Azimuthal asymmetries in pion pair production in UPCs and e^+e^- collisions

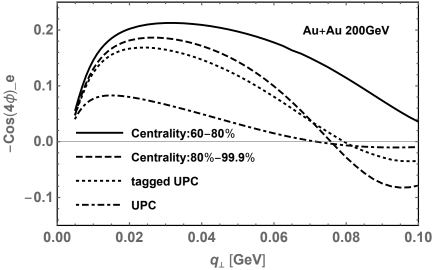
周雅瑾 in collaboration with 贾宇、周剑, arxiv:2404.xxxxx

for the related contents, collaborate also with 张成, 邢宏喜, Yoshikazu Hagiwara

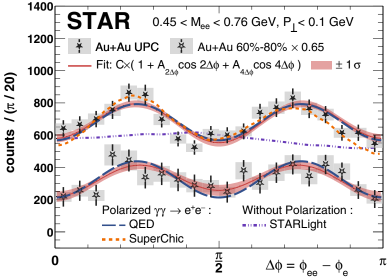
山东大学(青岛)
SHANDONG UNIVERSITY, QINGDAO

Linearly polarized photon in UPCs verified by STAR collaboration

Azimuthal asymmetries in $\gamma\gamma \to e^+e^-$



C. Li, J. Zhou and YZ, 2020



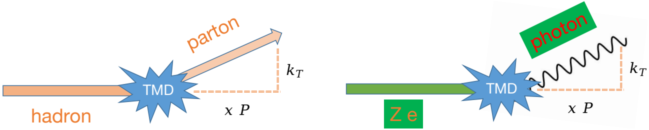
STAR collaboration, PRL127, 052302 (2021)

	Measured $\lvert A_4 \rvert$	QED calculation A_4
Tagged UPC	16.8% ± 2.5%	−16.5%
60%-80%	27% ± 6%	−34.5%

VERIFIED!

Linearly polarized photon

relativistically moving charged particles will introduce electromagnetic field
the photons are linearly polarized due to their transverse momentum



gluon/photon TMD factorization:

$$\int \frac{2dy^- d^2y_\perp}{xP^+(2\pi)^3} e^{ik\cdot y} \langle P|F_+^\mu(0)F_+^\nu(y)|P\rangle\Big|_{y^+=0} = \delta_\perp^{\mu\nu} f_1(x,k_\perp^2) + \left(\frac{2k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \delta_\perp^{\mu\nu}\right) h_1^\perp(x,k_\perp^2),$$

Mulders, Rodrigues, PRD63(2001)

$\boxed{f_1(x,k_\perp^2) = h_1^\perp(x,k_\perp^2)}$, small-x gluons/photons are highly linearly polarized

A. Metz and J. Zhou, 2011, C. Li, J. Zhou and YZ, 2019

Photon distribution

- induced by a large nucleus

$$xf_1^\gamma(x, k_\perp^2) = \frac{Z^2\alpha_e}{\pi^2} k_\perp^2 \left[\frac{F(k_\perp^2 + x^2M_p^2)}{(k_\perp^2 + x^2M_p^2)}\right]^2, \quad F(\vec{k}^2) = \int d^3r e^{i\vec{k}\cdot\vec{r}} \frac{\rho^0}{1 + \exp\left[(r - R_{WS})/d\right]}$$

with $F(\vec{k}^2)$ being the Woods-Saxon form factor.

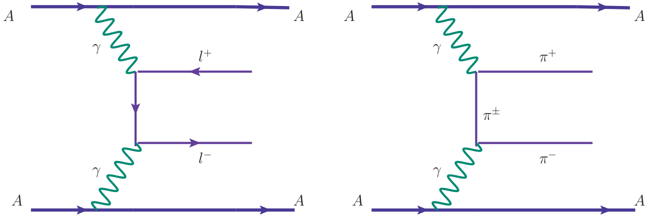
$h_1^\perp = f_1$ at small x, e.g., at RHIC Au-Au 200 GeV, LHC Pb-Pb 5020 GeV

- induced by a point-like particle, e.g., electron

$$xf(x, k_\perp^2) = \frac{\alpha_e}{2\pi^2}(1 + (1 - x)^2)\frac{k_\perp^2}{(k_\perp^2 + x^2m_e^2)^2}, \quad xh_1^\perp(x, k_\perp^2) = \frac{\alpha_e}{\pi^2}(1 - x)\frac{k_\perp^2}{(k_\perp^2 + x^2m_e^2)^2}$$

