中国科学技术大学
University of Science and Technology of China

USTC - Particle and Nuclear Physics
NPMiniWorkshop

The spin alignment of rho mesons in a pion gas

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Outline

1. Motivation

2. Spin Boltzmann equation

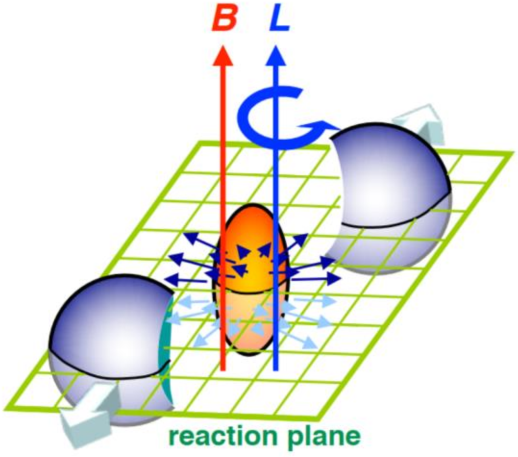
- (1) CTP Green’s function and KB equation
- (2) Effective Lagrangian
- (3) Collision terms

3. Numerical results

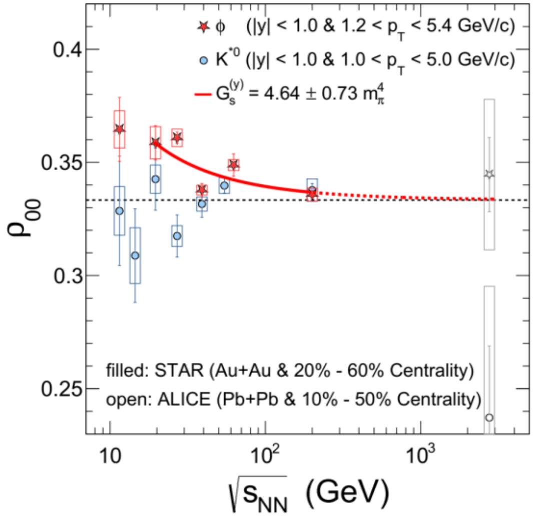
- (1) Initial condition without spin alignment
- (2) Initial condition with spin alignment
- (3) Conclusions

4. Summary and Outlook

Motivation



ϕ and K^{*0}
spin alignment



STAR, Nature volume 614, pages 244–248 (2023)

$$\rho_{00}^{\phi} \approx \frac{1}{3} + \boxed{c_\Lambda} + \boxed{c_\varepsilon} + \boxed{c_E} + \boxed{c_\phi}$$

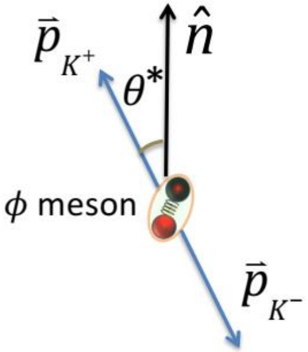
Quark coalescence vorticity & magnetic field

Vorticity tensor

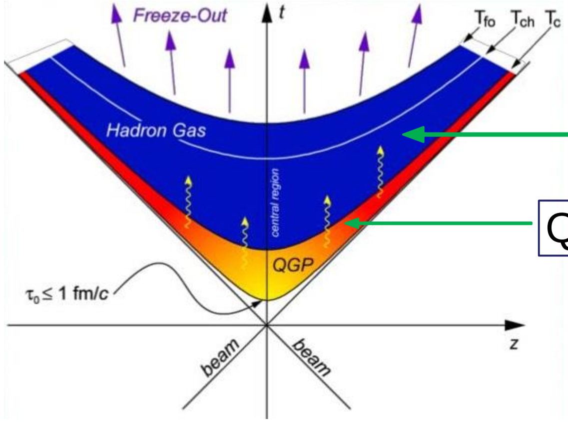
Electric field

Vector meson strong force field

Sheng el. al., Phys. Rev. D 101, 096005 (2020)



Motivation



Hadron interaction Kinetic theory and thermalization effect

Quark-gluon interaction $\delta\rho_{00} \approx c_\Lambda + c_\varepsilon + c_E + c_\phi + \ldots$

The width of ρ^0 meson is much larger than ϕ meson, so the coupling between ρ^0 meson and hadron gas must be considered.

