Goldstone Damping and Real-time Dynamics in QCD

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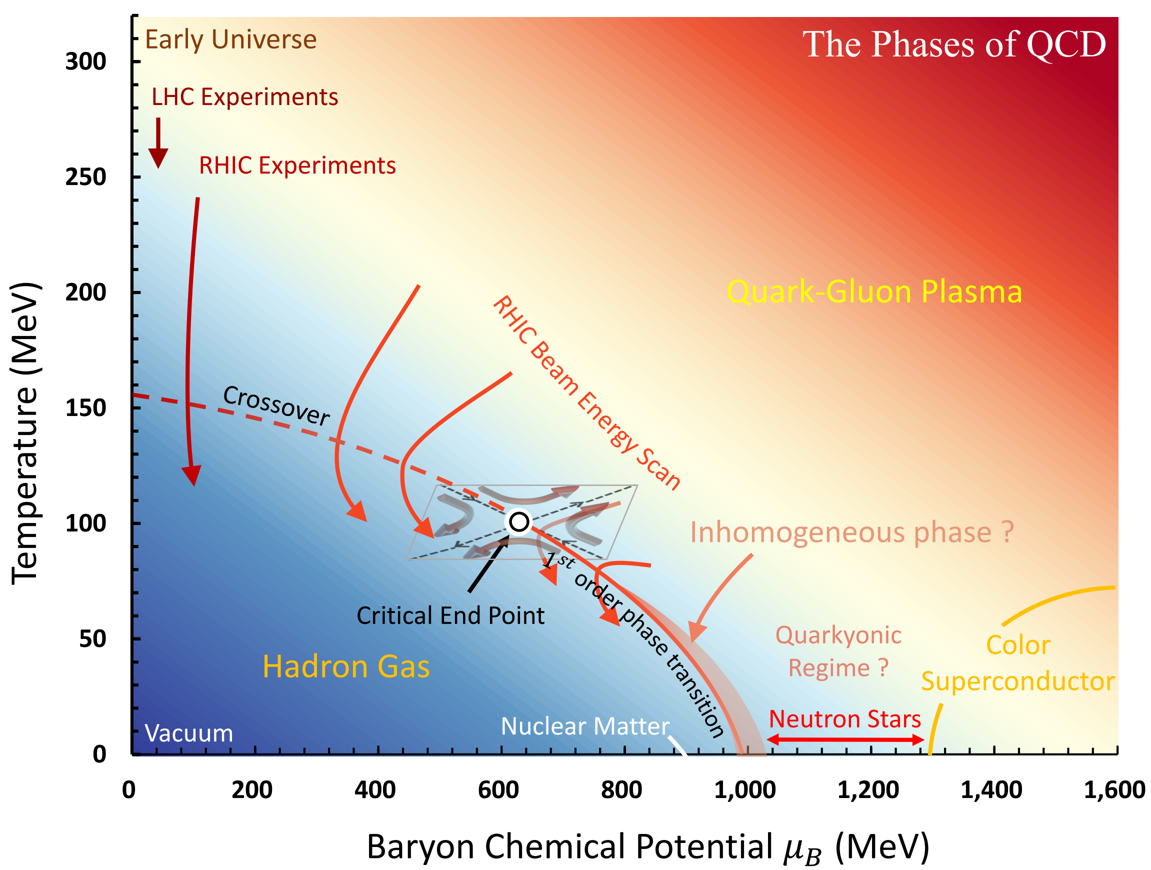
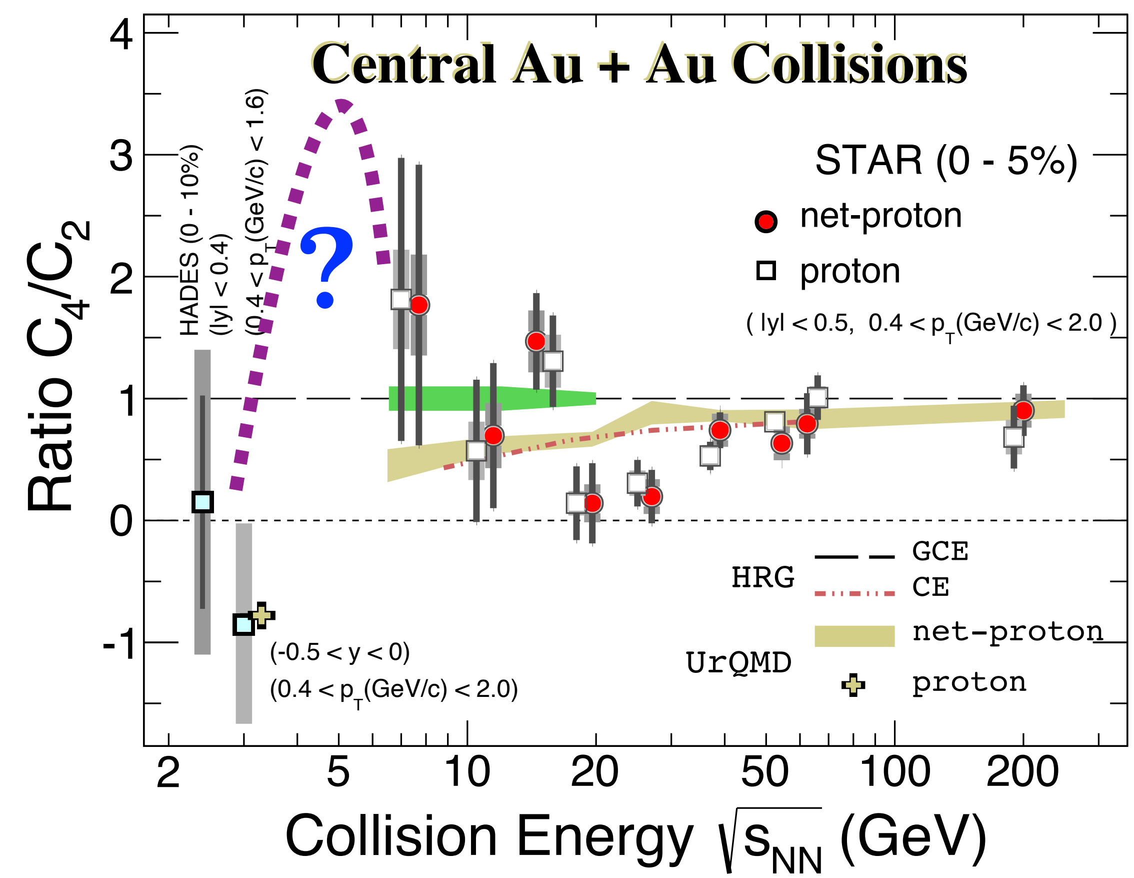
Based on :
Yang-yang Tan, Yong-rui Chen, WF, Wei-Jia Li, arXiv: 2403.03503;
Yang-yang Tan, Shi Yin, Yong-rui Chen, Chuang Huang, WF, in preparation.

