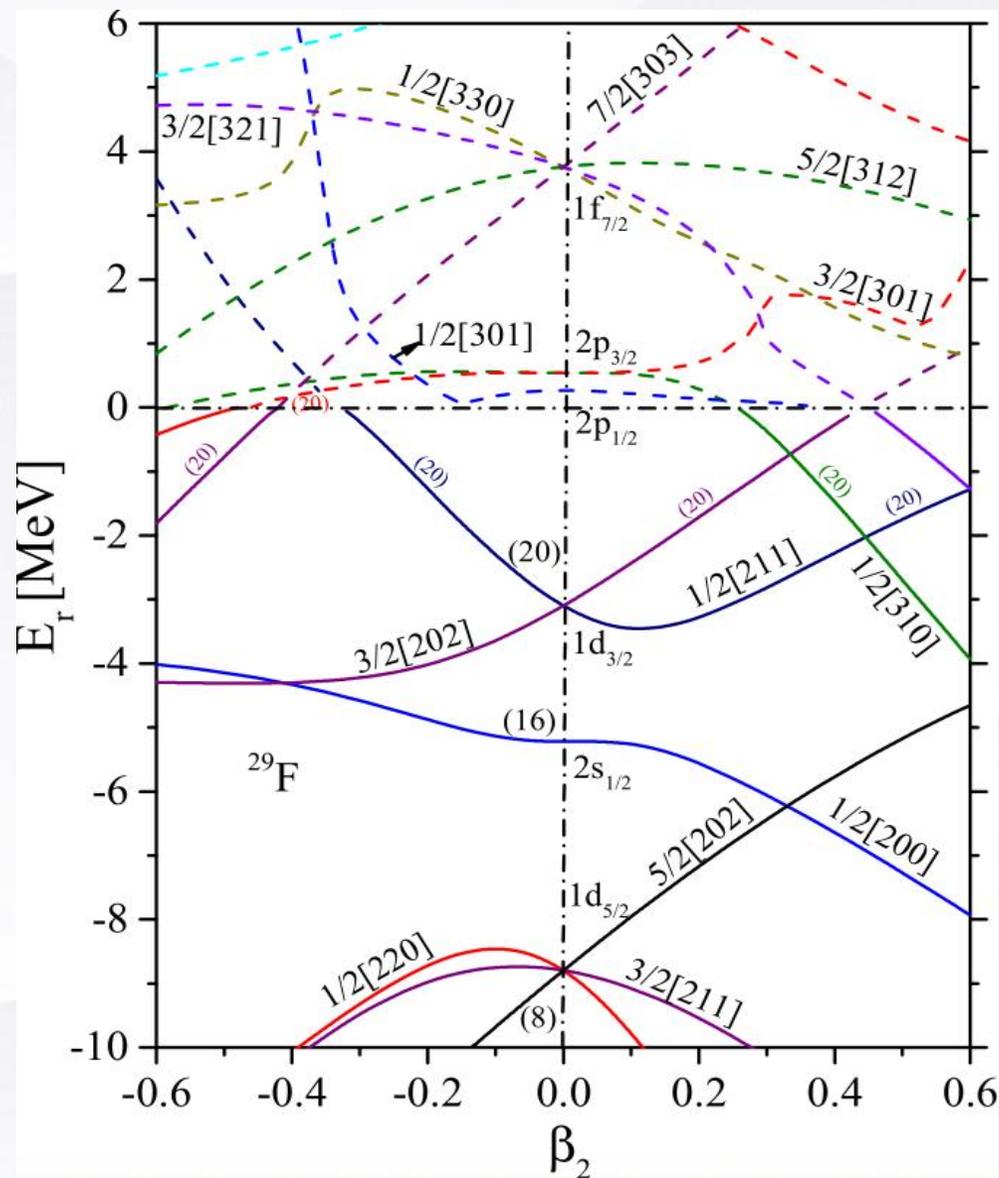


图1: 用RMF-CMR-GF方法在4种不同积分路径下计算得到 ^{28}F 的7/2[303]态在形变参数 $\beta_2 = -0.5$ 时的态密度 $\rho(\varepsilon)$, 连续态的能级密度 $\rho_0(\varepsilon)$, 以及连续谱的能级密度 $\Delta\rho(\varepsilon)$ 。