Difference between ρ^0 meson and ϕ meson

	$\boldsymbol{\rho^0}$	$\boldsymbol{\phi}$
Mass	$m \approx 770$MeV	$m \approx 1020$MeV
Width	$\Gamma \approx$147.4MeV	$\Gamma \approx$ 4.249MeV
Main decay channel	$\rho^0 \to \pi^+\pi^-$	$\phi \to K^+K^-$ $\phi \to K_L^0 K_S^0$ …
Quark constitution	$u\bar{u}, d\bar{d}$	$s\bar{s}$

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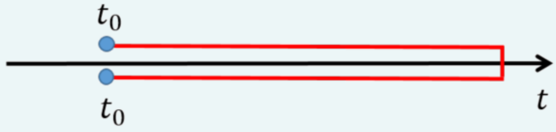
Spin Boltzmann equation

1. CTP Green's function and KB equation:

X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, and X.-N. Wang, 2206.05868

Two-point Green's functions on the Closed-time-path (CTP) for the vector mesons:

$$G^{\mu\nu}_{CTP}(x_1,x_2) = \langle T_C A^\mu(x_1) A^\nu(x_2)\rangle$$



$$\begin{aligned}
G^{<}_{\mu\nu}(x,p) &= 2\pi\hbar\sum_{\lambda_1,\lambda_2}\delta\left(p^2-m_\rho^2\right)\left\{\theta(p^0)\epsilon_\mu\left(\lambda_1,\mathbf{p}\right)\epsilon^*_\nu\left(\lambda_2,\mathbf{p}\right)f_{\lambda_1\lambda_2}(x,\mathbf{p})\right.\\
&\quad\left.+\theta(-p^0)\epsilon^*_\mu\left(\lambda_1,-\mathbf{p}\right)\epsilon_\nu\left(\lambda_2,-\mathbf{p}\right)\left[\delta_{\lambda_2\lambda_1}+f_{\lambda_2\lambda_1}(x,-\mathbf{p})\right]\right\},\\
G^{>}_{\mu\nu}(x,p) &= 2\pi\hbar\sum_{\lambda_1,\lambda_2}\delta\left(p^2-m_\rho^2\right)\left\{\theta(p^0)\epsilon_\mu\left(\lambda_1,\mathbf{p}\right)\epsilon^*_\nu\left(\lambda_2,\mathbf{p}\right)\left[\delta_{\lambda_1\lambda_2}+f_{\lambda_1\lambda_2}(x,\mathbf{p})\right]\right.
\end{aligned}$$

Kadanoff-Baym for the vector mesons:

$$\begin{aligned}
&p\cdot\partial_x G^{<,\mu\nu}(x,p)-\frac{1}{4}\left[p^\mu\partial^x_\eta G^{<,\eta\nu}(x,p)+p^\nu\partial^x_\eta G^{<,\mu\eta}(x,p)\right]\\
=&\ \frac{1}{4}\left[\Sigma^{<,\mu}{}_{\alpha}\left(x,p\right)G^{>,\alpha\nu}\left(x,p\right)-\Sigma^{>,\mu}{}_{\alpha}\left(x,p\right)G^{<,\alpha\nu}\left(x,p\right)\right]\\
&+\frac{1}{4}\left[G^{>,\mu}{}_{\alpha}\left(x,p\right)\Sigma^{<,\alpha\nu}\left(x,p\right)-G^{<,\mu}{}_{\alpha}\left(x,p\right)\Sigma^{>,\alpha\nu}\left(x,p\right)\right].
\end{aligned}$$

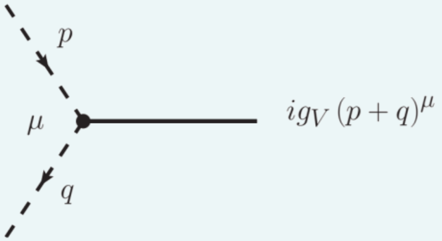
Spin Boltzmann equation

2. Effective Lagrangian:

T. Fujiwara *et al.*, Prog. Theor. Phys. **74**, 128 (1985)

We consider the chiral effective theory with SU(2) flavor symmetry.

