

**The 2nd Workshop on Ultra-Peripheral Collision Physics :  
Strong Electromagnetic fields, UPC and EIC/EicC**

**Hadron production in pp and pPb  
collisions at LHC energies**

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# outline

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- 1. Hadronization && phenomenological models**
- 2. Quark Number Scaling (QNS) property in production of light-flavor hadrons**
- 3. EVC mechanism of constituent quarks**
- 4. Systematic explanation of  $p_T$  spectra of light-flavor hadrons**
- 5. EVC explanation of  $v_2$  of hadrons**
- 6. EVC mechanism of charm hadron production**
- 7. Understand parton system before hadronization**
- 8. Summary**

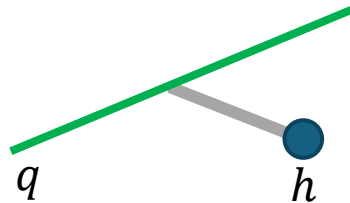
# 1. Hadronization

- the process of the formation of hadrons out of final-state quarks and/or gluons produced in high energy reactions
- Non-perturbative QCD process
- Currently modeled and/or parameterized in phenomenological methods.

## Two pictures:

### Fragmentation

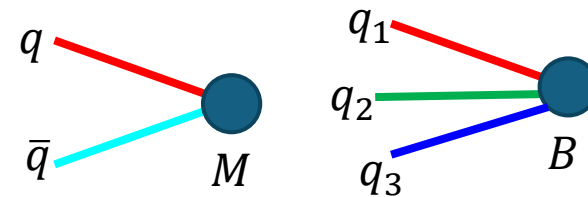
String fragmentation,  
cluster fragmentation,  
etc.



Probability  $D_{q \rightarrow h}(z)$

### Combination

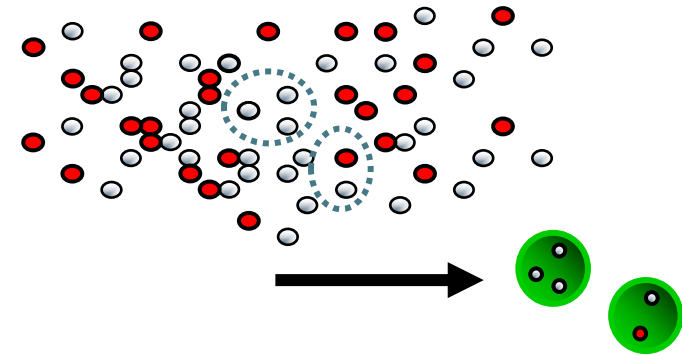
quark (re-)combination,  
parton coalescence,  
etc.



## Combination models on market

- **quark recombination model**  
R. J. Fries, B. Muller, C. Nonaka, S. A. Bass,  
e.g., Phys. Rev. C **68**, 044902 (2003)
- **parton coalescence model**  
V. Greco, C. M. Ko, P. Lévai, L.W. Chen, et al.  
e.g., Phys. Rev. C **68**, 034904 (2003)
- **quark recombination model**  
R. C. Hwa and C. B. Yang, e.g, Phys. Rev. C **70**, 024904 (2004)
- **resonance recombination model**  
L. Ravagli, R. Rapp, e.g., Phys.Lett. B 655,126 (2007)  
M. He, R.J. Fries, R. Rapp, e.g., Phys.Rev.C 82, 034907 (2010)
- **quark combination model (Shandong Group)**  
Q.B.Xie, F.L.Shao, et al., e.g., Phys. Rev. C71,044903 (2005)  
phenomenological combine rule
- **transport model**  
P.F.Zhuang, et al, e.g., Phys. Rev. C76, 014907(2009)
- **quark molecular dynamics model**  
M. Hofmann et al, e.g., Nucl. Phys. B 478,161(2000)
- **variational model**  
A.Alala, etal, e.g., Phys. Rev. C77,044901(2009)

....



which kind of quarks?

how to combine?