结果显示： ^{28}F 的共振能级发生了p-f反转，价核子占据低轨道角动量的p能级，这导致 ^{28}F 基态为负宇称态。

图2： ^{28}F 随四极形变参数 β_2 变化的单粒子能量。



► 实验表明²⁹F的基态有晕结构[S. Bagchi et al., 2020 PRL], 因此推断出氟同位素的 β_2 至少应为0.3。

图3: ²⁹F随四极形变参数 β_2 变化的单粒子能量。

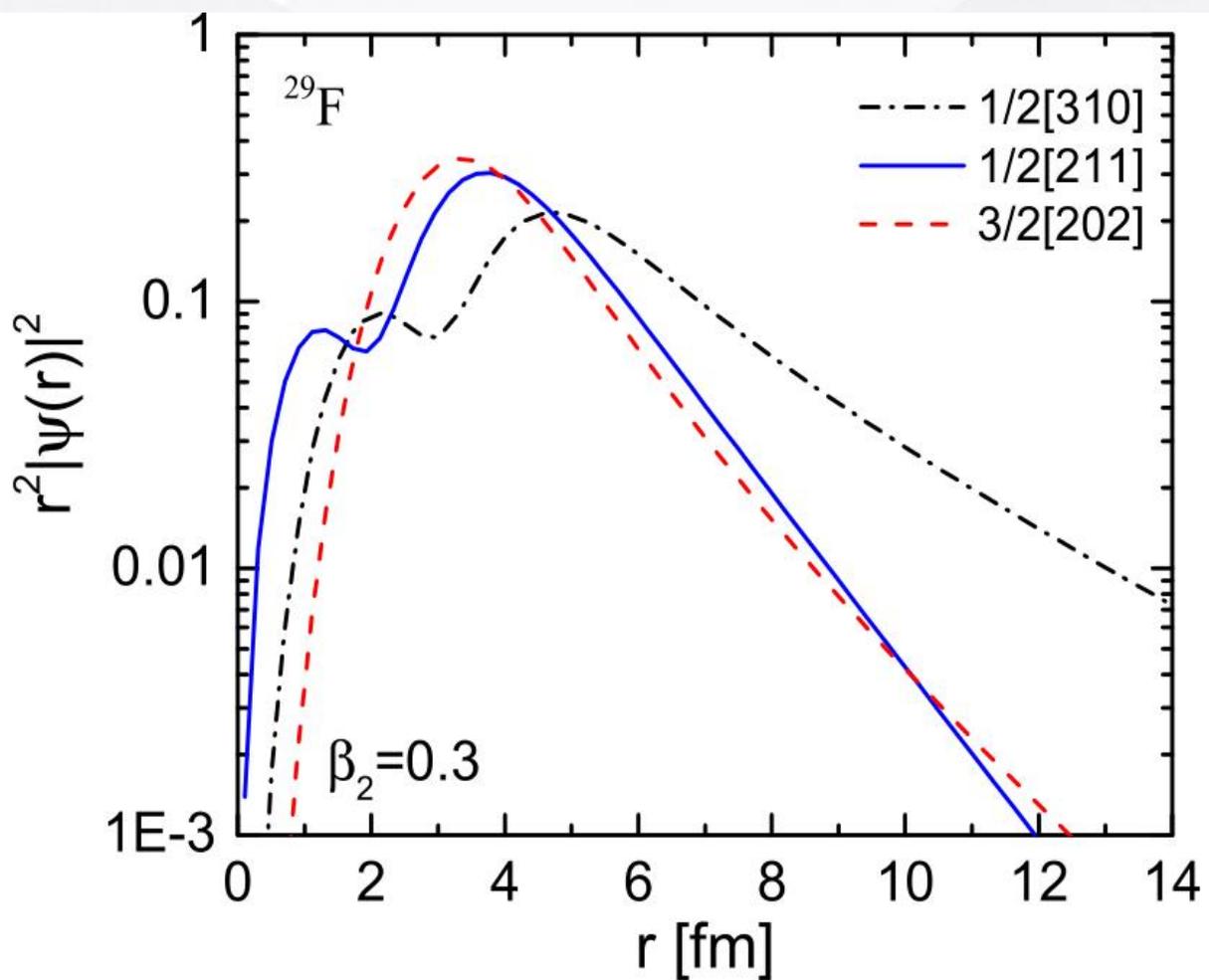


图4: $\beta_2=0.3$ 时, ^{29}F 的单粒子能级 $1/2[310]$, $1/2[211]$ 和 $3/2[202]$ 的径向密度乘以 r^2 随 r 的变化曲线。

