

# Core-crust transition density with symmetry energy in the nonlinear relativistic Hartree approximation



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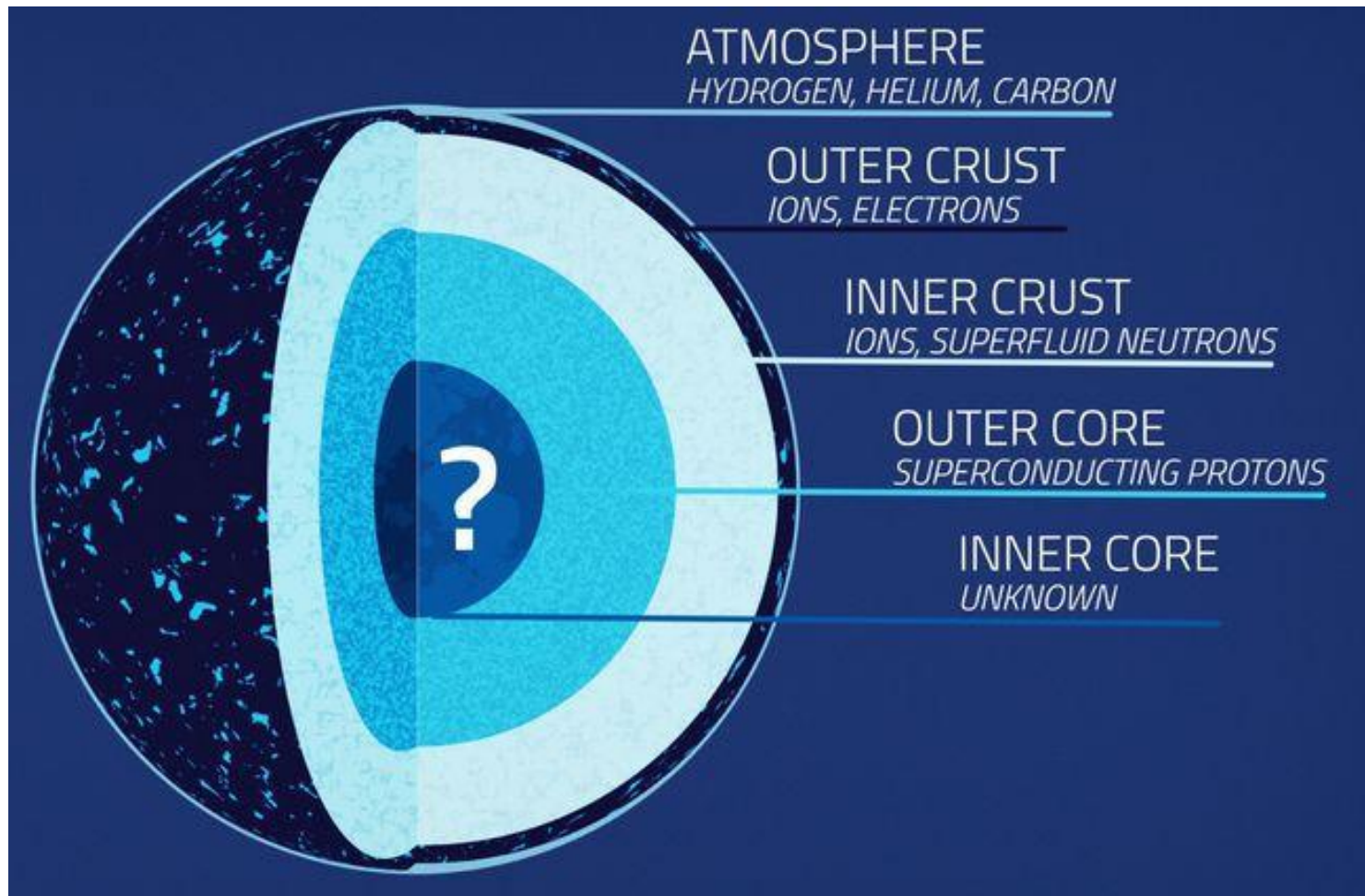
**1** Core-crust transition density

**2** RRPA method in nonlinear RHA

**3** Results and discussion

**4** Summary

# Core-crust transition density



➤ **core-crust transition density:** When the density is lower than a specific value, the outer core of a neutron star becomes unstable under the density fluctuation.

# Core-crust transition density

## ➤ The neutron star radius

M. Fortin, et al. Phys. Rev. C 94 (2016) 035804.

## ➤ Pulsar glitches

J.M. Lattimer, et al., Phys. Rep. 442 (2007) 109.

N. Chamel, et al., Phys. Rev. C 85 (2012) 035801.

N. Andersson, et al., Phys. Rev. Lett. 109 (2012) 241103.

J. Piekarewicz, et al., Phys. Rev. C 90 (2014) 015803.

S-N. Wei, et al., Chin. Phys. C 42 (2018) 74.

## ➤ Crust meltdown in inspiraling binary neutron stars

Z. Pan, et al., Phys. Rev. Lett. 125 (2020) 201102.

## ➤ Asteroseismology from giant magnetar flares

A.T. Deibel, et al., Phys. Rev. C 90 (2014) 025802.