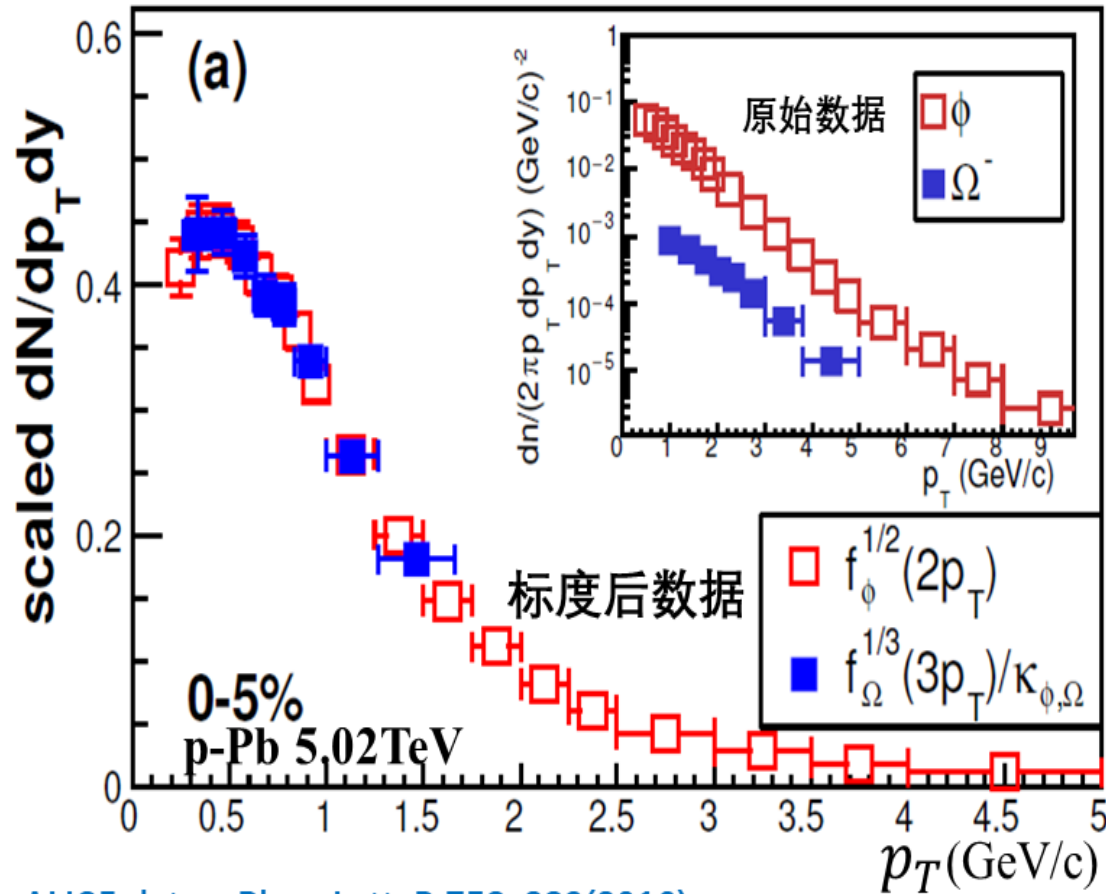
how to test in experiments?

## 2.

# **Quark Number Scaling (QNS) property in $p_T$ spectra of hadrons in pp and p-Pb collisions at LHC**

Refs:

- Jun Song, Hai-hong Li, and Feng-lan Shao, Phys. Rev. D105, 074027 (2022).
- Hai-hong Li, Feng-lan Shao, and Jun Song, Chin. Phys. C 45, 113105 (2021).
- Jian-wei Zhang, Hai-hong Li, Feng-lan Shao, and Jun Song, Chin.Phys. C44, 014101(2020).
- Jun Song, Xing-rui Gou, Feng-lan Shao, Zuo-tang Liang, Phys. Lett. B774, 516(2017).
- Xing-rui Gou, Feng-lan Shao, Rui-qin Wang, Hai-hong Li, Jun Song, Phys. Rev. D96,094010(2017).



