第二届超边缘碰撞物理研讨会
Strong Electromagnetic Fields, UPC and EIC/EicC

Perturbative calculation of spin polarization in central, peripheral and ultraperipheral heavy-ion collisions

Shanshan Cao
Shandong University

April 14, 2024 @ USTC

Outline

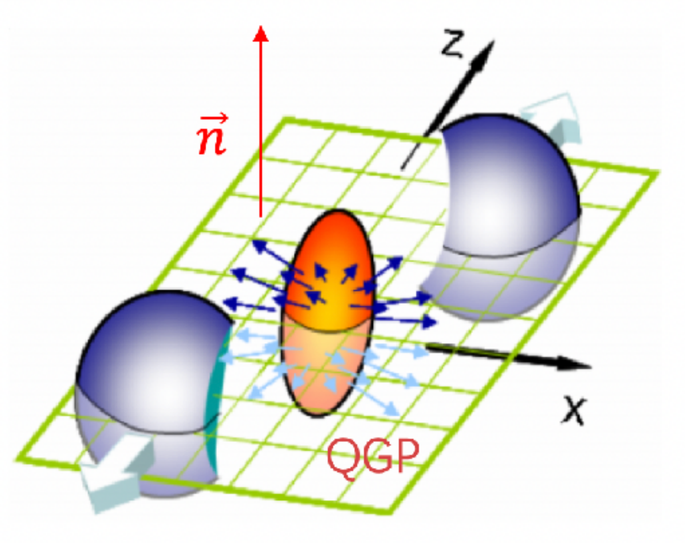
- **Introduction**
- **Production and evolution of the QGP global polarization**

  [X. Li, Z.-F. Jiang, SC, J. Deng, Eur. Phys. J. C 83 (2023) 1, 96]
- **Spin polarization from jet fragmentation**

  [X. Li, Z.-X. Chen, SC, S.-Y. Wei, Phys. Rev. D 109 (2024) 1, 014035]
- **Summary**

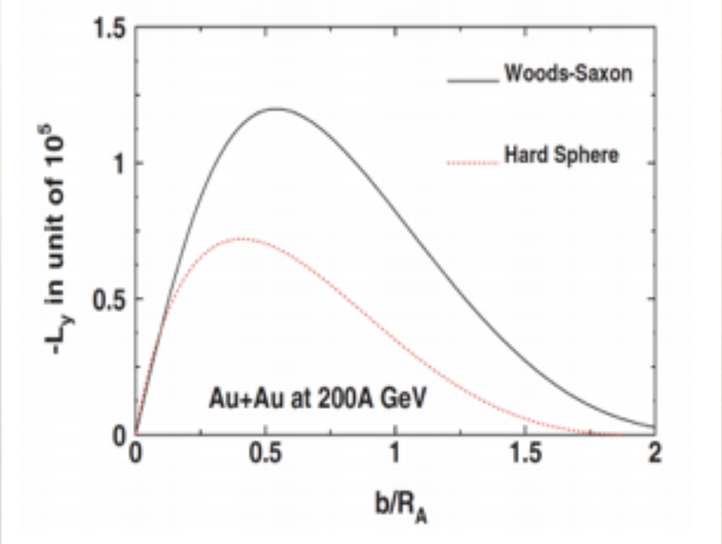
Spin-orbital coupling in a strongly interacting system

Non-central relativistic heavy-ion collisions



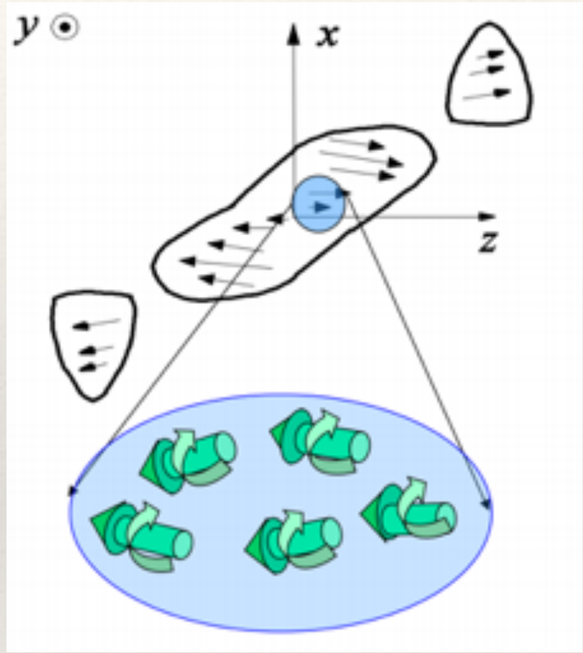
$$\hat{n} = \frac{\vec{b} \times \vec{P}}{\left| \vec{b} \times \vec{P} \right|} = -\hat{y}$$

Huge orbital angular momentum



J.-H. Gao, S.-W. Chen, W.-T. Deng, Z.-T. Liang, Q. Wang, X.-N. Wang, Phys. Rev. C 77 (2008) 044902

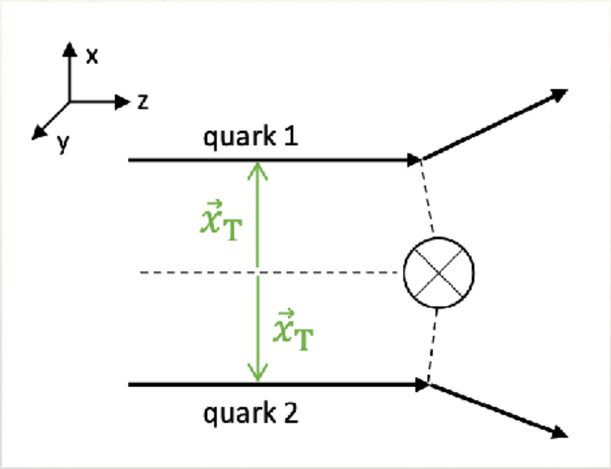
Global polarization



Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94 (2005) 102301

A brief review of the first calculation of global polarization

Quark-potential scattering model [Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94 (2005) 102301]



Consider a quark (quark 1) scattering with a potential

$$A_0(q_T) = g/(q_T^2 + \mu^2)$$

Initial momentum $(E, \vec{p})$, final spin $\lambda/2$ along $\hat{n}$ $(-\hat{y})$

Differential cross section w.r.t. $\vec{x}_T$:

$$\frac{d^2\sigma_\lambda}{d^2x_T} = C_T \int \frac{d^2q_T}{(2\pi)^2}\frac{d^2k_T}{(2\pi)^2} e^{i(\vec{k}_T - \vec{q}_T)\cdot\vec{x}_T} \mathcal{I}_\lambda$$

(Fourier transform into the coordinate space)

$$\mathcal{I}_\lambda = \frac{g^2}{2(2E)^2}\bar{u}_\lambda(p_q)\not{A}(q_T)(\not{p}+m)\not{A}(k_T)u_\lambda(p_k)$$

(Final spin not summed)

A brief review of the first calculation of global polarization

With small angle approximation

Spin-independent cross section:

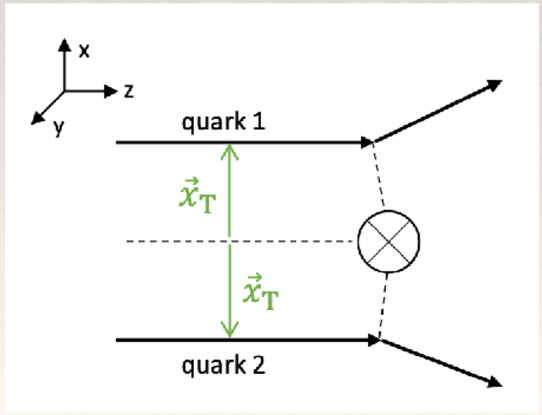
$$\frac{d\sigma}{d^2x_{\mathrm{T}}} = \frac{d\sigma_+}{d^2x_{\mathrm{T}}} + \frac{d\sigma_-}{d^2x_{\mathrm{T}}} = 4C_{\mathrm{T}}\alpha_{\mathrm{s}}^2 K_0^2(\mu x_{\mathrm{T}})$$

Spin-dependent cross section:

$$\frac{d\Delta\sigma}{d^2x_{\mathrm{T}}} = \frac{d\sigma_+}{d^2x_{\mathrm{T}}} - \frac{d\sigma_-}{d^2x_{\mathrm{T}}} = -4C_{\mathrm{T}}\alpha_{\mathrm{s}}^2\mu\frac{\vec{p}\cdot(\hat{x}_{\mathrm{T}}\times\hat{n})}{E(E+m)}K_0(\mu x_{\mathrm{T}})K_1(\mu x_{\mathrm{T}})$$

Polarization:

$$P = \frac{\Delta\sigma}{\sigma} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{\int_0^\infty dx \int_{-\infty}^\infty dy \frac{d\Delta\sigma}{dxdy}}{\int_0^\infty dx \int_{-\infty}^\infty dy \frac{d\sigma}{dxdy}} = -\frac{\pi\mu p}{2E(E+m)}$$

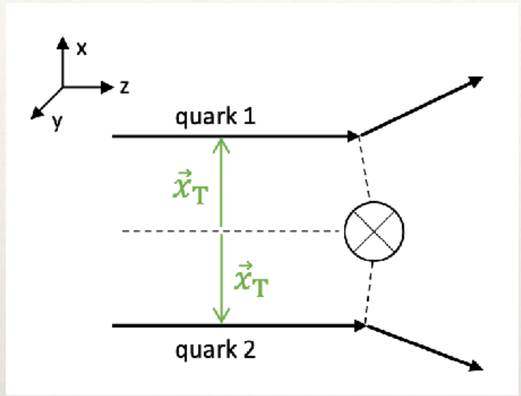


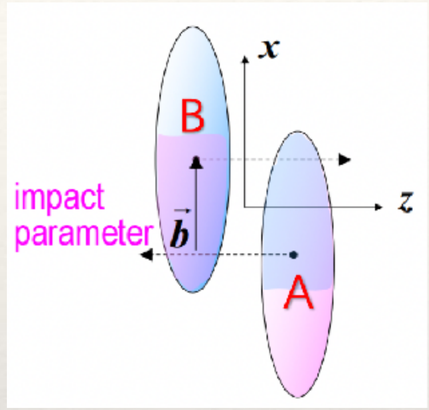
Caveats:

- Integration over only half plane (0 if over the whole plane)
- Approximating the whole nuclear matter with a single potential
- Only single scattering, no evolution

Model improvement

For a single quark from nucleus A scattering with the whole nucleus B





T(*x*,*y*): participant number distribution in nucleus B (Glauber)

$$P = \frac{\Delta\sigma}{\sigma} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{\int_0^\infty dx \int_{-\infty}^\infty dy \frac{d\Delta\sigma}{dxdy}}{\int_0^\infty dx \int_{-\infty}^\infty dy \frac{d\sigma}{dxdy}}$$

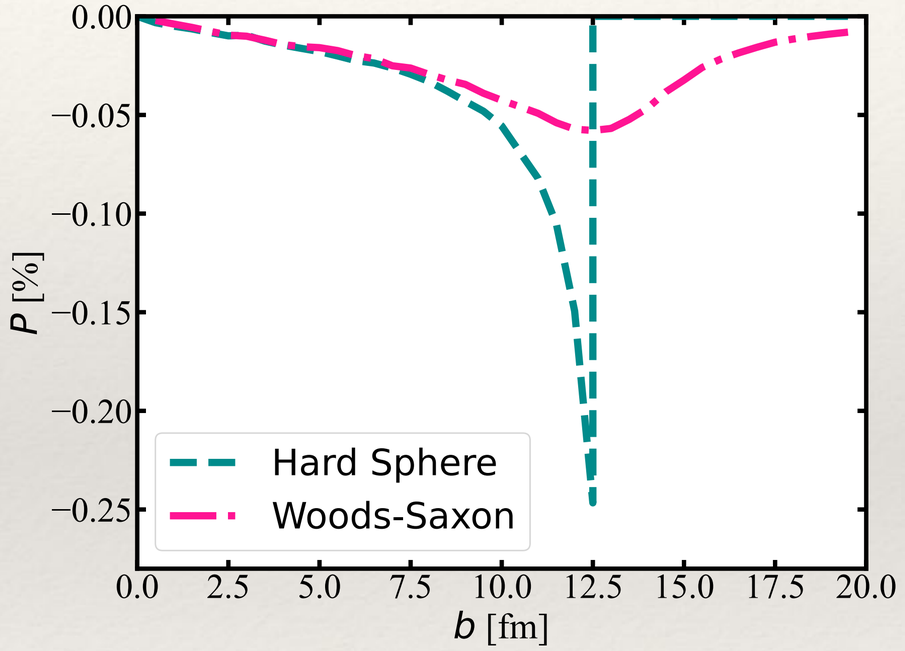
$$P = \frac{\int T(x,y) \frac{d\Delta\sigma(x,y)}{d^2x_T} d^2x_T}{\int T(x,y) \frac{d\sigma(x,y)}{d^2x_T} d^2x_T}$$

