Color superconductivity in sQGP from functional QCD method

Fei Gao

Beijing Institute of Technology, BIT, in fQCD collaboration

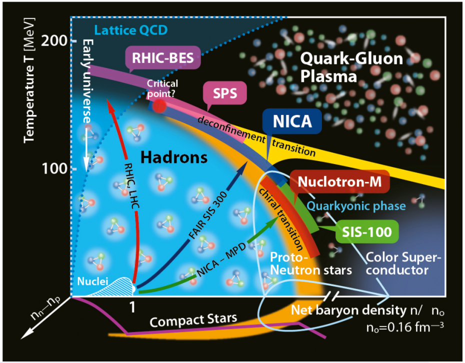
in collaboration with Yi Lu, Yuxin Liu, Jan M. Pawlowski

based on arXiv:2310.16345 and 2403.16816

Quantum ChromoDynamics

People have made many efforts on exploring the chiral phase transition and deconfinement. Besides of these phase transitions, QCD has rich phases above chiral phase transition T_c:

- Strongly coupled quark gluon plasma (sQGP) at low μ
- The counterparts of sQGP at large μ: inhomogeneous phase, quarkyonic phase, color superconductivity (CSC) phase
- sQGP acts as perfect fluid with small viscosities, while in the CSC phase, the Cooper pairing also leads to small viscosities.



How the sQGP evolves into CSC phase with increasing μ?
Is it possible that sQGP is the same phase as CSC?

The conventional CSC phase

The conventional CSC phase can only exist at high chemical potential and hence low temperature.

The Cooper pair Δ in conventional CSC is generated through the gap equation as:

$$\Delta = \int_q g^2 \frac{\Delta}{q^2 + \Delta^2} G(p-q)$$

This type of propagator gives a gap that is proportional to chemical potential μ as $\Delta \sim \mu\, e^{-\frac{\mathrm{const}}{g}}$ in weak coupling limit. (D. Son, PRD 59, 094019 (1999); R. Pisarski, D. Rischke, PRD 61, 074017 (2000))
The conventional CSC in QCD has been studied in the Abelian approximation and thus obtains the same type of pairing as in QED.

(M. G. Alford, et al, RMP 80, 1455 (2008). D. Nickel, et al, PRD 73, 114028 (2006))

The non Abelian feature of the interaction in QCD induces a completely different type of pairing.

The DSEs in Nambu Gorkov basis

To study the quark pairing in QCD, one needs to compute the gap equation i.e. the quark propagator Dyson-Schwinger equation, in the Nambu-Gorkov basis. It is to extend the fermion field as:

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \overline{\Psi} = \left(\overline{\psi}, \overline{\psi}_C\right),$$

with $\psi_C = C\psi^*$ the charge-conjugator spinor obtained through the charge conjugation matrix $C = \gamma^2\gamma^4$. The free quark propagator becomes:

$$\mathbf{S}_0^{-1}(p) = \begin{pmatrix} i\gamma \cdot p - \gamma_4\mu, & 0 \\ 0, & i\gamma \cdot p + \gamma_4\mu \end{pmatrix}$$

The general form of the quark self energy and quark propagator read:

$$\Sigma(p) \equiv \begin{pmatrix} \Sigma_+(p), & \Phi_-(p) \\ \Phi_+(p), & \Sigma_-(p) \end{pmatrix}, \qquad \mathbf{S} = \begin{pmatrix} S_+, & T_- \\ T_+, & S_- \end{pmatrix}$$

The off diagonal part of self energy defines the pairing gap as: $\Phi_\pm = \mathcal{M}\gamma_5\Delta_\pm$

The scheme of functional QCD methods

To solve the gap equation, the truncation is required as the equations are not closed. The information of gluon propagator and quark gluon vertex is required

QCD in vacuum:

Cyrol, Mitter, Pawlowski, Strodhoff, PRD 97 (2018) 5, 054006.

Binosi, Chang, Papavassiliou, Qin, Roberts, PLB 742, (2015) 183

Williams, Fischer, Heupel, PRD 93, (2016)034026.

Mitter, Pawlowski, Strodthoff, PRD 91, (2015)054035.