# Core-crust transition density

➤ Hydrodynamic method: Vlasov equation, Master equation

Chomaz et al PR389,263(04), Ducoin, et al.NPA 789, 403 (07),  
Zheng & Chen, PRD 85, 043013(12)

➤ Thermodynamic method:  $P$ & $\mu$  instability condition

Douchin & Haensel, PLB 485, 107 (00),  
Li, et al, NPA 699, 493 (02), Kubis, PRC 76, 025801 (07).

➤ Relativistic RPA: Dyson equation

Lim &Horowitz, NPA 501, 729(89),  
Carriere, Horowitz, et al, ApJ 593, 463 (03);  
Ma, et al, NPA 686, 173 (01)

**No vacuum effect or no self-consistent vacuum treatment**

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# RRPA method in nonlinear RHA

- The interacting Lagrangian is written as:

$$\mathcal{L}_{\text{int}} = \bar{\psi} [\gamma_{\mu} (i\partial^{\mu} - g_{\omega}\omega^{\mu} - g_{\rho}\boldsymbol{\tau} \cdot \mathbf{b}^{\mu} + \frac{e}{2}(1 + \tau_3)A^{\mu}) - (M - g_{\sigma}\phi)] \psi + 4g_{\rho}^2 b_{\mu} \cdot b^{\mu} \Lambda_v g_{\omega}^2 \omega_{\mu} \omega^{\mu} - U(\phi, \omega),$$

Where

$$U(\phi, \omega) = -\frac{1}{2}g_{\sigma\omega}\omega^2\phi^2 + g_2\phi^3/3! + g_3\phi^4/4!$$

The Lagrangian counterterms\* to renormalize the infinite nucleon self-energy is

$$\mathcal{L}_{CT} = \sum_{n=1}^4 \frac{\alpha_n}{n!} \phi^n$$

\* S. A. Chin, Ann. Phys. 108: 301 (1977)

# RRPA method in nonlinear RHA

$$U(\phi, \omega) = -\frac{1}{2}g_{\sigma\omega}\omega^2\phi^2 + g_2\phi^3/3! + g_3\phi^4/4!$$



$$\begin{aligned} U^{Ren.}(\phi, \omega) = & -\frac{1}{2}g_{\sigma\omega}\omega^2\phi^2 + \frac{1}{3!}g_2\phi^3 + \frac{1}{4!}g_3\phi^4 + \frac{1}{(8\pi)^2}\left[-\left(g_2\phi + \frac{g_3\phi^2}{2}\right)\tilde{m}_\sigma^2 \right. \\ & + \frac{1}{12}\frac{g_2^4\phi^4}{\tilde{m}_\sigma^4} + \left(\tilde{m}_\sigma^2 + g_2\phi + \frac{g_3\phi^2}{2}\right)^2 \ln\left(1 + \frac{g_2\phi + \frac{g_3\phi^2}{2}}{\tilde{m}_\sigma^2}\right) \\ & \left. - \frac{3}{2}\left(g_2\phi + \frac{g_3\phi^2}{2}\right)^2 - \frac{(g_2\phi)^2}{3\tilde{m}_\sigma^2}\left(g_2\phi + \frac{3g_3\phi^2}{2}\right)\right], \end{aligned}$$

where  $\tilde{m}_\sigma^2 = m_\sigma^2 - g_{\sigma\omega}\omega^2$ .

\* D. B. Serot and J. D. Walecka, Adv. Nucl. Phys. 16: 1 (1986)



# RRPA method in nonlinear RHA

- The interacting polarization is determined through the Dyson equation :

$$\tilde{\Pi}_L = \Pi_L + \tilde{\Pi}_L D_L \Pi_L$$

$$\tilde{\Pi}_L = (1 - D_L \Pi_L)^{-1} \Pi_L$$

Dielectric function:  $\epsilon_L = \det(1 - D_L \Pi_L)$

- The transition density  $\rho_t$ , the largest density determined by

$$\epsilon_L = \det[1 - D_L(q) \Pi_L(q, q_0 = 0)] \leq 0,$$

# RRPA method in nonlinear RHA

$$\Pi_L = \begin{pmatrix} \Pi_{00D}^e & 0 & 0 & 0 \\ 0 & \Pi_{sD}^n + \Pi_{sD}^p & \Pi_m^p & \Pi_m^n \\ 0 & \Pi_m^p & \Pi_{00D}^p & 0 \\ 0 & \Pi_m^n & 0 & \Pi_{00D}^n \end{pmatrix}$$



$$\Pi_L = \begin{pmatrix} \Pi_{00D}^e + \Pi_{00f}^e & 0 & 0 & 0 \\ 0 & \Pi_{sD}^n + \Pi_{sD}^p + \Pi_{sf}^n + \Pi_{sf}^p & \Pi_m^p & \Pi_m^n \\ 0 & \Pi_m^p & \Pi_{00D}^p + \Pi_{00f}^p & 0 \\ 0 & \Pi_m^n & 0 & \Pi_{00D}^n + \Pi_{00f}^n \end{pmatrix}$$