$$\mathcal{L} = \mathcal{L}_\rho + \mathcal{L}_\pi + \mathcal{L}_{\text{int}} \left\{ \begin{aligned} \mathcal{L}_\rho &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\rho^2 A_\mu A^\mu \\ \mathcal{L}_{\text{int}} &= i g_{\rho\pi\pi} A^\mu \left(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger\right) \\ \mathcal{L}_\pi &= \partial_\mu \phi^\dagger \partial^\mu \phi - m_\pi^2 \phi^\dagger \phi \end{aligned} \right.$$



Vertex

Spin Boltzmann equations:

X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, and X.-N. Wang, 2206.05868

$$\begin{aligned} p\cdot\partial_x f_{\lambda_1\lambda_2}(x,\mathbf{p}) = \boxed{\begin{aligned} & -\frac{1}{4}\delta_{\lambda_2\lambda_2'}\epsilon_\mu^*\left(\lambda_1,\mathbf{p}\right)\epsilon^\alpha\left(\lambda_1',\mathbf{p}\right) \\ & \times\left\{\left[\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right]\Sigma^{<,\mu}_{\ \ \ \alpha}\left(x,p\right) - f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\Sigma^{>,\mu}_{\ \ \ \alpha}\left(x,p\right)\right\} \\ & -\frac{1}{4}\delta_{\lambda_1\lambda_1'}\epsilon_\nu\left(\lambda_2,\mathbf{p}\right)\epsilon_\alpha^*\left(\lambda_2',\mathbf{p}\right) \\ & \times\left\{\left[\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right]\Sigma^{<,\alpha\nu}\left(x,p\right) - f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\Sigma^{>,\alpha\nu}\left(x,p\right)\right\} \end{aligned}} \end{aligned}$$

$$\boxed{\rho_{00} \equiv \frac{f_{00}}{\text{tr}\,f}}$$

→ Collision terms

(neglecting Possion bracket terms)

Spin Boltzmann equation

3. Collision terms

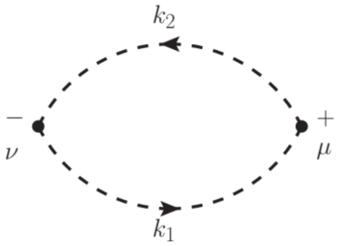
We decompose the collision terms into $C_{\text{coal/diss}}$ and C_{scat} for the coalescence-dissociation and scattering processes respectively,

$$\partial_t f_{\lambda_1\lambda_2}(x,\mathbf{p}) \;=\; C_{\text{coal/diss}} + C_{\text{scat}}$$

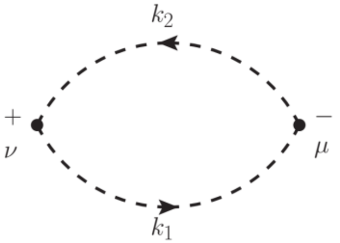
where we have assumed that the system is homogeneous in space.

A. Leading order

$$\begin{aligned} C^{(0)}_{\text{coal/diss}}\left(\rho^0 \leftrightarrow \pi^+\pi^-\right) \;=\; & \frac{g_V^2}{E_p^\rho}\int \frac{d^3k}{(2\pi\hbar)^3 4E_k^\pi E_{p-k}^\pi} 2\pi\hbar\delta\left(E_p^\rho - E_k^\pi - E_{p-k}^\pi\right) \\ & \times\left[\delta_{\lambda_2\lambda_2'} k\cdot\epsilon^*(\lambda_1,\mathbf{p}) k\cdot\epsilon(\lambda_1',\mathbf{p}) + \delta_{\lambda_1\lambda_1'} k\cdot\epsilon(\lambda_2,\mathbf{p}) k\cdot\epsilon^*(\lambda_2',\mathbf{p})\right] \\ & \times\left\{f_{\pi^+}(x,\mathbf{k}) f_{\pi^-}(x,\mathbf{p}-\mathbf{k})\left[\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right]\right. \\ & \left. - \left[1+f_{\pi^+}(x,\mathbf{k})\right]\left[1+f_{\pi^-}(x,\mathbf{p}-\mathbf{k})\right] f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right\}, \end{aligned}$$



