Spin Polarization, Anomalous Magnetic Moment and Transportation in the magnetized QCD background

1. The spin polarization and chiral phase transition with TSP and AMM under a magnetic field
2. The Anomalous Magnetic Moment and Transportation in the magnetized QCD background
3. Summary and Conclusions

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Based on:

(1) Y.-W. Qiu and S.-Q. Feng, Phys. Rev. D 107, 076004 (2023);
(2) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(3) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023).

1. The spin polarization and chiral phase transition with TSP and AMM under a magnetic field

The magnetic field of non-central heavy-ion collisions

By Considering the response of QGP in relativistic heavy-ion collisions

Here we stress the study of the magnetic field of non-central heavy-ion collisions, which comes from the laboratory of mankind.

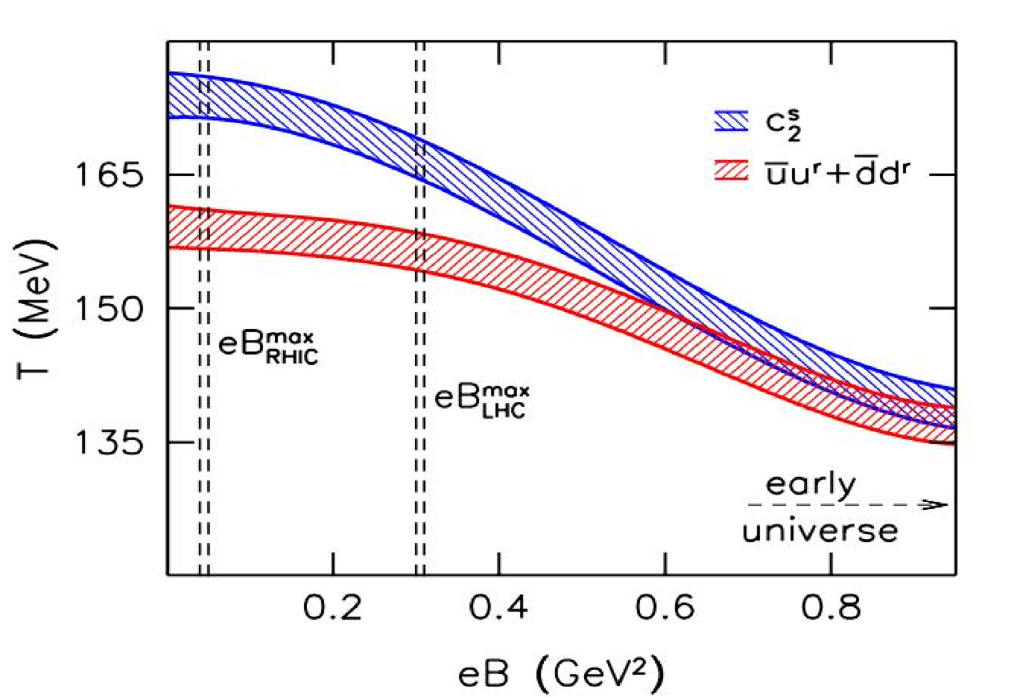
(1) The magnetic field reaches up to $\sqrt{eB} \sim 0.1$ GeV Au - Au collisions for RHIC and $\sqrt{eB} \sim 0.5$ GeV Pb - Pb collisions for LHC in non-central heavy-ion collisions. This magnetic field is external since it is generated by the spectators, and therefore it has a very short lifetime.

(2) The presence of **the quark-gluon plasma (QGP) medium response effect**, substantially **delays** the decay of these time-dependent magnetic fields. This is why in most cases, the effect of constant and uniform magnetic fields is taken.

1. W.-T. Deng and X.-G. Huang, Phys. Rev. C 85, 044907 (2012).
2. V. Skokov, A. Y. Illarionov, and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009).
3. D. She, S. -Q. Feng, European Physical Journal A 54: 48 (2018)
4. Y. Guo, S. Shi, S.-Q. Feng, and J. Liao, Phys. Lett. B 798, 134929 (2019).

Lattice Result in Phase Transition

G. Bali, et.al, PRD86, 071502; JHEP02,044(2012)



Lattice result: the QCD phase diagram in the magnetic field-temperature plane.

Some effective field theorices predict that the external magnetic field causes the magnetic catalytic feature of chiral symmetry breaking. However, lattice QCD theory provides different results, especially the corresponding inverse magnetic catalytic (IMC) of chiral symmetry breaking near the phase transition. This is an interesting question.

By introducing the **tensor spin polarization (TSP)** and **anomalous magnetic moment (AMM),** respectively, in the background magnetic field, we will analyze the magnetic catalytic characteristics of chiral symmetry breaking and phase transition with the magnetic field.

NJL model with Tensor Spin Polarization (TSP)

[1] E. J. Ferrer, V. de la Incera, I. Portillo etal., Phys. Rev. D 89, 085034 (2014).
[2] Yi-Wei Qiu and Sheng-Qin Feng, Phys. Rev. D 107, 076004 (2023)

The interaction term of NJL Lagrangian:

$$\mathcal{L}_{\rm int}^{\rm NJL} = \frac{g}{2\Lambda^2}\left(\bar{\psi}\gamma^{\mu}\psi\right)\left(\bar{\psi}\gamma_{\mu}\psi\right)$$

Anisotropic coupling →

$$\mathcal{L}_{\rm int}^{\rm NJL} = \frac{g_{\parallel}}{2\Lambda^2}\left(\bar{\psi}\gamma_{\parallel}^{\mu}\psi\right)\left(\bar{\psi}\gamma_{\mu}^{\parallel}\psi\right) + \frac{g_{\perp}}{2\Lambda^2}\left(\bar{\psi}\gamma_{\perp}^{\mu}\psi\right)\left(\bar{\psi}\gamma_{\mu}^{\perp}\psi\right)$$

The Fierz identities connecting the different Dirac ring elements:

$$\left(\Gamma_A\right)_{ij}\left(\Gamma^B\right)_{kl} = \frac{1}{16}Tr\left[\Gamma^A\Gamma_C\Gamma_B\Gamma_D\right]\left(\Gamma^D\right)_{il}\left(\Gamma^C\right)_{kj},$$

where $\left\{\Gamma^A\right\} = \left\{1, \gamma_5, \gamma^{\mu}, \gamma_5\gamma^{\mu}, \sigma^{\mu\upsilon}\right\}.$

For the particle-antiparticle channel the anisotropic Fierz identities in the presence of a constant and uniform magnetic field are

$$\begin{pmatrix} (1)_{ij}(1)_{kl} \\ (\gamma^{\parallel})_{ij}(\gamma_{\parallel})_{kl} \\ (\gamma^{\perp})_{ij}(\gamma_{\perp})_{kl} \\ (\sigma^{30})_{ij}(\sigma_{30})_{kl} \\ (\sigma^{\perp\parallel})_{ij}(\sigma_{\perp\parallel})_{kl} \\ \frac{1}{2}(\sigma^{\perp\perp})_{ij}(\sigma_{\perp\perp})_{kl} \\ (\gamma^{\parallel}\gamma_5)_{ij}(\gamma^{\parallel}\gamma_5)_{kl} \\ (\gamma^{\perp}\gamma_5)_{ij}(\gamma_{\perp}\gamma_5)_{kl} \\ (\gamma_5)_{ij}(\gamma_5)_{kl} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & -1 \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} (1)_{il}(1)_{kj} \\ (\gamma^{\parallel})_{il}(\gamma_{\parallel})_{kj} \\ (\gamma^{\perp})_{il}(\gamma_{\perp})_{kj} \\ (\sigma^{30})_{il}(\sigma_{30})_{kj} \\ (\sigma^{\perp\parallel})_{il}(\sigma_{\perp\parallel})_{kj} \\ \frac{1}{2}(\sigma^{\perp\perp})_{il}(\sigma_{\perp\perp})_{kj} \\ (\gamma^{\parallel}\gamma_5)_{il}(\gamma^{\parallel}\gamma_5)_{kj} \\ (\gamma^{\perp}\gamma_5)_{il}(\gamma_{\perp}\gamma_5)_{kj} \\ (\gamma_5)_{il}(\gamma_5)_{kj} \end{pmatrix}.$$

At last, Lagrangian density within (2+1)-flavor NJL model in the presence of an external magnetic field can be given by:

$$\mathcal{L}_{\rm TSP} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} + \gamma^0\mu - m\right)\psi + G_s\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma^5\lambda_a\psi\right)^2\right]$$

$$G_t\sum_{a=0}^{8}\left[\left(\bar{\psi}\Sigma_3\lambda_a\psi\right)^2 + \left(\bar{\psi}\Sigma_3 i\gamma^5\lambda_a\psi\right)^2\right] - K\left[\det\bar{\psi}\left(1+\gamma_5\right)\psi + \det\bar{\psi}\left(1-\gamma_5\right)\psi\right],$$

Where the quark field ψ_f^c carries a flavor iso-doublet ($f = u, d, s$) and three color ($c = r, g, b$), current quark mass m is considered as $m_u = m_d$ for maintain isospin symmetry, covariant derivative $D_{\mu} = \partial_{\mu} + iQA_{\mu}^{ext}$ introducing magnetic field, the charge matrix in flavor space .