$h_1^\perp \neq f_1$, not small x, e.g., 10.579 GeV@Belle, 3.770 GeV@BES

$\pi^+\pi^-$ production in UPCs

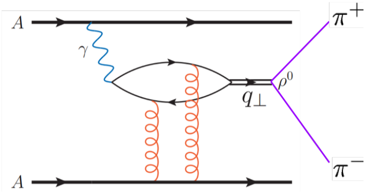


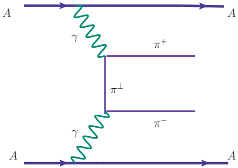
similar to l^+l^- production process but more complicated. Motivations:

- the first attempt to study the azimuthal asymmetry in heavy composite particles
- studying the loop effect in chiral perturbation theory
- may shed light on Light-By-Light (LBL) scattering process mediated by the pion loop

Research on azimuthal asymmetries of $\pi\pi$ production in UPCs

① $\rho^0 \to \pi\pi$, ρ^0 produced by photo-nucleus interaction;
② direction production, $\gamma\gamma \to \pi\pi$.





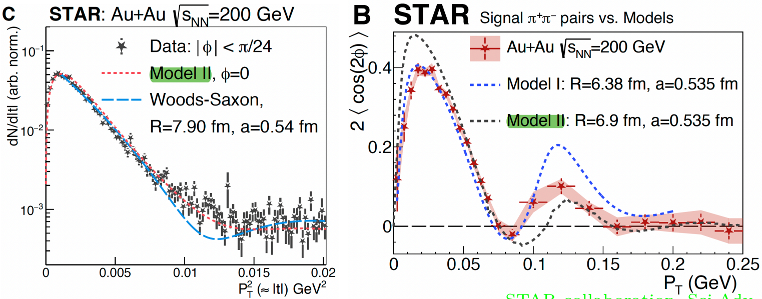
Examples:

- $\cos(2\phi)$ in $\rho^0 \to \pi\pi$ photoproduction [H.X. Xing, C. Zhang, J. Zhou and YZ, JHEP10(2020)064]
- $\cos(4\phi)$ in $\rho^0 \to \pi\pi$ photoproduction [Y. Hagiwara, C. Zhang, J. Zhou and YZ, PRD104, 094021 (2021)]
- $\cos(\phi)$ and $\cos(3\phi)$, $\rho^0 \to \pi\pi$ and $\gamma\gamma \to \pi\pi$ interference [Y. Hagiwara, C. Zhang, J. Zhou and YZ, PRD103, 074013 (2021)]

- $\cos(2\phi)$ azimuthal asymmetry in $\rho^0 \to \pi\pi$ photoproduction,

$$\frac{d\sigma_{\rho\to\pi\pi}}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2\tilde{b}_\perp} = \frac{1}{2(2\pi)^7}\frac{P_\perp^2}{(Q^2-M_\rho^2)^2+M_\rho^2\Gamma_\rho^2} f_{\rho\pi\pi}^2 \int d^2\Delta_\perp d^2k_\perp d^2k'_\perp \delta^2(k_\perp+\Delta_\perp-q_\perp)(\hat{P}_\perp\cdot\hat{k}_\perp)(\hat{P}_\perp\cdot\hat{k}'_\perp)$$
$$\times\Big\{\int d^2b_\perp e^{i\tilde{b}_\perp\cdot(k'_\perp-k_\perp)}\left[T_A(b_\perp)\mathcal{A}_{in}(x_2,\Delta_\perp)\mathcal{A}^*_{in}(x_2,\Delta'_\perp)\mathcal{F}(x_1,k_\perp)\mathcal{F}(x_1,k'_\perp)+(A\leftrightarrow B)\right]$$
$$+\left[e^{i\tilde{b}_\perp\cdot(k'_\perp-k_\perp)}\mathcal{A}_{co}(x_2,\Delta_\perp)\mathcal{A}^*_{co}(x_2,\Delta'_\perp)\mathcal{F}(x_1,k_\perp)\mathcal{F}(x_1,k'_\perp)\right]\left[e^{i\tilde{b}_\perp\cdot(\Delta'_\perp-k_\perp)}\mathcal{A}_{co}(x_2,\Delta_\perp)\mathcal{A}^*_{co}(x_1,\Delta'_\perp)\mathcal{F}(x_1,k_\perp)\mathcal{F}(x_2,k'_\perp)\right]+\ldots\Big\} \quad (1)$$

consistent with STAR:

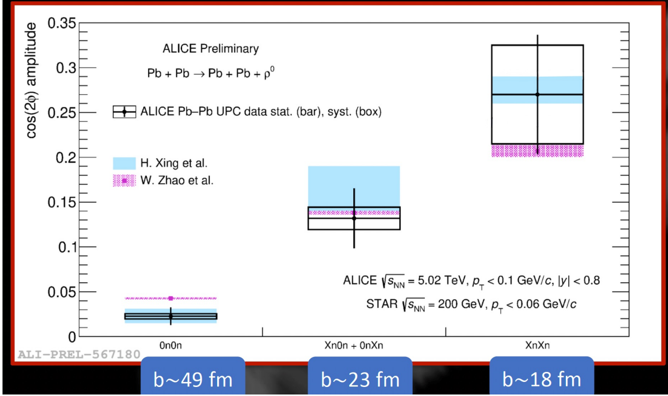


STAR collaboration, Sci.Adv. 9, eabq3903 (2023)

Model II: H.X. Xing, C. Zhang, J. Zhou and YZ, JHEP10(2020)064

Fermi-scale interference effect, linearly polarized photon

The prediction for the impact parameter dependence is verified by ALICE



First measurement of the azimuthal anisotropy of the ρ^0 yield as a function of the impact parameter

The **modulation strength strongly increases as *b* decreases**

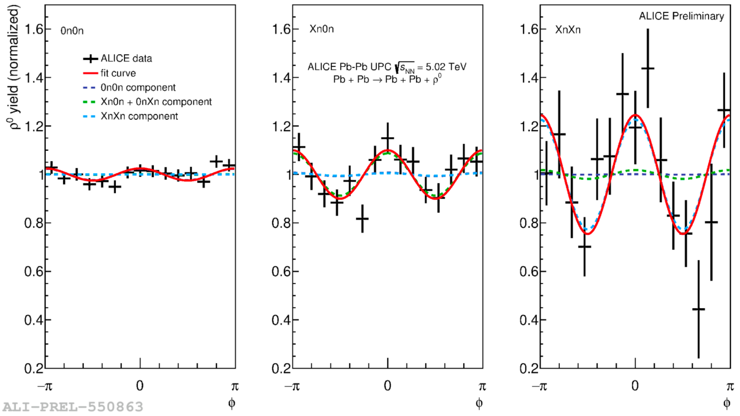
Compatible with **theory** [6,7], XnXn amplitude compatible with **STAR** results [8] for Au-Au and U-U collisions at lower energy

It is not possible to constrain models yet → goal with Run 3 data!

ALICE Collaboration, talk by A. G. Riffero on UPC 2023 (Mexico)

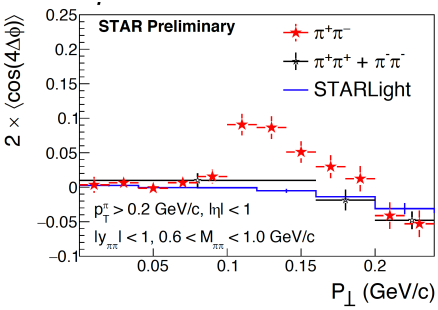
- (blue box) Hongxi Xing, Cheng Zhang, Jian Zhou, Ya-Jin Zhou *JHEP* 10 (2020) 064
- (magenta box) Heikki Mäntysaari, Farid Salazar, Björn Schenke, Chun Shen, Wenbin Zhao *Phys.Rev.C* 109 (2024) 2, 024908

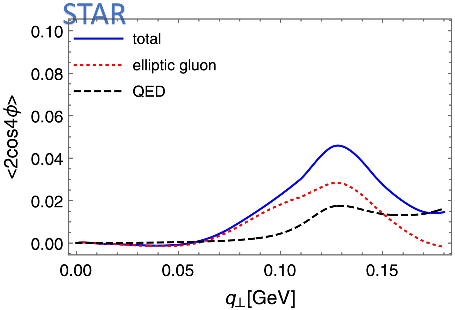
ALICE data



ALICE Collaboration, talk by A. G. Riffero on UPC 2023 (Mexico)

- $\cos(4\phi)$ azimuthal asymmetry in $\rho^0 \to \pi\pi$ photoproduction





Daniel Brandenburg, QM 2019

elliptic gluon Wigner distribution is important to describe the STAR data

- $\cos(\phi)$ and $\cos(3\phi)$ azimuthal asymmetry, interference between $\rho^0 \to \pi\pi$ photoproduction and $\gamma\gamma \to \pi\pi$
  - $\rho^0 \to \pi\pi$, QCD