(a)



(b)

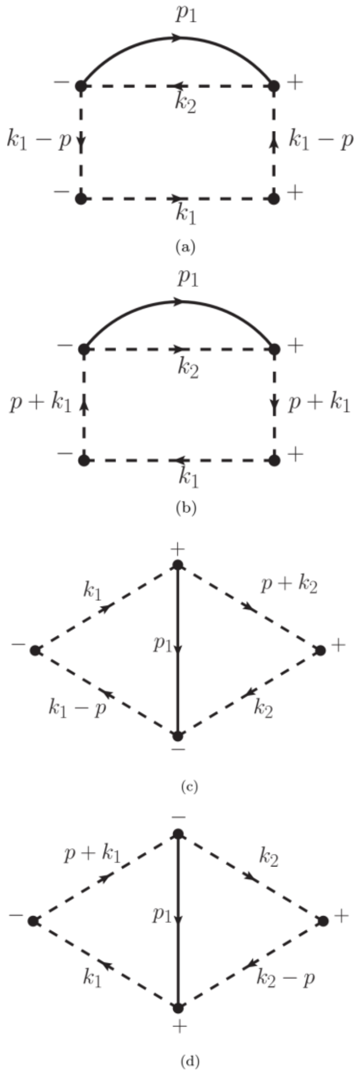
Spin Boltzmann equation

3. Collision terms

B. Next-to-leading order

$$
\begin{aligned}
C^{(1)}_{\mathrm{coal/diss}}\left(\rho^0\rho^0 \leftrightarrow \pi^+\pi^-\right) \;=\;& \frac{4g_V^4}{E_p^\rho}\int\frac{d^3k_1}{(2\pi\hbar)^3\,2E^\pi_{k_1}}\int\frac{d^3k_2}{(2\pi\hbar)^3\,2E^\pi_{k_2}}\int\frac{d^3p_1}{(2\pi\hbar)^3\,2E^\rho_{p_1}}\\
&\times(2\pi\hbar)^4\,\delta^{(4)}\left(p+p_1-k_1-k_2\right)\\
&\times\left[\delta_{\lambda_2\lambda_2'}D_{(2)}(s_1,\lambda_1')D^*_{(2)}(s_2,\lambda_1)+\delta_{\lambda_1\lambda_1'}D_{(2)}(s_1,\lambda_2)D^*_{(2)}(s_2,\lambda_2')\right]\\
&\times\left[f_{\pi^+}(x,\mathbf{k}_1)f_{\pi^-}(x,\mathbf{k}_2)\left(\delta_{s_1s_2}+f_{s_1s_2}(x,\mathbf{p}_1)\right)\left(\delta_{\lambda_1'\lambda_2'}+f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right)\right.\\
&\left.-\left(1+f_{\pi^+}(x,\mathbf{k}_1)\right)\left(1+f_{\pi^-}(x,\mathbf{k}_2)\right)f_{s_1s_2}(x,\mathbf{p}_1)f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right],\\
C_{\mathrm{scat}}\left(\rho^0\pi^\pm \leftrightarrow \rho^0\pi^\pm\right) \;=\;& \frac{4g_V^4}{E_p^\rho}\int\frac{d^3k_1}{(2\pi\hbar)^3\,2E^\pi_{k_1}}\int\frac{d^3k_2}{(2\pi\hbar)^3\,2E^\pi_{k_2}}\int\frac{d^3p_1}{(2\pi\hbar)^3\,2E^\rho_{p_1}}\\
&\times(2\pi\hbar)^4\,\delta^{(4)}\left(p+k_2-p_1-k_1\right)\\
&\times\left[\delta_{\lambda_2\lambda_2'}D_{(1)}(s_1,\lambda_1)D^*_{(1)}(s_2,\lambda_1')+\delta_{\lambda_1\lambda_1'}D_{(1)}(s_1,\lambda_2')D^*_{(1)}(s_2,\lambda_2)\right]\\
&\times\left[f_{s_1s_2}(x,\mathbf{p}_1)f_{\pi^\pm}(x,\mathbf{k}_1)\left(1+f_{\pi^\pm}(x,\mathbf{k}_2)\right)\left(\delta_{\lambda_1'\lambda_2'}+f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right)\right.\\
&\left.-\left(\delta_{s_1s_2}+f_{s_1s_2}(x,\mathbf{p}_1)\right)\left(1+f_{\pi^\pm}(x,\mathbf{k}_1)\right)f_{\pi^\pm}(x,\mathbf{k}_2)f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right],
\end{aligned}
$$