The Lagrangian density of the (2 + 1)-flavor NJL model by considering TSP

$$\mathcal{L}_{\text{TSP}} = \bar{\psi}(i\gamma^{\mu}D_{\mu} + \gamma^{0}\mu - m)\psi + G_s\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma^5\lambda_a\psi)^2] + G_t\sum_{a=0}^{8}\{(\bar{\psi}\Sigma_3\lambda_a\psi)^2 + (\bar{\psi}\Sigma_3 i\gamma^5\lambda_a\psi)^2\} - K\{\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\},$$

The coupling constant G_s in the scalar/pseudoscalar channel is closely related to the spontaneously chiral symmetry breaking, which produces a dynamical quark mass, and the tensor/pseudotensor channels term $G_t\sum_{a=0}^{8}[(\bar{\psi}_f^c\Sigma^3\lambda_a\psi_f^c)^2+(\bar{\psi}_f^c i\Sigma^3\gamma^5\lambda_a\psi_f^c)^2]$ is closely related to the spin-spin interaction, which causes spin-polarization condensation.

[1] E. J. Ferrer, V. de la Incera, I. Portillo etal., Phys. Rev. D 89, 085034 (2014).
[2] Yi-Wei Qiu and Sheng-Qin Feng, Phys. Rev. D 107, 076004 (2023)

The 2 + 1 Flavors NJL Model with TSP under a Magnetic Field

The effective potential of using a standardized process is given

$$\Omega_{\mathrm{TSP}} = G_s \sum_{f=u,d,s} \langle\bar{\psi}\psi\rangle_f^2 + G_t\langle\bar{\psi}\lambda_3\Sigma^3\psi\rangle^2 + G_t\langle\bar{\psi}\lambda_8\Sigma^3\psi\rangle^2 - \frac{N_c}{2\pi}\sum_{f=u,d,s}|q_f B|\sum_{l=0}^{\infty}\alpha_l\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}$$
$$\times\left\{\varepsilon_{f,l,\eta} + T\ln\left[1+\exp\left(\frac{-\varepsilon_{f,l,\eta}-\mu}{T}\right)\right] + T\ln\left[1+\exp\left(\frac{-\varepsilon_{f,l,\eta}+\mu}{T}\right)\right]\right\}$$
$$+ 4K\langle\bar{\psi}\psi\rangle_u\langle\bar{\psi}\psi\rangle_d\langle\bar{\psi}\psi\rangle_s$$

Note that the breaking of energy spectrum degeneracy caused by spin known as Zeeman effect. Therefore, the contributions of spin come not only from the ground state of Landau level, but also from the whole excited states of Landau level.

$$\varepsilon^2_{u,l=0} = p_z^2 + \left(M_f + \left(F_3 + \frac{F_8}{\sqrt{3}}\right)\right)^2,$$

$$\varepsilon^2_{u,l\neq0,\eta} = p_z^2 + \left(\left(M_f^2 + 2|q_f B|l\right)^{1/2} + \eta\left(F_3 + \frac{F_8}{\sqrt{3}}\right)\right)^2,$$

$$\varepsilon^2_{d,l=0} = p_z^2 + \left(M_f + \left(F_3 - \frac{F_8}{\sqrt{3}}\right)\right)^2,$$

$$\varepsilon^2_{d,l\neq0,\eta} = p_z^2 + \left(\left(M_f^2 + 2|q_f B|l\right)^{1/2} + \eta\left(F_3 - \frac{F_8}{\sqrt{3}}\right)\right)^2,$$

$$\varepsilon^2_{s,l=0} = p_z^2 + \left(M_f + \left(\frac{2F_8}{\sqrt{3}}\right)\right)^2,$$

$$\varepsilon^2_{s,l\neq0,\eta} = p_z^2 + \left(\left(M_f^2 + 2|q_f B|l\right)^{1/2} + \eta\left(\frac{2F_8}{\sqrt{3}}\right)\right)^2.$$

[1] E. J. Ferrer, V. de la Incera, I. Portillo etal., Phys. Rev. D 89, 085034 (2014).
[2] Yi-Wei Qiu and Sheng-Qin Feng, Phys. Rev. D 107, 076004 (2023)

The 2 + 1 Flavors NJL Model with TSP under a Magnetic Field

The tensor condensate parameter F_3 and F_8 are self-consistently satisfied the minimum of the thermodynamic potential, five coupling gap equations as:

$$\frac{\partial\Omega_{TSP}\left(M_f, F_3, F_8\right)}{\partial M_f} = 0.$$

and

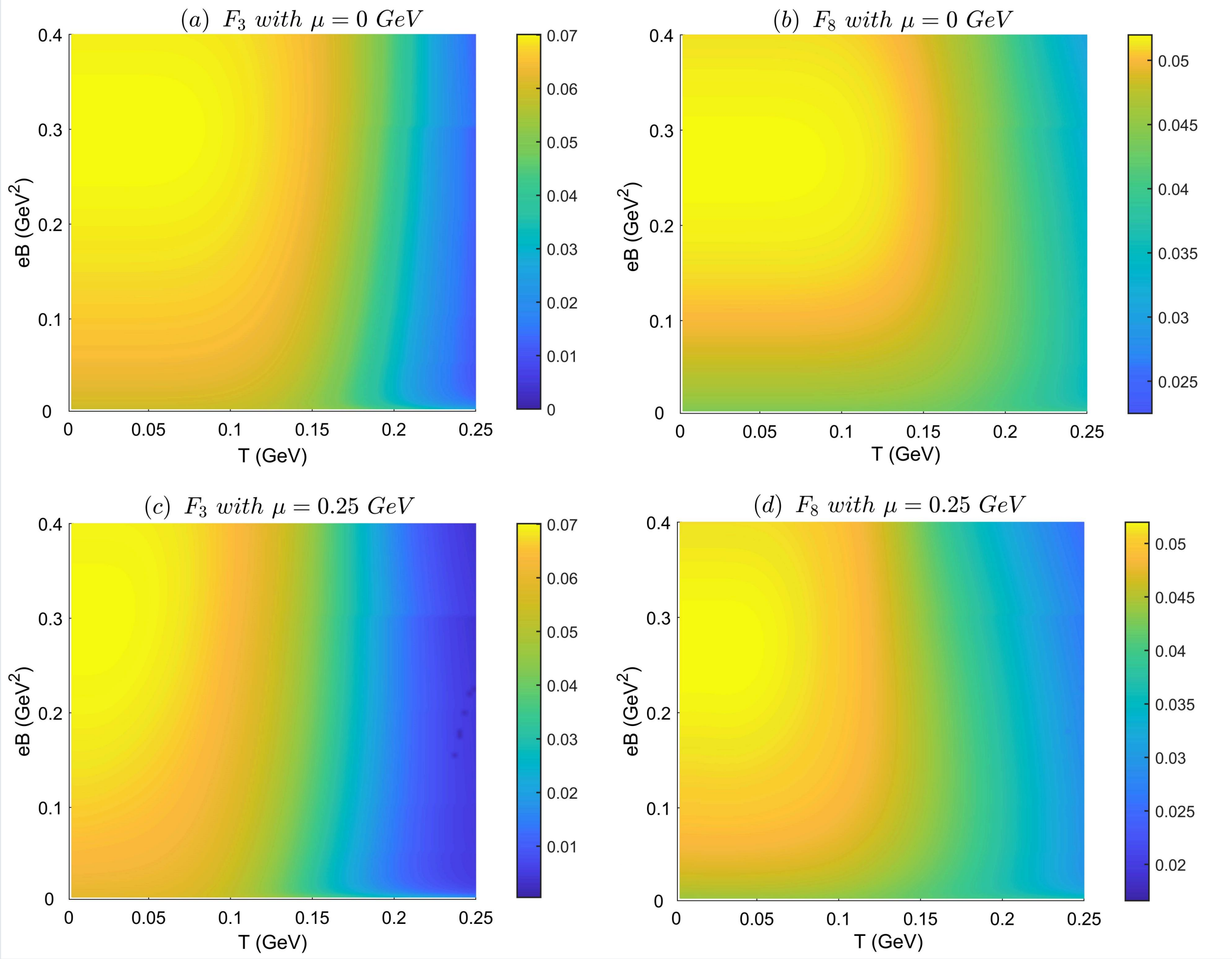
$$\frac{\partial\Omega_{TSP}\left(M_f, F_3, F_8\right)}{\partial F_3} = \frac{\partial\Omega_{TSP}\left(M_f, F_3, F_8\right)}{\partial F_8} = 0$$

$$F_3 = -2G_t\left\langle\bar{\psi}\Sigma^3\lambda_3\psi\right\rangle,$$
$$F_8 = -2G_t\left\langle\bar{\psi}\Sigma^3\lambda_8\psi\right\rangle.$$

The scheme dealing with the sums of all Landau levels within the integrals by means of the Hurwitz zeta function.