Qin, Chang, Liu, Roberts, Schmidt, PLB 722 (2013) 384

Chang, Roberts, PRL 106 (2011) 072001 ...

Yang-Mills sector:

Eichmann, Pawlowski, Silva, PRD 104 (2021) 11, 114016

Aguilar, Ferreira, Papavassiliou, PRD 105 (2022) 1, 014030

Huber, PR 879, 1 (2020)

Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Aguilar, Binosi, Papavassiliou, PRD 86 (2012) 014032 ...

Phase Structure:

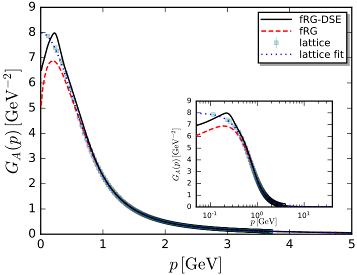
Fu, Pawlowski, Renneke, PRD 101 (2020) 5, 054032; Gao, Chen, Liu, Roberts, Schmidt, PRD 93 (2016) 9, 094019; Fischer, PPNP 105,(2019)1;Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022 Isserstedt, Buballa, Fischer, PRD 100 (2019) 7, 074011; Qin, Chang, Chen, Liu, Roberts, PRL106 (2011) 172301...

The minimal requirements for a truncation scheme that describes QCD:

- *Describe the running mass of quark and gluon*
- *Describe the running of the coupling*

Gluon propagator:

The Yang-Mills sector is relatively separable. One can apply the data of gluon propagator in vacuum:



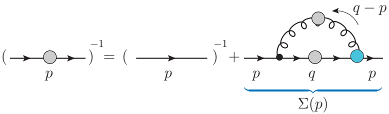
Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

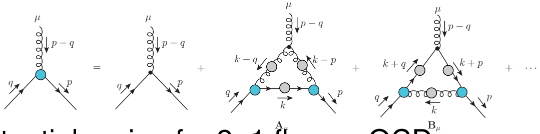
Compute the difference between finite T/μ and vacuum:

$$G^{-1}_{\mu\nu}(k)|_{T,\boldsymbol{\mu}} = G^{-1}_{\mu\nu}(k)|_{0,\mathbf{0}} + \Delta\Pi^{\text{gauge}}_{\mu\nu}(k) + \Delta\Pi^{\text{qrk}}_{\mu\nu}(k)$$

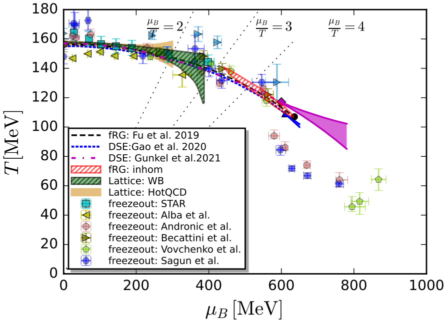
Coupled DSEs of quark propagator and quark gluon vertex

Coupled DSEs of quark propagator and quark gluon vertex can be solved:





Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



The fQCD computations of chiral phase transition are converging:

- $T_c = 155\,\mathrm{MeV}$ and $\kappa \sim 0.016$
- Estimated range of CEP: $T \in (100, 110)$ MeV $\mu_B \in (600, 700)$ MeV

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and Jan M. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

A further simplification on the quark gluon vertex:

Quark gluon vertex In Landau gauge:

$$\Gamma^{\mu}(q,p)=\sum_{i=1}^{8}t_i(q,p)P^{\mu\nu}(q-p)\mathcal{T}_i^{\nu}(q,p)\,,$$

The dominant structures are Dirac and Pauli term:

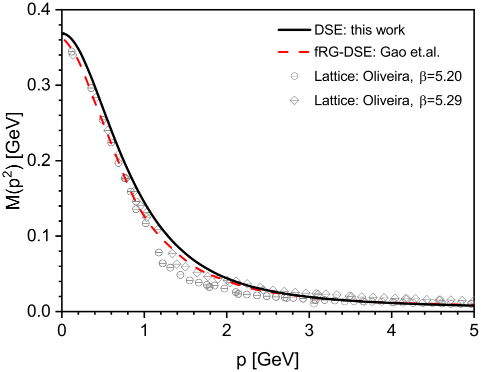
$$\mathcal{T}_1(p,q)=-i\gamma^{\mu}\,,\mathcal{T}_4^{\mu}(p,q)=\sigma_{\mu\nu}(p-q)^{\nu}\,,$$

$$t_1(p,q)=F(k^2)\frac{A(p^2)+A(q^2)}{2}$$

$$t_4(p,q)=\left[Z(k^2)\right]^{-1/2}\frac{B(p^2)-B(q^2)}{p^2-q^2}$$

L. Chang, YX Liu, and C. D. Roberts, PRL 106, 072001(2011)
SX Qin, L. Chang, YX Liu, C. D. Roberts, S. M. Schmidt, PLB 722, 384(2013)
Y, Lu, **FG**, YX Liu, J. Pawlowski, arXiv:2310.16345.

All quantities are expressed by the running of two point functions.
The Quark Mass function:



The quark gluon vertex in NG basis

The quark gluon vertex in NG basis is:

$$\Gamma^a_\nu = \begin{pmatrix} \Gamma^a_{\nu+}, & \Xi^a_{\nu-} \\ \Xi^a_{\nu+}, & \Gamma^a_{\nu-} \end{pmatrix}.$$

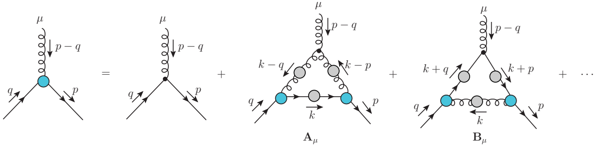
Focus on the chiral symmetric phase above T_c, and one has for the diagonal part:

$$\Gamma^a_\nu(p,q) = \frac{\Lambda^a}{2} F((p-q)^2)\gamma_\nu,$$

For off- diagonal part, only the Pauli term:

$$\Xi^a_{\mu\pm}(p,q) = t_4(p,q)\,\mathcal{K}^a_\pm \sigma_{\mu\nu}(p-q)^\nu\gamma_5, \quad a = 1,2,3.$$

In the vertex DSE,
diagram A is non Abelian diagram and
diagram B is the Abelian diagram similar
to QED.



The quark gluon vertex in NG basis

The dynamics related to diagram A is very different from that from diagram B.

We apply the NJL type approximation for Δ and coefficients t_4 as;

$\Delta(p) = \Delta(\vec{p} = 0; p_4^+ = \pi T + i\mu) = \Delta; t_4(p, q) = t_4(\vec{q}, \vec{p} = 0; q_4^+, p_4^+ = \pi T + i\mu) = t_4$.

With diagram A, the DSE yields for the off-diagonal part $\Xi^a_{\mu\pm}$:

$$t_4 = Z_1\Delta + Zt_4^2\Delta,$$

- In ultraviolet region with for instance $p \to \infty$, the term $Z_1\Delta$ is dominant as Z_1 is proportional to $1/p^2$ and leads to $t_4 \sim \Delta/p^2$.
- In the infrared limit, Z_1 and Z are finite constants. Considering Δ to be small, the two solutions become $t_4 = Z_1\Delta$ and $t_4 = \frac{1}{Z\Delta}$.
- With diagram B, one only gets the first solution $t_4 \propto \Delta$ since there is no t_4^2 term there, it gives the conventional CSC;

- *The second solution is unique in non-Abelian theory.*

FG, Y, Lu, YX Liu, arXiv:2403.16816.

A new type of pairing

- With $t_4 \propto \frac{1}{\Delta}$, the gap equation becomes:

$$\Delta = -\delta_m \Delta + \frac{K}{Z\Delta},$$

$$K = \frac{3}{2} g^2 \int_k \frac{k_4^+ \bar{k}_4 + \vec{k}^2}{k_+^2 + \Delta^2} \left(G_L(\bar{k}^2) + 2G_T(\bar{k}^2)\right),$$

A simple expression for the pairing gap if neglecting quadratic contribution:

$$\Delta = \sqrt{\frac{K}{Z(1+\delta_m)}}\,.$$

$\frac{K}{Z(1+\delta_m)} > 0$, a finite solution for Δ; $\frac{K}{Z(1+\delta_m)} < 0$, the trivial solution as $t_4 = \Delta = 0$.