    Breit-Wigner distribution, $\mathcal{M}_{\rho\to\pi^+\pi^-} = \mathcal{M}_\rho \; f_{\rho\pi\pi} \frac{P_\perp \cdot \epsilon_\perp^V}{Q^2 - M_\rho^2 + iM_\rho \Gamma_\rho}$

$$\frac{d\sigma_{\rho\to\pi\pi}}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2\tilde{b}_\perp} = \frac{1}{2(2\pi)^7}\frac{P_\perp^2}{(Q^2-M_\rho^2)^2+M_\rho^2\Gamma_\rho^2} f_{\rho\pi\pi}^2 \int d^2\Delta_\perp d^2k_\perp d^2k'_\perp \delta^2(k_\perp+\Delta_\perp-q_\perp)(\hat{P}_\perp\cdot\hat{k}_\perp)(\hat{P}_\perp\cdot\hat{k}'_\perp)\cdots$$

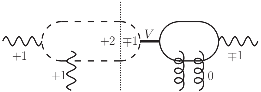
  - $\gamma\gamma \to \pi\pi$, QED. small $M_{\pi\pi}$ region, π point-like approximation

$$\frac{d\sigma_{\gamma\gamma\to\pi\pi}}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2\tilde{b}_\perp} = \frac{\alpha_e^2}{Q^4}\frac{1}{(2\pi)^2}\int d^2k_\perp d^2\Delta_\perp d^2k'_\perp \delta^2(q_\perp-k_\perp-\Delta_\perp)e^{i(k_\perp-k'_\perp)\cdot\tilde{b}_\perp}$$

$$\times 4\left[\hat{k}_\perp\cdot\hat{\Delta}_\perp - \frac{2P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_\perp\cdot\hat{P}_\perp)(\hat{\Delta}_\perp\cdot\hat{P}_\perp)\right]\left[\hat{k}'_\perp\cdot\hat{\Delta}'_\perp - \frac{2P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}'_\perp\cdot\hat{P}_\perp)(\hat{\Delta}'_\perp\cdot\hat{P}_\perp)\right]\mathcal{F}(x_1,k_\perp^2)\mathcal{F}^*(x_1,k'^2_\perp)\mathcal{F}(x_2,\Delta_\perp^2)\mathcal{F}^*(x_2,\Delta'^2_\perp)$$

  - interference term

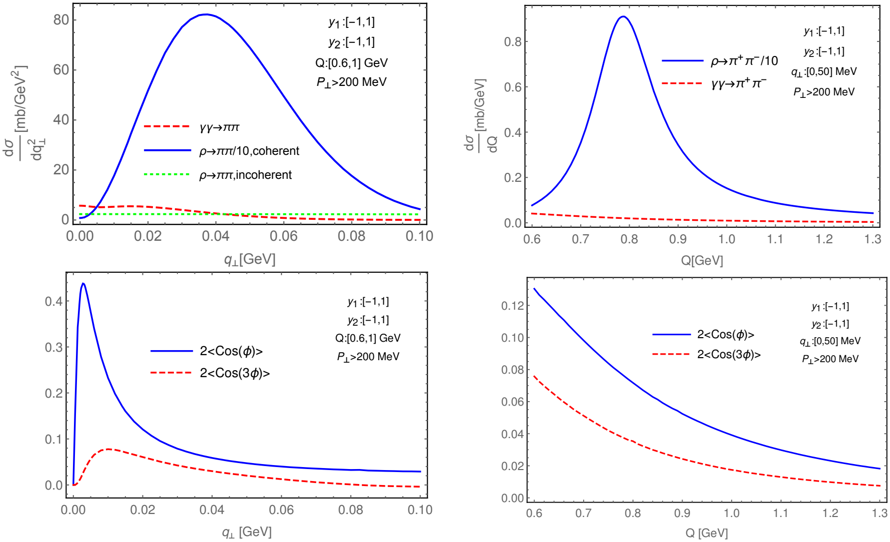
$$\frac{d\sigma_I}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2\tilde{b}_\perp} = \frac{\alpha_e}{Q^2}\frac{1}{(2\pi)^4}\frac{1}{\sqrt{4\pi}}\frac{2M_\rho\Gamma_\rho|P_\perp|f_{\rho\pi\pi}}{(Q^2-M_\rho^2)^2+M_\rho^2\Gamma_\rho^2}\int d^2\Delta_\perp d^2k_\perp d^2k'_\perp$$



$$\times\delta^2(k_\perp+\Delta_\perp-q_\perp)\left[\hat{k}_\perp\cdot\hat{\Delta}_\perp - \frac{2P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_\perp\cdot\hat{P}_\perp)(\hat{\Delta}_\perp\cdot\hat{P}_\perp)\right](\hat{P}_\perp\cdot\hat{k}'_\perp)$$