[1] D. P. Menezes et.al., Phys. Rev. C 79, 035807(2009).
[2] R. M. Aguirre, Phys. Rev. D 102, 096025 (2020).

The two tapes of spin polarization with the TSP



$$F_3 = -2G_t \left\langle \bar{\psi}\Sigma^3\lambda_3\psi \right\rangle ,$$
$$F_8 = -2G_t \left\langle \bar{\psi}\Sigma^3\lambda_8\psi \right\rangle .$$

(1) Both F_3 and F_8 become stronger at low temperatures, especially with the increase of the magnetic field.

(2) F_3 is almost zero at high temperature, and F_8 is very small but not zero at high temperature.

(3) The polarizations become weak at high temperatures. **It thus can be concluded that it is more difficult to be polarized in the hot QGP background. It is characterized by ferromagnetism, and ferromagnetic spin polarization is zero at high temperature.**

Yi-Wei Qiu and Sheng-Qin Feng, Phys. Rev. D 107, 076004 (2023)

Intrducing Anomalous Magnetic Moment (AMM)

In the case of strong magnetic field, the anomalous magnetic moment (AMM) effect of quarks has aroused great interest, and the inverse catalytic property (IMC) effect appears [1-5]. Dynamic chiral symmetry breaking is one of the most important characteristics of QCD, which enables quarks to obtain the dynamic mass of QCD. The AMM of quarks can also be generated like the dynamic quark mass M. Therefore, once quarks obtain kinetic mass due to condensation, they should also obtain kinetic AMM $\frac{1}{2}q\kappa F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi\ (\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}])$. The coefficient κ of quark AMM in a magnetic field is introduced to study the magnetic catalytic effect at finite temperature. For QCD, spontaneous chiral symmetry breaking results in quark AMM, also known as dynamic AMM.

[1] S. Fayazbakhsh and N. Sadooghi, Phys. Rev. D 90, 105030(2014).
[2] N. Chaudhuri, S. Ghosh, S. Sarkar, and P. Roy, Phys. Rev. D 99, 116025 (2019).
[3] S. Ghosh, N. Chaudhuri, S. Sarkar, and P. Roy, Phys. Rev. D 101, 096002 (2020).
[4] K. Xu, J. Chao, and M. Huang, Phys. Rev. D 103, 076015 (2021).
[5] Yi-Wei Qiu and Sheng-Qin Feng, Phys. Rev. D 107, 076004 (2023).

The（2 + 1） Flavors NJL model with AMM

The effect Lagrangian density of the (2 + 1)- flavor NJL model with AMM is given as

$$\mathcal{L}_{\rm AMM}=\bar{\psi}\left(i\gamma^{\mu}D_{\mu}+\gamma^{0}\mu-m+\frac{1}{2}q_f\kappa\sigma^{\mu\nu}F_{\mu\nu}\right)\psi+G_s\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2+\left(\bar{\psi}i\gamma^5\lambda_a\psi\right)^2\right]-K\left[\det\bar{\psi}(1+\gamma_5)\psi+\det\bar{\psi}(1-\gamma_5)\psi\right],$$

The effective potential with AMM can be taken as

$$\Omega_{\rm AMM}=G_s\sum_{f=u,d,s}\left\langle\bar{\psi}\psi\right\rangle_f^2+4K\left\langle\bar{\psi}\psi\right\rangle_u\left\langle\bar{\psi}\psi\right\rangle_d\left\langle\bar{\psi}\psi\right\rangle_s-\frac{N_c\sum_{f=u,d,s}|q_fB|}{2\pi}\sum_{l=0}^{\infty}\sum_{t=\pm1}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\left\{E_{f,l,t}+T\ln\left[1+\exp\left(\frac{-E_{f,l,t}-\mu}{T}\right)\right]+T\ln\left[1+\exp\left(\frac{-E_{f,l,t}+\mu}{T}\right)\right]\right\},$$

where $E_{f,l,t}=\sqrt{p_z^2+\left(\left(M_f^{\,2}+2|q_fB|l\right)^{1/2}-t\kappa_f q_f eB\right)^2}$ is the energy spectrum under different Landau energy levels and corresponds to the two kinds of spin direction of quark-antiquark pair.

The（2 + 1） Flavors NJL model with AMM

$$\frac{\partial \Omega_{AMM}}{\partial M_f} = 0,$$

where $f = u, d, s$ for the three different flavors. We can obtain three dynamical quark mass as:

$$M_u = m_u - 4G_s\langle\bar{\psi}\psi\rangle_u + 2K\langle\bar{\psi}\psi\rangle_d\langle\bar{\psi}\psi\rangle_s,$$

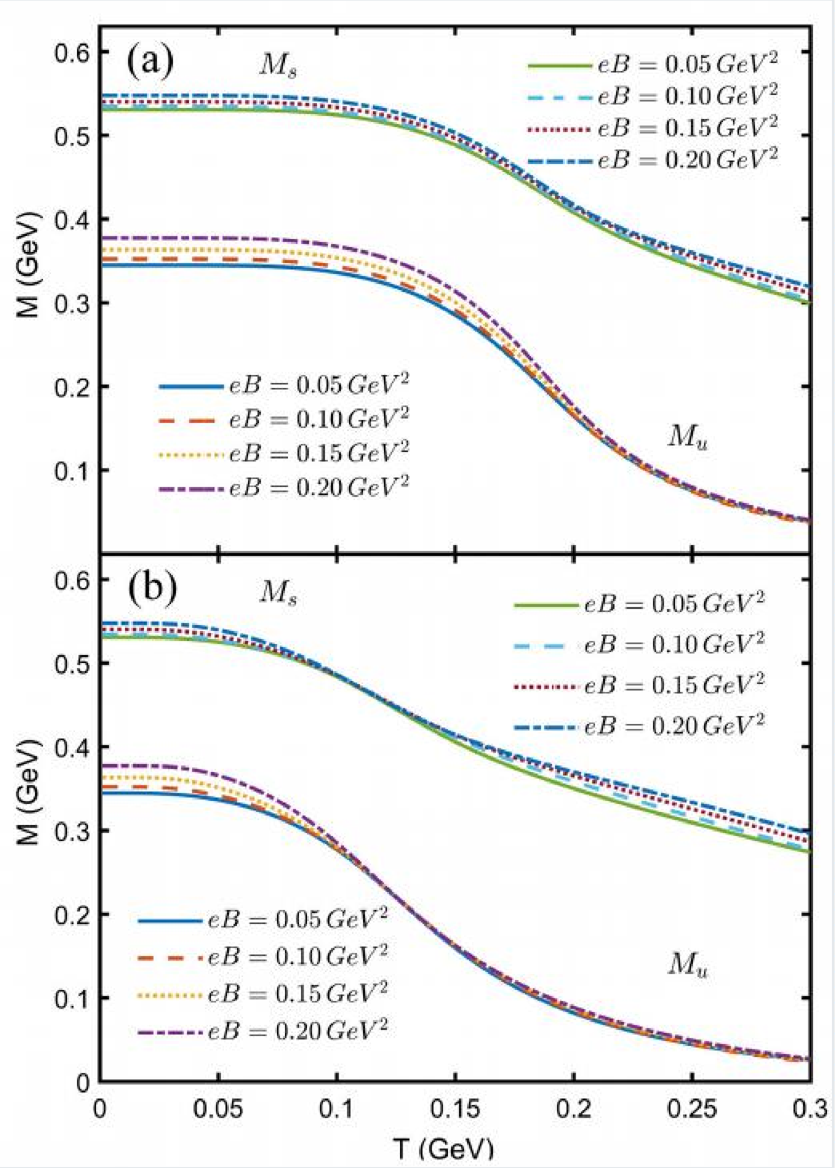
$$M_d = m_d - 4G_s\langle\bar{\psi}\psi\rangle_d + 2K\langle\bar{\psi}\psi\rangle_u\langle\bar{\psi}\psi\rangle_s,$$

$$M_s = m_s - 4G_s\langle\bar{\psi}\psi\rangle_s + 2K\langle\bar{\psi}\psi\rangle_u\langle\bar{\psi}\psi\rangle_d,$$

and

$$\langle\bar{\psi}\psi\rangle_f = \frac{N_c G_s}{2\pi}\sum_{l=0}^{\infty}\alpha_l\,|q_f B|\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi}\frac{M_f}{\varepsilon_{f,l,t}}\left(1-\frac{s\kappa_f q_f B}{\hat{M}_{f,l,t}}\right)\left\{1-\frac{1}{e^{\frac{\varepsilon_{f,l,t}+\mu}{T}}+1}-\frac{1}{e^{\frac{\varepsilon_{f,l,t}-\mu}{T}}+1}\right\}.$$

corresponds to chiral condensation of different quark flavors ($f = u, d, s$).