ALICE data : Phys. Lett. B 758, 389(2016).  
Eur. Phys. J. C76, 245 (2016)

## QNS formula

$$f_\Omega^{1/3}(3p_T) = \kappa_{\phi,\Omega} f_\phi^{1/2}(2p_T)$$

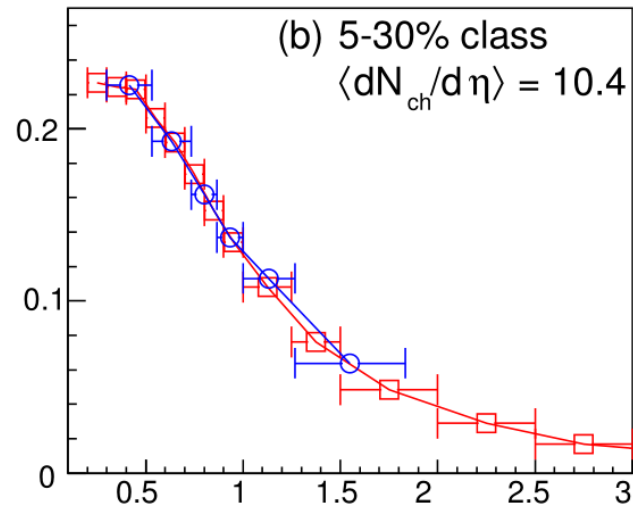
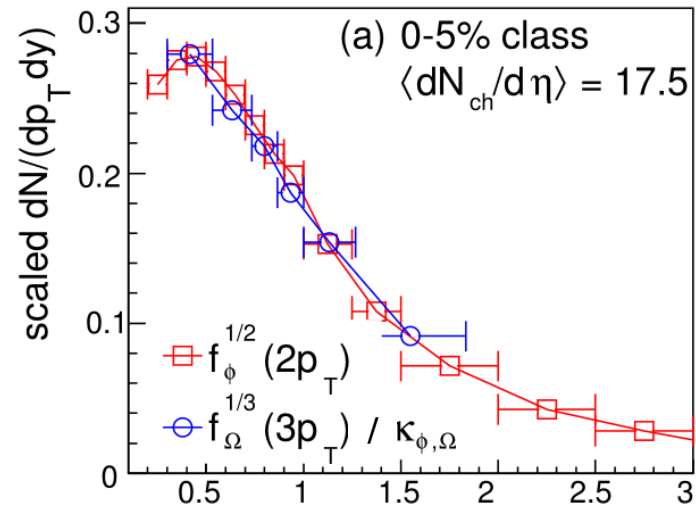
## another form