Interact with one scattering center in a half plane

Interact with multiple scattering centers distributed according to nuclear density

Transverse plane averaged polarization from initial collisions

$$\overline{P} = \frac{\int [T_1(x,y)\Delta\mathcal{P}_{\rm A}(x,y) + T_2(x,y)\Delta\mathcal{P}_{\rm B}(x,y)]dxdy}{\int [T_1(x,y)\mathcal{P}_{\rm A}(x,y) + T_2(x,y)\mathcal{P}_{\rm B}(x,y)]dxdy}$$



- Overall negative value of polarization (along $-\hat{y}$ direction)
- Noticeable difference between HS and WS calculations only at large impact parameter

Multiple scatterings inside the QGP

For polarized initial quark state [X.-G. Huang, P. Huovinen, X.-N. Wang, Phys. Rev. C 84 (2011) 054910]

$$\frac{d\sigma_\lambda}{dx_T^2} = \frac{1}{2}g^4 C_T\left[\frac{1}{4\pi^2}(1+\lambda_f P_i)K_0^2(\mu x_T) - \frac{\mu}{4\pi^2}K_0(\mu x_T)K_1(\mu x_T)\frac{1}{E(E+m)}(\lambda_f + P_i)\ \vec{p}\cdot(\vec{n}\times\vec{x_T})\right]$$

Spin-independent cross section:

$$\frac{d\sigma}{d^2x_{\mathrm{T}}} = \frac{d\sigma_+}{d^2x_{\mathrm{T}}} + \frac{d\sigma_-}{d^2x_{\mathrm{T}}}$$

Spin-dependent cross section:

$$\frac{d\Delta\sigma}{d^2x_{\mathrm{T}}} = \frac{d\sigma_+}{d^2x_{\mathrm{T}}} - \frac{d\sigma_-}{d^2x_{\mathrm{T}}}$$

Polarization of final quark state:

$$P_f(x,y) = \frac{\int dx_1 dy_1\, s(x_1,y_1)\frac{d\Delta\sigma(x,y,x_1,y_1)}{dx_1 dy_1}}{\int dx_1 dy_1\, s(x_1,y_1)\frac{d\sigma(x,y,x_1,y_1)}{dx_1 dy_1}}$$

(*s*: entropy density of the QGP, from hydrodynamic model)

Time evolution of polarization:

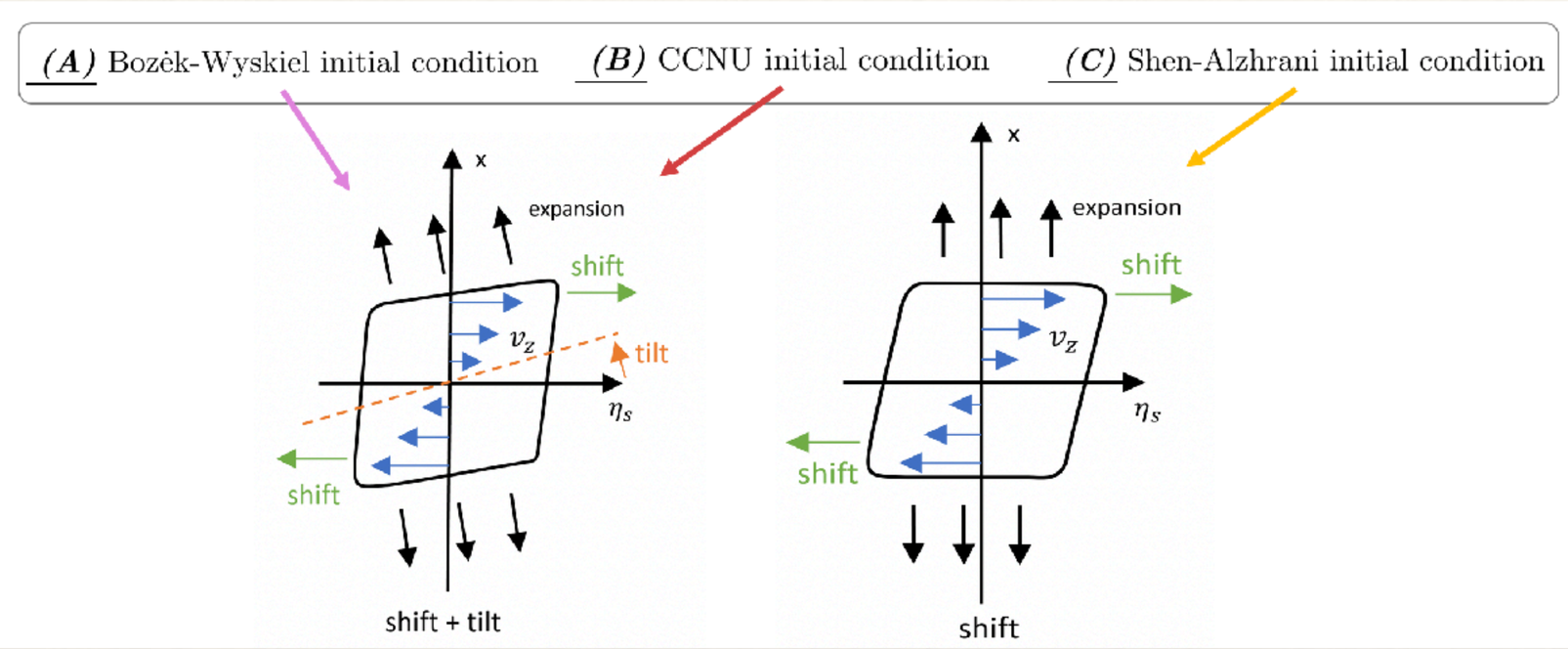
$$\frac{dP(x,y)}{dt} = \frac{\Delta P(x,y)}{\tau_q} = \frac{4T\rho}{9s}\frac{s}{\eta_v}\Delta P(x,y)$$

$(\Delta P = P_f - P_i)$

Application of hydrodynamic model

- (3+1)-D CLVisc hydrodynamic model [L.-G. Pang, H. Petersen, X.-N. Wang, Phys. Rev. C 97 (2018) 064918; X.-Y. Wu, L.-G. Pang, G.-Y. Qin, X.-N. Wang, Phys. Rev. C 98 (2018) 024913]

- Compare three different initial geometries



(A) P. Bozek and I. Wyskiel, Phys. Rev. C 81 (2010) 054902

(B) Z.-F. Jiang, C.-B. Yang, Q. Peng, Phys. Rec. C 104 (2021) 064903

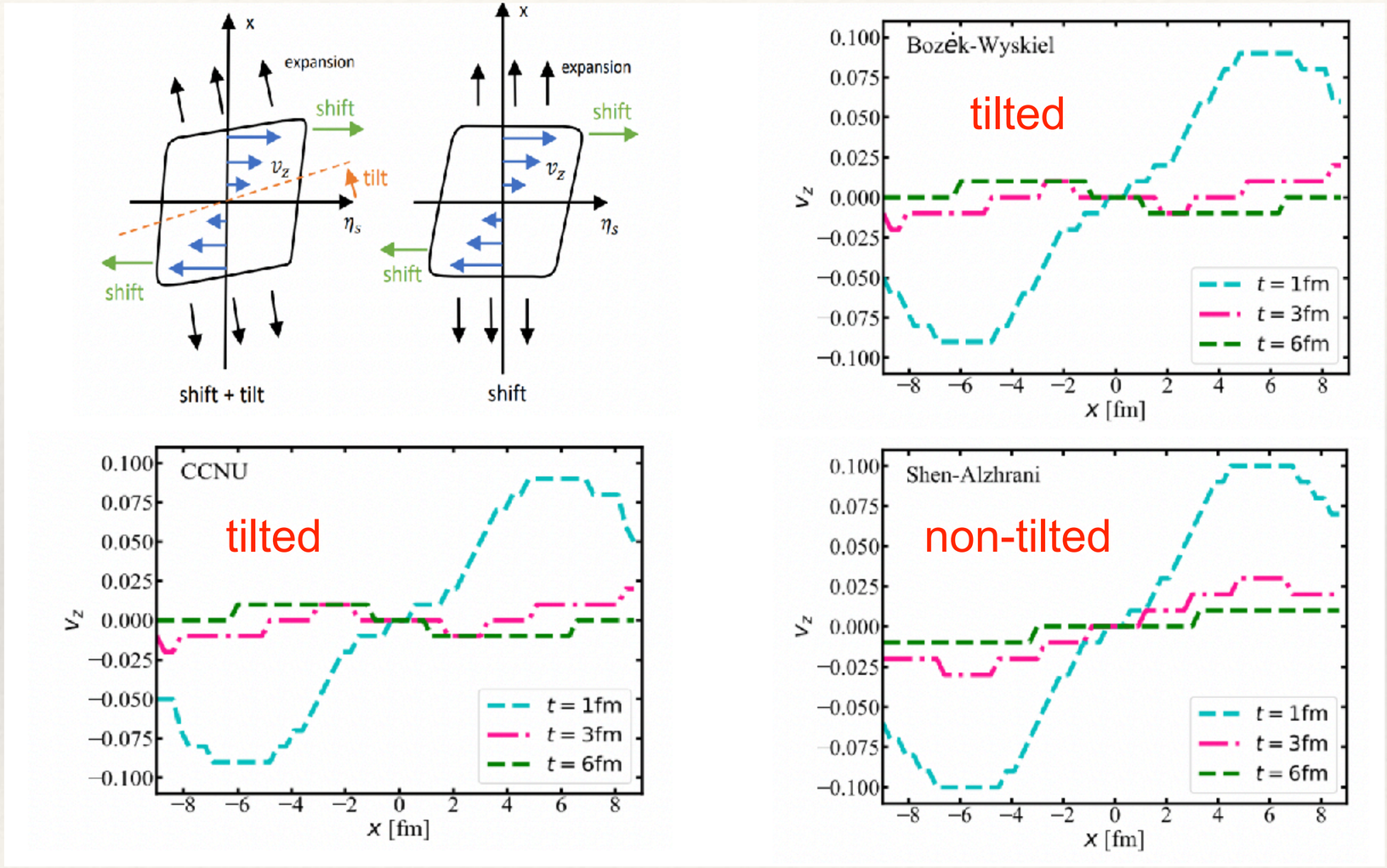
(C) S. Ryu, V. Jupic, C. Shen, Phys. Rev. C 104 (2021) 054908

- Introduce initial longitudinal flow $v_{\eta_s} = T^{\tau\eta}/T^{\tau\tau}$

$$T^{\tau\tau}(x, y, \eta_s) = \epsilon(x, y, \eta_s)\cosh(y_L) \qquad T^{\tau\eta}(x, y, \eta_s) = \frac{1}{\tau_0}\epsilon(x, y, \eta_s)\sinh(y_L)$$

[S. Ryu, V. Jupic, C. Shen, Phys. Rev. C 104 (2021) 054908]

Evolution of longitudinal velocity at $z = 0$ plane



Sensitive to the initial geometry: tilted geometry reduces the longitudinal flow faster, even flip its sign

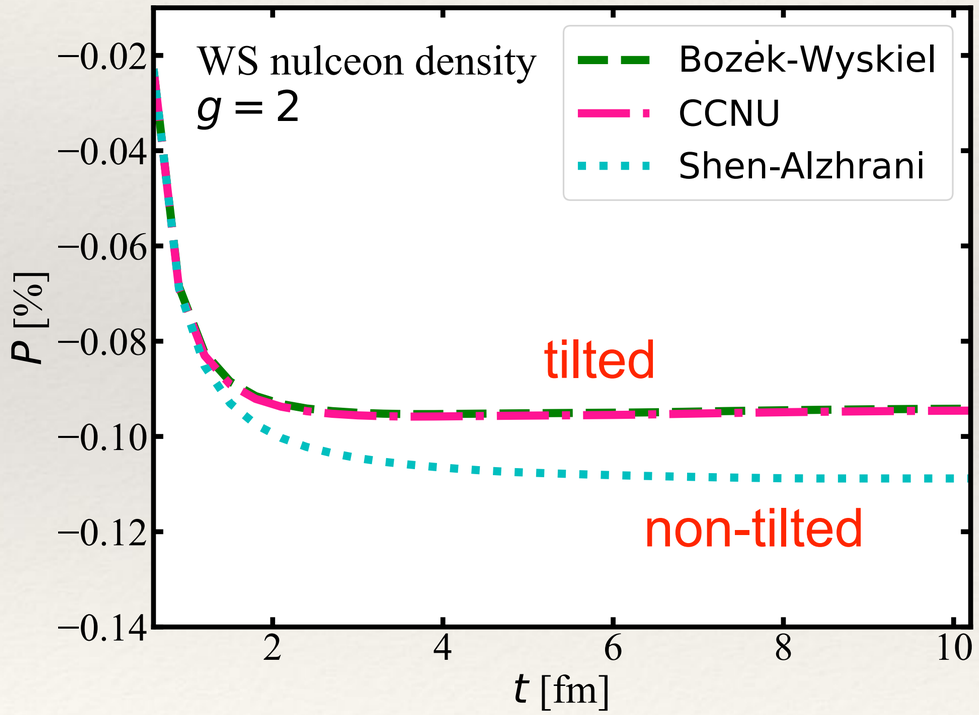
Time evolution of the average global polarization