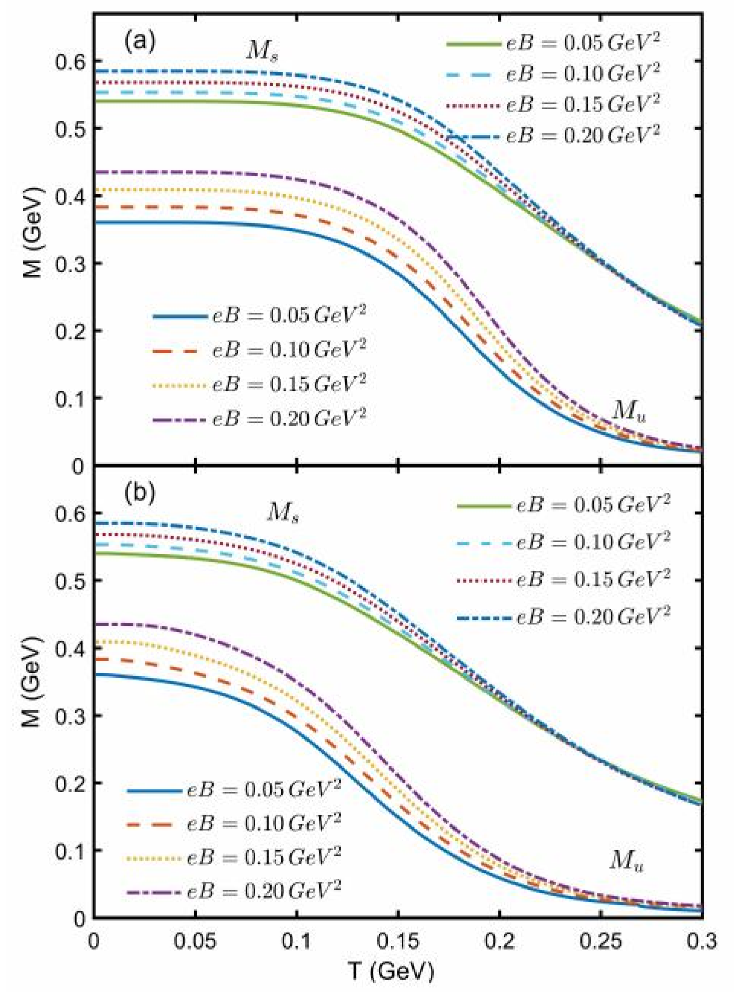
Without considering AMM and TSP

The dynamical quark masses M of *u*, *d* and *s* quarks without considering AMM and TSP are manifested as decreasing *smooth functions* of temperatures at μ = 0 GeV (a) and μ = 0.25 GeV (b), which indicates a *chiral crossover*. The dynamical mass *M* is apparently enhanced by increasing the magnetic field.

The larger the magnetic field is, the larger the corresponding chiral condensation is. ***This phenomenon is manifested as magnetic catalysis,*** which accounts for the magnetic field has a strong tendency to enhance (or catalyze) spin-zero quark-antiquark condensates.

The dynamical quark masses M of *u*, d and s quarks ***by without considering AMM and TSP***

The influence of dynamical mass M by considering TSP of quarks

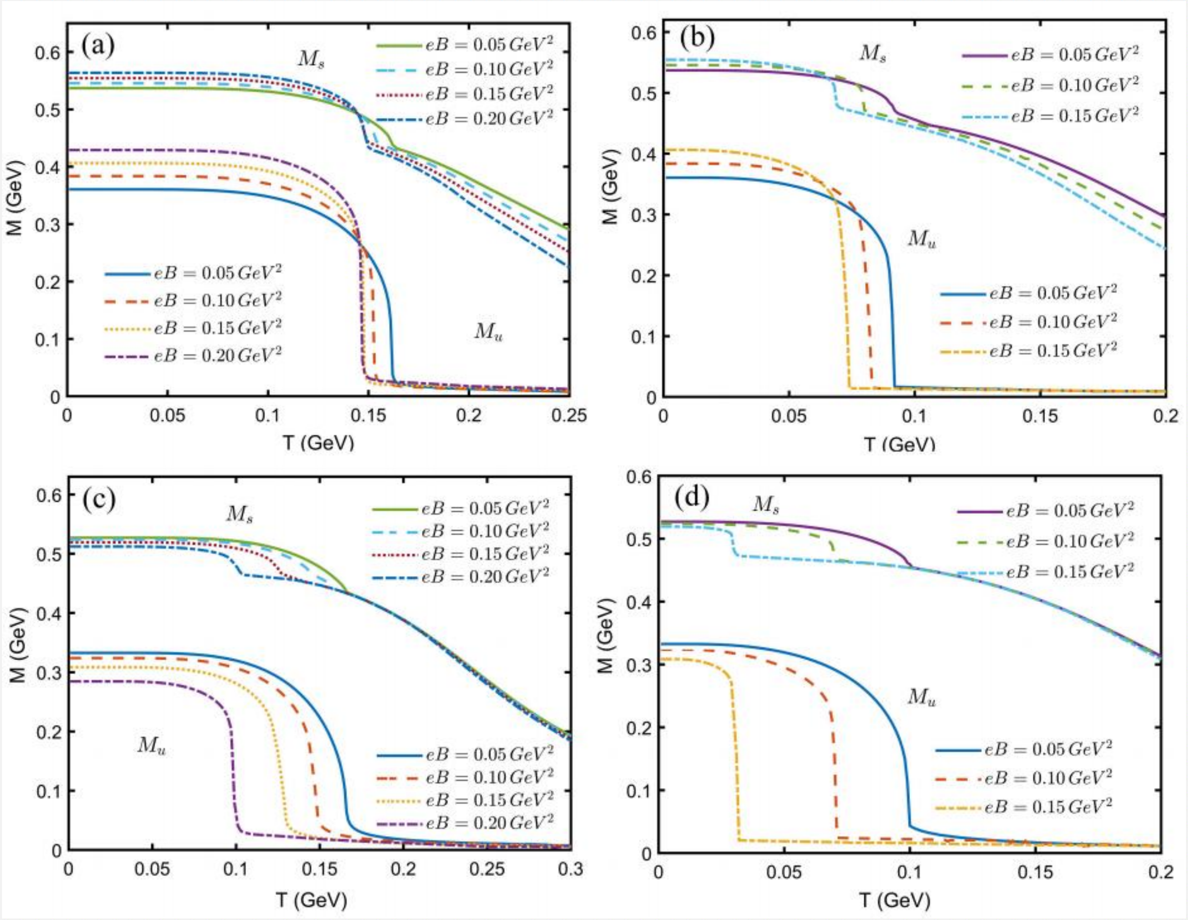


(1) The dynamical mass M by considering TSP of quarks is manifested as **a decreasing smooth function of temperatures** for different magnetic fields and chemical potentials, which corresponds to **a chiral crossover.**

(2) It is found that considering TSP will **enhance magnetic catalytic effect.**

$\mu = 0$ (a) and $\mu = 0.25$ GeV (b) , respectively *by considering TSP of quarks, .*