$$\begin{aligned} \Pi_{sf}^0 = & \frac{1}{\pi^2} \sum_{i=p,n} \left[ \frac{m_\sigma^2 - q^2}{8} - \frac{3}{4} \int_0^1 dx (M_i^{*2} + x(x-1)q^2) \ln \frac{x(x-1)q^2 + M_i^{*2}}{x(x-1)m_\sigma^2 + M_i^2} \right. \\ & \left. - \frac{3}{4} \int_0^1 dx (M_i^{*2} - M_i^2) \ln \left( 1 + \frac{x(x-1)m_\sigma^2}{M_i^2} \right) - \frac{1}{2} (3M_i \Sigma_s - \frac{9}{2} \Sigma_s^2) \right] \end{aligned}$$

where  $\Sigma_s = M_i - M_i^*$ . **Renormalization at  $q^2 = 0$ ,  $m_\sigma^2$ ?**

# RRPA method in nonlinear RHA

$$D_L^0 = \begin{pmatrix} d_g & 0 & -d_g & 0 \\ 0 & -d_s^0 & 0 & 0 \\ -d_g & 0 & d_g + d_v^0 + d_\rho^0 & d_v^0 - d_\rho^0 \\ 0 & 0 & d_v^0 - d_\rho^0 & d_v^0 + d_\rho^0 \end{pmatrix}$$



$$D_L = \begin{pmatrix} d_g & 0 & -d_g & 0 \\ 0 & -d_\sigma & d_{\sigma\omega} & d_{\sigma\omega} \\ -d_g & d_{\sigma\omega} & d_g + d_V + 2d_{\omega\rho} & d_I \\ 0 & d_{\sigma\omega} & d_I & d_V - 2d_{\omega\rho} \end{pmatrix},$$

with  $d_V = d_\omega + d_\rho$ , and  $d_I = d_\omega - d_\rho$ .

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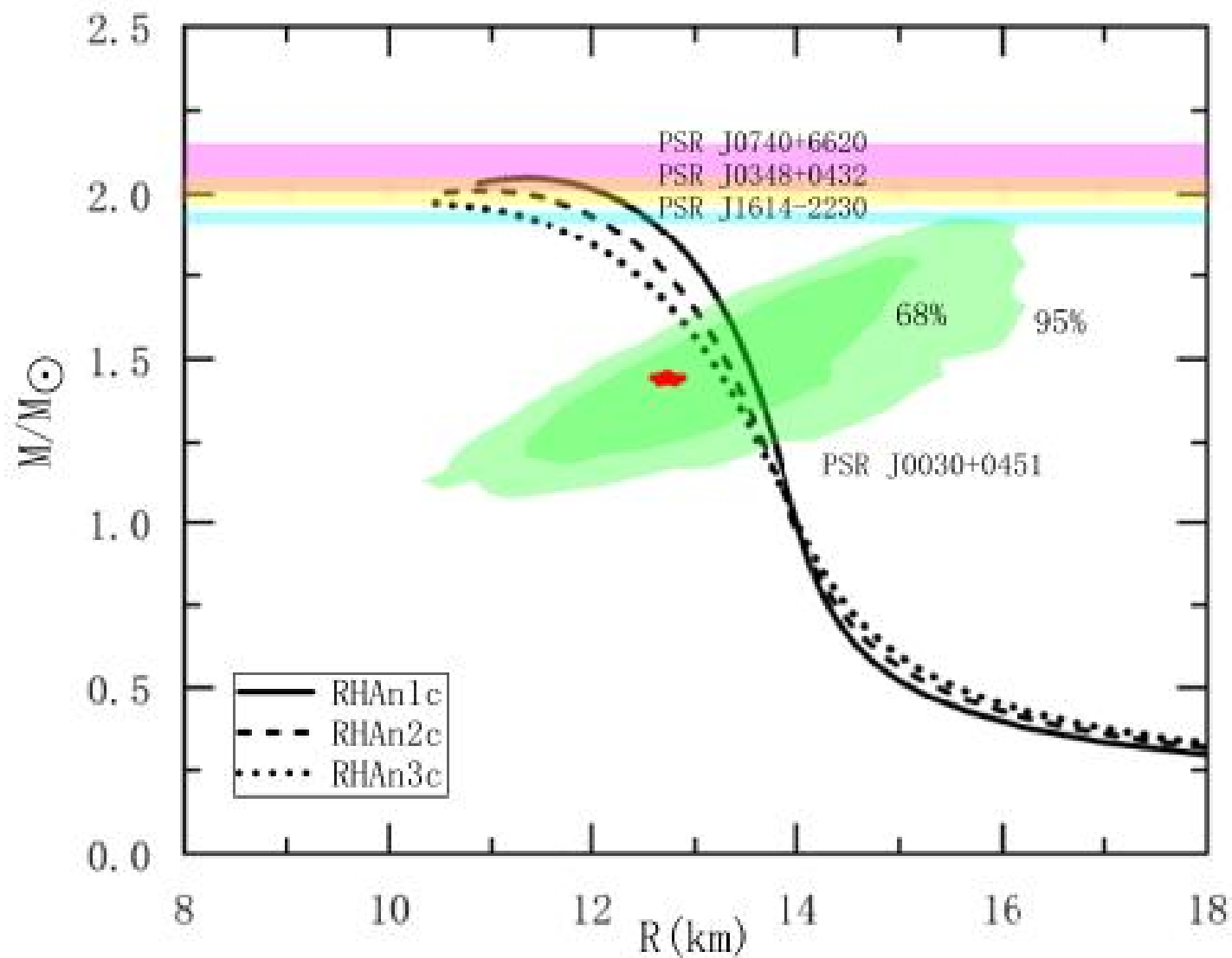
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# Results and discussion