Increment of global polarization during one time step:

$$\overline{\Delta P}(t) = \frac{\int_{T>T_{\rm frz}} dxdys(x,y,t)\Delta P(x,y,t)}{\int dxdys(x,y,t)}$$

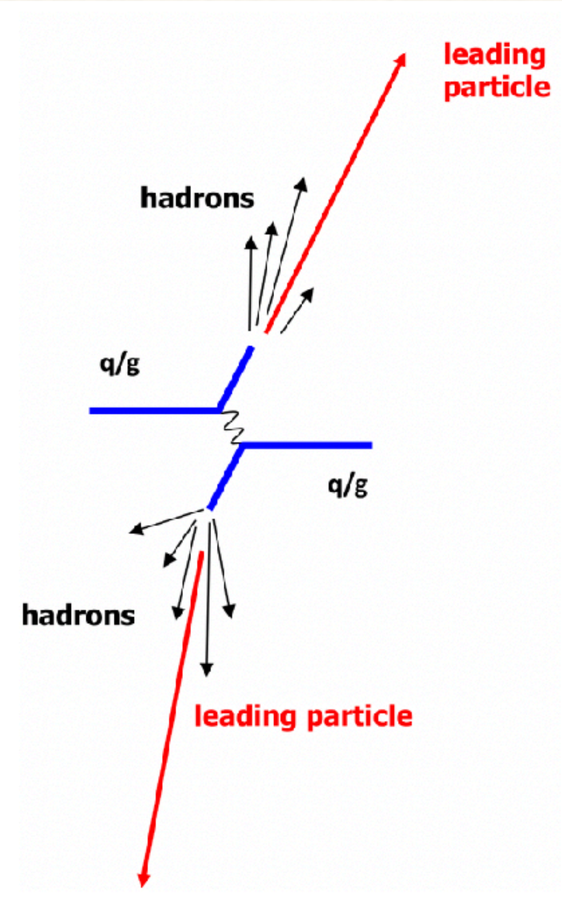
Update of polarization:

$$\overline{P}(t+\Delta t) = \overline{P}(t) + \overline{\Delta P}(t)$$



- Increase (in magnitude) of polarization from an initial value given by initial hard scatterings
- Evolution of polarization in QGP depends on the QGP initial condition
- About 15% difference in the final global polarization due to different initial condition

Another source of hadron polarization — jet fragmentation



- In pp collisions:

$$d\sigma_h = \sum_{abjd} f_{a/p} \otimes f_{b/p} \otimes d\sigma_{ab\to jd} \otimes D_{h/j}$$

- In AA collisions (QGP modification of jet parton):

$$d\tilde{\sigma}_h = \sum_{abjdj'} f_{a/A} \otimes f_{b/B} \otimes d\sigma_{ab\to jd} \otimes P_{j\to j'} \otimes D_{h/j'}$$

- Fragmentation function (FF):

$$D_{q\to h}(z, S, S_q) = D_1(z) + \lambda_q \lambda_h G_{1\mathrm{L}}(z) + \vec{S}_{q\perp} \cdot \vec{S}_{\mathrm{T}} H_{1\mathrm{T}}(z)$$

unpolarized, polarized (longitudinal and transverse)

Constraints on longitudinal spin transfer

- Good constraints on unpolarized FFs, early stage for polarized FFs
- Example: DSV FFs to Λ hyperons [de Florian, Stratmann, Vogelsang, Phys. Rev. D 57 (1998) 5811]

Unpolarized parts: $D_1^{u\to\Lambda} = D_1^{d\to\Lambda} = D_1^{s\to\Lambda}$

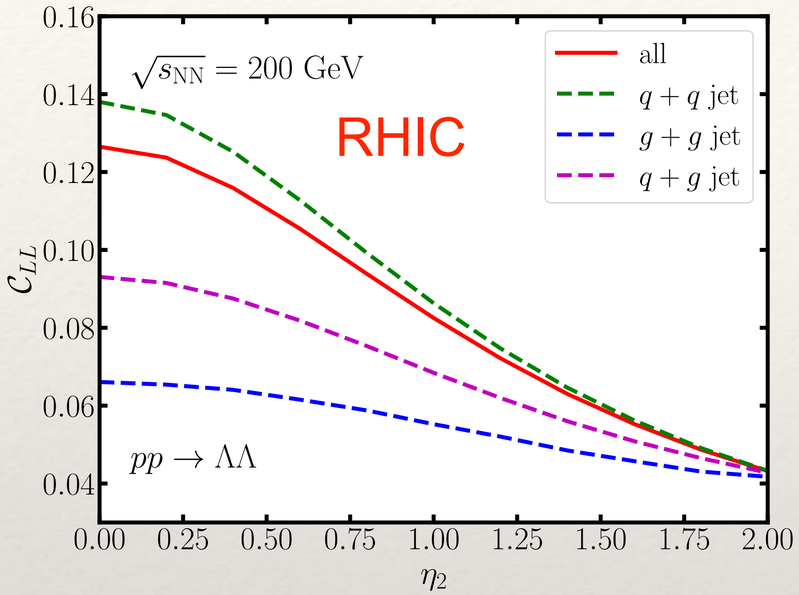
Polarized parts: $G_{1\mathrm{L}}^{u\to\Lambda} = G_{1\mathrm{L}}^{d\to\Lambda} = \xi G_{1\mathrm{L}}^{s\to\Lambda}$ $\quad (G_{1\mathrm{L}}^{g\to\Lambda} = G_{1\mathrm{L}}^{c\to\Lambda} = 0$ at initial scale)

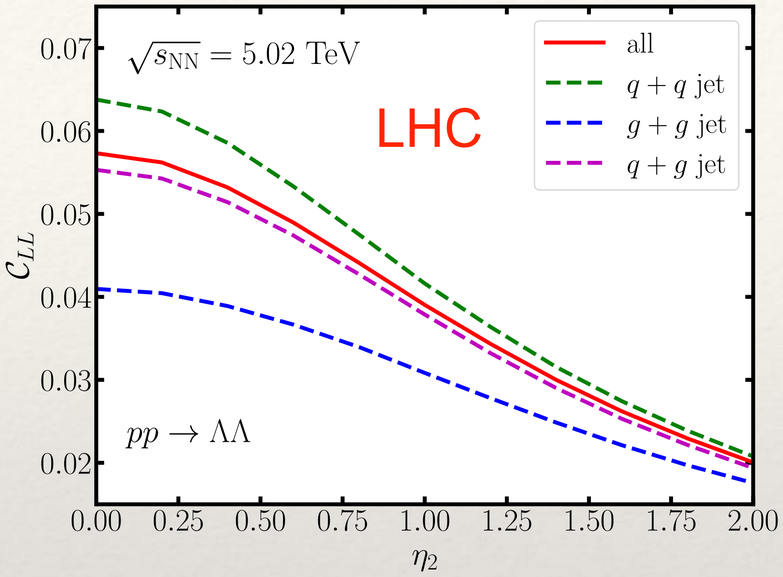
Scenario 1: $\xi = 0$; Scenario 2: $\xi = -0.2$; Scenario 3: $\xi = 1$

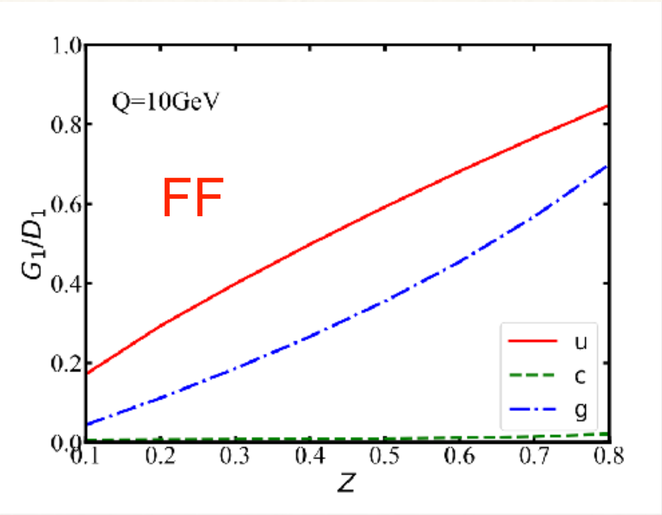
- Usual measurements of $G_{1\mathrm{L}}$ with polarized beams or weak-interaction-dominating reactions
- Proposal on constraining $G_{1\mathrm{L}}$ using the correlation of dihadron polarization (neither polarized beam nor weak interaction process) [H.-C. Zhang and S.-Y. Wei, Phys. Lett. B 839 (2023) 137821]

$$\mathscr{C}_{\mathrm{LL}}(\eta_1,\eta_2) = \left(\frac{d\sigma_{\lambda_1=\lambda_2}}{d\eta_1 d\eta_2} - \frac{d\sigma_{\lambda_1=-\lambda_2}}{d\eta_1 d\eta_2}\right) \Bigg/ \left(\frac{d\sigma_{\lambda_1=\lambda_2}}{d\eta_1 d\eta_2} + \frac{d\sigma_{\lambda_1=-\lambda_2}}{d\eta_1 d\eta_2}\right)$$

Correlation of dihadron polarization in pp collisions







- $\Lambda\Lambda$ production: *t*-channel dominating, prefers same sign of partonic λ_1 and λ_2
- $\mathscr{C}_{\rm LL}^{q+q} > \mathscr{C}_{\rm LL}^{q+g} > \mathscr{C}_{\rm LL}^{g+g}$: smaller longitudinal spin transfer from *g* than from *q* to Λ
- $\mathscr{C}_{\rm LL}({\rm RHIC}) > \mathscr{C}_{\rm LL}({\rm LHC})$: larger *z* probed at RHIC and larger *q* contribution at RHIC

From pp to AA collisions

- Motivation: increase luminosity for experimental measurement
- Dihadron cross section in AA collisions at the leading order:

$$\frac{d\sigma^{\mathrm{AA}}_{\lambda_1\lambda_2}}{d\eta_1 d^2\vec{p}_{\mathrm{T}1} d\eta_2 d^2\vec{p}_{\mathrm{T}2}} = \int dxdy T_{\mathrm{A}}(x-\tilde{b}/2,y) T_{\mathrm{A}}(x+\tilde{b}/2,y) \int \frac{dz_1}{z_1^2}\frac{dz_2}{z_2^2} \sum_{ab\to cd}\sum_{\lambda_c\lambda_d} \frac{1}{\pi} x_a f_{\mathrm{A},a}(x_a) x_b f_{\mathrm{A},b}(x_b)$$
$$\times \frac{d\hat{\sigma}^{ab\to cd}_{\lambda_c\lambda_d}}{dt} \mathcal{D}_c(z_1,\lambda_1;\lambda_c)\mathcal{D}_d(z_2,\lambda_2;\lambda_d)\,\delta^2\left(\frac{\vec{p}_{\mathrm{T}1}}{z_1} + \Delta\vec{p}_{\mathrm{T}1} + \frac{\vec{p}_{\mathrm{T}2}}{z_2} + \Delta\vec{p}_{\mathrm{T}2}\right) + \{c\leftrightarrow d\}.$$

- Nuclear modification:

  Nuclear modified PDF (cold nuclear matter effect) — CTEQ PDF + EPPS shadowing

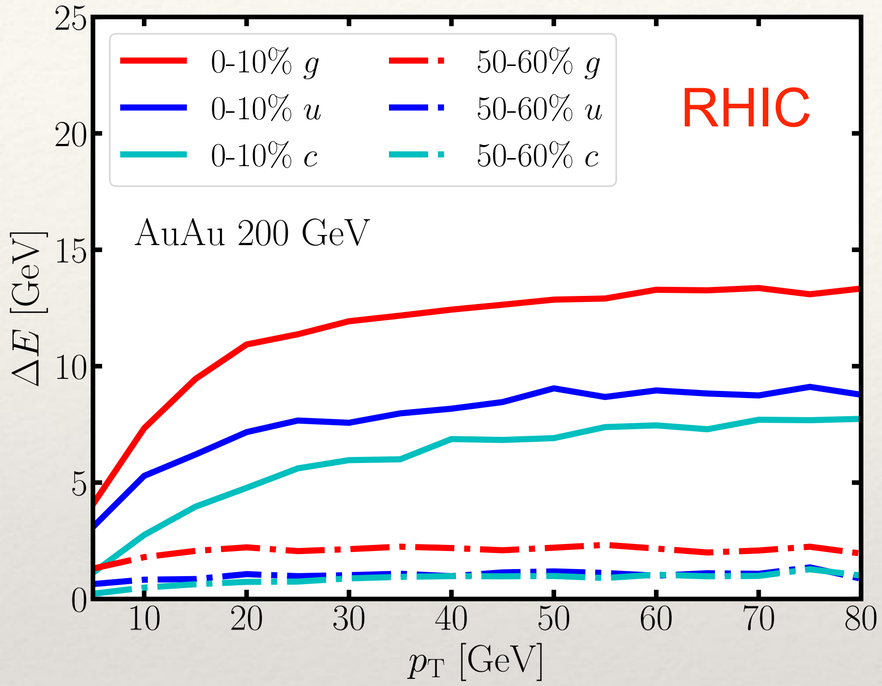
  Parton energy loss in QGP: calculated using transport model (production location dependent)

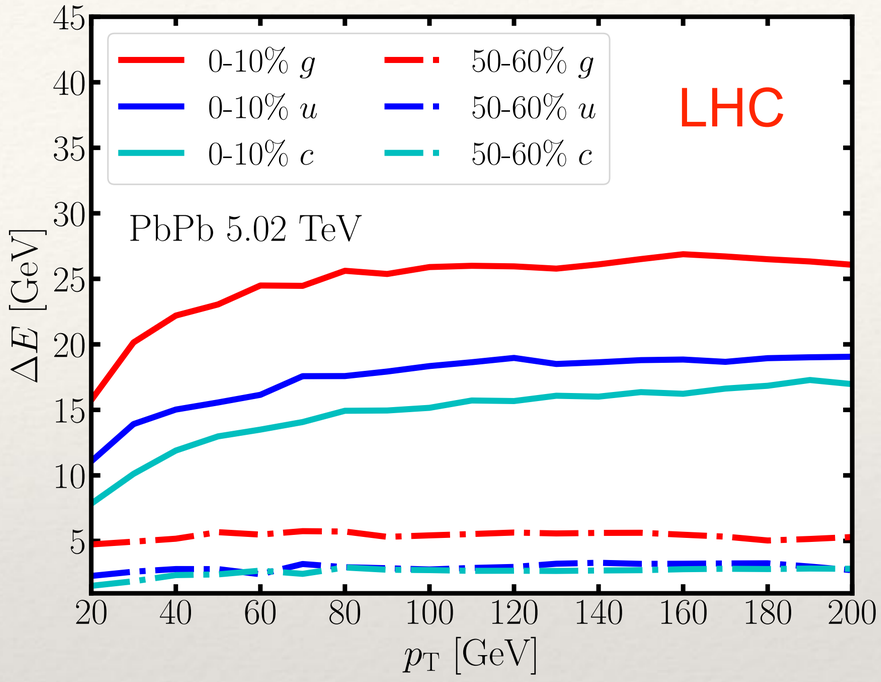
  Production location — nuclear thickness function from Woods-Saxon distribution

- In the end:

$$\mathscr{C}_{\mathrm{LL}}(\eta_1,\eta_2) = \left(\frac{d\sigma_{\lambda_1=\lambda_2}}{d\eta_1 d\eta_2} - \frac{d\sigma_{\lambda_1=-\lambda_2}}{d\eta_1 d\eta_2}\right) \Big/ \left(\frac{d\sigma_{\lambda_1=\lambda_2}}{d\eta_1 d\eta_2} + \frac{d\sigma_{\lambda_1=-\lambda_2}}{d\eta_1 d\eta_2}\right)$$

Energy loss of different flavors of partons





- Calculated using linear Boltzmann transport (LBT) model + hydrodynamic model

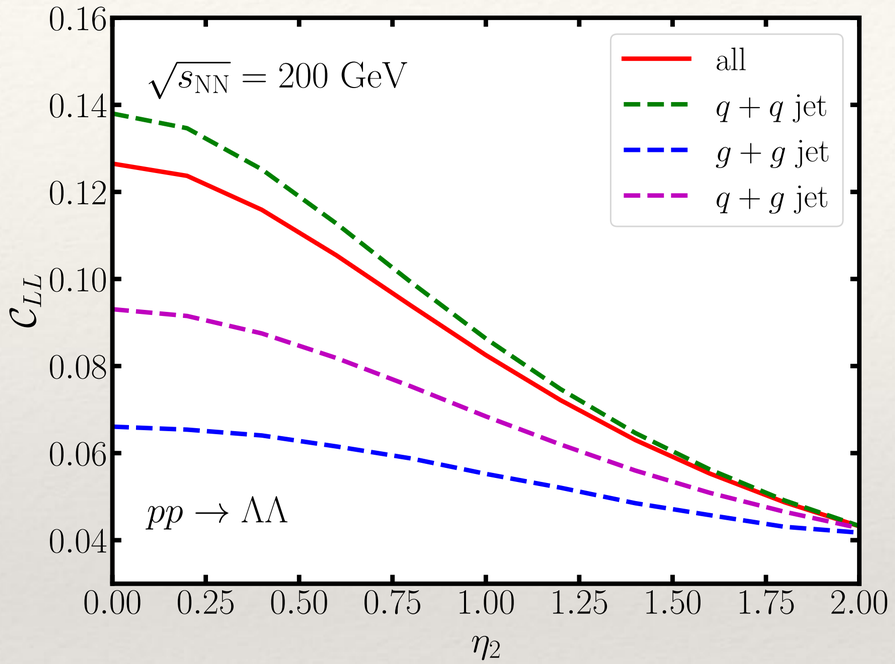
  [SC, T. Luo, G.-Y. Qin, X.-N. Wang, Phys. Rev. C 94 (2016) 1, 014909]

- Flavor hierarchy of parton energy loss: $\Delta E_g > \Delta E_{u/d/s} > \Delta E_c$

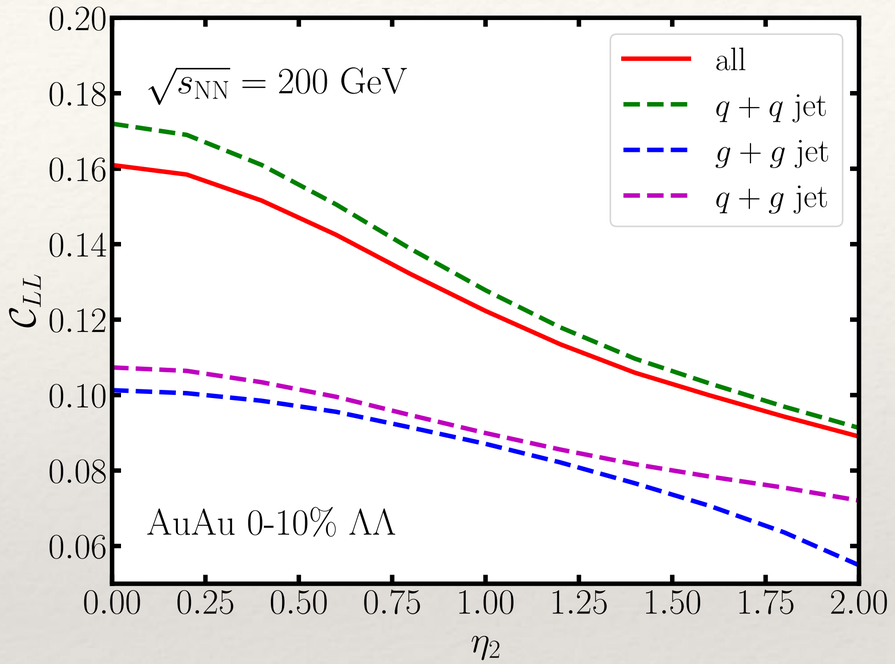
- Stronger energy loss is more central collisions, and in more energetic collisions

Correlation of dihadron polarization in pp *vs.* AA

pp



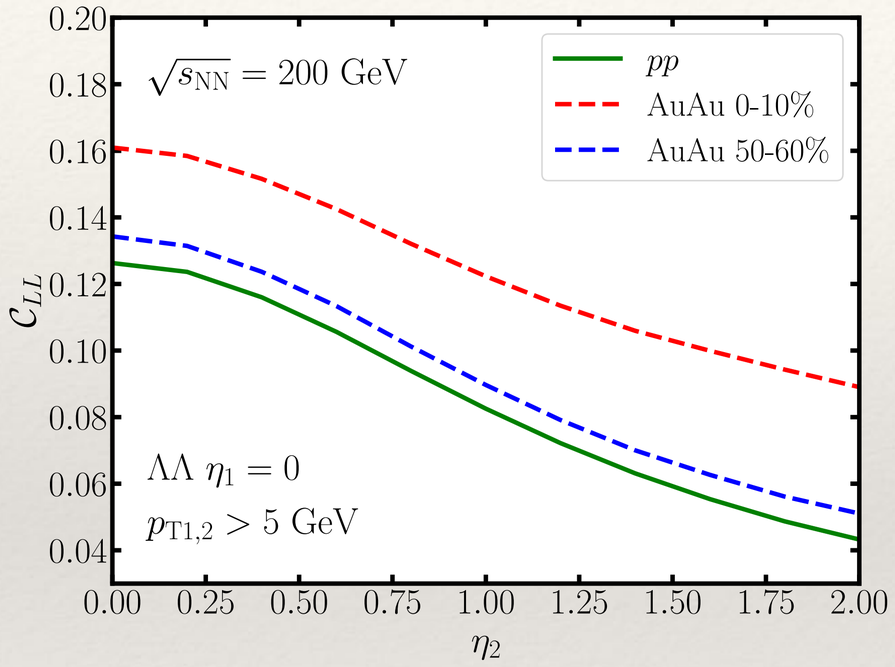
AuAu



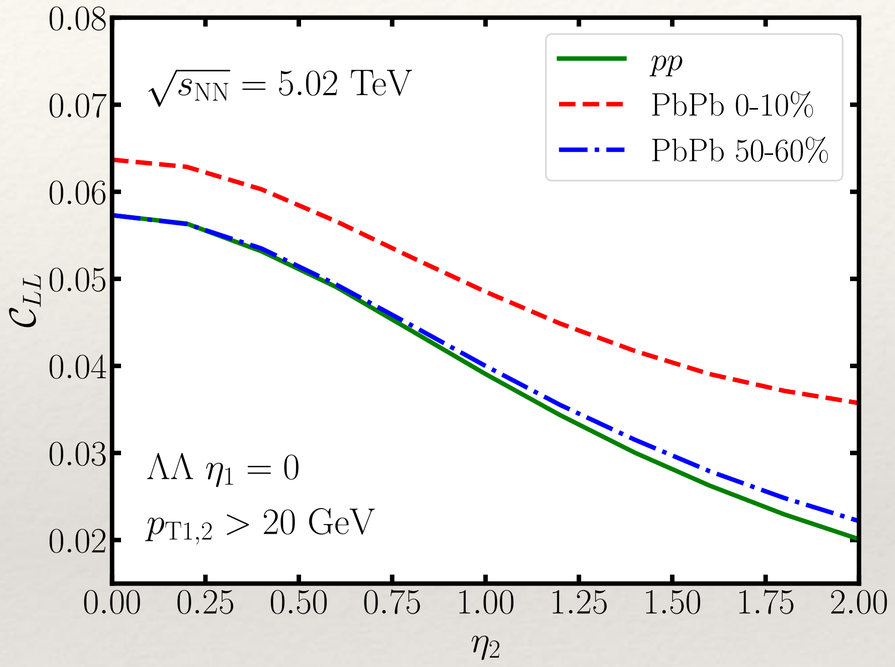
- Stronger spin correlation in AuAu than in pp — parton energy loss, hadron with the same final p_T probes larger z region of FFs
- Stronger enhancement for gluons than for quarks
- At the same final p_T, energy loss increases quark *vs.* gluon fraction → enhance the total $\mathscr{C}_{\rm LL}$