$$f_\Omega(3p_T) = \kappa_\Omega f_s^3(p_T)$$

$$f_\phi(2p_T) = \kappa_\phi f_s^2(p_T)$$

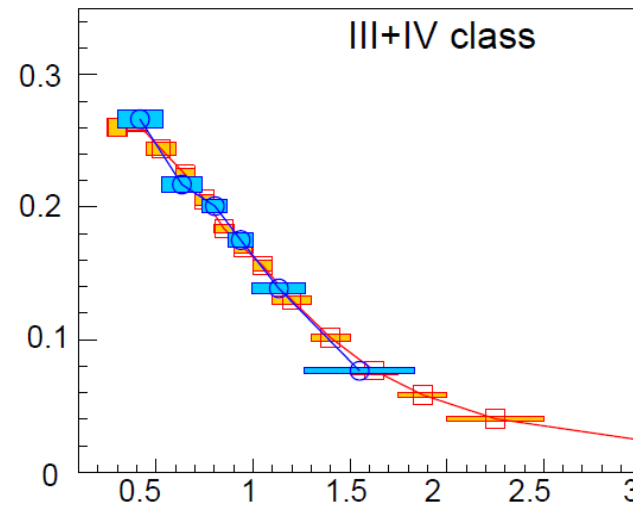
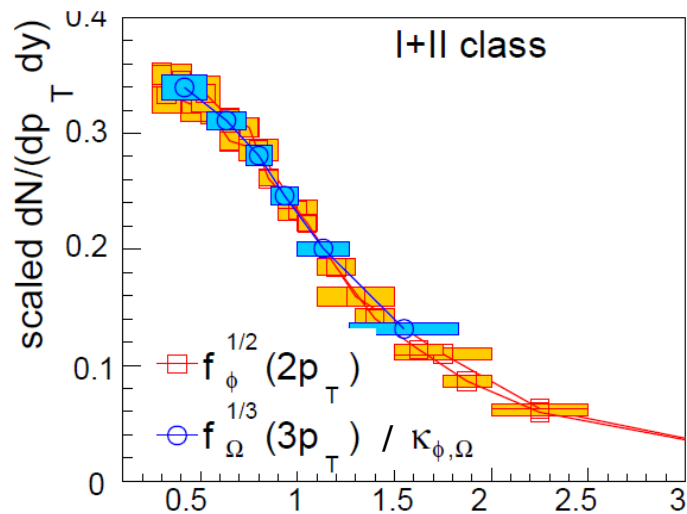
$$f(p_T) \equiv dN_h/dp_T dy$$

$\kappa_\phi, \kappa_\Omega, \kappa_{\phi,\Omega}$  are independent of  $p_T$

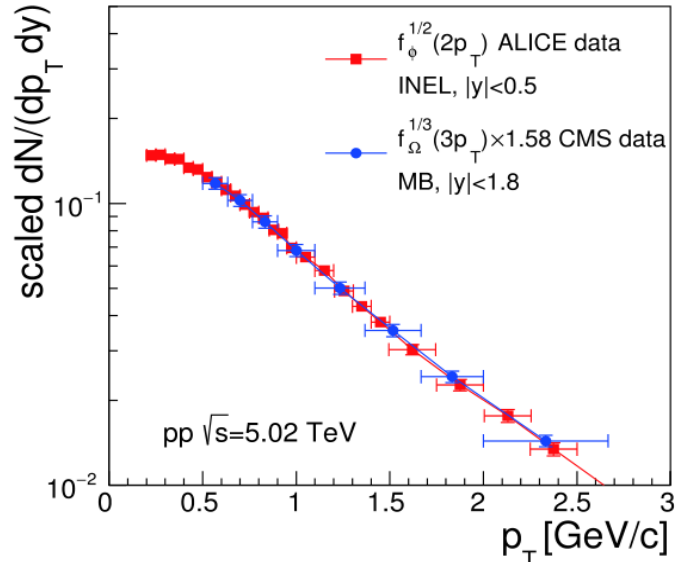


pp at  $\sqrt{s} = 7\text{TeV}$

Zhang,Shao,Song, *Chin.Phys. C44*, 014101(2020)