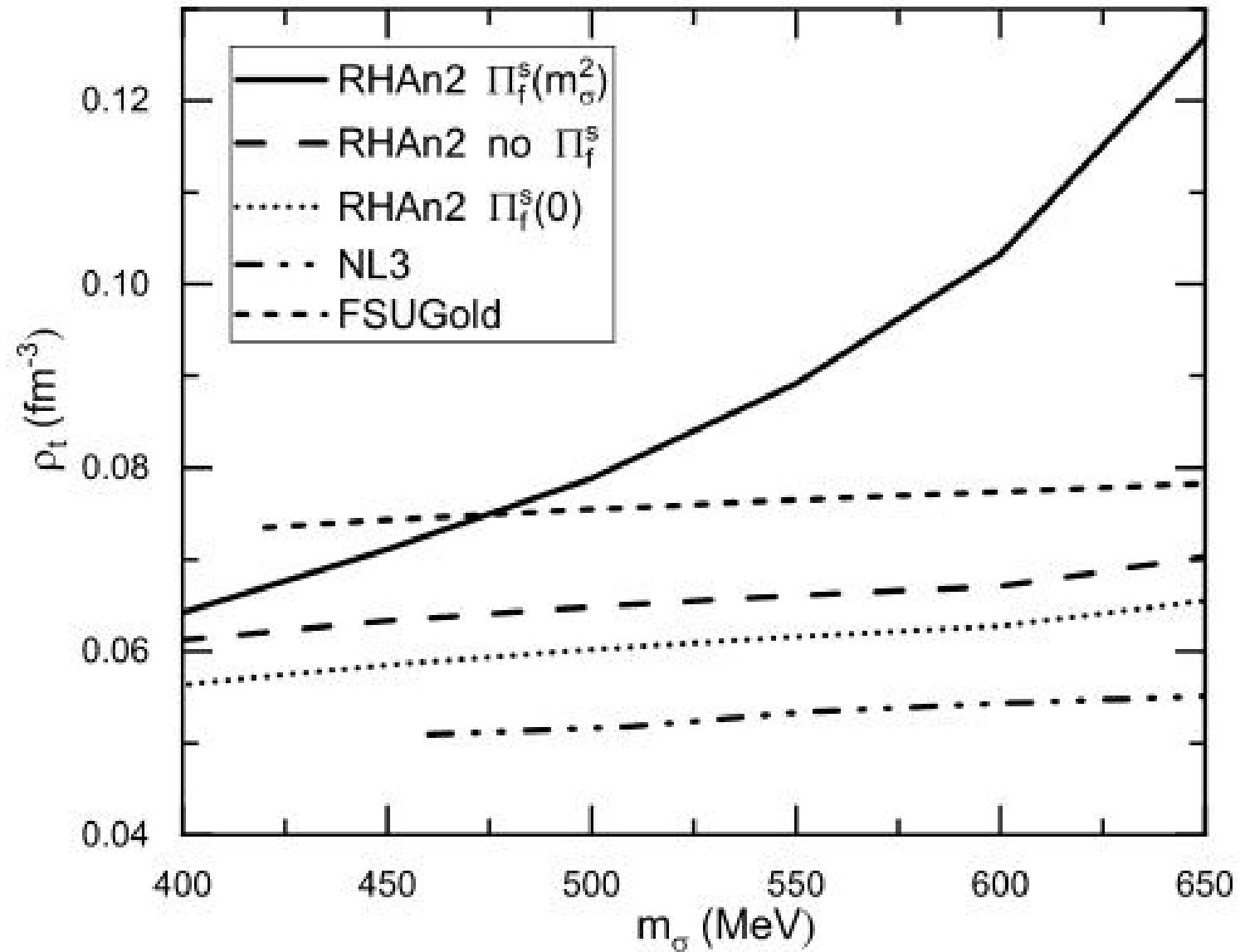
Model	$g_\sigma$	$g_\rho$	$g_\omega$	$g_2$	$g_3$	$g_{\sigma\omega}$	$c_3$	$M^*/M$	$\rho_t$	$\kappa$	$R_{1.4}$	$M_{NS}^{max}/M_\odot$	$M_{NS}^{max*}/M_\odot$
RHAn1c	7.72	4.03	8.45	25.19	21.16	0.5	0.0	0.782	0.081	300	13.62	2.048	2.03
RHAn2c	7.59	4.06	7.96	35.58	39.24	0.5	-4.3	0.800	0.081	270	13.44	2.012	1.91
RHAn3c	7.47	4.10	7.39	51.11	88.90	7.0	-9.0	0.820	0.079	240	13.35	1.975	1.72

: Main parameters of RHAnC Models and properties of nuclear matters and neutron stars.  $M_{NS}^{max*}$  is the maximum neutron star mass in RHAn Models without  $g_{\sigma\omega}$ . The masses of  $\sigma$ ,  $\omega$ , and  $\rho$  mesons are 512, 783, and 770 MeV, respectively. The saturation density  $\rho_0 = 0.16 fm^{-3}$  and binding energy per nucleon  $E_b = -16$  MeV at saturation. The transition density  $\rho_t$ , incompressibility  $\kappa$ , and radius  $R_{1.4}$  of  $1.4M_\odot$  star are in units of  $fm^{-3}$ , MeV, and km, respectively.