From pp to peripheral to central AA collisions

AuAu

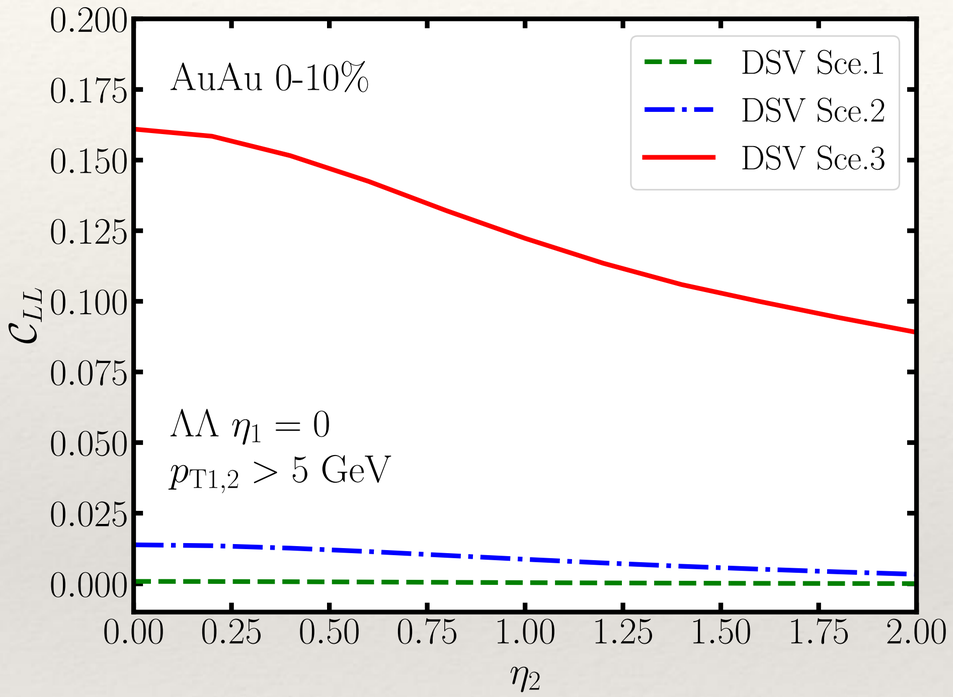


PbPb



- Increase of $\mathscr{C}_{\rm LL}$ from pp to peripheral to central AA collisions

Constraint on the spin-dependent FFs



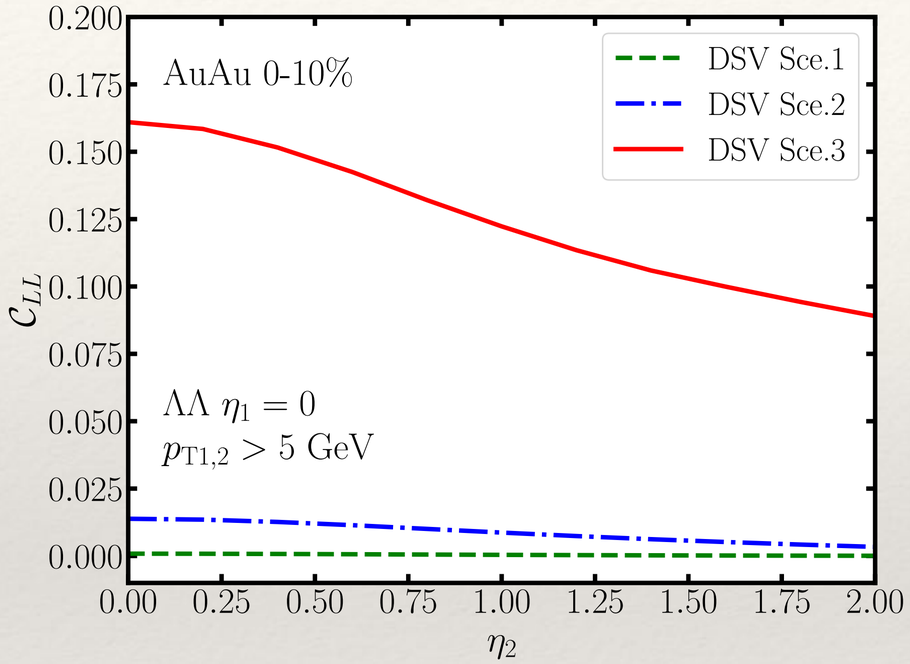
Comparison between 3 scenarios of DSV FFs (all tuned to spin-independent data)

- Scenario 1: only *s* transfers longitudinal spin to Λ
- Scenario 2: *s* has positive contribution, *u* and *d* have small negative contributions
- Scenario 3: *u*, *d*, *s* contribute equally to the polarized parts of FFs

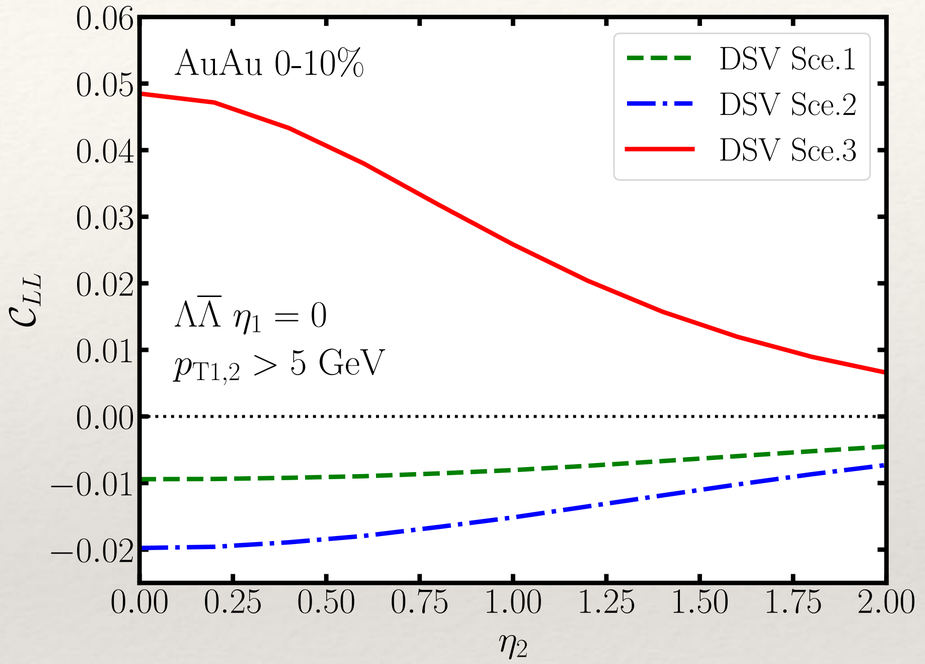
C_{LL} can be used to constrain the longitudinal spin transfer in FFs

Comparison between $\Lambda\Lambda$ and $\Lambda\overline{\Lambda}$ correlations

$\Lambda\Lambda$



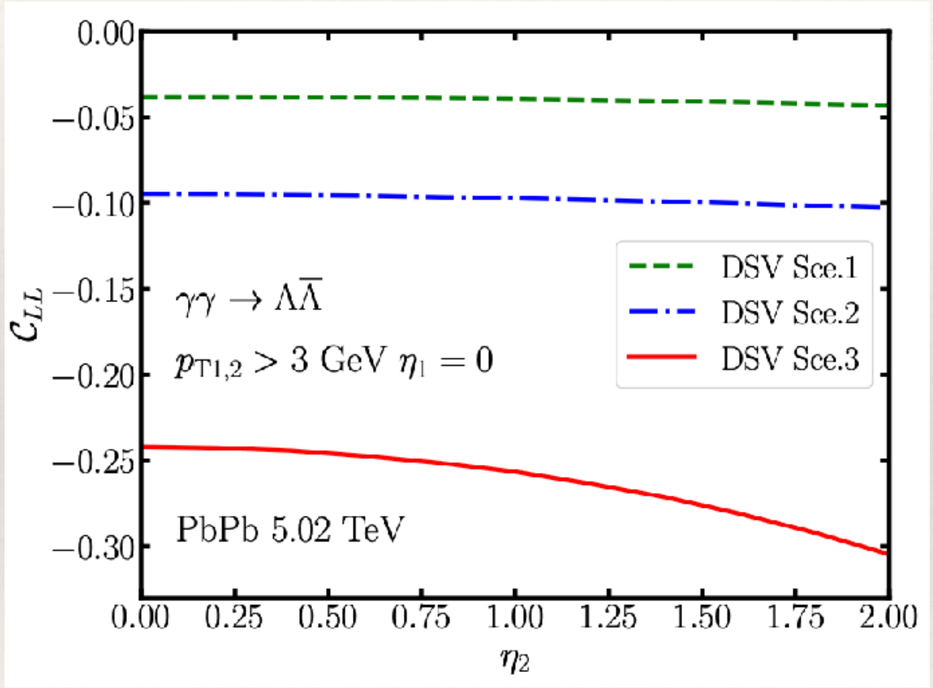
$\Lambda\overline{\Lambda}$



- Scenario 1 reflects contributions from *s*-channel $s\bar{s}$ production, negative $\mathscr{C}_{\mathrm{LL}}$
- $\mathscr{C}_{\mathrm{LL}}$ of $\Lambda\overline{\Lambda}$ is also sensitive to the selection of spin-dependent FFs

$\Lambda\bar{\Lambda}$ correlation in ultraperipheral collisions

- $\gamma\gamma \to \Lambda\bar{\Lambda}$ process



$$\frac{d\sigma^{\gamma\gamma}}{d\eta_1 d\eta_2} = \int dz_1 dz_2 \int \frac{d^2\vec{p}_{T1}}{z_1^2} x_1 f_\gamma(x_1) x_2 f_\gamma(x_2) \sum_q \frac{1}{\pi} \frac{d\hat{\sigma}^{\gamma\gamma\to q\bar{q}}}{dt} \times \left[D^{\Lambda}_{1,q}(z_1) D^{\bar{\Lambda}}_{1,\bar{q}}(z_2) - \lambda_1 \lambda_2 G^{\Lambda}_{1L,q}(z_1) G^{\bar{\Lambda}}_{1L,\bar{q}}(z_2) \right].$$

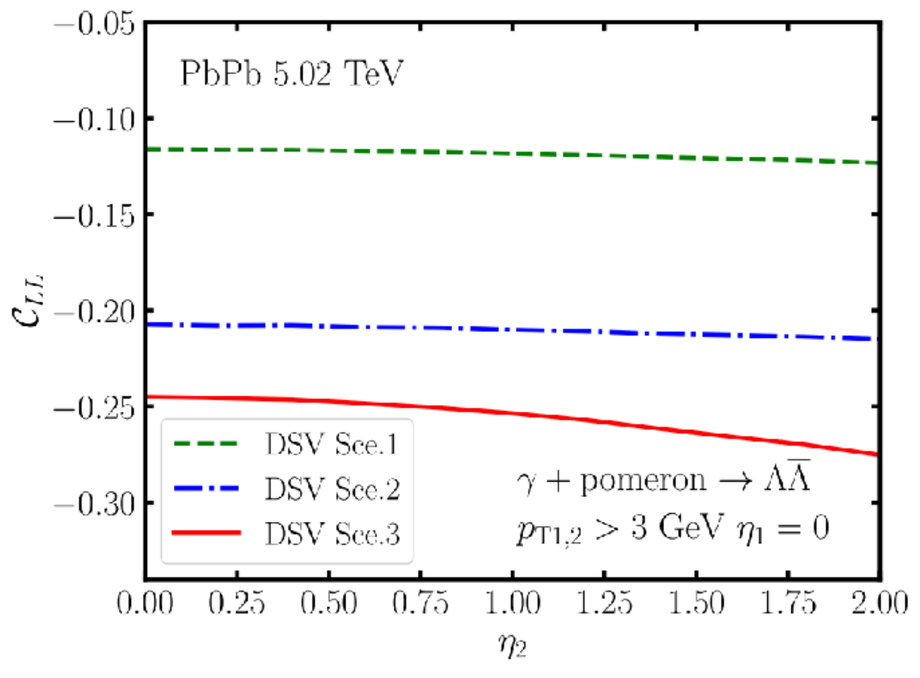
Hard cross section:

$$\frac{d\hat{\sigma}^{\gamma\gamma\to q\bar{q}}}{dt} = \frac{6\pi\alpha^2 e_q^4}{s^2} \frac{t^2+u^2}{tu}$$

- Different flavor dependence from $e^+e^- \to q\bar{q}$ process (e_q^2)
- Stronger contribution from *u* than *s* quarks
- Correlation from Scenario 3 is almost an order of magnitude larger than correlation from Scenario 1

$\Lambda\bar{\Lambda}$ correlation in ultraperipheral collisions

- $\gamma\mathbb{P} \to \Lambda\bar{\Lambda}$ process



$$\frac{d\sigma^{\gamma\mathbb{P}}}{d\eta_1 d\eta_2} = \int \frac{dx}{x} \int dz_1 dz_2 \int \frac{d^2\vec{p}_{T1}}{z_1^2} x_1 f_\gamma(x_1) x g_{\mathbb{P}}(x, Q^2) \sum_q \frac{1}{\pi} \frac{d\hat{\sigma}^{\gamma\mathbb{P}\to q\bar{q}}}{dt} \times \left[D^{\Lambda}_{1,q}(z_1) D^{\bar{\Lambda}}_{1,\bar{q}}(z_2) - \lambda_1 \lambda_2 G^{\Lambda}_{1\mathrm{L},q}(z_1) G^{\bar{\Lambda}}_{1\mathrm{L},\bar{q}}(z_2) \right].$$