- With $t_4 \propto \Delta$, the gap equation becomes:

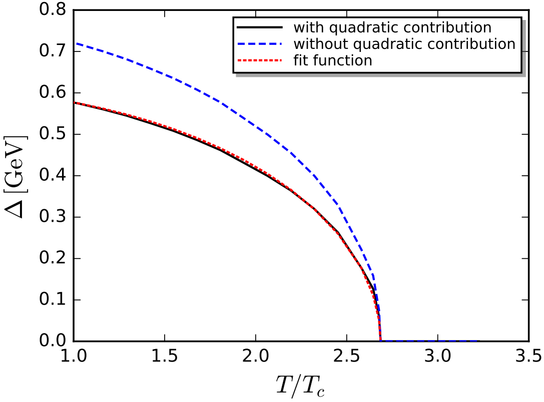
$$\Delta \propto \int_k \frac{\Delta}{k_+^2 + \Delta^2} \left(G_L(\bar{k}^2) + 2G_T(\bar{k}^2)\right),$$

which gives the conventional CSC gap and proportional to chemical potential μ.

A new type of pairing at zero chemical potential

In the gap, Z is always positive and δ_m is small, one can expand K as:

$$K = \frac{3}{2}\langle g^2 A^2 \rangle - \frac{3}{2}\langle g^2 \frac{k_4^+ p_4^+}{k_+^2}(G_L(\bar{k}^2) + 2G_T(\bar{k}^2))\rangle,$$

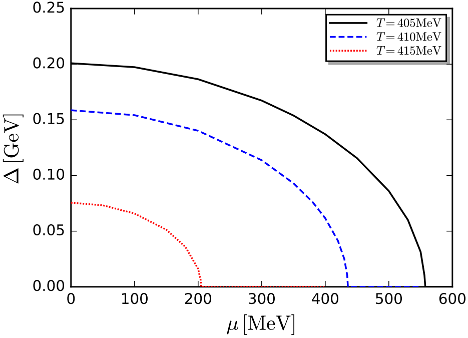


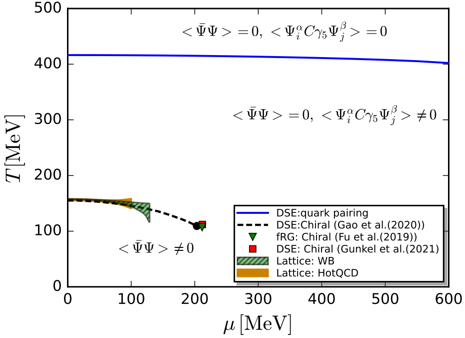
- The quark pairing gap is related to the dimensional 2 gluon condensate and thus dominant by the glue dynamics.
- A second order phase transition at temperature T_Δ, as one has $\Delta = 0$ above T_Δ, and near below T_Δ:

  $$\Delta^2 \propto 1 - (T/T_\Delta)^a,$$

  with the best fit as $a = 2.16$.
- The relation then yields a mean filed critical exponent as $\beta = 1/2$.

Phase diagram of the pairing





The pairing phase in $T-\mu$ plane:

- Represents a color deconfined phase above the chiral phase transition;
- Quarks are confined into colored bound states as a partial deconfined phase;
- Temperature range $T\in[T_c, T_\Delta\approx 2-3\,T_c]$, overlaping with Chiral Spin Symmetric phase and the other proposed strongly coupled states in sQGP.

Outlook:

Momentum dependence of the gap and the vertex coefficients:

- The gluon condensate contains a quadratic divergence that is artificial due to the neglect of the momentum dependence of t_4.
- After incorporating the momentum dependence as $\Delta(p^2)$ and $t_4(p^2, q^2)$, a finite gap can be generated without the bothering of the divergence.
- Further investigations in T-μ_B plane will be done in our future work.

Fermionic mode:

- A fermion mode emerges from an inhomogeneous pairing as $\Delta(\vec{r})$ considering the vortex dynamics .
- The proposed fermionic field with the string-like interaction in chiral spin symmetric phase.

Thank you!