QCD phase structure

QCD phase diagram



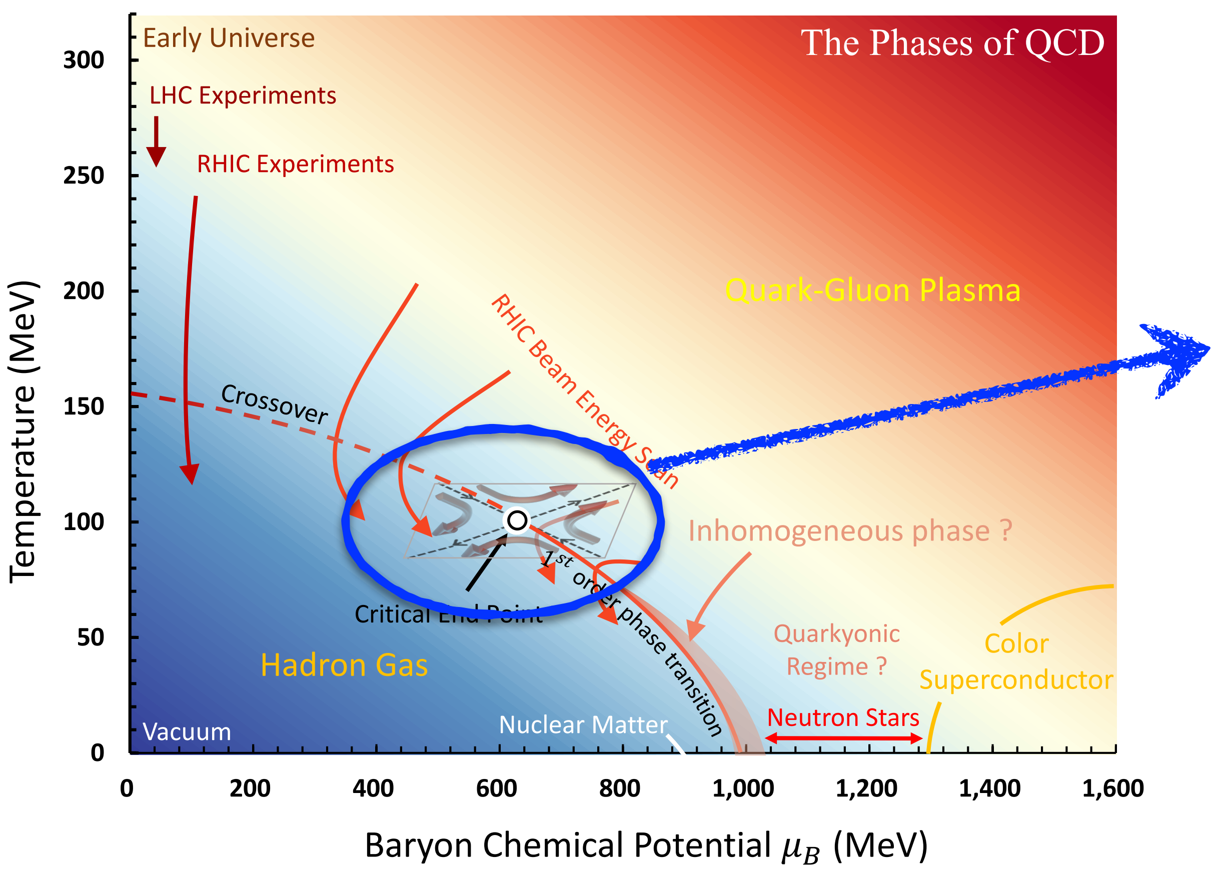
Fluctuations measured by STAR

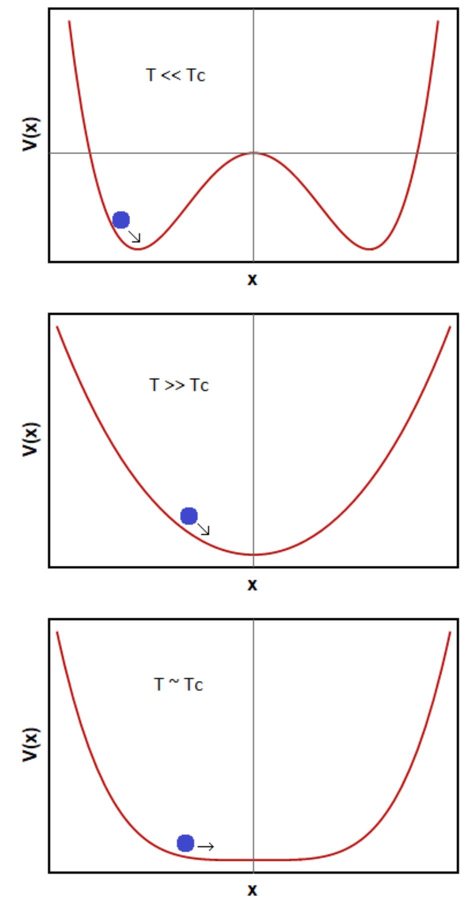


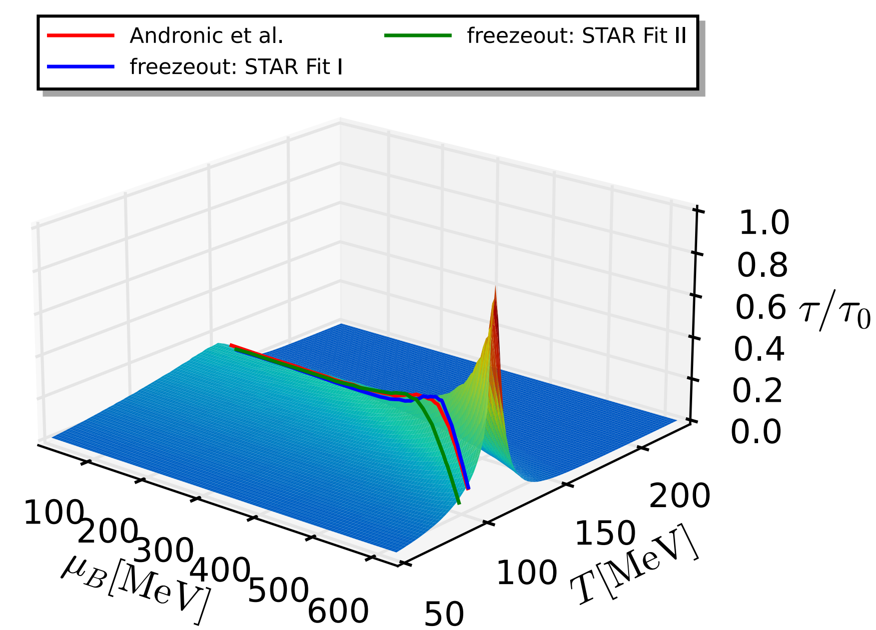
J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301;
M. Abdallah *et al.* (STAR), *PRC* 104 (2021), 024902;
M. Abdallah *et al.* (STAR), *PRL* 128 (2022) 20, 202303

- The non-monotonicity of the kurtosis is observed with 3.1σ significance.
- Is there a “peak” structure in the regime of low colliding energy?

Critical slowing down near QCD critical point







Tan, Yin, Chen, Huang, WF, in preparation

Relaxation time:

$$\tau = \xi^{z} f(k\xi)$$

z: **dynamic critical exponent**

Call for:

- Real-time description of strongly interacting systems.
- Nonperturbative approach of QCD.