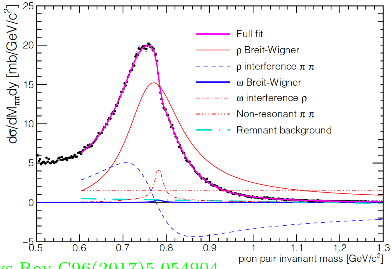
$$\times 2\left\{\left[e^{i\tilde{b}_\perp\cdot(k'_\perp-k_\perp)}\mathcal{F}(x_1,k_\perp)\mathcal{F}(x_2,\Delta_\perp)\mathcal{F}(x_1,k'_\perp)\mathcal{A}^*_{co}(x_2,\Delta'_\perp)\right]\left[e^{i\tilde{b}_\perp\cdot(\Delta'_\perp-k_\perp)}\mathcal{F}(x_2,k_\perp)\mathcal{F}(x_1,\Delta_\perp)\mathcal{F}(x_2,k'_\perp)\mathcal{A}^*_{co}(x_1,\Delta'_\perp)\right]\right\}$$

$\cos(\phi)$ and $\cos(3\phi)$ azimuthal asymmetry, numerical results



How about direct production at lower invariant mass and how about e^+e^- collider

The discussions above were based on Au200@RHIC and Pb5020@LHC with $M_{\pi\pi}$ about 0.6 to 1 GeV, and mostly focused on ρ^0 decay.



STAR, Phys.Rev.C96(2017)5,054904

to study polarization effect of poin pair production from $\gamma\gamma$ fusion,

- in UPCs, lower invariant mass is better
- e^+e^- collider is even better

Theoretical calculation:

- ChPT,
- dispersion relation technique

Fruit of UPC2023@Fudan

Chiral perturbative theory (ChPT or χPT)

S. Weinberg, Physica A 96 (1979) ,327

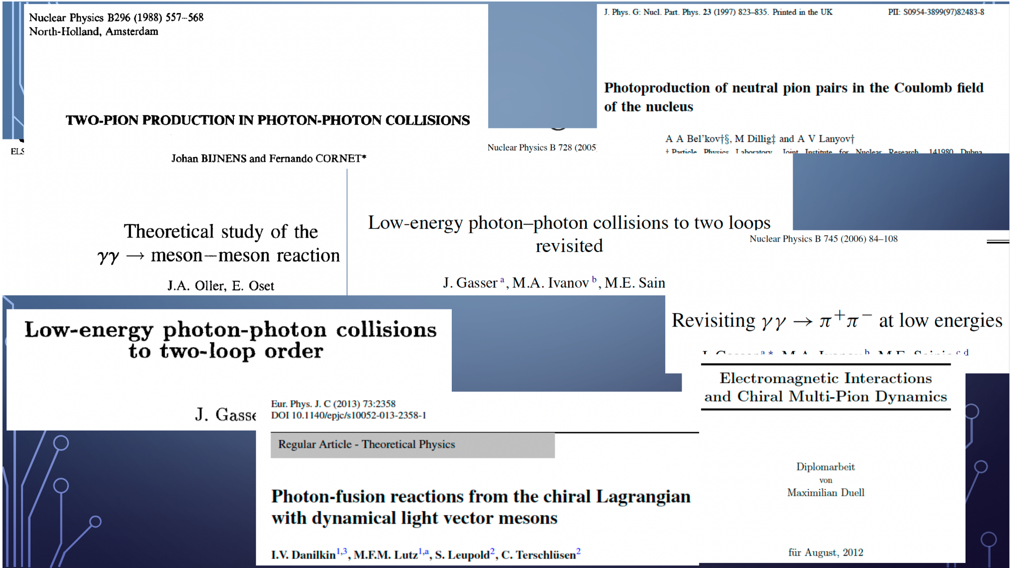
Founder of effective theory

J. Gasser, H. Leutwyler, *Nucl.Phys.B* 250 (1985) 465

- based on the global $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R} \times \mathrm{U(1)_V}$ symmetry of the QCD Lagrangian in the limit of massless u, d, and s quarks, and spontaneously broken down to $\mathrm{SU(3)_V} \times \mathrm{U(1)_V}$ giving rise to eight massless Goldstone bosons
- systematic expansion of the Greens functions in powers of small momenta and small quark masses
- an effective field theory for QCD at low energies