Hard cross section:

$$\frac{d\hat{\sigma}^{\gamma\mathbb{P}\to q\bar{q}}}{dt} = \frac{\pi\alpha\alpha_s e_q^2}{s^2} \frac{t^2 + u^2}{tu}$$

- Same flavor dependence as $e^+e^- \to q\bar{q}$ process
- Less difference between different scenarios of FFs compared to the previous $\gamma\gamma \to \Lambda\bar{\Lambda}$ process
- Supplementary channel to e^+e^- for studying polarized FFs

Summary

- Developed a perturbative approach for both initial production and spacetime evolution of quark polarization in relativistic heavy-ion collisions
- Found sensitivity of global polarization to the initial geometry of the QGP
- Studied the correlation of dihadron polarization ($\mathscr{C}_{\mathrm{LL}}$) from jet fragmentation
- Found enhancement of $\mathscr{C}_{\mathrm{LL}}$ from pp to peripheral AA and then to central AA collisions due to the increasing parton energy loss inside the QGP, and sensitivity of $\mathscr{C}_{\mathrm{LL}}$ to the longitudinal spin transfer in polarized FFs

Spatial distribution of quark polarization

Spin-independent probability for a quark from nucleus A

$$\mathcal{P}_{\rm A}(x_{\rm A},y_{\rm A}) = \int T_2(x_{\rm B},y_{\rm B}) \frac{d\sigma(x_{\rm A},y_{\rm A},x_{\rm B},y_{\rm B})}{dx_{\rm B}dy_{\rm B}} dx_{\rm B}dy_{\rm B}$$

Spin-dependent probability for a quark from nucleus A

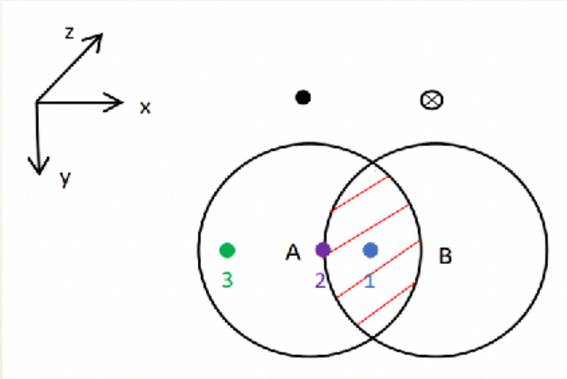
$$\Delta\mathcal{P}_{\rm A}(x_{\rm A},y_{\rm A}) = \int T_2(x_{\rm B},y_{\rm B}) \frac{d\Delta\sigma(x_{\rm A},y_{\rm A},x_{\rm B},y_{\rm B})}{dx_{\rm B}dy_{\rm B}} dx_{\rm B}dy_{\rm B}$$

Spatial distribution of quark polarization

$$P(x,y) = \frac{T_1(x,y)\Delta\mathcal{P}_{\rm A}(x,y)+T_2(x,y)\Delta\mathcal{P}_{\rm B}(x,y)}{T_1(x,y)\mathcal{P}_{\rm A}(x,y)+T_2(x,y)\mathcal{P}_{\rm B}(x,y)}$$

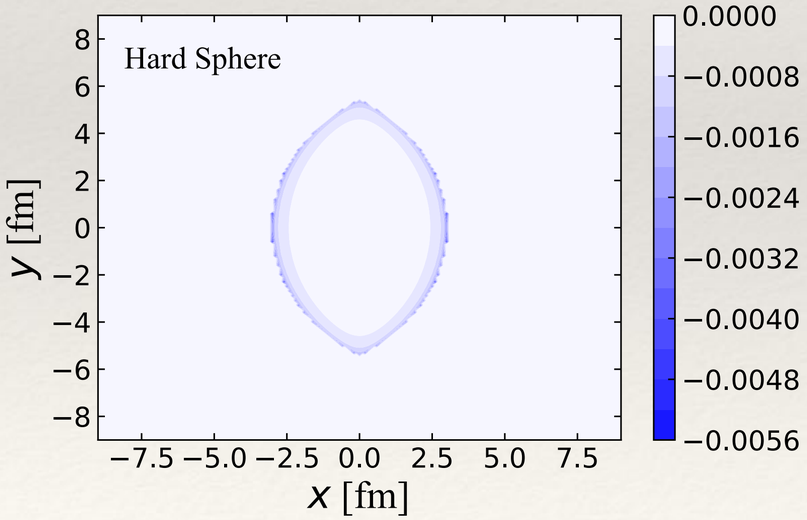
Spatial distribution of polarization from initial hard collisions

$$P(x,y) = \frac{T_1(x,y)\Delta\mathcal{P}_{\rm A}(x,y)+T_2(x,y)\Delta\mathcal{P}_{\rm B}(x,y)}{T_1(x,y)\mathcal{P}_{\rm A}(x,y)+T_2(x,y)\mathcal{P}_{\rm B}(x,y)}$$

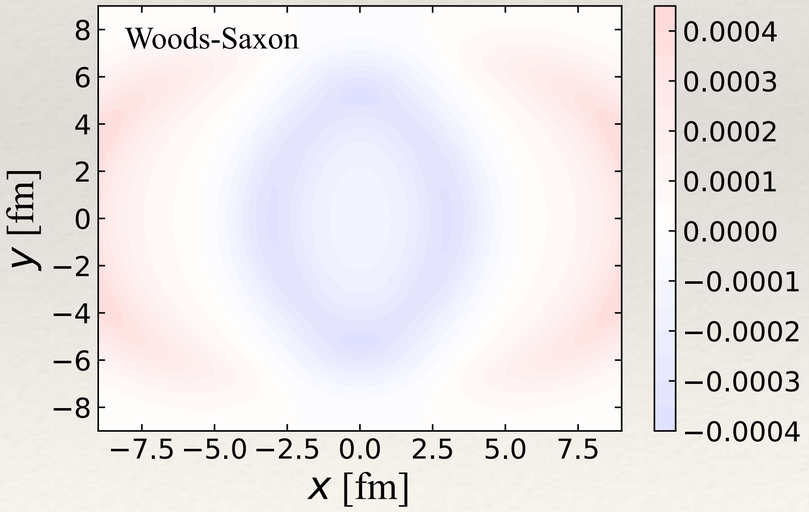


For 5-40% Au+Au @ 200 GeV

From Hard Sphere distribution

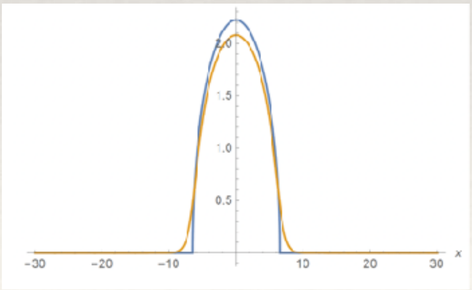


From Woods-Saxon distribution

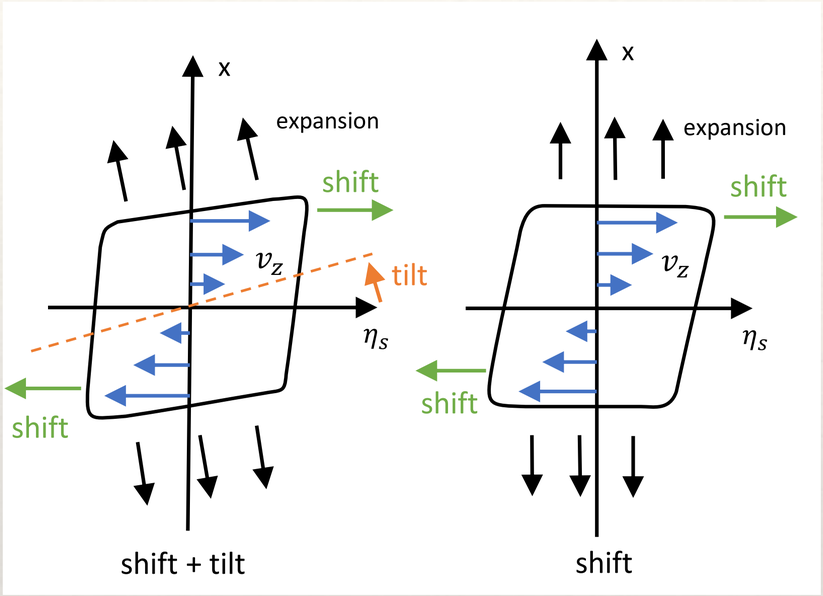


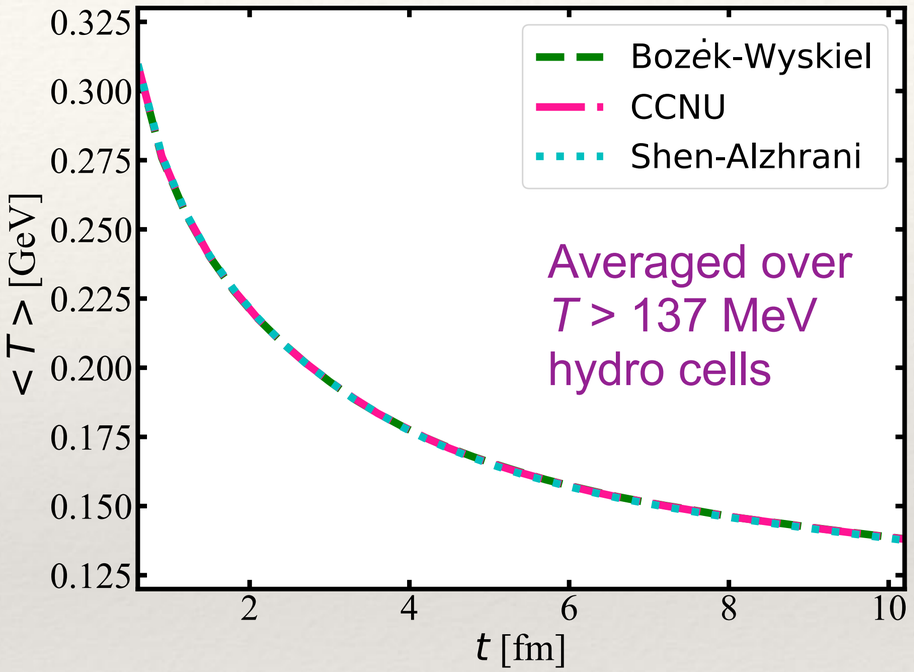
Hard Sphere (HS)

Woods-Saxon (WS)