Outline

* Introduction
* Brief review about real-time fRG
* Universality of Pseudo-Goldstone damping
* Relaxation dynamics of QCD in phase diagram
* Summary

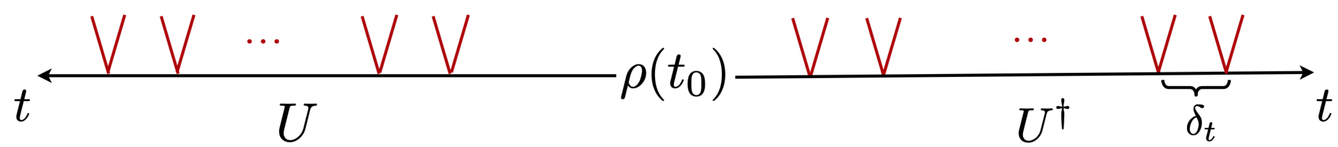
Schwinger-Keldysh path integral

- **Schrödinger equation:**

$$U(t,t_0) = e^{-iH(t-t_0)}$$

$$i\partial_t|\psi(t)\rangle = H|\psi(t)\rangle \longrightarrow |\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle,$$

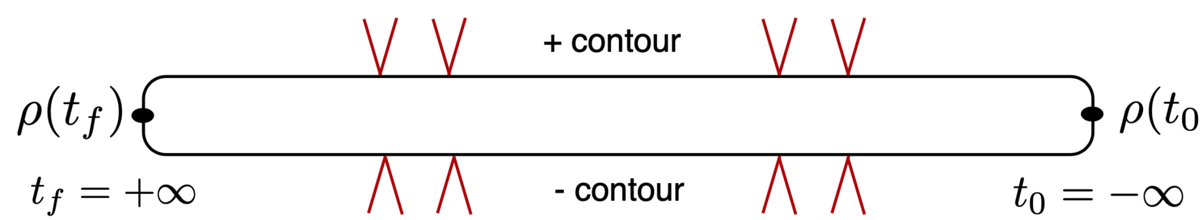
- **von Neumann equation:**



$$\partial_t\rho(t) = -i[H,\rho(t)] \longrightarrow \rho(t) = U(t,t_0)\rho(t_0)U^\dagger(t,t_0),$$

- **Keldysh partition function:**

$$Z = \mathrm{tr}\,\rho(t),$$



- **two-point closed time-path Green's function:**

$$G(x,y) \equiv -i\mathrm{tr}\{T_p\big(\phi(x)\phi^\dagger(y)\rho\big)\}$$

$$\equiv -i\langle T_p\big(\phi(x)\phi^\dagger(y)\big)\rangle\,,$$

$$G(x,y) = \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix}$$

$$\equiv \begin{pmatrix} G_F & G_+ \\ G_- & G_{\tilde{F}} \end{pmatrix}\,,$$

$$G_F(x,y) \equiv -i\langle T\big(\phi(x)\phi^\dagger(y)\big)\rangle\,,$$

$$G_+(x,y) \equiv -i\langle \phi^\dagger(y)\phi(x)\rangle\,,$$

$$G_-(x,y) \equiv -i\langle \phi(x)\phi^\dagger(y)\rangle\,,$$

$$G_{\tilde{F}}(x,y) \equiv -i\langle \tilde{T}\big(\phi(x)\phi^\dagger(y)\big)\rangle\,,$$

Schwinger, J. Math. Phys. 2, 407 (1961);
Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964);
Chou, Su, Hao, Yu, Phys. Rept. 118, 1 (1985).

Functional renormalization group

Functional integral with an IR regulator

$$Z_k[J] = \int (\mathscr{D}\hat{\Phi}) \exp\left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

flow of the Schwinger function:

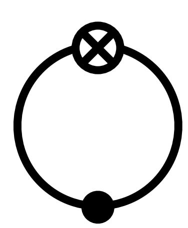
$$\partial_t W_k[J] = -\frac{1}{2}\mathrm{STr}\Big[\big(\partial_t R_k\big) G_k\Big] - \frac{1}{2}\Phi_a \partial_t R_k^{ab} \Phi_b$$

Legendre transformation:

$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

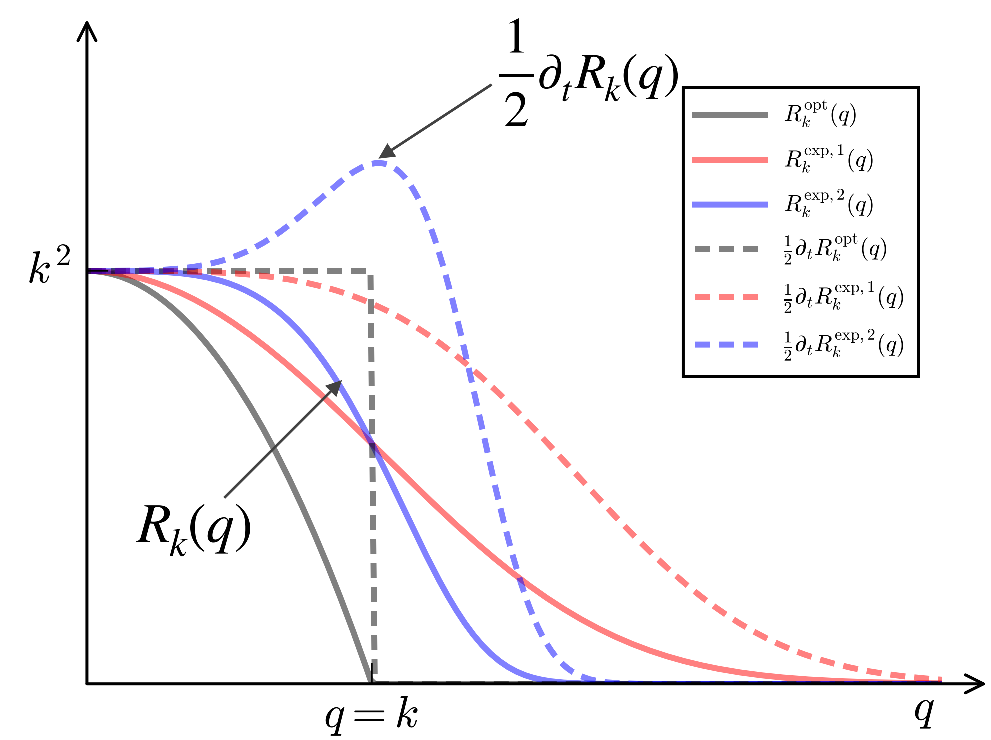
flow of the effective action:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\Big[\big(\partial_t R_k\big) G_k\Big] = \frac{1}{2}$$