Some ChPT calculations for $\gamma\gamma \to \pi\pi$

e.g.,



Nuclear Physics B296 (1988) 557–568
North-Holland, Amsterdam

TWO-PION PRODUCTION IN PHOTON-PHOTON COLLISIONS

Johan BIJNENS and Fernando CORNET*

J. Phys. G: Nucl. Part. Phys. 23 (1997) 823–835. Printed in the UK PII: S0954-3899(97)82483-8

Photoproduction of neutral pion pairs in the Coulomb field of the nucleus

A A Bel'kov†§, M Dillig‡ and A V Lanyov†

Nuclear Physics B 728 (2005

Theoretical study of the
$\gamma\gamma \to$ meson – meson reaction

J.A. Oller, E. Oset

Low-energy photon–photon collisions to two loops revisited

Nuclear Physics B 745 (2006) 84–108

J. Gasser [a], M.A. Ivanov [b], M.E. Sain

Low-energy photon-photon collisions to two-loop order

J. Gasse

Revisiting $\gamma\gamma \to \pi^+\pi^-$ at low energies

Eur. Phys. J. C (2013) 73:2358
DOI 10.1140/epjc/s10052-013-2358-1

Regular Article - Theoretical Physics

Photon-fusion reactions from the chiral Lagrangian with dynamical light vector mesons

I.V. Danilkin[1,3], M.F.M. Lutz[1,a], S. Leupold[2], C. Terschlüsen[2]

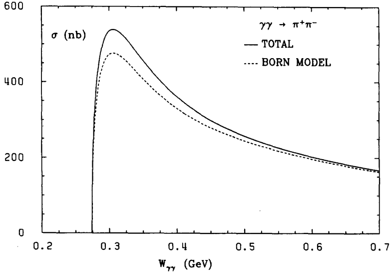
Electromagnetic Interactions and Chiral Multi-Pion Dynamics

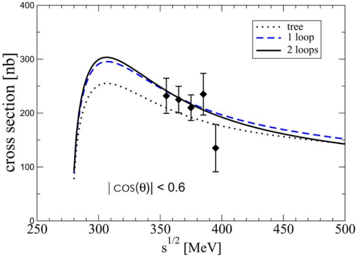
Diplomarbeit
von
Maximilian Duell

für August, 2012

Some ChPT calculations for $\gamma\gamma \to \pi\pi$

e.g.,





J.BIJNENS and F. CORNET, NPB296(1988)557-568

J. Gasser et al., NPB745(2006)84–108

$$A_{\gamma\gamma\to\pi^+\pi^-} = 2ie^2\left\{C\varepsilon_1\cdot\varepsilon_2 - \frac{p_1\cdot\varepsilon_1 p_2\cdot\varepsilon_2}{p_1\cdot k_1} - \frac{p_1\cdot\varepsilon_2 p_2\cdot\varepsilon_1}{p_1\cdot k_2}\right\}$$

$$C = 1 + \frac{4Q^2}{f_\pi^2}(L_9^r + L_{10}^r) - \frac{1}{16\pi^2 f_\pi^2}\left(\frac{3}{2}Q^2 + \frac{1}{2}m_\pi^2\ln^2 g_\pi(Q^2) + m_K^2\ln^2 g_K(Q^2)\right)$$

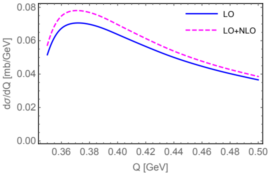
Dipoin production in UPCs, ChPT

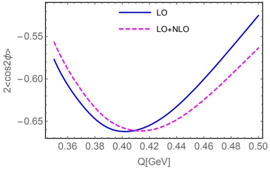
$$\frac{d\sigma_{AA\to\gamma\gamma\to\pi^+\pi^-}}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2d^2\tilde{b}_\perp} = \frac{\alpha_e^2}{Q^4}\frac{1}{(2\pi)^2}\int d^2k_{1\perp}d^2k_{2\perp}d^2k'_{1\perp}\delta^2(q_\perp - k_{1\perp} - k_{2\perp})\, e^{i(k_{1\perp}-k'_{1\perp})\cdot\tilde{b}_\perp}$$

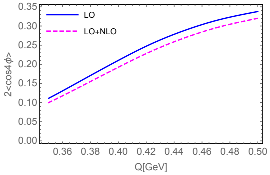
$$\times 4\left|\mathcal{C}\,\hat{k}_{1\perp}\cdot\hat{k}_{2\perp} - \frac{2P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_{1\perp}\cdot\hat{P}_\perp)(\hat{k}_{2\perp}\cdot\hat{P}_\perp)\right|^2 \mathcal{F}(x_1,k_{1\perp}^2)\mathcal{F}^*(x_1,k_{1\perp}'^2)\mathcal{F}(x_2,k_{2\perp}^2)\mathcal{F}^*(x_2,k_{2\perp}'^2)$$