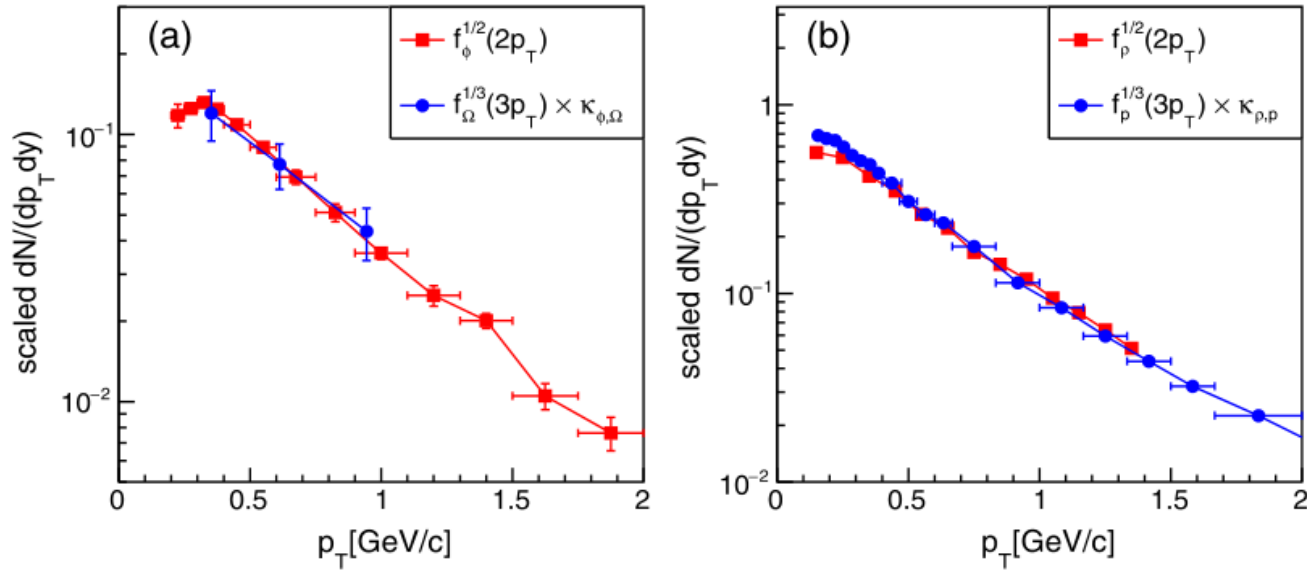


pp at  $\sqrt{s} = 13\text{TeV}$



pp at  $\sqrt{s} = 5.02\text{TeV}$

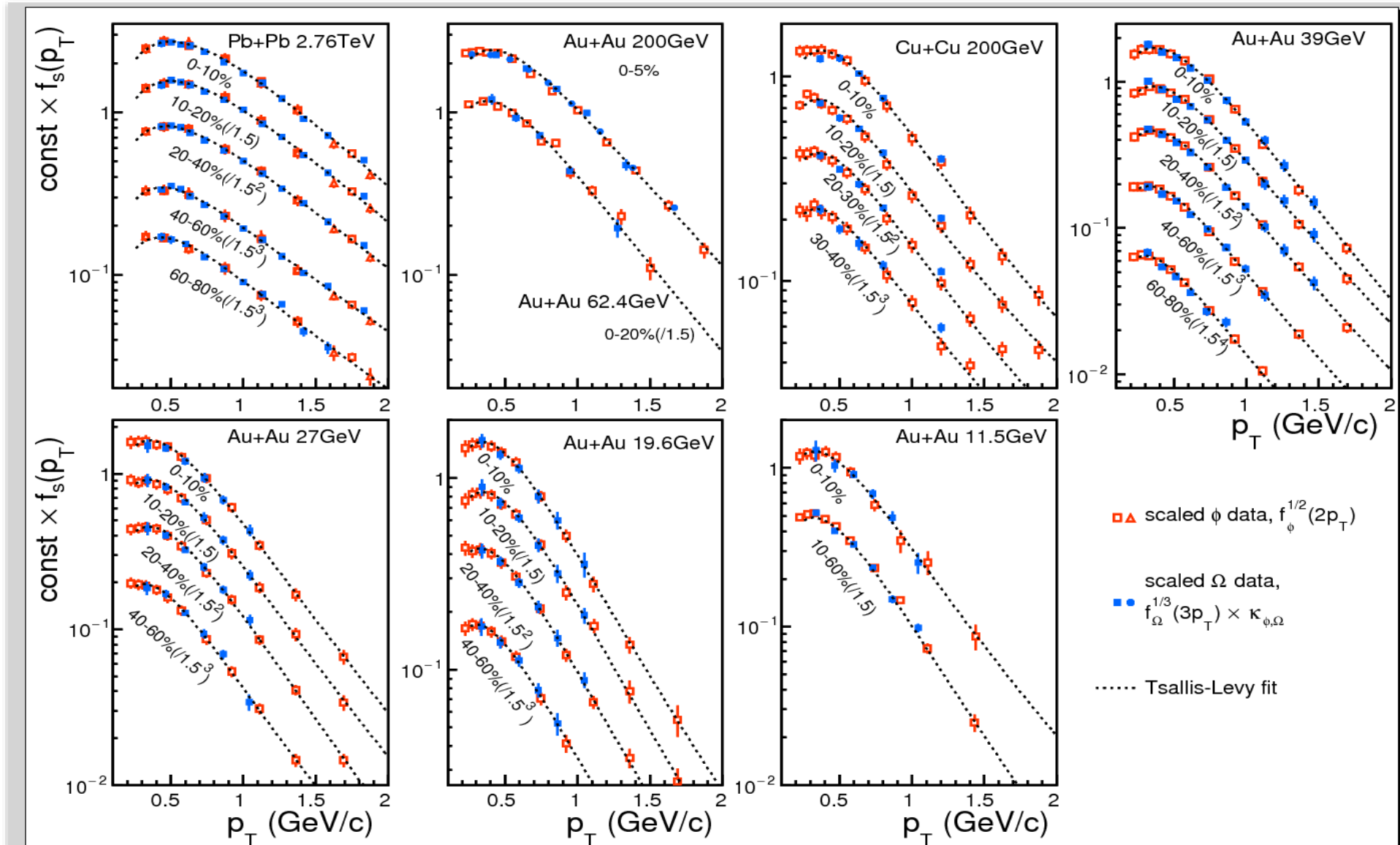
Song, Li, Shao, PRD105, 074027 (2022)