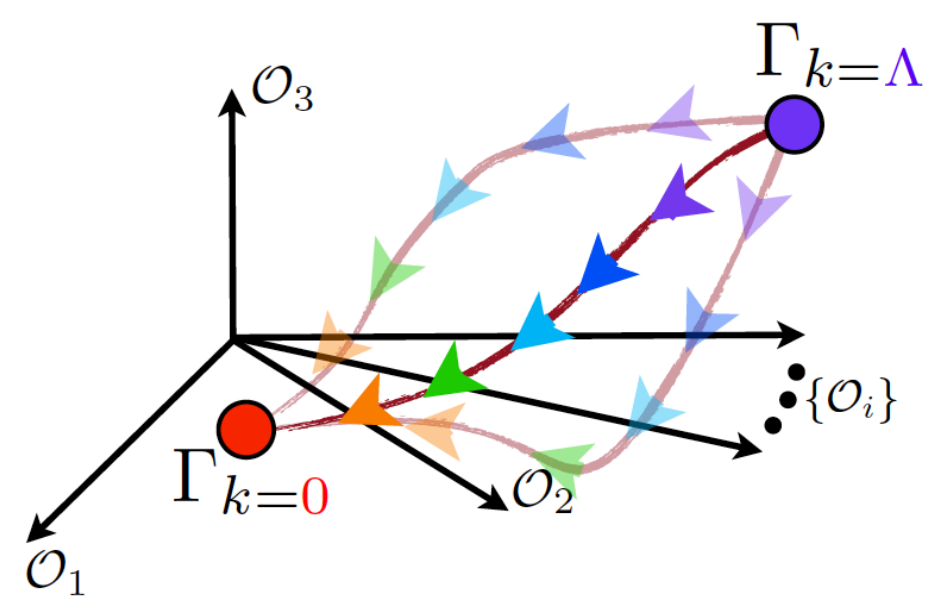


Wetterich equation

C. Wetterich, *PLB*, 301 (1993) 90



$$G_{k,ab} = \gamma^c{}_a \Big(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi]\Big)^{-1}_{cb},$$



Review: WF, *CTP* 74 (2022) 097304, arXiv: 2205.00468 [hep-ph]

FRG in Keldysh path integral

- **Implement the formalism of fRG in the two time branches:**

$$Z_k[J_c, J_q] = \int (\mathscr{D}\varphi_c \mathscr{D}\varphi_q) \exp\left\{ i\left(S[\varphi] + \Delta S_k[\varphi] + (J_q^i \varphi_{i,c} + J_c^i \varphi_{i,q}) \right) \right\},$$

with

$$\Delta S_k[\varphi] = \frac{1}{2}(\varphi_{i,c}, \varphi_{i,q}) \begin{pmatrix} 0 & R_k^{ij} \\ (R_k^{ij})^* & 0 \end{pmatrix} \begin{pmatrix} \varphi_{j,c} \\ \varphi_{j,q} \end{pmatrix}$$

$$= \frac{1}{2}\left(\varphi_{i,c} R_k^{ij} \varphi_{j,q} + \varphi_{i,q} (R_k^{ij})^* \varphi_{j,c} \right),$$

Keldysh rotation:

$$\begin{cases} \varphi_{i,+} = \frac{1}{\sqrt{2}}(\varphi_{i,c} + \varphi_{i,q}), \\ \varphi_{i,-} = \frac{1}{\sqrt{2}}(\varphi_{i,c} - \varphi_{i,q}), \end{cases}$$

- **Then we derive the flow equation in the closed time path:**

$$\partial_\tau \Gamma_k[\Phi] = \frac{i}{2} \mathrm{STr}\left[(\partial_\tau R_k^*) G_k \right],$$

$$R_k^{ab} \equiv \begin{pmatrix} 0 & R_k^{ij} \\ (R_k^{ij})^* & 0 \end{pmatrix},$$

$$iG(x, y) = \begin{pmatrix} iG^K(x, y) & iG^R(x, y) \\ iG^A(x, y) & 0 \end{pmatrix},$$

$$iG^R(x, y) = \theta(x^0 - y^0) \langle [\phi(x), \phi^*(y)] \rangle,$$
$$iG^A(x, y) = \theta(y^0 - x^0) \langle [\phi^*(y), \phi(x)] \rangle,$$
$$iG^K(x, y) = \langle \{ \phi(x), \phi^*(y) \} \rangle,$$

Tan, Chen, WF, *SciPost Phys*. 12 (2022) 026,
arXiv: 2107.06482

A relaxation critical O(N) model

- **The effective action on the Schwinger-Keldysh contour reads**

Hohenberg and Halperin, *Rev. Mod. Phys*. 49 (1977) 435.

Model A

$$\Gamma[\phi_c, \phi_q] = \int d^4x \left(Z_a^{(t)} \phi_{a,q} \partial_t \phi_{a,c} - Z_a^{(i)} \phi_{a,q} \partial_i^2 \phi_{a,c} + V'(\rho_c) \phi_{a,q} \phi_{a,c} - 2 Z_a^{(t)} T \phi_{a,q}^2 - \sqrt{2} c \, \sigma_q \right)$$

$\Gamma = 1/Z_a^{(t)}$: relaxation rate

$Z_a^{(i)}$: wave function

$V'(\rho_c)$: potential $\rho_c \equiv \phi_c^2/4$

c: explicit breaking

Gaussian white noise with coefficient determined by fluctuation-dissipation theorem

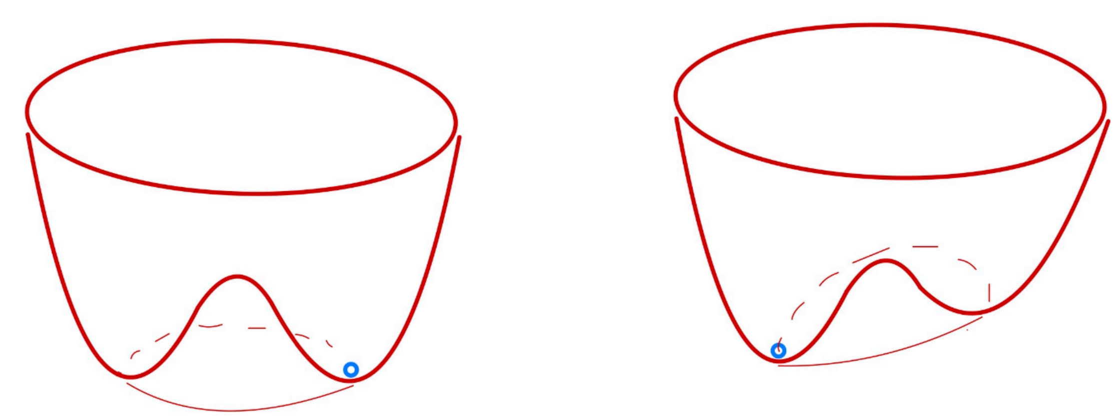
- **Retarded propagator**

$$G_{ab}^R = \left(\frac{\delta^2 \Gamma[\phi_c, \phi_q]}{\delta \phi_{a,q} \, \delta \phi_{b,c}} \right)^{-1}$$

Retarded propagator of Goldstone

$$G_{\varphi\varphi}^R(\omega, q) = \frac{1}{-\mathrm{i} Z_\varphi^{(t)} \omega + Z_\varphi^{(i)} \left(q^2 + m_\varphi^2 \right)}$$

pseudo-Goldstone:



Mass of pseudo-Goldstone

$$m_\varphi^2 = \frac{V'(\rho_0)}{Z_\varphi^{(i)}} = \frac{c}{\sigma_0 Z_\varphi^{(i)}}$$

Gell-Mann--Oakes--Renner (GMOR) relation

Universal damping or not?