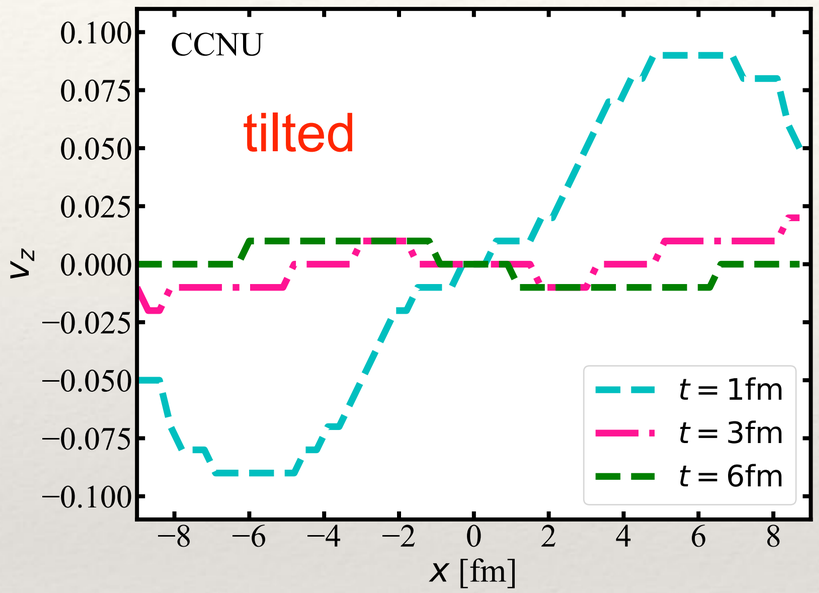
Evolution of average temperature



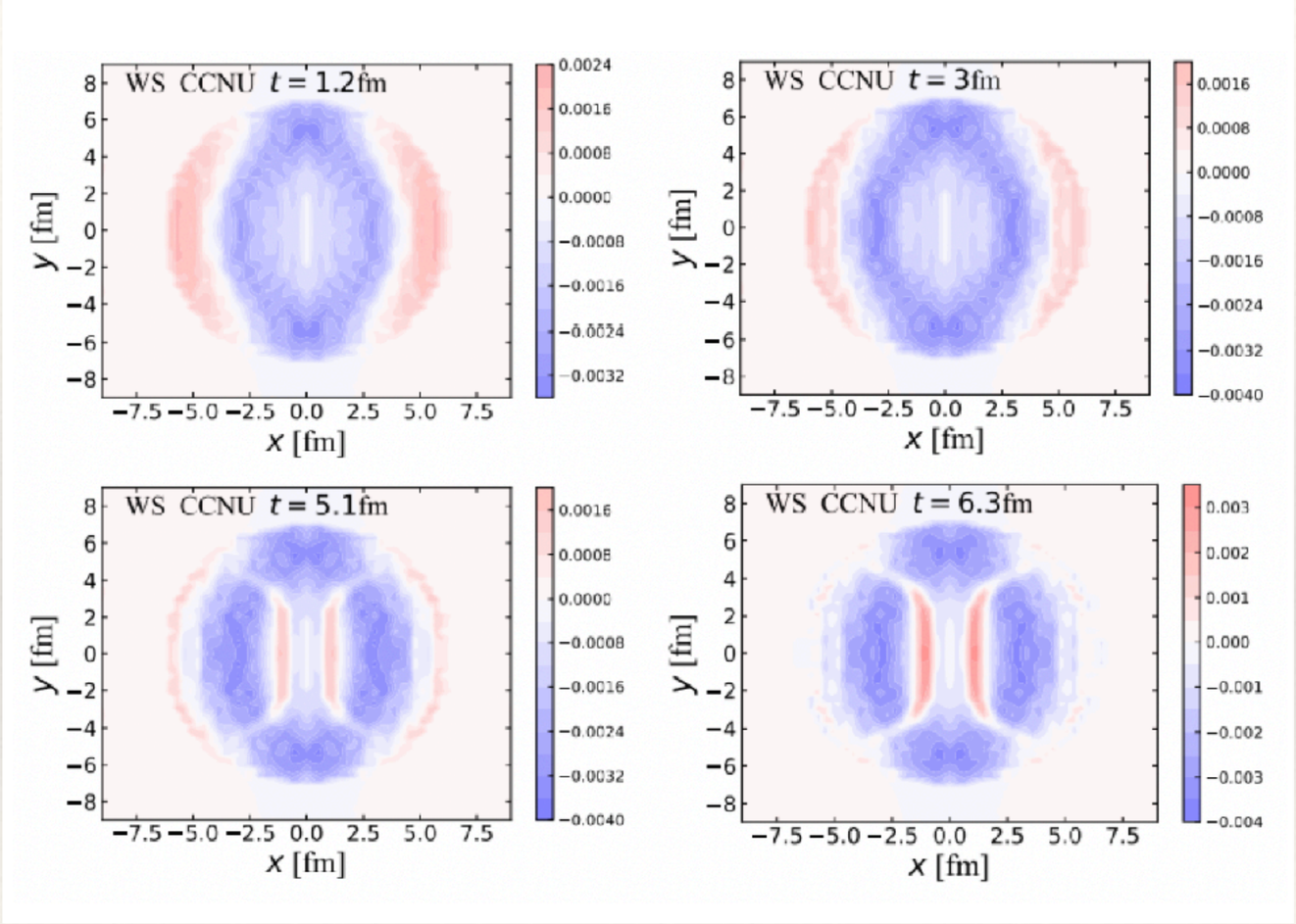


Consistent temperature evolution profiles between the three initial condition models

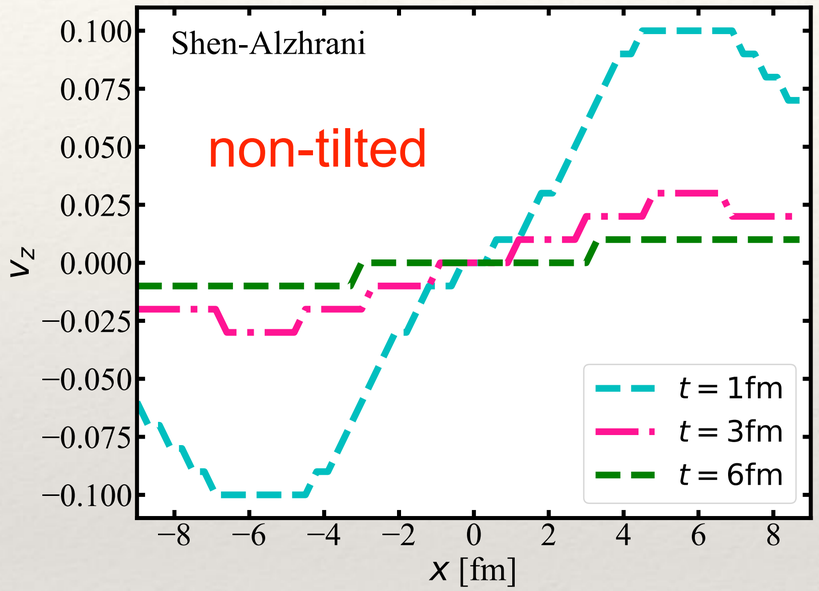
Spatial distribution of polarization with tilted initial geometry



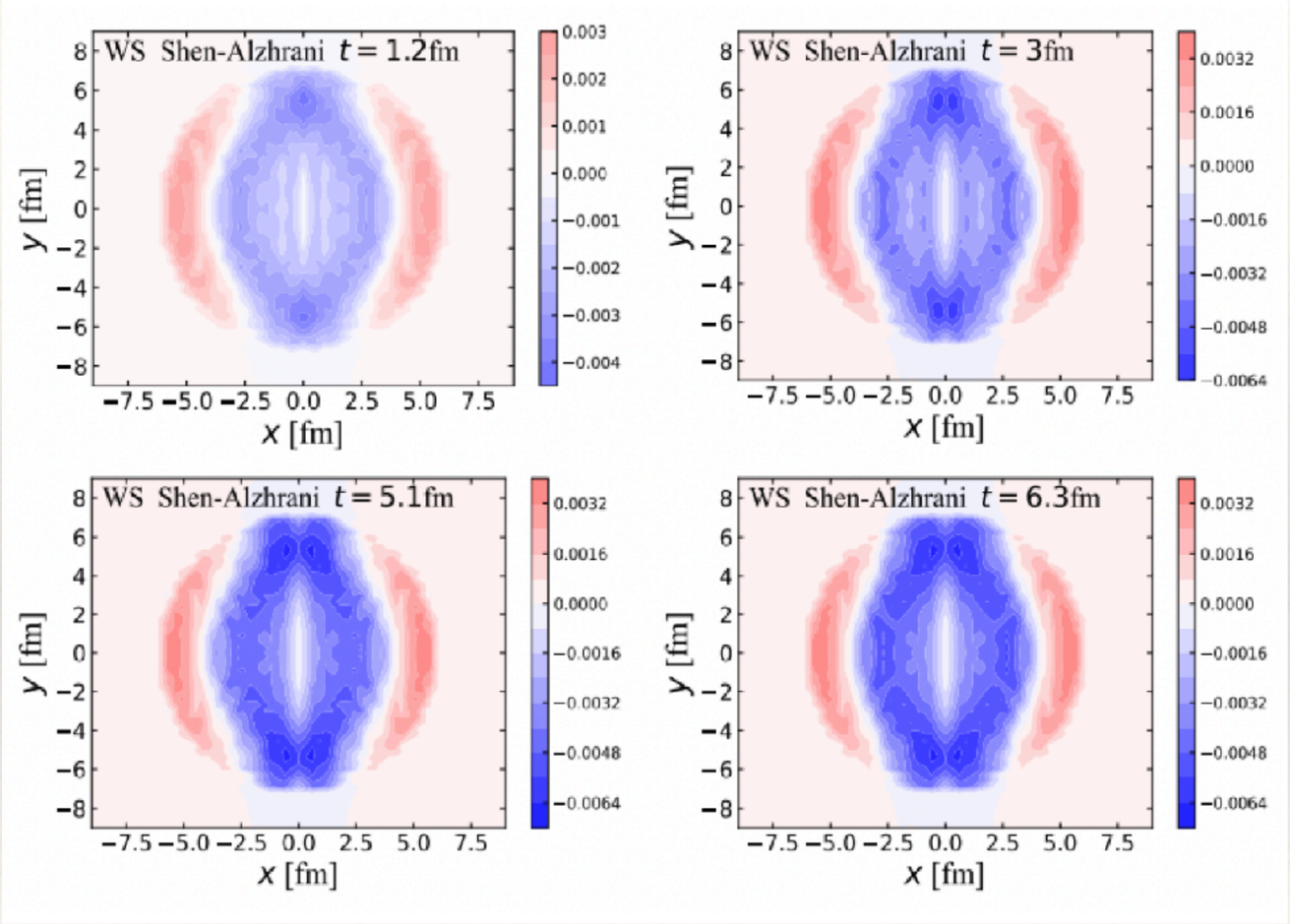
- The velocity gradient is the main origin of polarization generated inside the QGP



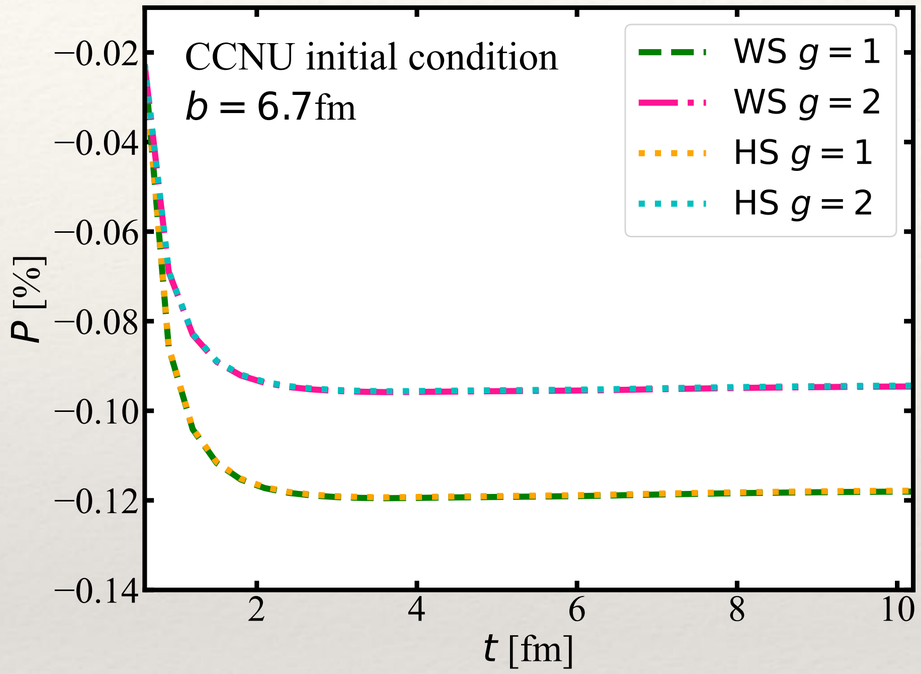
Spatial distribution of polarization with non-tilted geometry