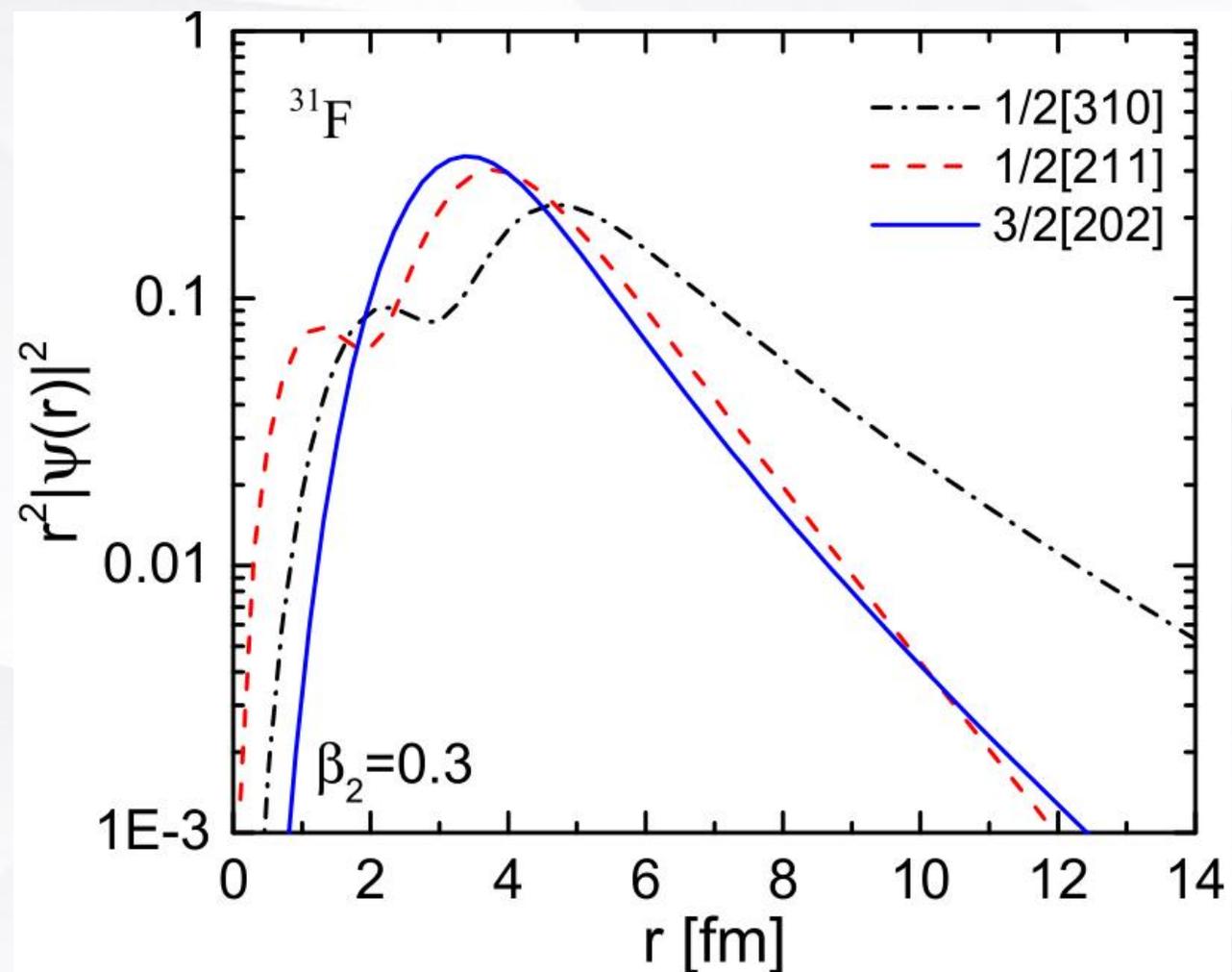


图5: $\beta_2=0.3$ 时, ^{31}F 的单粒子能级 $1/2[310]$, $1/2[211]$ 和 $3/2[202]$ 的径向密度乘以 r^2 随 r 的变化曲线。

3

DRMF-CMR-BCS理论及其结果

Lagrangian密度为: $L = \bar{\psi}[i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \vec{\tau} \vec{\rho}_\mu] \psi$

$$\omega^{\mu\nu} \equiv \partial^\mu \omega^\nu - \partial^\nu \omega^\mu,$$

$$\vec{\rho}^{\mu\nu} \equiv \partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu,$$

$$A^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu.$$

$$\begin{aligned} & -\bar{\psi} \left[\frac{1}{2} e \gamma^\mu (1 - \tau_3) A_\mu \right] \psi + \frac{1}{2} (\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) \\ & - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} \vec{\rho}^{\mu\nu} \vec{\rho}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \vec{\rho}_\mu \\ & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{4} c_3 (\omega^\mu \omega_\mu)^2 \end{aligned} \quad (1)$$

核子的Dirac方程: $[\vec{\alpha} \cdot \vec{p} + V(\vec{r}) + \beta(M + S(\vec{r}))] \psi(\vec{r}) = \varepsilon \psi(\vec{r}) \quad (2)$

矢量势和标量势:
$$\begin{cases} V(\vec{r}) = g_\omega \omega^0(\vec{r}) + g_\rho \tau_3 \rho^0(\vec{r}) + e A^0(\vec{r}) \\ S(\vec{r}) = g_\sigma \sigma(\vec{r}) \end{cases} \quad (3)$$

狄拉克旋量可以根据球面构型展开空间:

$$\psi_{m_j}(\vec{r}) = \frac{1}{r} \sum_{lj} \begin{pmatrix} iG^{lj}(r) Y_{jm_j}^l(\Omega_r) \\ -F^{lj}(r) Y_{jm_j}^{\tilde{l}}(\Omega_r) \end{pmatrix} \quad (4)$$

对于具有反射对称性的轴对称形变系统，用Legendre多项式展开原子核的密度和势场：

$$f(r) = \sum_{\lambda} f_{\lambda}(r) P_{\lambda}(\cos \theta), \quad \lambda = 0, 2, 4, \dots, \quad (5)$$

其中：

$$P_{\lambda}(\Omega) = \sqrt{\frac{4\pi}{2\lambda+1}} Y_{\lambda}^0(\Omega),$$

$$f_{\lambda}(r) = \frac{2\lambda+1}{4\pi} \int d\Omega f(r) P_{\lambda}(\Omega),$$

为了获得共振态，我们将Dirac方程转到动量空间：

$$\int d\vec{k}' \langle \vec{k} | H | \vec{k}' \rangle \psi(\vec{k}') = \varepsilon \psi(\vec{k}) \quad (6)$$

这里： $H = \vec{\alpha} \cdot \vec{p} + V(\vec{r}) + \beta(M + S(\vec{r}))$ (7)

$$\psi(\vec{k}) = \psi_{m_j}(\vec{k}) = \sum_{lj} \begin{pmatrix} f^{lj}(k) \phi_{ljm_j}(\Omega_k) \\ g^{lj}(k) \phi_{ljm_j}(\Omega_k) \end{pmatrix}, \quad (\tilde{l} = 2j - l) \quad (8)$$

将波函数代入式(6), Dirac方程简化为:

$$\begin{aligned} Mf^{lj}(k) - kg^{lj}(k) + \sum_{l'j'} \int k'^2 dk' V^+(l', j', p, q, l, j, m_j, k, k') f^{l'j'}(k') &= \varepsilon f^{lj}(k), \\ -kf^{lj}(k) - Mg^{lj}(k) + \sum_{l'j'} \int k'^2 dk' V^-(\tilde{l}', j', p, q, \tilde{l}, j, m_j, k, k') g^{l'j'}(k') &= \varepsilon g^{lj}(k), \end{aligned} \quad (9)$$