From the pole of the retarded propagator of Goldstone

$$G^R_{\varphi\varphi}(\omega, q) = \frac{1}{-\mathrm{i}Z^{(t)}_\varphi \omega + Z^{(i)}_\varphi \left(q^2 + m_\varphi^2\right)}$$

One obtains the dispersion relation of a damped mode

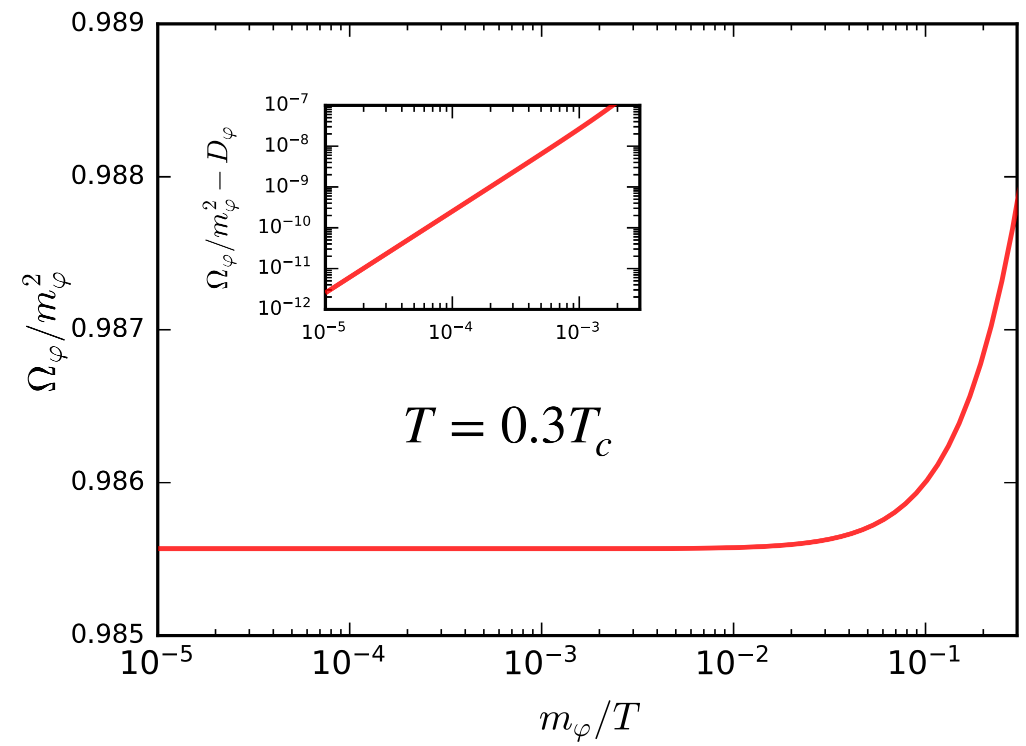
$$\omega(q) = -\,\mathrm{i}\,\frac{Z^{(i)}_\varphi}{Z^{(t)}_\varphi}\left(m_\varphi^2 + q^2\right)$$

The relaxation rate at zero momentum reads

$$\Omega_\varphi \equiv -\,\mathrm{Im}\,\omega(q=0) = \frac{Z^{(i)}_\varphi}{Z^{(t)}_\varphi}\, m_\varphi^2$$

- **If** $T \ll T_c$

$$\frac{\Omega_\varphi}{m_\varphi^2} \simeq D_\varphi(T) + \mathcal{O}\left(\frac{m_\varphi^2}{T^2}\right) \quad \textbf{with} \quad D_\varphi(T) \equiv \frac{Z^{(i)}_\varphi(T, c=0)}{Z^{(t)}_\varphi(T, c=0)}$$



Tan, Chen, WF, Li, arXiv: 2403.03503

This seemingly appears as a **universal** relation that was also observed in Holographics, Hydrodynamics, and EFT

Holographics:
Amoretti, Areán, Goutéraux, Musso, *PRL* 123 (2019) 211602;
Amoretti, Areán, Goutéraux, Musso, *JHEP* 10 (2019) 068;
Ammon *et al*., *JHEP* 03 (2022) 015;
Cao, Baggioli, Liu, Li, *JHEP* 12 (2022) 113

Hydrodynamics:
Delacrétaz, Goutéraux, Ziogas, *PRL* 128 (2022) 141601

EFT:
Baggioli, *Phys. Rev. Res.* 2 (2020) 022022;
Baggioli, Landry, *SciPost Phys*. 9 (2020) 062

Breaking down of the universal damping in the critical region

In the critical region, the two wave function renormalizations read

$$Z_\varphi^{(i)} = t^{-\nu\eta} f^{(i)}(z)\,, \qquad Z_\varphi^{(t)} = t^{-\nu\eta_t} f^{(t)}(z)$$

Here $f^{(i)}(z), f^{(t)}(z)$: scaling functions; $z \equiv tc^{-1/(\beta\delta)}$: scaling variable; $t \equiv (T_c - T)/T_c$: reduced temperature.The static and dynamic anomalous dimensions are

$$\eta = -\frac{\partial_\tau Z_\varphi^{(i)}}{Z_\varphi^{(i)}}\,, \qquad \eta_t = -\frac{\partial_\tau Z_\varphi^{(t)}}{Z_\varphi^{(t)}}$$