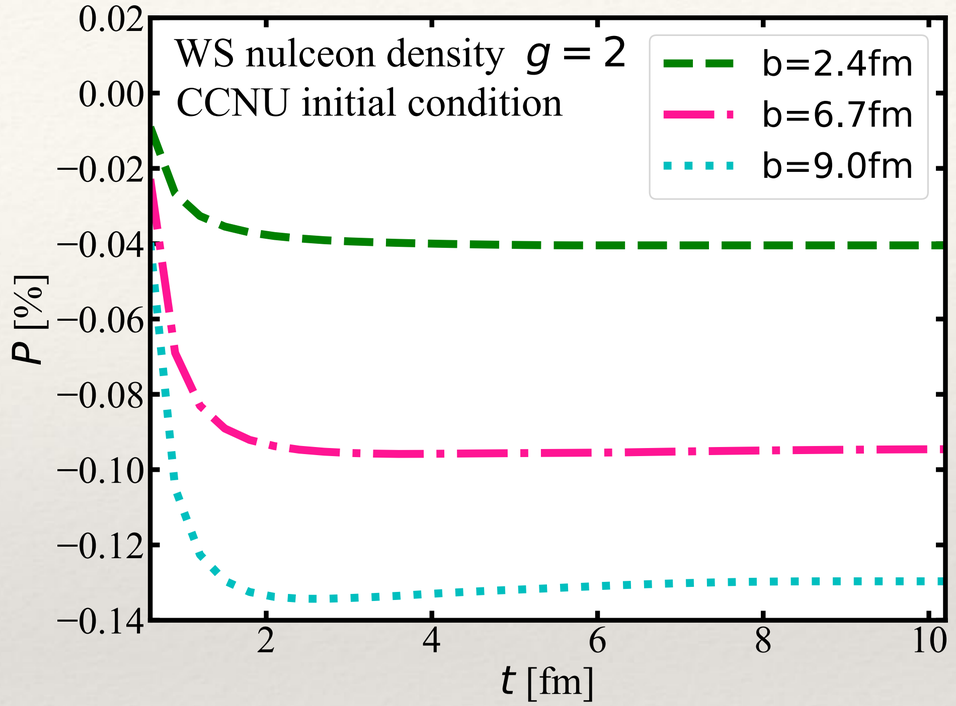


- No sign flip for polarization with non-tilted initial geometry of QGP



Time evolution of the average global polarization

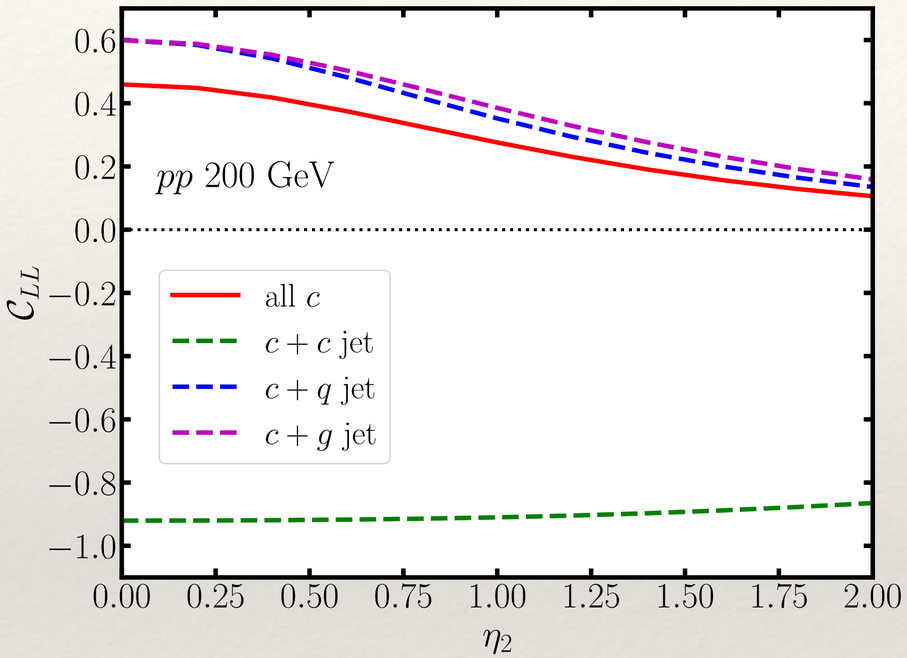




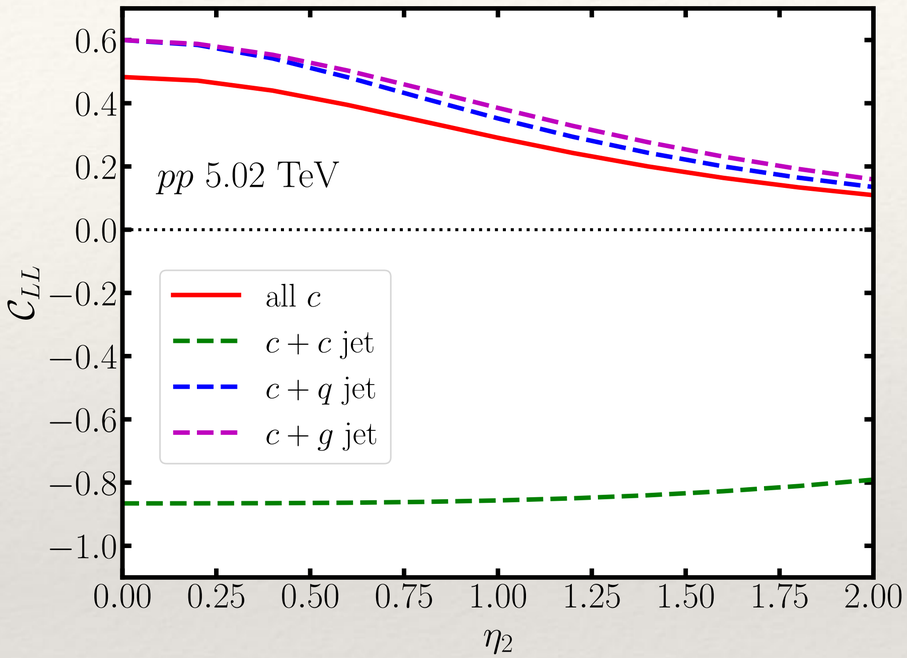
- Negligible dependence on the nuclear density distribution for the average polarization
- Decrease of polarization with increase of the coupling strength (via μ_{D})
- Significant increase of polarization with impact parameter

Correlation between charm quark and another parton

pp 200 GeV

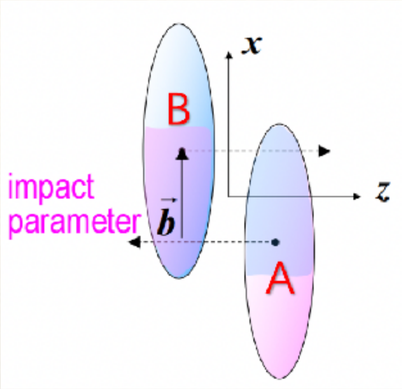


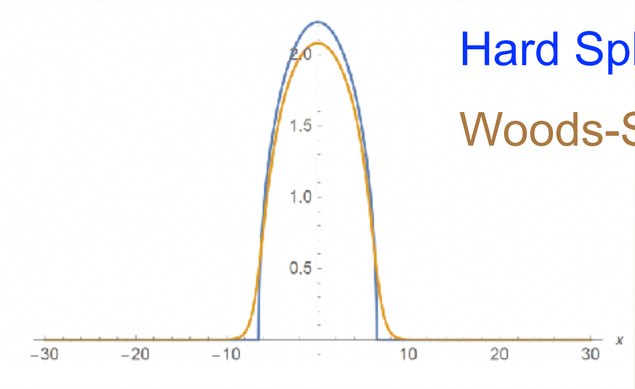
pp 5.02 TeV



- Results at partonic level (lack of spin-dependent FFs for *c*)
- Different charm production mechanism (pair production, flavor excitation, gluon splitting)
- Strong negative $\mathscr{C}_{\mathrm{LL}}$ of back-to-back $c\bar{c}$ may help constrain FFs to Λ_c in the future

Comparison between two distributions





Hard Sphere (HS)

Woods-Saxon (WS)

Distribution in one nuclei

Hard Sphere:

$$T_{\mathrm{A,B}}^{\mathrm{HS}}(x,y) = \frac{3A}{2\pi R^3}\sqrt{R^2-(x\pm b/2)^2-y^2}\,\Theta(R-\sqrt{(x\pm b/2)^2+y^2})$$

Woods-Saxon:

$$T_{\mathrm{A,B}}^{\mathrm{WS}}(x,y) = C_0\int(1+\exp\frac{\sqrt{(x\pm b/2)^2+y^2+z^2}-R}{a})^{-1}dz$$

Distribution of participants from one nuclei

Hard Sphere:

$$T_{1,2}^{\mathrm{HS}}(x,y) = T_{\mathrm{A,B}}^{\mathrm{HS}}\Theta(R-\sqrt{(x\mp b/2)^2+y^2})$$

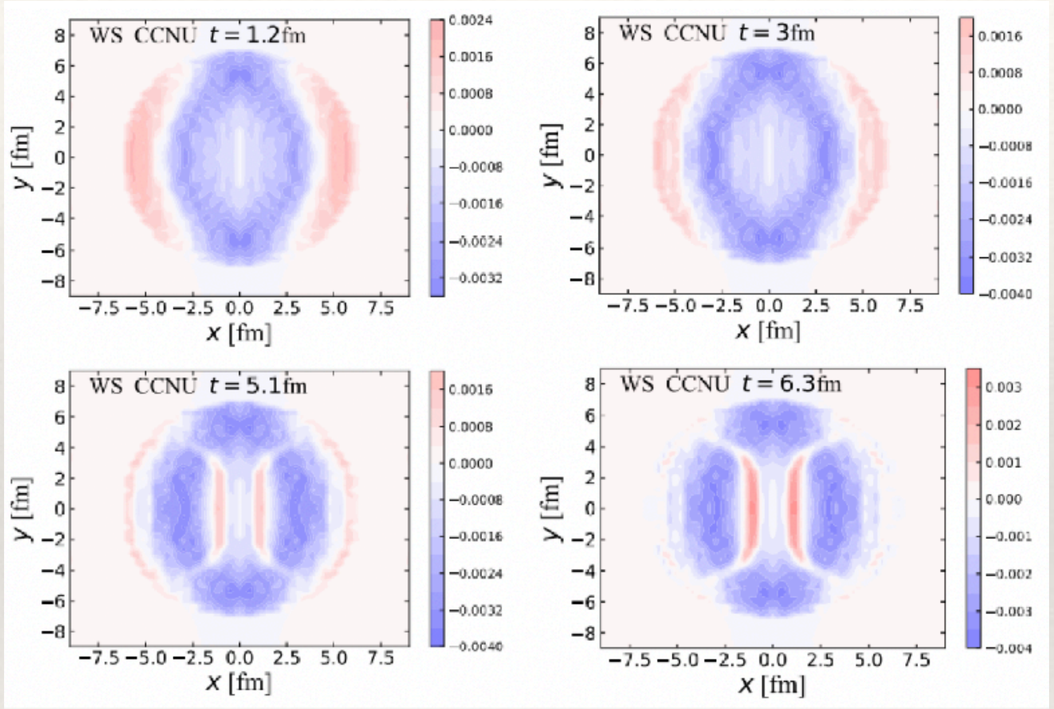
Woods-Saxon:

$$T_{1,2}^{WS}(x,y) = T_{\mathrm{A,B}}^{\mathrm{WS}}[1-\exp(-\sigma_{\mathrm{NN}}\times T_{\mathrm{B,A}}^{\mathrm{WS}})]$$

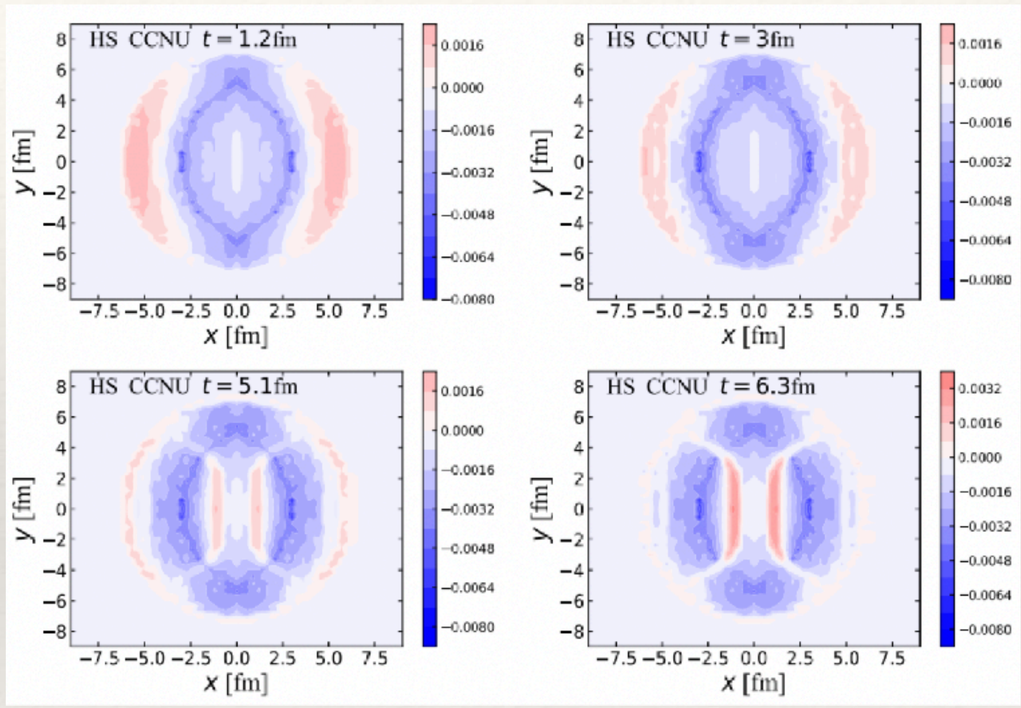
$$P = \frac{\int T(x,y)\frac{d\Delta\sigma(x,y)}{d^2x_{\mathrm{T}}}d^2x_{\mathrm{T}}}{\int T(x,y)\frac{d\sigma(x,y)}{d^2x_{\mathrm{T}}}d^2x_{\mathrm{T}}}$$

Effects of nuclear density distribution on polarization

Woods-Saxon



Hard Sphere



- Smoother distribution of quark polarization across the transverse plane from Woods-Saxon than from the Hard Sphere density distribution