这里:

$$\begin{aligned} V^+(l', j', p, q, l, j, m_j, k_a, k_b) &= (-1)^{l'+l} \frac{2}{\pi} \int r^2 dr [V(r) + S(r)] j_l(k'r') j_l(kr) \sum_{m_s} \langle lm | Y_{pq}(\Omega_r) | l'm \rangle \langle lm \frac{1}{2} m_s | jm_j \rangle \langle l'm \frac{1}{2} m_s | j'm_j \rangle \\ V^-(\tilde{l}', j', p, q, \tilde{l}, j, m_j, k_a, k_b) &= (-1)^{\tilde{l}'+\tilde{l}} \frac{2}{\pi} \int r^2 dr [V(r) - S(r)] j_{\tilde{l}}(k'r') j_{\tilde{l}}(kr) \sum_{m_s} \langle \tilde{l}m | Y_{pq}(\Omega_r) | \tilde{l}'m \rangle \langle \tilde{l}m \frac{1}{2} m_s | jm_j \rangle \langle \tilde{l}'m \frac{1}{2} m_s | j'm_j \rangle \end{aligned} \quad (10)$$

利用Gauss-Legendre积分公式, 公式(9)可以转化为:

$$\begin{cases} \sum_b [M \delta_{ab} \mathbf{f}^{lj}(k_b) + \sum_{l'j'} \sqrt{\omega_a \omega_b} k_a k_b V^+(l', j', p, q, l, j, m_j, k_a, k_b) \mathbf{f}^{l'j'}(k_b) - k_a \delta_{ab} \mathbf{g}^{lj}(k_b)] = \varepsilon \mathbf{f}^{lj}(k_a) \\ \sum_b [-k_a \delta_{ab} \mathbf{f}^{lj}(k_b) - M \delta_{ab} \mathbf{g}^{lj}(k_b) + \sum_{l'j'} \sqrt{\omega_a \omega_b} k_a k_b V^-(\tilde{l}', j', p, q, \tilde{l}, j, m_j, k_a, k_b) \mathbf{g}^{l'j'}(k_b)] = \varepsilon \mathbf{g}^{lj}(k_a) \end{cases} \quad (11)$$

至此，求解Dirac方程变为求解对称矩阵的本征解。所有束缚态和共振态可通过对角化哈密顿量得到，为了获得坐标空间中的密度分布，将波函数转换为式(8)中上下分量的形式：

$$\begin{aligned} G^{lj}(\mathbf{r})/r &= -i^{l+1} \sqrt{\frac{2}{\pi}} \int k^2 dk j_l(kr) f^{lj}(k) \\ F^{lj}(\mathbf{r})/r &= -i^{\tilde{l}} \sqrt{\frac{2}{\pi}} \int k^2 dk j_{\tilde{l}}(kr) g^{lj}(k) \end{aligned} \quad (12)$$

当考虑共振时，可以用gap方程和粒子数方程来处理配对关系：

$$\begin{aligned} \sum_b \frac{1}{\sqrt{(\varepsilon_b - \lambda)^2 + \Delta^2}} + \sum_r \int \frac{g_r(\varepsilon)}{\sqrt{(\varepsilon - \lambda)^2 + \Delta^2}} d\varepsilon &= \frac{2}{G}, \\ \sum_b \left(1 - \frac{\varepsilon_b - \lambda}{\sqrt{(\varepsilon_b - \lambda)^2 + \Delta^2}} \right) + \sum_r \int g_r(\varepsilon) \left(1 - \frac{\varepsilon - \lambda}{\sqrt{(\varepsilon - \lambda)^2 + \Delta^2}} \right) d\varepsilon &= N, \end{aligned} \quad (13)$$