$$
\begin{aligned}
D_{(1)}(s,\lambda)=&\frac{\left[k_1\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_2\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{(p+k_2)^2-m_\pi^2}+\frac{\left[k_2\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_1\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{(p-k_1)^2-m_\pi^2},\\
D_{(2)}(s,\lambda)=&\frac{\left[k_1\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_2\cdot\epsilon\left(\lambda,\mathbf{p}\right)\right]}{(p-k_2)^2-m_\pi^2}+\frac{\left[k_2\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_1\cdot\epsilon\left(\lambda,\mathbf{p}\right)\right]}{(p-k_1)^2-m_\pi^2}.
\end{aligned}
$$



Spin Boltzmann equation

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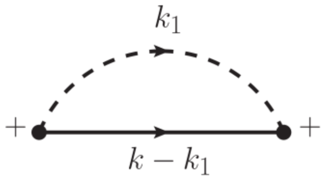
3. Collision terms

C. Regulation of pion propagators

$$D_{(1)}(s,\lambda) = \frac{\left[k_1\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_2\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{\left(p+k_2\right)^2 - m_\pi^2} + \frac{\left[k_2\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_1\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{\boxed{\left(p-k_1\right)^2 - m_\pi^2}},$$

$$D_{(2)}(s,\lambda) = \frac{\left[k_1\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_2\cdot\epsilon\left(\lambda,\mathbf{p}\right)\right]}{\left(p-k_2\right)^2 - m_\pi^2} + \frac{\left[k_2\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_1\cdot\epsilon\left(\lambda,\mathbf{p}\right)\right]}{\left(p-k_1\right)^2 - m_\pi^2}.$$

→ Divergent



Introduce self-energy corrections with medium effects:

$$S^F(k) = \frac{i}{k^2 - m_\pi^2} \qquad\qquad S^F(k) = \frac{i}{k^2 - m_\pi^2 - \Sigma^F(k)}$$

$$S^{\overline{F}}(k) = \frac{-i}{k^2 - m_\pi^2} \quad \Longrightarrow \quad S^{\overline{F}}(k) = \frac{-i}{k^2 - m_\pi^2 + \Sigma^{\overline{F}}(k)}$$

$$\begin{aligned}\Gamma(k) \equiv \mathrm{Im}\Sigma^F(k) = & \, 2g_V^2\theta(k^0)\int\frac{d^3k_1}{(2\pi\hbar)^3 2E^\pi_{k_1}}\int\frac{d^3p}{(2\pi\hbar)^3 2E^\rho_p} \\ & \times(2\pi\hbar)^4\delta^{(4)}\left(k+k_1-p\right)f_{\pi^-}(\mathbf{k}_1)\left[m_\pi^2 - \frac{\left(k_1\cdot p\right)^2}{m_\rho^2}\right] \\ & +2g_V^2\theta(-k^0)\int\frac{d^3k_1}{(2\pi\hbar)^3 2E^\pi_{k_1}}\int\frac{d^3p}{(2\pi\hbar)^3 2E^\rho_p} \\ & \times(2\pi\hbar)^4\delta^{(4)}\left(k-k_1+p\right)f_{\pi^+}(\mathbf{k}_1)\left[m_\pi^2 - \frac{\left(k_1\cdot p\right)^2}{m_\rho^2}\right]\end{aligned}$$

we only consider the imaginary part of the self-energy since the mass correction from the real part is much smaller.