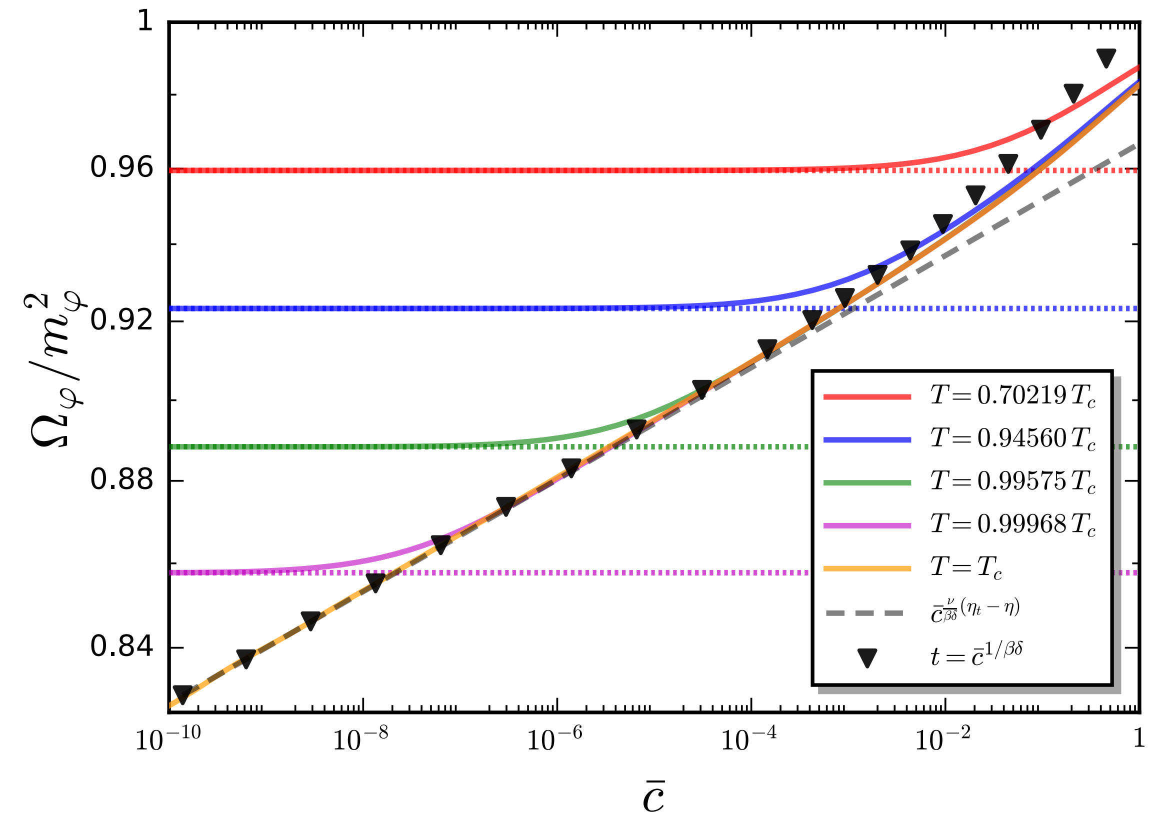
RG time $\tau = \ln(k/\Lambda)$

- In the case of $c \to 0$

$$\frac{Z_\varphi^{(i)}}{Z_\varphi^{(t)}} \propto t^{\nu(\eta_t - \eta)}$$

- In the other case of $t \to 0$

$$\frac{Z_\varphi^{(i)}}{Z_\varphi^{(t)}} \propto c^{\frac{\nu}{\beta\delta}(\eta_t - \eta)} \propto m_\varphi^{(\eta_t - \eta)} \quad \text{with} \quad m_\varphi^2 \propto c^{\frac{2\nu}{\beta\delta}}$$



Tan, Chen, WF, Li, arXiv: 2403.03503

From the fixed-point equation we determine in the O(4) symmetry

$$\eta \approx 0.0374, \quad \eta_t \approx 0.0546$$

Thus

$$\Delta_\eta \equiv \eta_t - \eta \approx 0.0172$$

Estimate of size of the dynamic critical region:

$$m_{\pi 0} \lesssim 0.1 \sim 1 \text{ MeV}$$

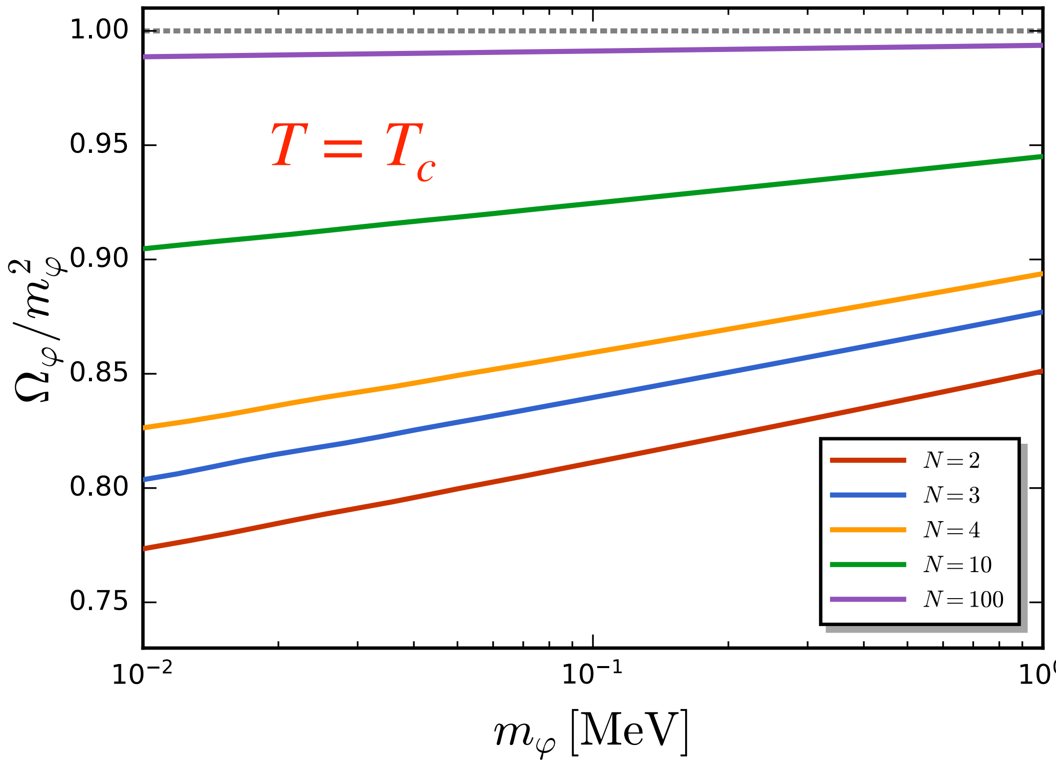
Large N limit

In the large N limit, the static and dynamic anomalous dimensions can be solved analytically

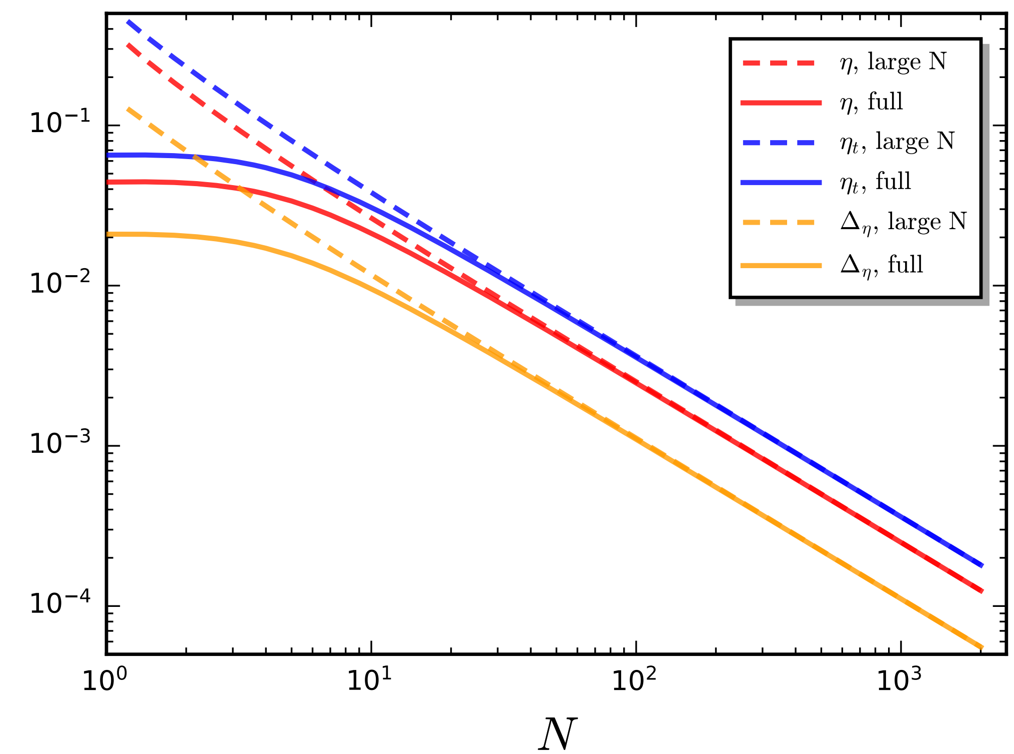
$$\eta = \frac{5}{N-1}\frac{(1+\eta)(1-2\eta)^2}{(5-\eta)(2-\eta)^2}$$

and

$$\eta_t = \frac{1}{9(N-1)}\frac{(1-2\eta)^2(13+15\eta-2\eta^3)}{(2-\eta)^2}$$



Tan, Chen, WF, Li, arXiv: 2403.03503



- In the limit $N \to \infty$, the breaking down of the universal damping disappears.
- One should not expect that the anomalous scaling regime can be observed in classical holographic models.