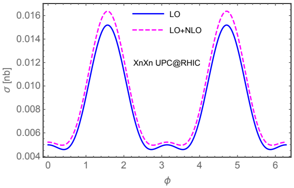
$$\mathcal{F}(x,k_\perp) = \frac{Z\sqrt{\alpha_e}}{\pi}|k_\perp|\frac{F(k_\perp^2+x^2M_p^2)}{(k_\perp^2+x^2M_p^2)}$$

numerical results:









Dipoin production in e^+e^- collider, ChPT

$$\frac{d\sigma_{e^+e^-\to\gamma\gamma\to\pi^+\pi^-}}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2} = \frac{4\alpha_e^2}{Q^4}\int d^2k_{1\perp}d^2k_{2\perp}\delta^2(q_\perp - k_{1\perp} - k_{2\perp})x_1x_2$$

$$\times\Bigg\{\left|\mathcal{C}\,\hat{k}_{1\perp}\cdot\hat{k}_{2\perp} - \frac{2\,P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_{1\perp}\cdot\hat{P}_\perp)(\hat{k}_{2\perp}\cdot\hat{P}_\perp)\right|^2 h_1^\perp(x_1,k_{1\perp}^2)\,h_1^\perp(x_2,k_{2\perp}^2)$$

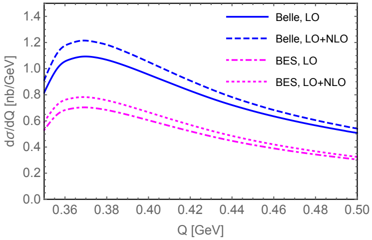
$$+\left[\frac{1}{2}|\mathcal{C}|^2 + \frac{2\,P_\perp^4(\hat{k}_{1\perp}\cdot\hat{P}_\perp)^2}{(P_\perp^2+m_\pi^2)^2} - \frac{(\mathcal{C}+\mathcal{C}^*)\,P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_{1\perp}\cdot\hat{P}_\perp)^2\right]h_1^\perp(x_1,k_{1\perp}^2)\left(f(x_2,k_{2\perp}^2) - h_1^\perp(x_2,k_{2\perp}^2)\right)$$

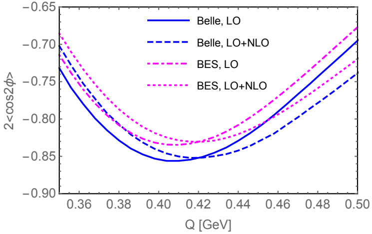
$$+\left[\frac{1}{2}|\mathcal{C}|^2 + \frac{2\,P_\perp^4(\hat{k}_{2\perp}\cdot\hat{P}_\perp)^2}{(P_\perp^2+m_\pi^2)^2} - \frac{(\mathcal{C}+\mathcal{C}^*)\,P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_{2\perp}\cdot\hat{P}_\perp)^2\right]h_1^\perp(x_2,k_{2\perp}^2)\left(f(x_1,k_{1\perp}^2) - h_1^\perp(x_1,k_{1\perp}^2)\right)$$

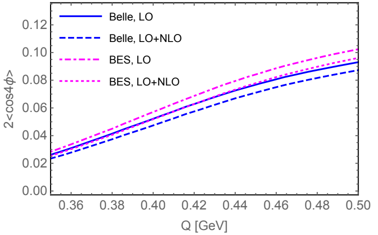
$$+\left[\frac{P_\perp^4}{(P_\perp^2+m_\pi^2)^2} + \frac{1}{2}|\mathcal{C}|^2 - \frac{(\mathcal{C}+\mathcal{C}^*)P_\perp^2}{2(P_\perp^2+m_\pi^2)}\right]\left(f(x_1,k_{1\perp}^2) - h_1^\perp(x_1,k_{1\perp}^2)\right)\left(f(x_2,k_{2\perp}^2) - h_1^\perp(x_2,k_{2\perp}^2)\right)\Bigg\}$$

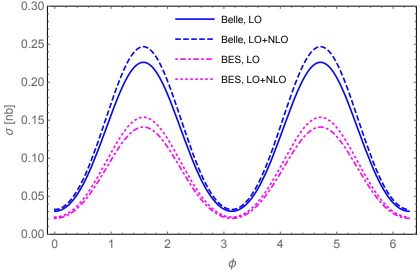
$$xf(x,k_\perp^2) = \frac{\alpha_e}{2\pi^2}(1+(1-x)^2)\frac{k_\perp^2}{(k_\perp^2+x^2m_e^2)^2}, \qquad xh_1^\perp(x,k_\perp^2) = \frac{\alpha_e}{\pi^2}(1-x)\frac{k_\perp^2}{(k_\perp^2+x^2m_e^2)^2}$$

numerical results:









Dipoin production in e$^+$e$^-$ collider, with the aid of dispersion relation

Ling-Yun Dai and M.R. Pennington, "Comprehensive Amplitude Analysis of $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ and $\overline{\text{K}}$K below 1.5 GeV", PRD90, 036004 (2014)

$$\frac{d\sigma}{d\Omega} = \frac{\rho(s)}{128\pi^2 s}\left[|M_{+-}|^2 + |M_{++}|^2\right],$$

partial wave expansions of the amplitudes:

$$M_{++}(s,\theta,\phi) = e^2\sqrt{16\pi}\sum_{J\geq 0} F_{J0}(s)Y_{J0}(\theta,\phi),$$

$$M_{+-}(s,\theta,\phi) = e^2\sqrt{16\pi}\sum_{J\geq 2} F_{J2}(s)Y_{J2}(\theta,\phi).$$

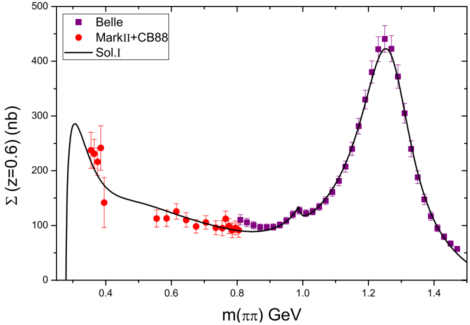
isospin decomposition of the amplitudes:

$$\mathcal{F}_\pi^{+-}(s) = -\sqrt{\frac{2}{3}}\mathcal{F}_\pi^{I=0}(s) - \sqrt{\frac{1}{3}}\mathcal{F}_\pi^{I=2}(s),$$

$$\mathcal{F}_\pi^{00}(s) = -\sqrt{\frac{1}{3}}\mathcal{F}_\pi^{I=0}(s) + \sqrt{\frac{2}{3}}\mathcal{F}_\pi^{I=2}(s),$$

$$\mathcal{F}_K^{+-}(s) = -\sqrt{\frac{1}{2}}\mathcal{F}_K^{I=0}(s) - \sqrt{\frac{1}{2}}\mathcal{F}_K^{I=1}(s),$$

$$\mathcal{F}_K^{00}(s) = -\sqrt{\frac{1}{2}}\mathcal{F}_K^{I=0}(s) + \sqrt{\frac{1}{2}}\mathcal{F}_K^{I=1}(s).$$

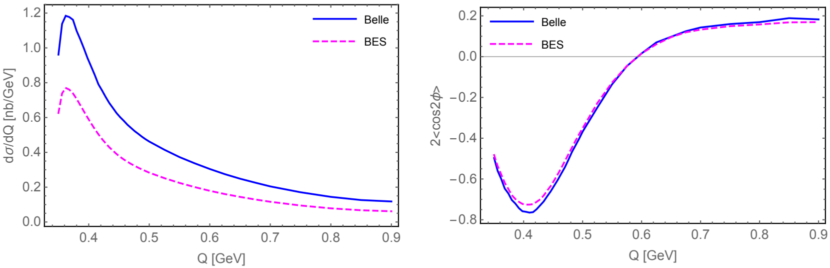


cross section in helicity amplitude form with linealy polarized photon:

$$\frac{d\sigma_{e^+e^-\to\gamma\gamma\to\pi^+\pi^-}}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2} = \frac{1}{16\pi^2Q^4}\int d^2k_{1\perp}d^2k_{2\perp}\delta^2(q_\perp - k_{1\perp} - k_{2\perp})x_1x_2\left\{\frac{1}{2}\left(|M_{+-}|^2 + |M_{++}|^2\right)f(x_1,k_{1\perp}^2)f(x_2,k_{2\perp}^2)\right.$$

$$- \cos 2(\phi_1)\mathrm{Re}[M_{++}M_{+-}^*]f(x_2,k_{2\perp}^2)h_1^\perp(x_1,k_{1\perp}^2) - \cos 2(\phi_2)\mathrm{Re}[M_{++}M_{+-}^*]f(x_1,k_{1\perp}^2)h_1^\perp(x_2,k_{2\perp}^2)$$

$$\left.+\frac{1}{2}\left[\cos 2(\phi_1 - \phi_2)|M_{++}|^2 + \cos 2(\phi_1 + \phi_2)|M_{+-}|^2\right]h_1^\perp(x_1,k_{1\perp}^2)h_1^\perp(x_2,k_{2\perp}^2)\right\}$$



Summary

Linearly polarized photon introduce significant azimuthal asymmetries in dipoin photoproduciton in UPCs and e^+e^- collisions at low energy.