The final results:

$$D_{\pi^+(1)}(s,\lambda) = \frac{\left[k_1\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_2\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{\left(p+k_2\right)^2 - m_\pi^2 + i\Gamma(p+k_2)} + \frac{\left[k_2\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_1\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{\left(p-k_1\right)^2 - m_\pi^2 + i\Gamma(-p+k_1)},$$

$$D_{\pi^-(1)}(s,\lambda) = \frac{\left[k_1\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_2\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{\left(p+k_2\right)^2 - m_\pi^2 + i\Gamma(-p-k_2)} + \frac{\left[k_2\cdot\epsilon\left(s,\mathbf{p}_1\right)\right]\left[k_1\cdot\epsilon^*\left(\lambda,\mathbf{p}\right)\right]}{\left(p-k_1\right)^2 - m_\pi^2 + i\Gamma(p-k_1)}.$$

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Numerical results

Assumptions and methods

1. Pions reach equilibrium much faster than ρ^0 mesons, so we assume that pions always obey the Bose-Einstein distribution.

2. The initial condition of ρ^0 meson distribution should be determined by other simulations, such as coalescence model or relativistic hydrodynamic model. Since we do not know the exact initial condition, we try some conditions and see the evolution.

3. We use Monte Carlo method to calculate the integral in the collision terms.

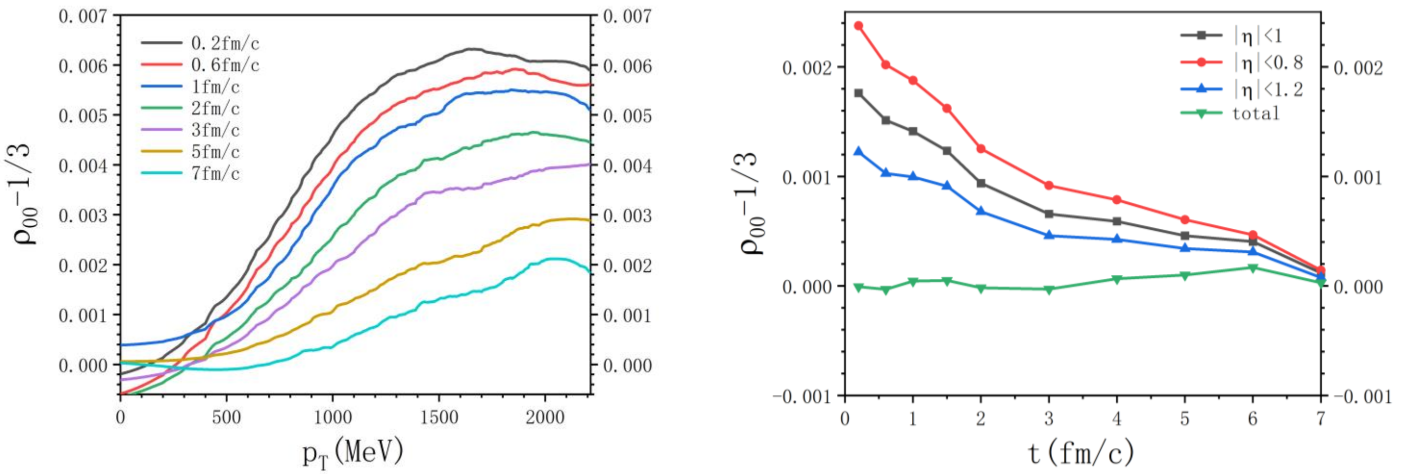
Numerical results

1. Initial condition without spin alignment

We assume that $\pi^{\pm}$ are in global thermal equilibrium, so they obey the Bose-Einstein distribution

$$f_{\pi^\pm}(x,\mathbf{p}) = f_{\pi^\pm}(\mathbf{p}) = \frac{1}{\exp\left[\beta\left(E_p \mp \mu_\pi\right)\right]-1},$$

and we choose $\mu_\pi = 0$ and $T = 156.5$MeV.