Relaxation dynamics in QCD phase diagram

- **Langevin equation of the sigma mode**

$$\bar{Z}_\sigma^{(t)}\partial_t\bar{\sigma}_c - \partial_i^2\bar{\sigma}_c + V'(\bar{\rho}_c)\bar{\sigma}_c - \sqrt{2}\,\bar{c} = \bar{\xi}$$

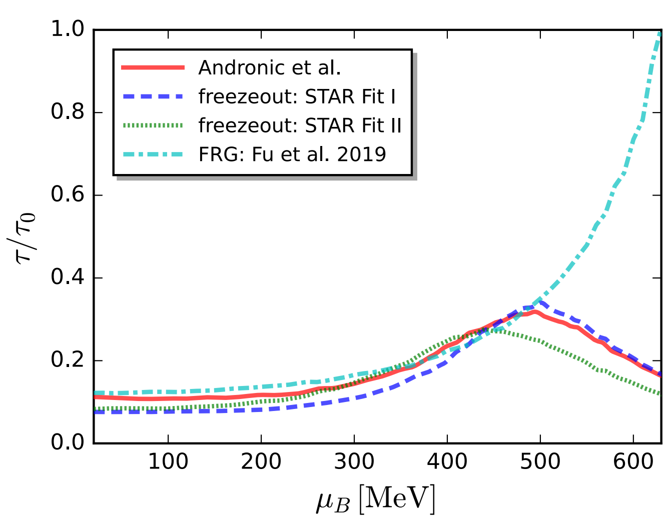
Correlation of white noise:

$$\langle\bar{\xi}(t,\vec{x})\bar{\xi}(t',\vec{x}')\rangle = 4\bar{Z}_\sigma^{(t)}T\delta(t-t')\delta(\vec{x}-\vec{x}')$$

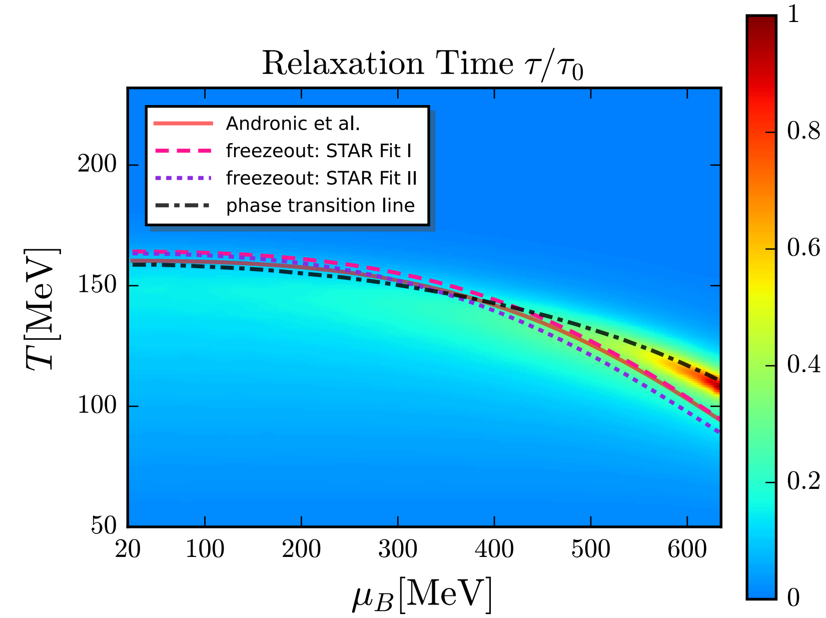
Transport coefficient:

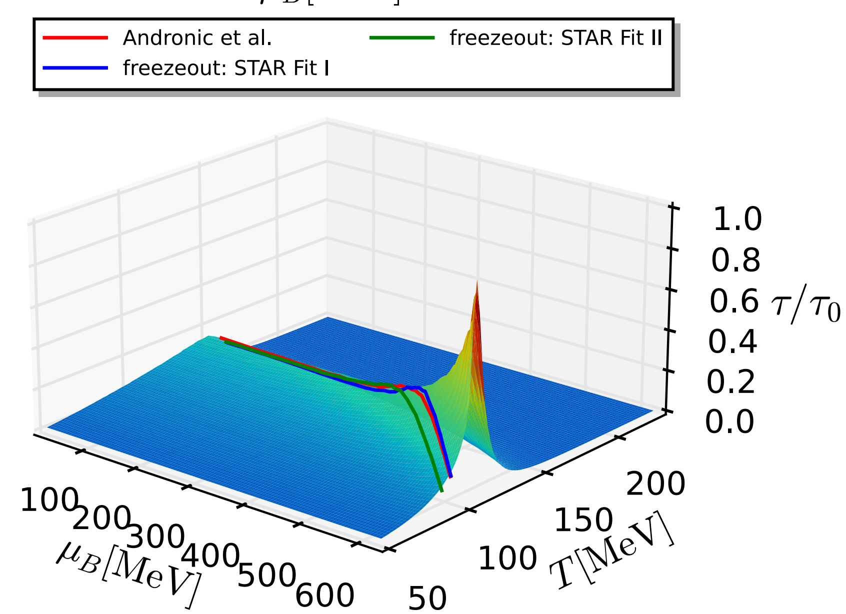
$$\bar{Z}_\sigma^{(t)} = \frac{Z_\sigma^{(t)}}{Z_\sigma^{(i)}} = \frac{1}{Z_\sigma^{(i)}}\lim_{|\boldsymbol{p}|\to 0}\lim_{p_0\to 0}\frac{\partial}{\partial p_0}\Im\Gamma^{(2)}_{\sigma_q\sigma_c}\left(p_0, |\boldsymbol{p}|\right)$$

Both $\bar{Z}_\sigma^{(t)}$ and the potential $V'(\bar{\rho}_c)$ can be input from the first-principles functional QCD at finite temperature and density (WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032).