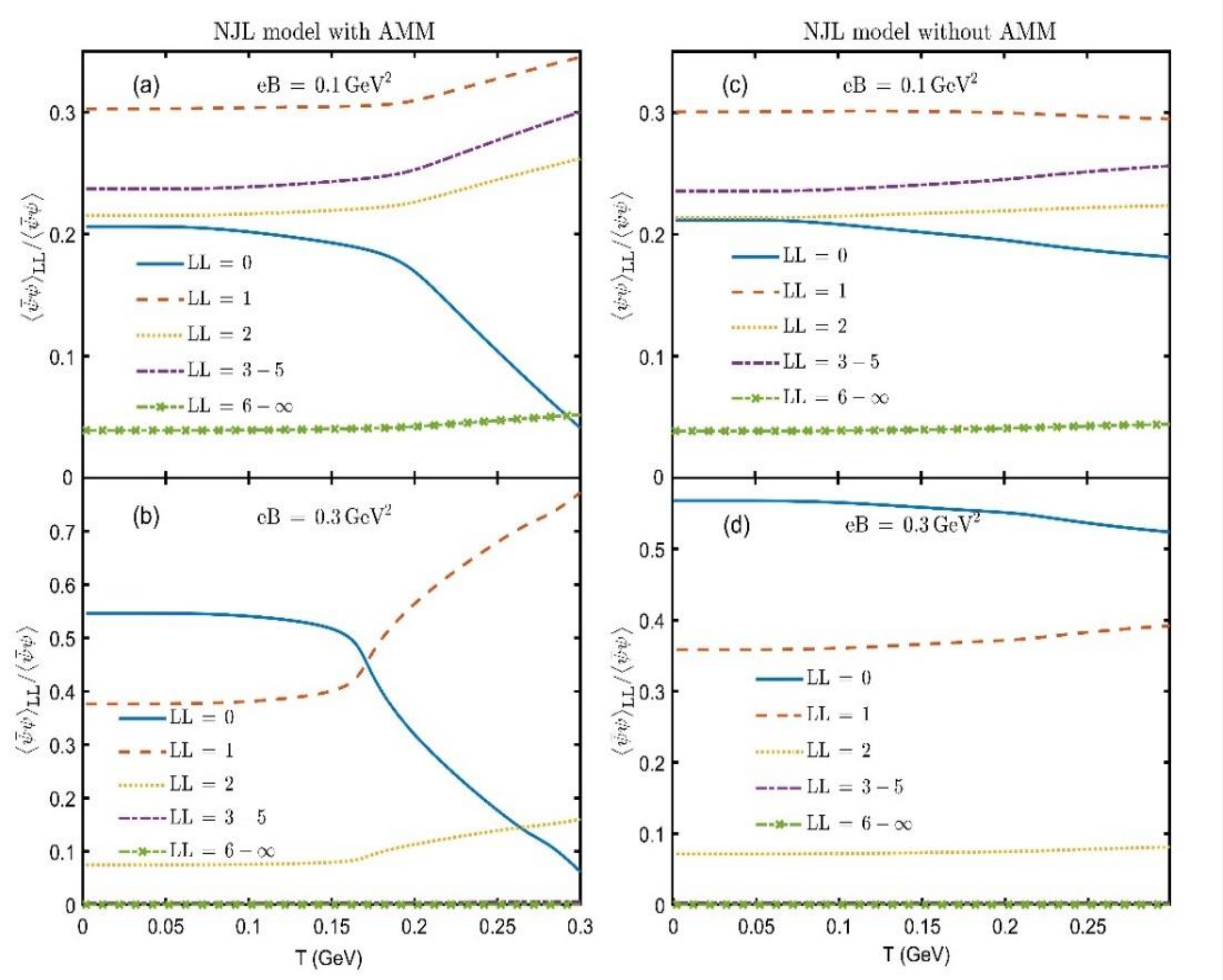
The influence of dynamical mass M by considering AMM of quarks



The generation of dynamical quark mass from the dimensional reduction from 3 + 1 D to 1 + 1 D is predominated by LLL at the low temperature region. That effect can be reflected in AMM1 obviously. If the role of the AMM item is enough to alter the nature of the medium in the $T = 0$ like the case of AMM2, the IMC effect characteristics of the AMM2 will be more significant.

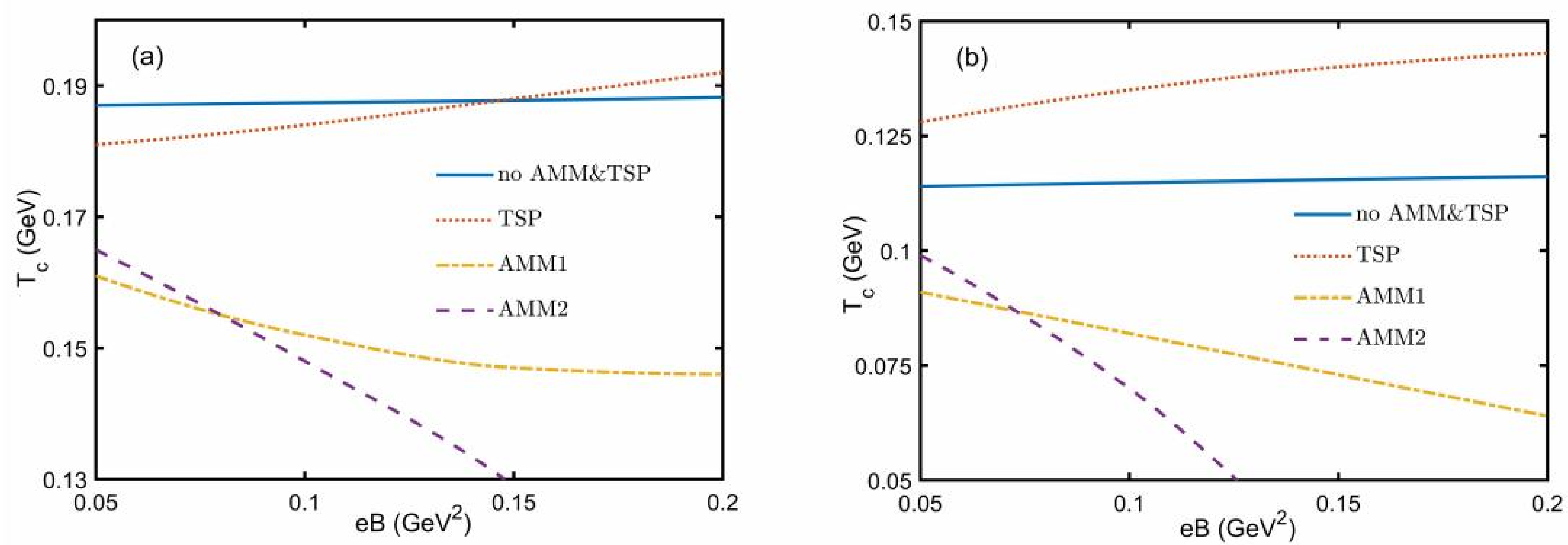
by considering the different sets of AMM. (a, b) are for $\mu = 0$ and $\mu = 0.25$ GeV respectively with AMM1 set as $\kappa_u = \kappa_d = 0.38$, $\kappa_s = 0.25$. (c, d) for AMM2 set as $\kappa_u = 0.123$, $\kappa_d = 0.555$, $\kappa_s = 0.329$.

The dependent of ratio of chiral condensate on Landal Energy Levels by considering AMM of quarks



It is found that as the temperature increases, after reaching the chiral phase transition temperature, the contribution of the lowest Landau level to chiral condensation decreases significantly by considering AMM as shown in (a, b), while the contribution of the excited Landau level increases accordingly. Therefore, we can believe that the effect of AMM can make particles with the lowest Landau level more easily excited to higher excited energy level at high temperatures. And as the magnetic field increases, this effect becomes more pronounced. It is believed that this may be the reason for the formation of inverse magnetic catalysis (IMC) at high temperatures after the introduction of AMM.

The comparison of chiral phase transition by considering the TSP and the AMM



The critical temperature of *u* and *d* quarks as a function of the magnetic field at $\mu = 0$ (a) and $\mu = 0.25$ GeV (b).

It is thus found that the critical temperature decreases with the magnetic field for the AMM1 and AMM2 sets, which indicates an inverse magnetic catalysis. On the contrary, with the TSP, T_C enhances as a function of the magnetic field, which is the extension of the magnetic catalysis effect from vacuum to finite temperature.

2. Anomalous Magnetic Moment and Transportation in the magnetized QCD background

(1) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(2) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023).

Dissipative QCD fluid and shear viscosity coefficient

The shear viscosity coefficient η is essential parameters for quantifying the dissipation process in QCD fluid dynamics evolution under the external magnetic field background. It is particularly noteworthy that η is very sensitive to the QCD phase transition characteristics in a magnetized media.

The total energy-momentum tensor of the fluid is given as

$T_{\mu\nu} = T^{0}_{\mu\nu} + T^{D}_{\mu\nu}$, $\quad T^{0}_{\mu\nu} = -pg^{\mu\nu} + wu^{\mu}u^{\nu}$ is for the ideal fluid, and

$T^{D}_{\mu\nu} = \eta(D^{\mu}u^{\nu} + D^{\nu}u^{\mu} + \frac{2}{3}\Delta^{\mu\nu}\partial_{\alpha}u^{\alpha}) - \zeta\partial_{\alpha}u^{\alpha}$ **is for the dissipative fluid.**

The shear viscosity coefficient in the magnetic field background

(1) **We will study the shear viscosity of magnetized quark matter near the quark chiral phase transition point by using a dynamical quasiparticle NJL model.** The effective quark masses are determined by the in-medium quark condensate.