# Results and discussion



# Results and discussion



# Results and discussion

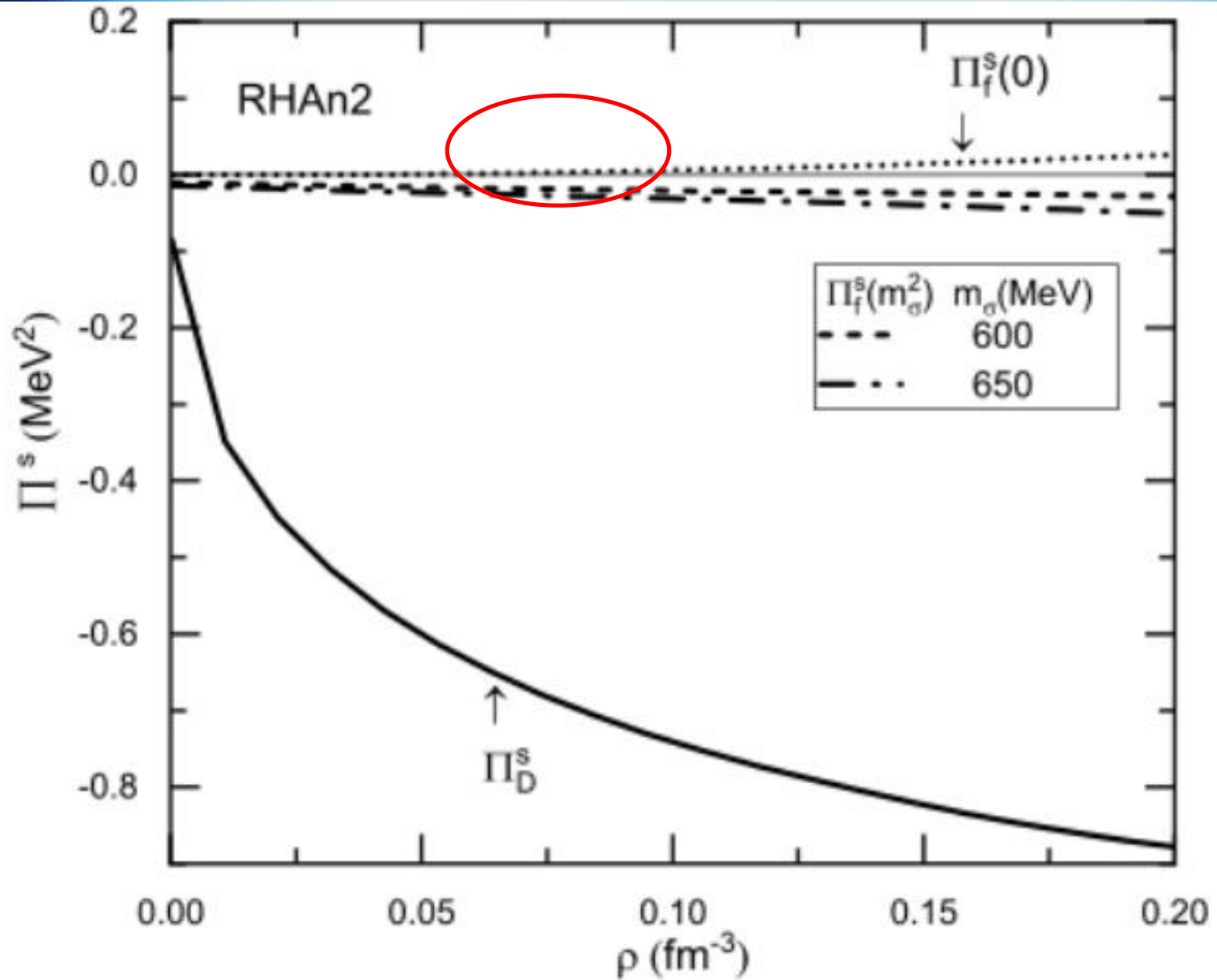
- A simplified  $\epsilon_L$  by neglecting the Coulomb interaction , photon polarization and the crossing coupling terms

$$\epsilon_L = [1 + d_s(\Pi_D^s + \Pi_f^s)] \cdot [1 + 4d_\omega d_\rho \Pi_L^2 + 2(d_\omega + d_\rho)\Pi_L] - 4(1 + 2d_\rho \Pi_L)d_s d_\omega \Pi_M^2,$$