G , $\Delta = \frac{12}{\sqrt{\Lambda}}$, N 分别是对力强度，能隙和粒子数。

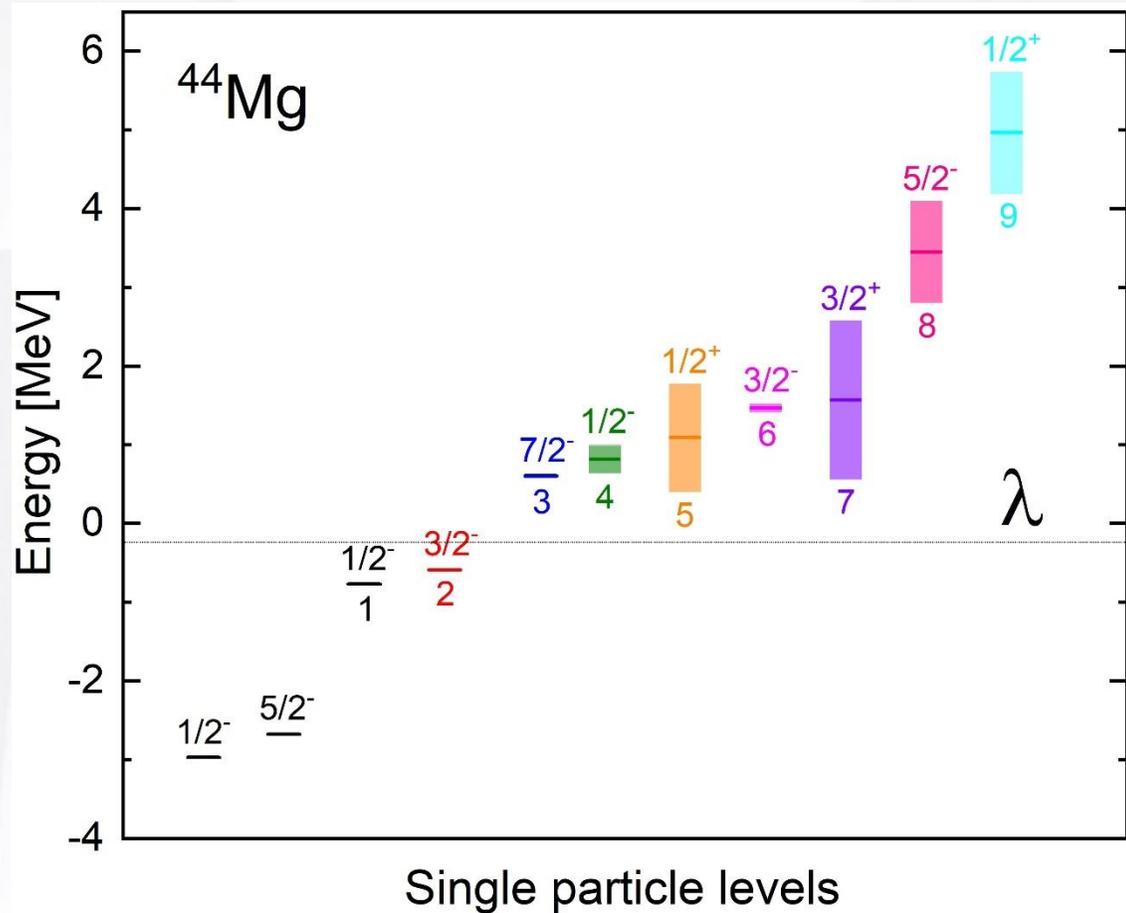


图6: ^{44}Mg 的费米面周围所有相关的单中子能级, 包括束缚态和共振态。共振态的能级用矩形表示。角动量和宇称的量子数用 Ω^π 标记。化学势 λ 用虚线标记, n 代表所考虑的能级数。