pp at  $\sqrt{s} = 200\text{ GeV}$



# QNS in heavy-ion collisions



### 3.

## **Equal-Velocity Combination (EVC) of constituent quarks and antiquarks at hadronization**

Refs:

- Jun Song, Xing-rui Gou, Feng-lan Shao, Zuo-tang Liang, Phys. Lett. B774, 516(2017).
- Xing-rui Gou, Feng-lan Shao, Rui-qin Wang, Hai-hong Li, Jun Song, Phys. Rev. D96,094010(2017).
- Jun Song, Hai-hong Li, and Feng-lan Shao, Eur. Phys. J. C 78:344 (2018).
- Hai-hong Li, Feng-lan Shao, and Jun Song, Phys. Rev. C 97, 064915 (2018).

## Start from general formula in momentum space

$$f_{B_j}(p_B) = \int dp_1 dp_2 dp_3 R_{B_j}(p_1, p_2, p_3; p_B) f_{q_1 q_2 q_3}(p_1, p_2, p_3)$$
$$f_{M_j}(p_M) = \int dp_1 dp_2 R_{M_j}(p_1, p_2; p_M) f_{q_1 \bar{q}_2}(p_1, p_2)$$

## Assume independent distribution of (anti-)quarks

$$f_{q_1 q_2 q_3}(p_1, p_2, p_3) = f_{q_1}(p_1) f_{q_2}(p_2) f_{q_3}(p_3)$$
$$f_{q_1 \bar{q}_2}(p_1, p_2) = f_{q_1}(p_1) f_{\bar{q}_2}(p_2)$$

## equal velocity combination (EVC) approximation

$$R_{B_j}(p_1, p_2, p_3; p_B) = \kappa_{B_j} \prod_{i=1}^3 \delta(p_i - x_i p_B)$$
$$R_{M_j}(p_1, p_2; p_M) = \kappa_{M_j} \prod_{i=1}^2 \delta(p_i - x_i p_M)$$