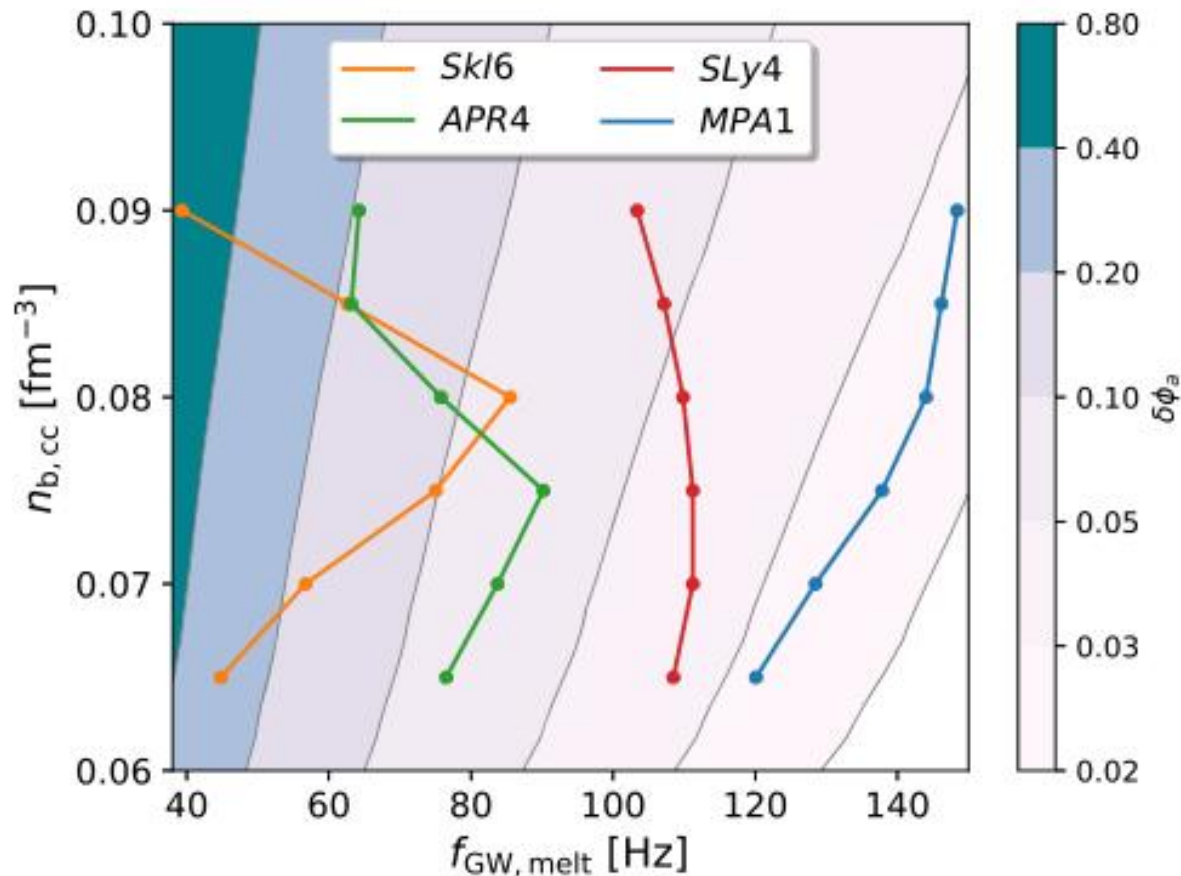
- The difference between  $\Pi_f^s(q^2)$  at  $q^2=0$  and  $m_\sigma^2$  is not small, their effects on the  $\rho_t$  depart in sharp contrast.
- $\Pi_f^s(m_\sigma^2)$ , which is of the same sign of  $\Pi_D^s$ , shifts the zero point of the dielectric function to a larger density, that is, the transition density  $\rho_t$  grows clearly with the rise of  $m_\sigma$ .



# Results and discussion



# Results and discussion



Z. Pan, et al., Phys. Rev. Lett. 125 (2020) 201102.

The crust melting induced phase change  $\delta\phi_a$  in GWs of a BNS merger with each star of  $M^*=1.3/M_\odot$  and  $R=12.5/11.3/11.7/12.7$  km for the EOS SkI6/APR4/SLy4/MPA1. The core-crust transition density is  $n_{b;cc}$ .

# Results and discussion

- The fraction of the crustal moment of inertia  $\Delta I/I$  can be derived as\*:

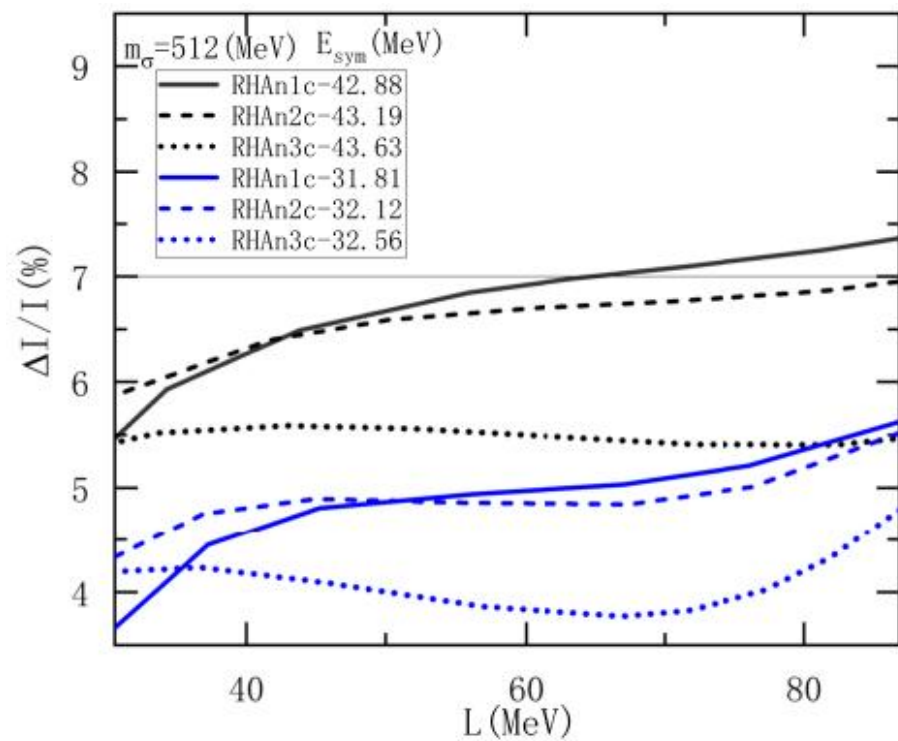
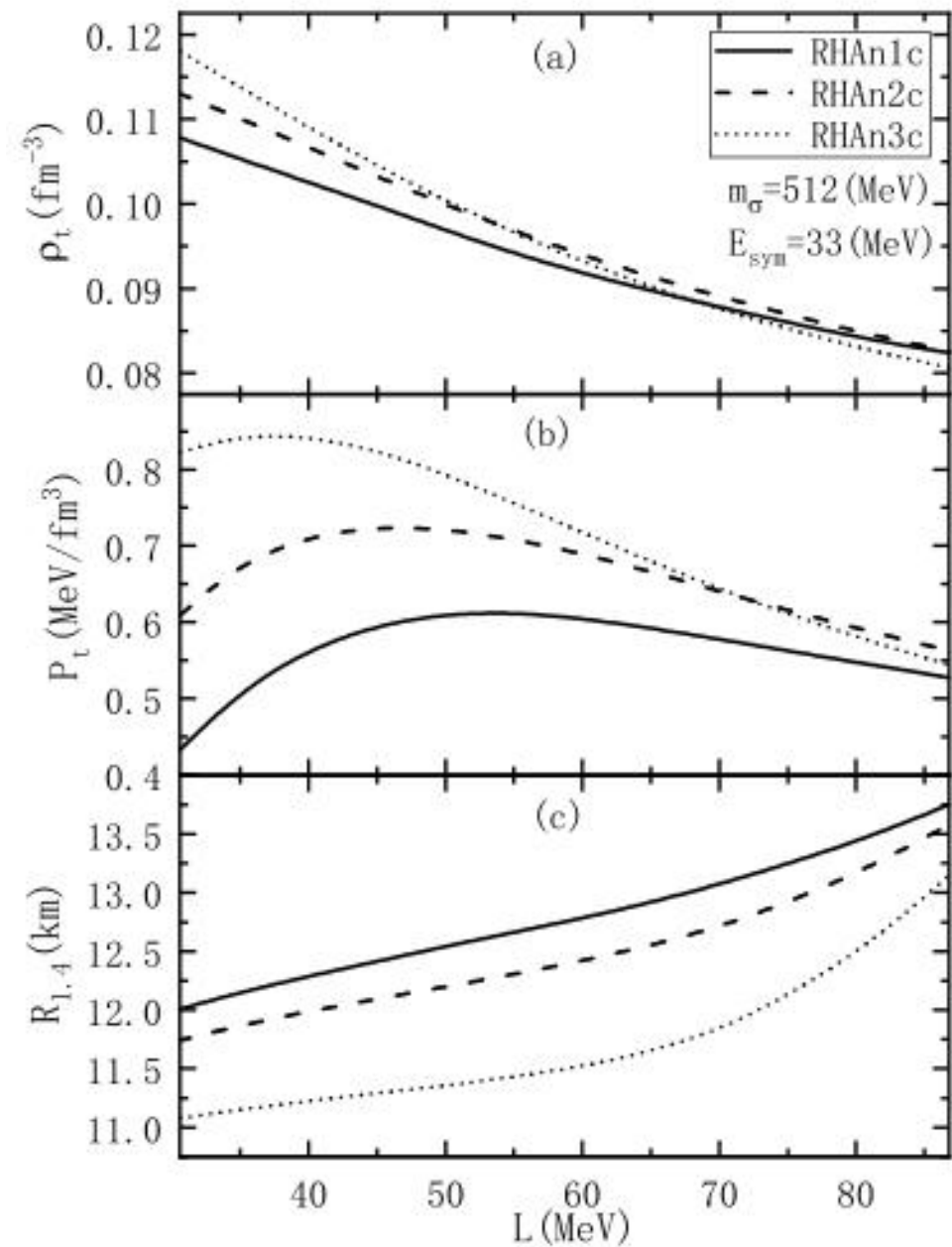
$$\frac{\Delta I}{I} \simeq \frac{28\pi P_t R^3}{3M_{NS} \cdot c^2} \frac{(1 - 1.67\beta - 0.6\beta^2)}{\beta} \quad (7)$$
$$\times \left[ 1 + \frac{2P_t(1 + 5\beta - 14\beta^2)}{\rho_t M c^2 \beta^2} \right]^{-1},$$

where  $\Delta I$  is the crustal moment of inertia,  $I$  is the total moment of inertia,  $\beta = GM_{NS}/Rc^2$  is the compactness parameter, and  $M_{NS}$  and  $R$  are the neutron star mass and radius, respectively.