Relaxation time:

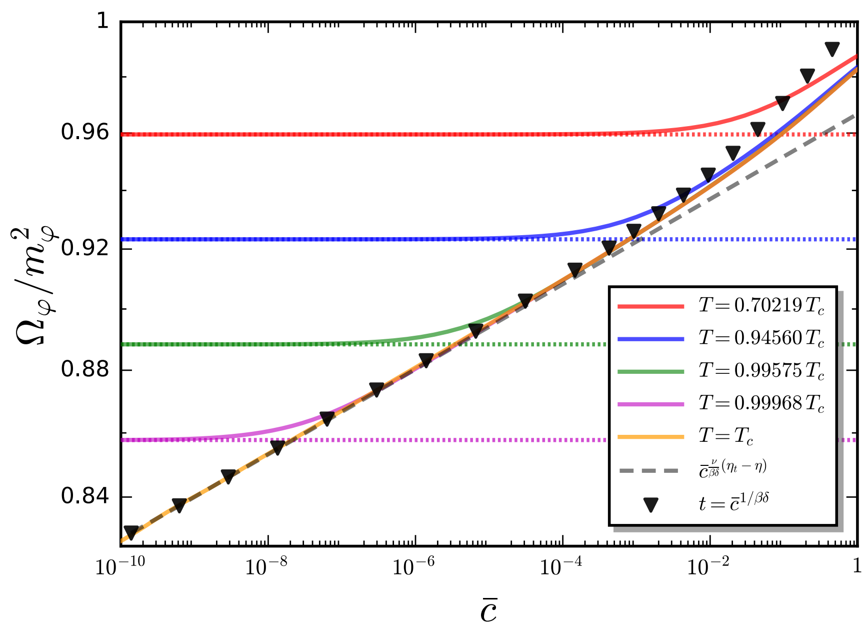


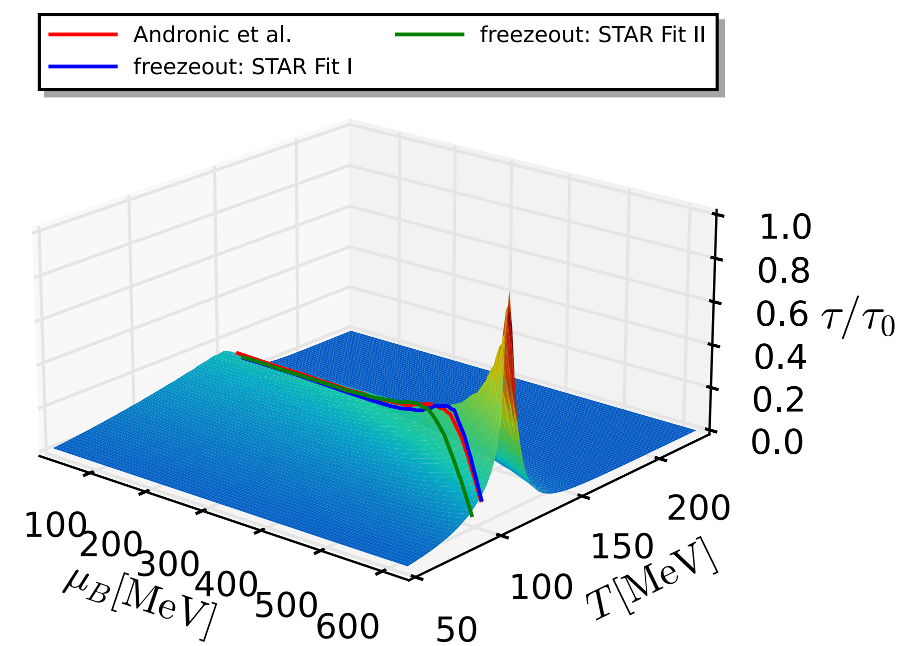


Tan, Yin, Chen, Huang, WF, in preparation

- Relaxation time drops quickly once the system is away from the critical regime.

Summary

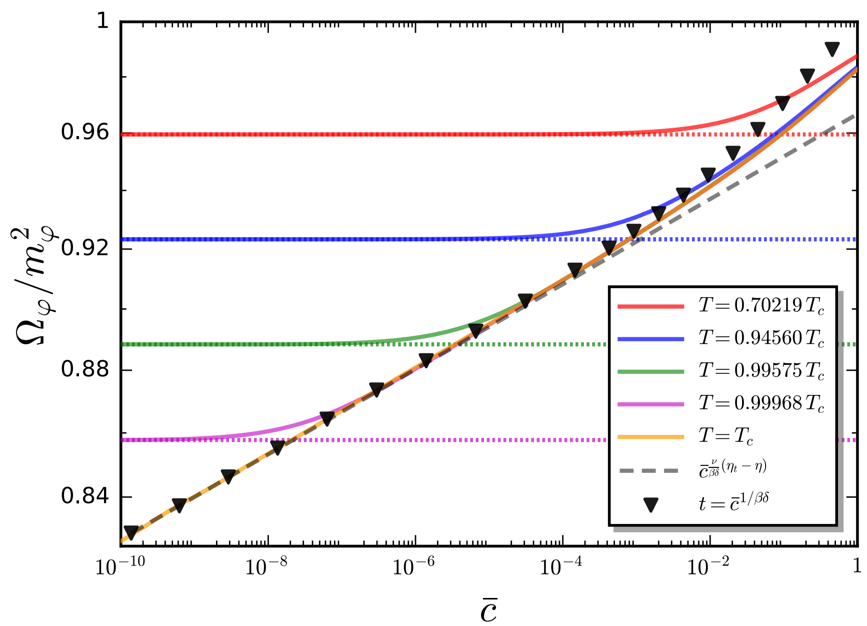


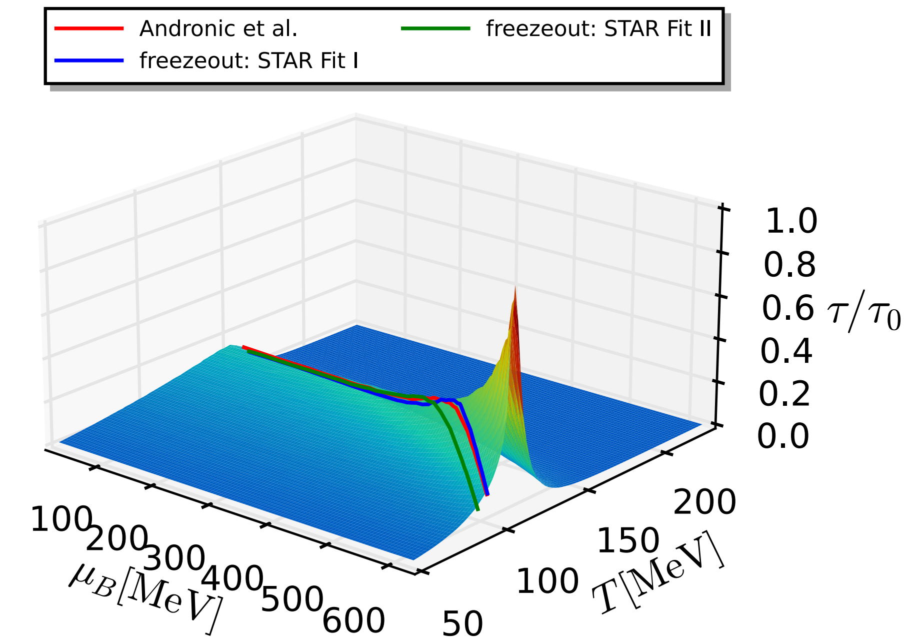


★ Two different universalities of pseudo-Goldstone damping are found.

★ Relaxation time drops quickly once the system is away from the critical end point.

Summary





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Thank you very much for your attentions!