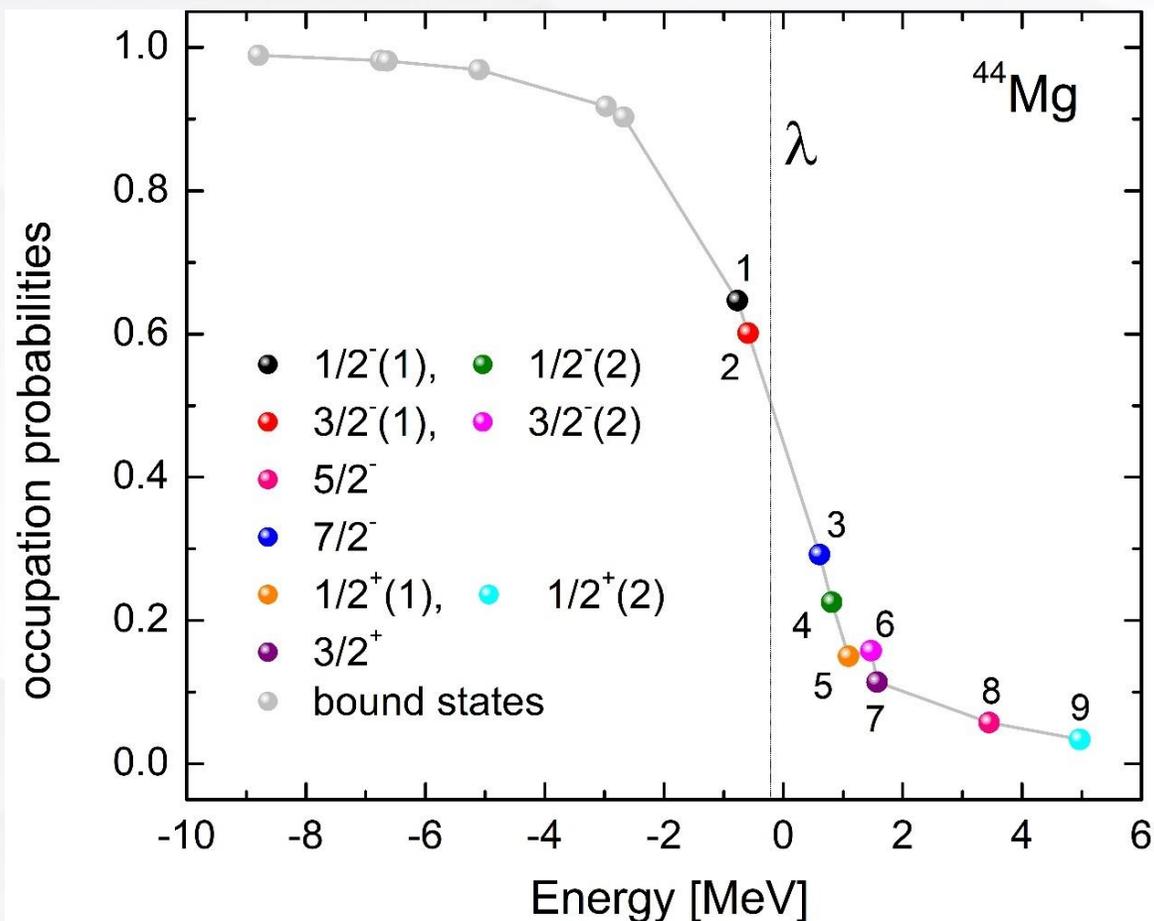


图7: ^{44}Mg 的费米面周围的单粒子能级的占据几率。灰色圆圈表示深束缚能级。用颜色不同的实心圆表示弱束缚能级和共振能级。化学势 λ 代表费米面的位置。

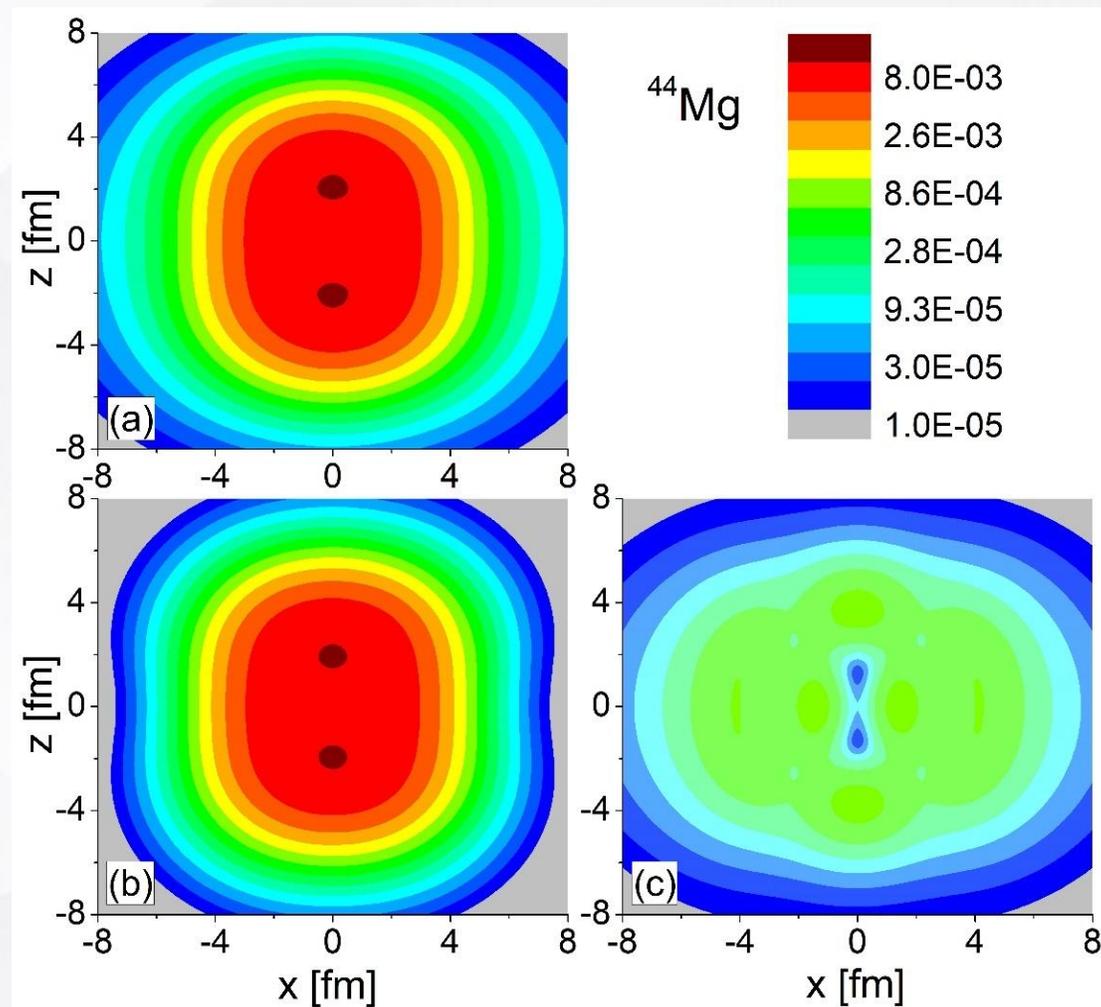


图8: ^{44}Mg 的中子的物质密度分布。子图 (a) 代表总的中子密度分布。子图 (b) 和 (c) 代表这些中子占据在深束缚能级以及弱束缚能级和共振能级上的密度分布。

4

总结



总结

- 我们已经将复动量 (CMR) 方法和格林函数 (GF) 方法相结合, 发展了RMF-CMR-GF方法。研究了四极形变和连续谱效应对 $^{28,29,31}\text{F}$ 单粒子结构的影响, 并展示了这两种现象如何导致 ^{28}F 的负宇称基态和 $^{29,31}\text{F}$ 的晕结构。
- 在复动量表象 (CMR) 下发展了形变RMF理论, 并考虑对关联, 建立的DRMF-CMR-BCS理论, 解决了统一描述束缚态和共振态、窄共振和宽共振的难题, 获得了其它方法难以获得的连续阈附近低轨道角动量宽共振的信息。探究了丰中子核 ^{44}Mg 的形变晕结构。



谢谢各位老师 and 同学们的倾听！

- 原子核： ^{28}F
- 相互作用：NL3
- 利用在相对论平均场(RMF)计算得到自洽的球形平均势场，再通过拟合的方法获得WoodsSaxon势的参数。
- Woods-Saxon势场：
$$f(r) = \frac{1}{1 + e^{\frac{r-R}{a}}}$$
- 弥散参数： $a = 0.75 \text{ fm}$.
- 半径： $R = r_0 A^{1/3} = 3.32 \text{ fm}$.
- 调整势场的深度以再现实验中获得的分离能 $S_n(^{28}\text{F}) = -199(6) \text{ keV}$ [Revel, et al. 2020 PRL]。势场深度为： $V_0 = 374 \text{ MeV}$, $S_0 = -436 \text{ MeV}$ 。
- 在改变四极形变的情况下，通过计算CLD可以提取共振参数。因此，我们首先检查 ^{28}F 计算得到的CLD对积分路径的依赖性。