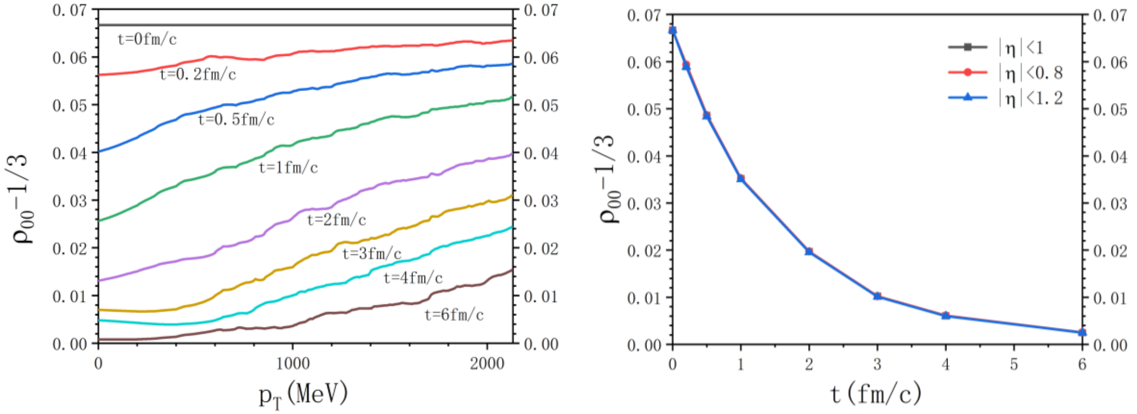


The ρ^0 spin alignment with initial condition $f_{\lambda_1\lambda_2}(t=0)=0$.

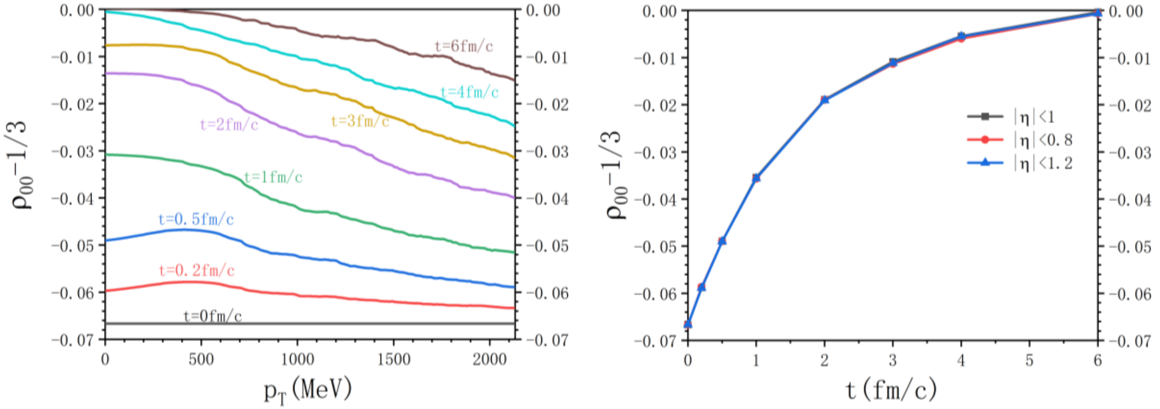
Momentum range: -2.5 ~ 2.5 GeV
Lattice: 100×100×100 MeV^3
Time step: 10^{-3} fm/c

Numerical results

2. Initial condition with spin alignment



The ρ^0 spin alignment with initial condition $f_{\lambda_1\lambda_2} = \mathrm{diag}(0.9, 1.2, 0.9) \times f_{\mathrm{BE}}$.



The ρ^0 spin alignment with initial condition $f_{\lambda_1\lambda_2} = \mathrm{diag}(1.1, 0.8, 1.1) \times f_{\mathrm{BE}}$.

Time step:
0.01 fm/c

Numerical results

3. Conclusions

(1) ρ_{00} is slightly larger than 1/3 in the central rapidity region of ρ^0 mesons for the initial condition without ρ^0 mesons. It is because that we choose +y to be the spin quantization direction, which is different from x and z.

(2) The spin alignment decreases rapidly because of the strong interaction between ρ^0 and $\pi^{\pm}$, especially for low p_T region. The initial value of the spin alignment can be washed in about 6 fm/c.

Summary and Outlook

1. We derived the spin Boltzmann equations for ρ^0 meson with the LO and NLO collision terms. We assumed that the system is homogeneous, and considered the regulation of pion propagators with medium effects.

2. We numerically simulate the evolution of spin alignment of ρ^0 meson with different initial conditions. It is found that all the alignment of ρ^0 meson will decreases rapidly.

3. For future works, the simulation can be improved more precise by loosening some restrictions. For example, we can consider the spacial derivative and the distribution of $\pi^\pm$ during the process.

4. The spectral of ρ^0 meson may be considered.

Thanks

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