(2) **We use the Boltzmann equation's dynamic model and relaxation time approximation to study the features of the viscosity coefficient components** in the first-order phase transition and critical endpoint (CEP) phase transition regions under the strong magnetic field background.

The shear viscosity coefficient η can be obtained by comparing the dissipative part of the energy momentum tensor and possess five independent components in the presence of an external magnetic field:

$$\eta = \frac{1}{15T}\sum_a g_a \int \frac{d^3\vec{k}_a}{(2\pi)^3}\frac{\vec{k}_a^4}{E_a{}^2}\tau_a^c\left(f_a^0\left(1-f_a^0\right)\right)$$

The ramaining four components can be written as:

$$\eta_{n(n=1,2,3,4)} = \frac{1}{15T}\sum_a g_a \int \frac{d^3\vec{k}_a}{(2\pi)^3}\frac{\vec{k}_a^4}{E_a{}^2}\tau_{a,n}^{eff}\left(f_a^0\left(1-f_a^0\right)\right)$$

g_a is the degeneration of collisional particle type a, the $\tau_{a,n}^{eff}$ is the effective relaxation time, which τ_a^B is related to the magnetic relaxation time and collision relaxation time τ_a^c .

The effective relaxation time in the magnetic field background

$$\tau_{a,1}^{eff} = \tau_a^c \frac{1}{4\{\frac{1}{4}+(\tau_a^c/\tau_a^B)^2\}}, \quad \tau_{a,2}^{eff} = \tau_a^c \frac{1}{\{1+(\tau_a^c/\tau_a^B)^2\}}$$

$$\tau_{a,3}^{eff} = \tau_a^c \frac{\tau_a^c/\tau_a^B}{2\{\frac{1}{4}+(\tau_a^c/\tau_a^B)^2\}}, \quad \tau_{a,4}^{eff} = \tau_a^c \frac{\tau_a^c/\tau_a^B}{\{1+(\tau_a^c/\tau_a^B)^2\}}.$$

The approximation of the field-induced relaxation time $\tau_a^B = E_a/(q_a B)$ has been applied in the strong magnetic field limit, since the deviation of equilibrium state mostly results from the field rather than the particle collisions.

The effective relaxation time in the magnetic field background

The two-body collision $a + b \to c + d$

$$u u \to u u,\ u d \to u d,\ u \bar{u} \to u \bar{u},\ u \bar{u} \to d \bar{d},$$
$$u \bar{d} \to u \bar{d},\ d d \to d d,\ \bar{u} d \to \bar{u} d,\ d \bar{d} \to d \bar{d},$$
$$d \bar{d} \to u \bar{u},\ \bar{u} \bar{u} \to \bar{u} \bar{u},\ \bar{u} \bar{d} \to \bar{u} \bar{d},\ \bar{d} \bar{d} \to \bar{d} \bar{d}.$$

One assumes the particle a is a probe particle, and the collision relaxation time of the system could be evaluated $\tau_a^c = \frac{1}{\Gamma_a^c}$. The collision width Γ_a^c is given as:

$$\Gamma_a^c(p_a) = \sum_a \int \frac{d^3 p_b}{(2\pi)^3} \sigma_{ab}(p_a, p_b) \nu_{ab}(p_a, p_b) f_b.$$

(1) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(2) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023)

The effective relaxation time in the magnetic field background

$$\frac{d\sigma_{ab}}{dt} = \frac{1}{16\pi s}\frac{1}{p_{ab}^2}\left|\tilde{M}_{ab}\right|^2,$$

$$\nu_{ab}\left(p_a, p_b\right) = \frac{\left(E_a + E_b\right)\sqrt{s^2 - 4M^2}}{2E_a E_b},$$

The scattering matrix are:

They can be simplified from 12 situations to only two independent cases by the isospin symmetry, charge conjugation and crossing symmetries.

$$\begin{aligned}\left|\tilde{M}_{u\bar{u}\to u\bar{u}}\right|^2 =& s^2\left|D_\pi\left(\sqrt{s},0\right)\right|^2 + t^2\left|D_\pi\left(0,\sqrt{-t}\right)\right|^2\left(s-4m^2\right)^2\left|D_\sigma\left(\sqrt{s},0\right)\right|^2 + \left(t-4m^2\right)^2\left|D_\sigma\left(0,\sqrt{-t}\right)\right|^2\\ &+\frac{1}{N_c}\mathrm{Re}\left[stD_\pi^*\left(\sqrt{s},0\right)D_\pi\left(0,\sqrt{-t}\right) + s\left(4m^2-t\right)D_\pi^*\left(\sqrt{s},0\right)D_\sigma\left(0,\sqrt{-t}\right)\right.\\ &\left.+t\left(4m^2-s\right)D_\pi\left(0,\sqrt{-t}\right)D_\sigma^*\left(\sqrt{s},0\right) + \left(4m^2-s\right)\left(4m^2-t\right)D_\sigma^*\left(\sqrt{s},0\right)D_\sigma^*\left(0,\sqrt{-t}\right)\right].\end{aligned}$$

P. Zhuang, J. Hufner, S. P. Klevansky, and L. Neise, Phys. Rev. D 51, 3728 (1995).

$$\begin{aligned}\left|\tilde{M}_{u\bar{d}\to u\bar{d}}\right|^2 =& 4s^2\left|D_\pi\left(\sqrt{s},0\right)\right|^2 + t^2\left|D_\pi\left(0,\sqrt{-t}\right)\right|^2\left(s-4m^2\right)^2\left|D_\sigma\left(\sqrt{s},0\right)\right|^2 + \left(t-4m^2\right)^2\left|D_\sigma\left(0,\sqrt{-t}\right)\right|^2\\ &+\frac{1}{N_c}\mathrm{Re}\left[-2stD_\pi^*\left(\sqrt{s},0\right)D_\pi\left(0,\sqrt{-t}\right) + 2s\left(4m^2-t\right)D_\pi^*\left(\sqrt{s},0\right)D_\sigma\left(0,\sqrt{-t}\right)\right],\end{aligned}$$

The effective relaxation time in the magnetic field background

In the random phase approximation (RPA), the effective meson propagator D_M is

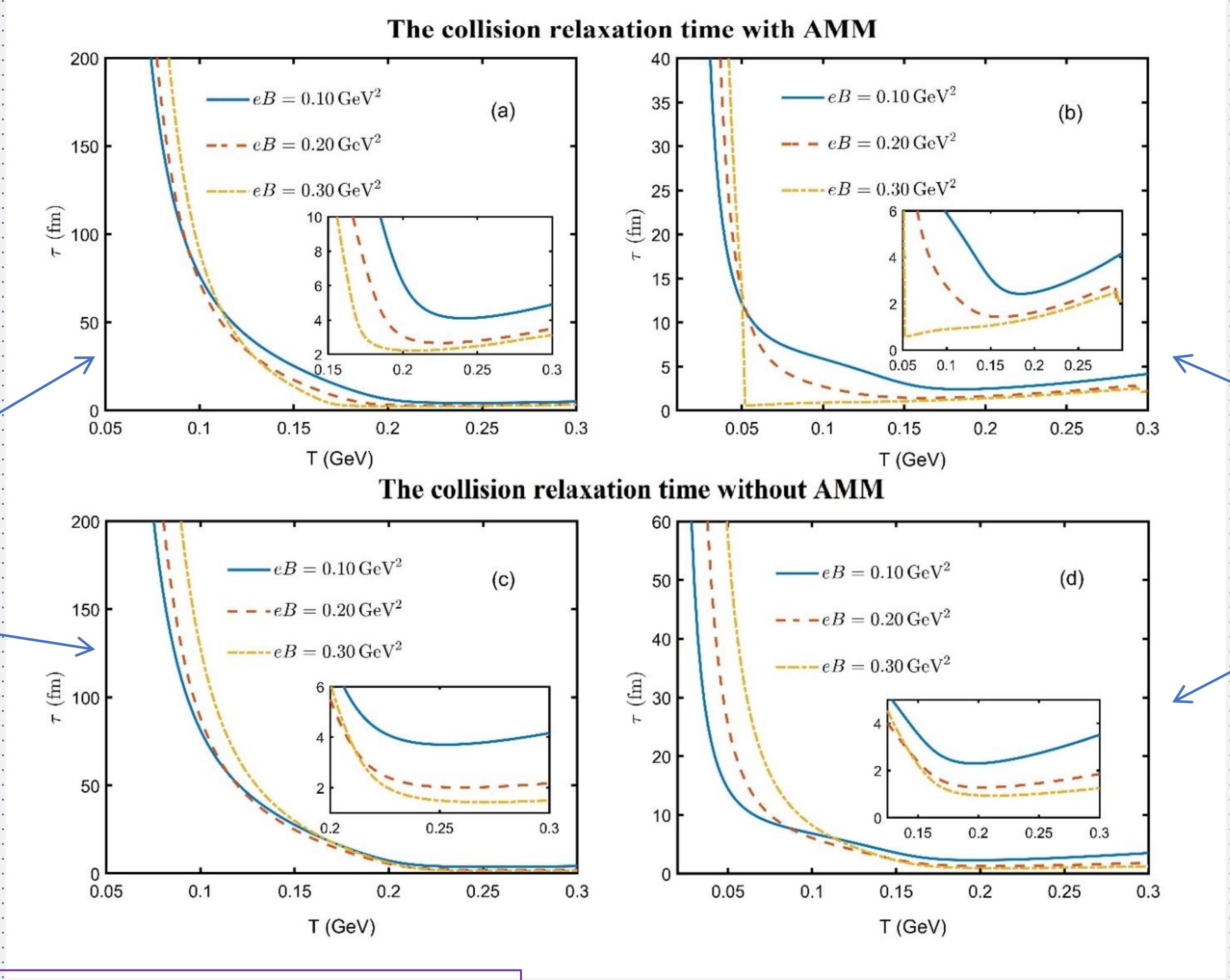
$$D_M(\omega, \mathbf{p}) = \frac{2iG}{1 - 2G\Pi_M(\omega, \mathbf{p})},$$