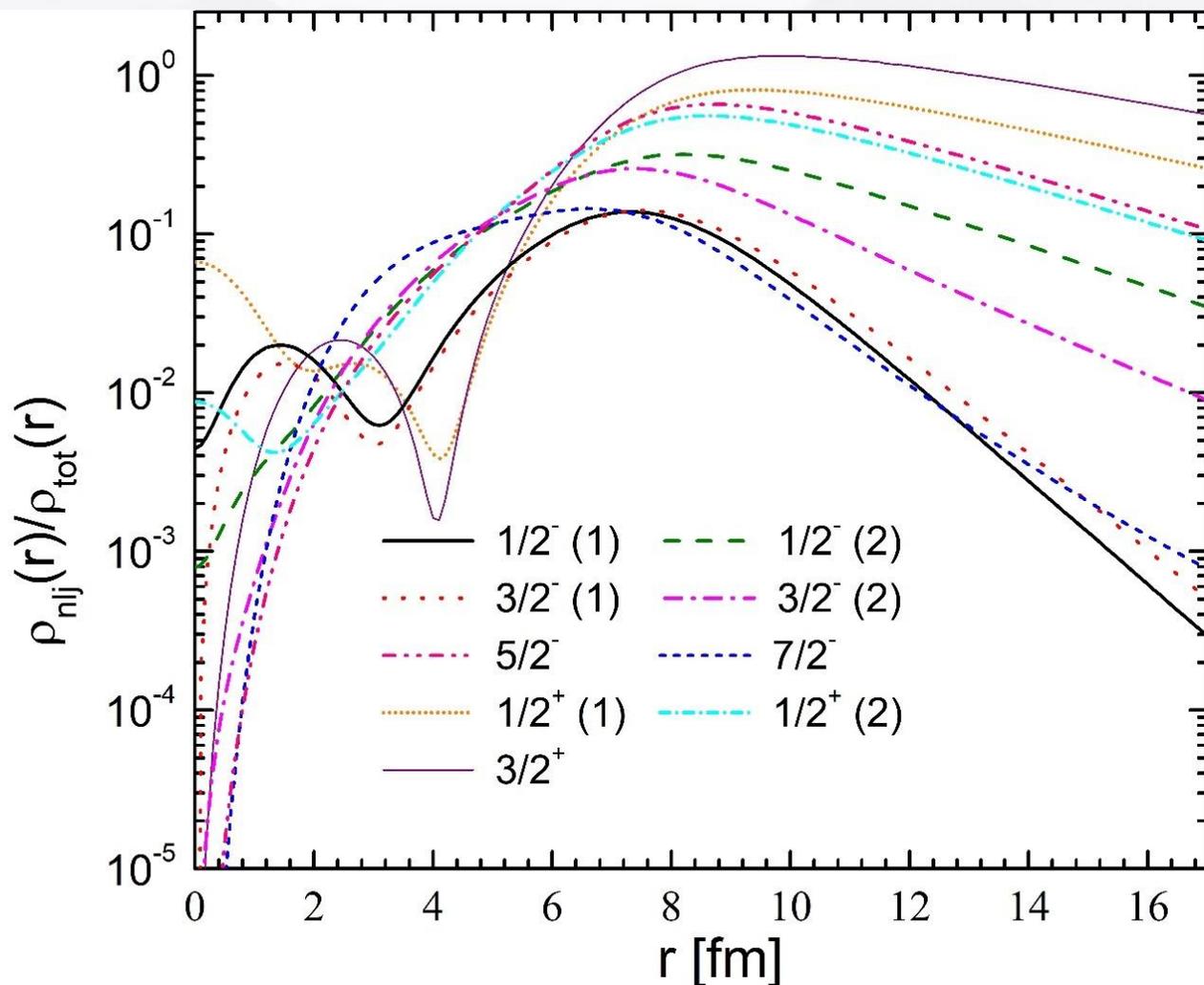


图9: ^{44}Mg 的弱束缚能级和共振能级的密度分布与总的密度分布的比值。这些能级与图7中的能级相同。

- 弱束缚能级 $1/2^-(1)$ 和 $3/2^-(1)$ 的密度分布不太弥散。
- 虽然共振态 $7/2^-$ 有着较低的能量和较大的中子占据几率，但是它具有较大的轨道角动量和较高的离心势垒，阻碍了中子晕的形成。
- 窄共振能级 $1/2^-(2)$ 和 $3/2^-(2)$ （图7中的第4个能级和第6个能级）的密度分布是比较弥散的，占据几率相对较大，对奇特的晕现象的形成有一定的贡献。
- 宽共振态 $5/2^-$ 和 $1/2^+(2)$ （图7中的第8个能级和第9个能级）的密度分布是弥散的，但占据几率相对较小，对总密度分布的贡献相对较小。
- 宽共振态 $3/2^+$ 和 $1/2^+(1)$ 的密度分布非常弥散，占据几率相对较大，在中子晕的形成中占主要贡献。