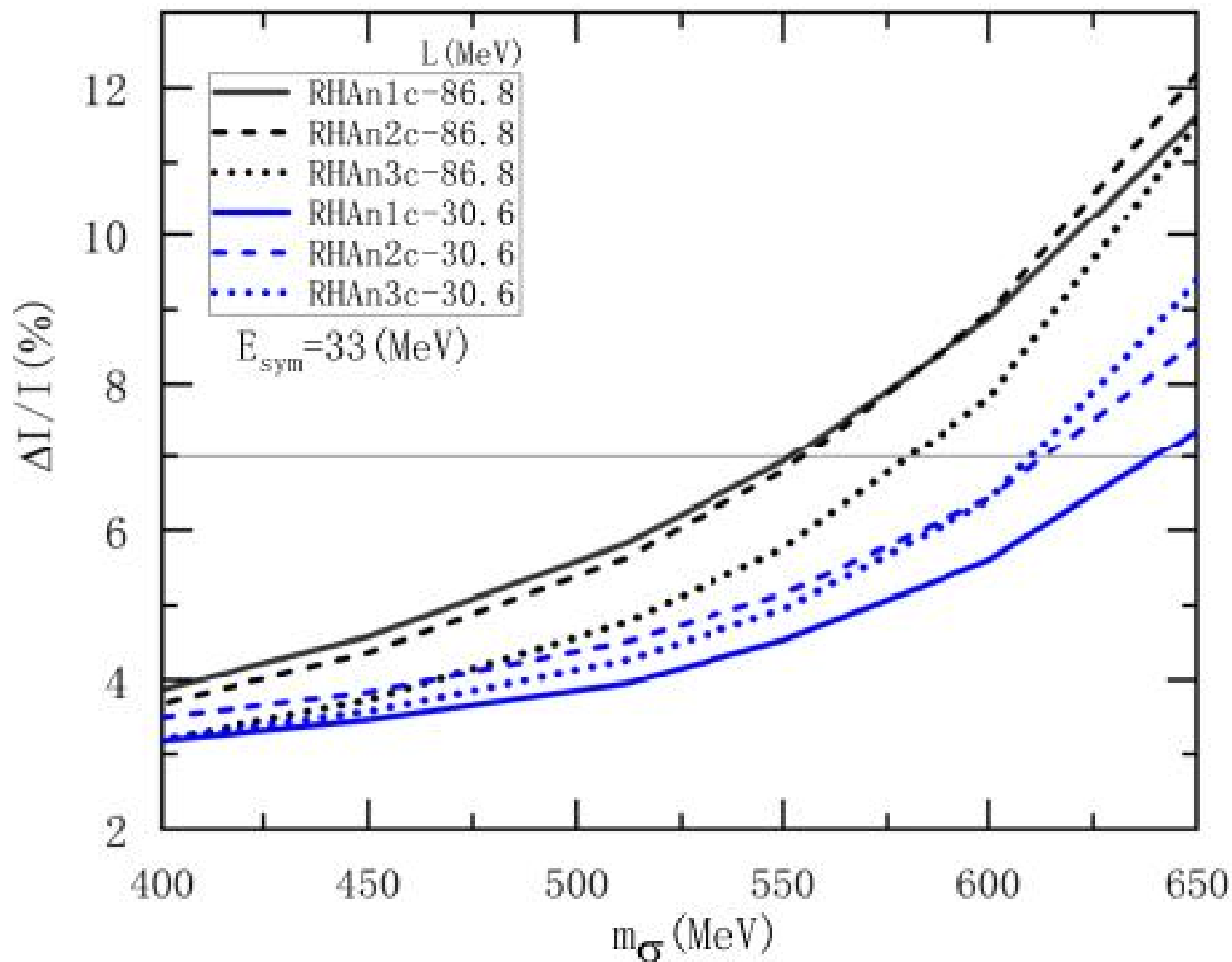
Reproducing the pulsar glitch condition:  $\Delta I/I \geq 7\%$

\* J. M. Lattimer and M. Prakash. Phys. Rep., 333: 121 (2000)

# Results and discussion



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# Summary

- A strong correlation between  $m_\sigma$  and  $\rho_t$  overwhelming the uncertainty of the nuclear EOS in the RHA models;
- Our research indicates that merely adjusting the symmetry energy and/or slope within reasonable regions is difficult to achieve  $\Delta I/I \geq 7\%$ ; Increasing  $m_\sigma$  makes it easier to achieve this goal.
- The nonlinear RHA models generally consistent with astrophysical observations and can be further improved.

**Thank you !**