The above $M = \pi$, or σ meson, and the Π_M is the polarization function corresponding the M mesonic channel with AMM under the magnetic field.

P. Zhuang, J. Hufner, S. P. Klevansky, and L. Neise, Phys. Rev. D 51, 3728 (1995).

(1) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(2) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023).

The dependence of collision relaxation time on temperature with AMM

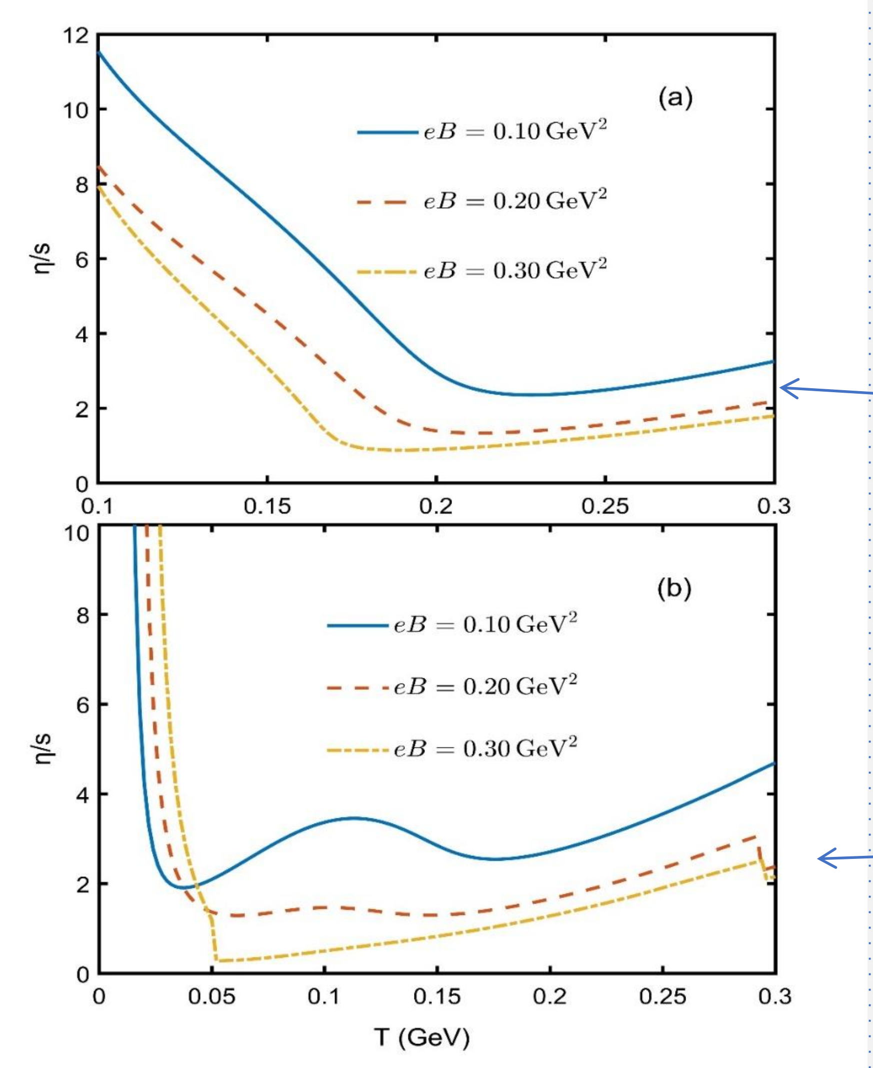


$\mu = 0$ GeV

$\mu = 0.225$ GeV

(1) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(2) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023).

The dependence of η/s on temperature T with different magnetic field with AMM

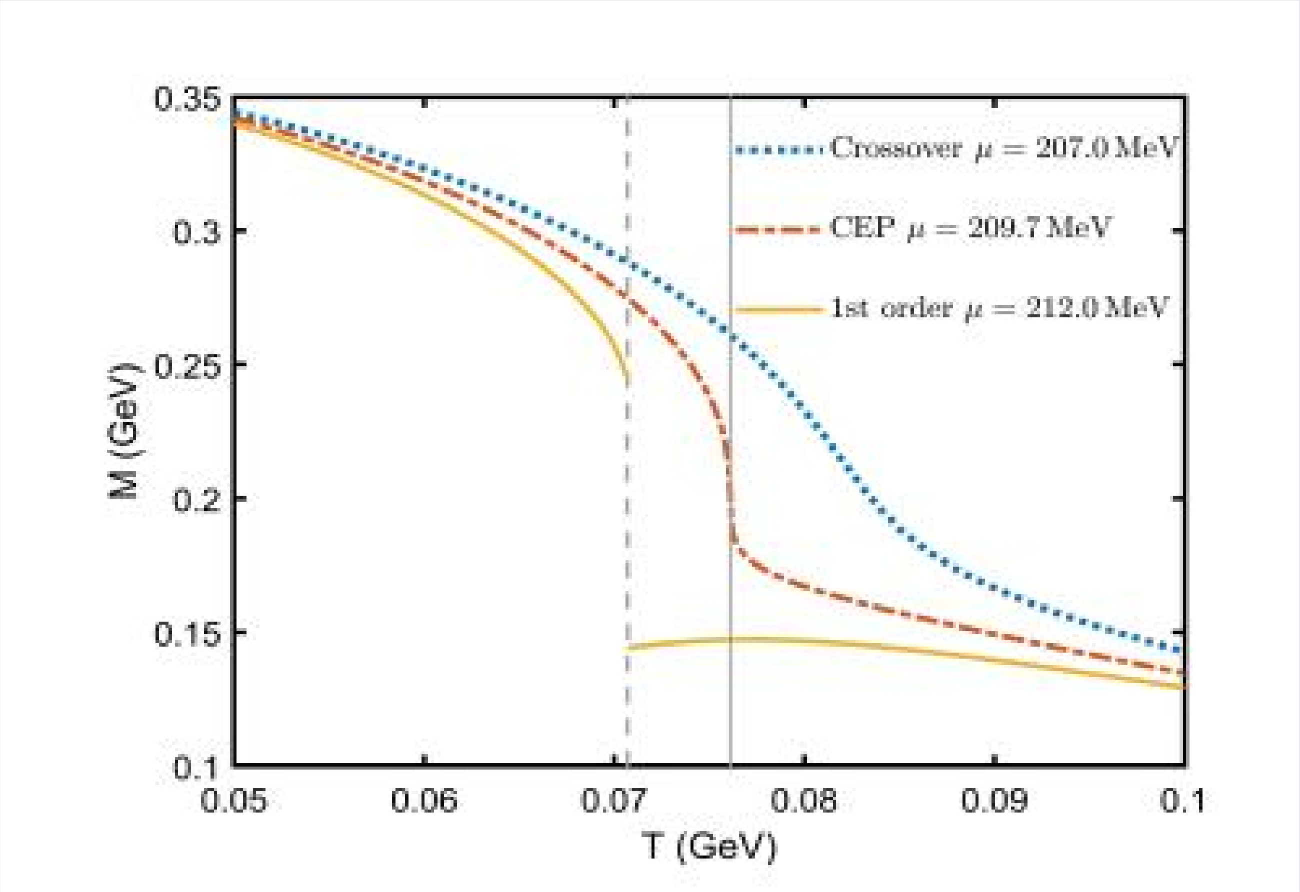


(1) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(2) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023).