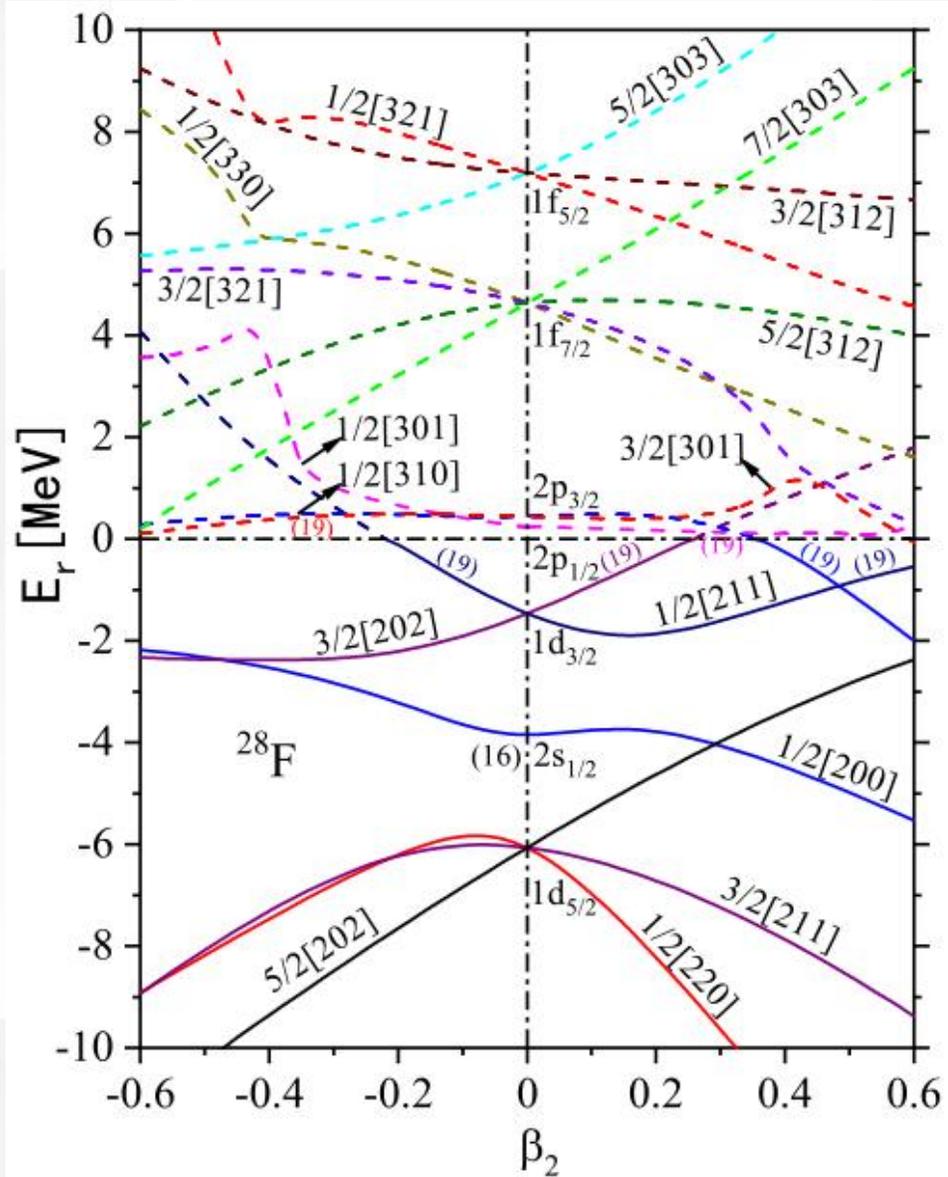


图2: ^{28}F 随四极形变参数 β_2 变化的单粒子能量。

结果显示： ^{28}F 的共振能级发生了p-f反转，价核子占据低轨道角动量的p能级，这些导致 ^{28}F 基态为负宇称态。

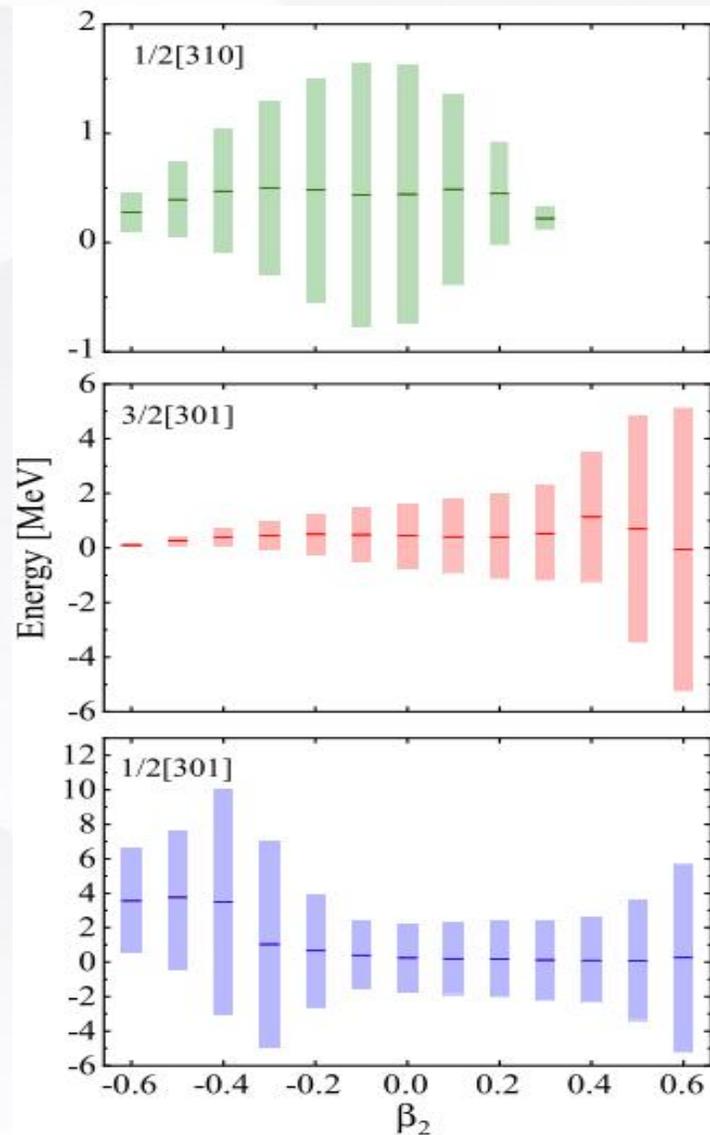


图3: $1/2[310]$, $3/2[301]$, $1/2[301]$ 能级的能量和宽度是 β_2 的函数。矩形的高度代表共振态的宽度。 30