$\mu = 0.0$ GeV

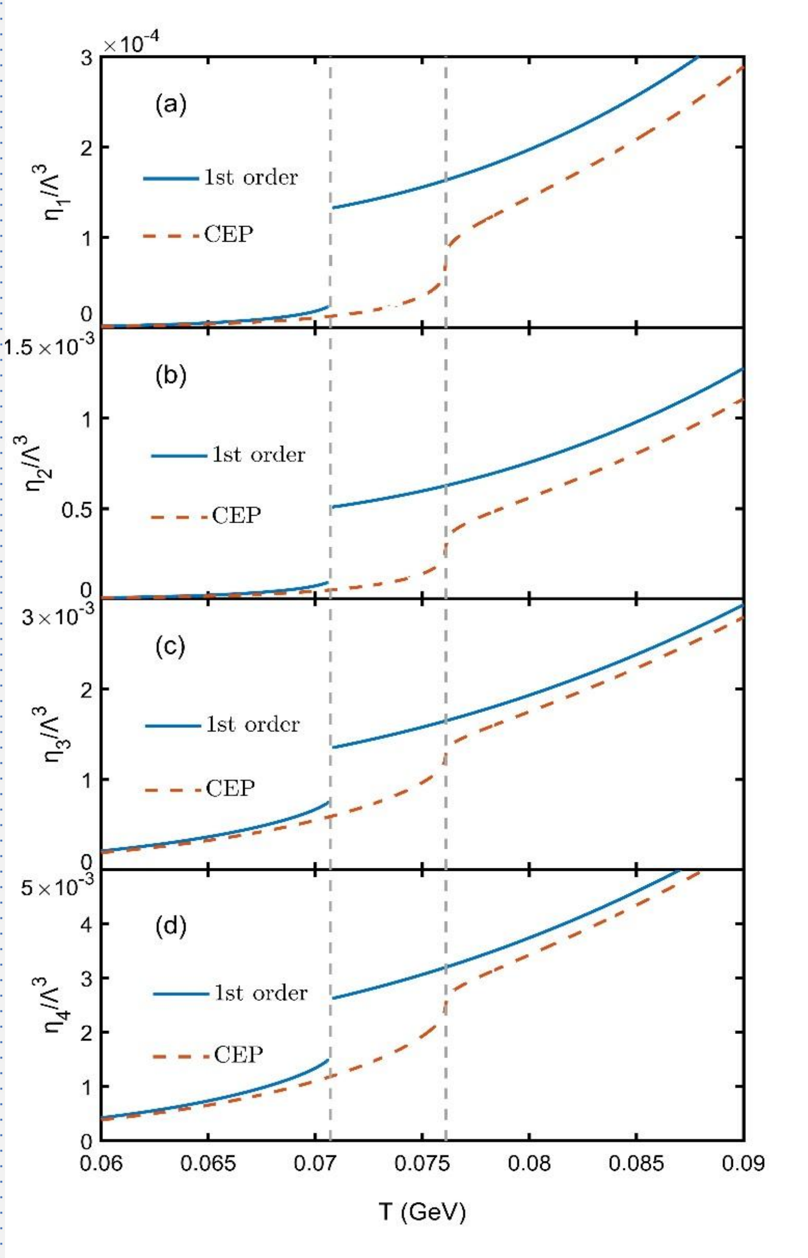
$\mu = 0.225$ GeV

The dependencies of the dynamic quark mass *M* on the temperature of the crossover, the first order phase transition, and the CEP phase transition



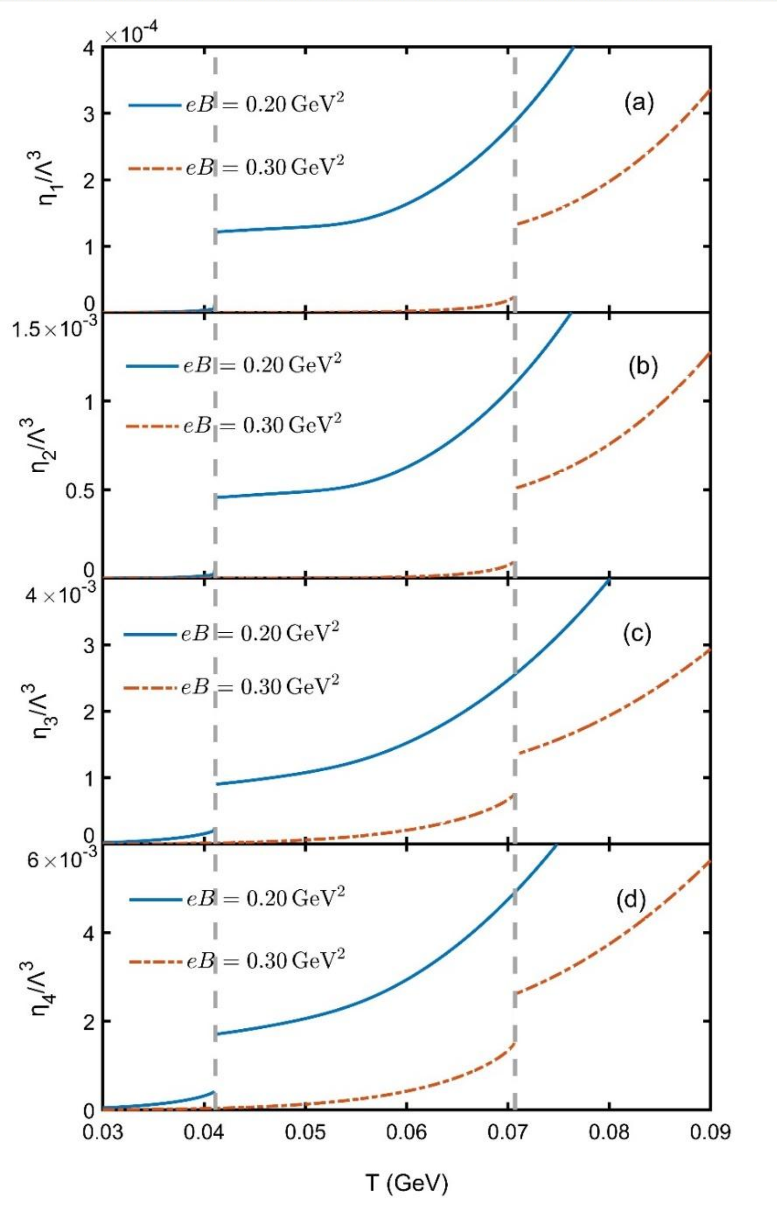
(1) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(2) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023).

The dependencies of renormalized shear viscosity coefficient components on the temperature of CEP phase transition and first order phase transition



$(eB = 0.2\ \mathrm{GeV}^2)$.

The dependencies of renormalized shear viscosity coefficient components on the temperature of first order phase transition with different magnetic fields



(1) X.-Q. Zhu and S.-Q. Feng, Phys. Rev. D 107, 016018 (2023);
(2) Y.-W. Qiu and S.-Q. Feng, X.-Q. Zhu, Phys. Rev. D 108, 116022 (2023).

4. Summary and Conclusions

Summary and conclusions

1. In the TSP case, since the dynamical quark mass is increased by the spin condensate, which is generated by an extra tensor channel independently as well as enhanced by the magnetic field, the pseudocritical temperature is increased by a rising magnetic field. Quark Spin polarization is ferromagnetic, and ferromagnetic Spin polarization approachs zero at high temperature.

2. While in the AMM case, the AMM term $1/2\, q_f \kappa \delta^{\mu\nu} F_{\mu\nu}$ does not directly produce a new condensate to impact the dynamical mass. Instead, it changes the energy spectrum of all Landau levels. As the result, it has been found that the AMM term will reduce the dynamical mass, once the temperature is high enough to excite much more particles to jump to higher Landau levels.

3. The shear viscosity component $\eta_1, \eta_2, \eta_3, \eta_4$ all increase with temperature. Discontinuities of $\eta_1, \eta_2, \eta_3, \eta_4$ for the first order phase transition point, and there is an upward discontinuous jump from the chiral broken phase of the low temperature to the chiral restoration phase of the high temperature. The shear viscosity coefficient of magnetized QCD medium is not a function of smooth transition for the first order phase transition.

Thanks！