## momentum fraction

$$\text{meson } \mathbf{x}_{1,2} = \frac{m_{1,2}}{m_1 + m_2}, \quad \text{baryon } \mathbf{x}_{1,2,3} = \frac{m_{1,2,3}}{m_1 + m_2 + m_3}$$

$$m_s = 500 \text{ MeV}, m_u = m_d = 330 \text{ MeV}.$$

Constituent  
quark model  
of hadron

**We obtain**

$$f_{B_j}(p_B) = \kappa_{B_j} f_{q_1}(x_1 p_B) f_{q_2}(x_2 p_B) f_{q_3}(x_3 p_B)$$

$$f_{M_j}(p_M) = \kappa_{M_j} f_{q_1}(x_1 p_M) f_{\bar{q}_2}(x_2 p_M)$$

**The combination of  $s$  and  $\bar{s}$**

$$f_{\Omega}(3p_T) = \kappa_{\Omega} f_s^3(p_T)$$

$$f_{\phi}(2p_T) = \kappa_{\phi} f_s^2(p_T)$$



$$f_{\Omega}^{\frac{1}{3}}(3p_T) = \kappa_{\phi, \Omega} f_{\phi}^{\frac{1}{2}}(2p_T)$$

$f_s(p_T) = f_{\bar{s}}(p_T)$  is taken at LHC

**The combination of  $u(d)$  and  $s$ , denote  $\frac{x_u}{x_s} = \frac{m_u}{m_s} = r$**

$$f_{\Xi^{*0}}((2+r)p_T) = \kappa_{\Xi^{*0}} f_s^2(p_T) f_u(r p_T)$$

$$f_{K^{*0}}((1+r)p_T) = \kappa_{K^{*0}} f_s(p_T) f_{\bar{d}}(r p_T)$$



$$\frac{f_{\Xi^{*0}}((2+r)p_T)}{f_{K^{*0}}((1+r)p_T)} = \kappa_{\phi, K^{*0}, \Xi^{*0}} f_{\phi}^{\frac{1}{2}}(2p_T)$$

Consider stochastic combination and flavor-blind of strong interaction

$$\frac{\kappa_{Mj}}{A_{Mj}} = C_{Mj} P_{q\bar{q} \rightarrow M}$$

$$\frac{\kappa_{Bj}}{A_{Bj}} = C_{Bj} N_{iter} P_{qqq \rightarrow B}$$

$A_{Bj} = 1/\int dp_T \prod_{i=1}^3 f_{q_i}^{(n)}(x_i p_T)$ ,  $A_{Mj} = 1/\int dp_T f_{q_1}^{(n)}(x_1 p_T) f_{\bar{q}_2}^{(n)}(x_2 p_T)$  are normalization coefficients of jointed quark distributions.

$$P_{q\bar{q} \rightarrow M} \approx \frac{2}{x(1-z^2)} \left[ 1 - z \frac{(1+z)^a + (1+z)^a}{(1+z)^a - (1+z)^a} \right], \text{ averaged probability of } q\bar{q} \rightarrow M$$

$$P_{qqq \rightarrow B} \approx \frac{8}{3x^2(1+z)^3} \frac{(1+z)^a}{(1+z)^a - (1+z)^a}, \text{ averaged probability of } qqq \rightarrow B$$

Song, Shao, PRC 88, 027901(2013)

$N_{iter}=1,3,6$  for three identical, two identical and three different flavors

$x = N_q + N_{\bar{q}}$ ,  $z = (N_q - N_{\bar{q}})/x$ ,  $a = 1 + \frac{1}{3} \left( \overline{N}_M / \overline{N}_B \right)_{z=0} \approx 4.86 \pm 0.1$  for light-flavor sector

$N_q$  number of all quarks;  $N_{\bar{q}}$  that of all antiquarks

$C_{M_j}$  and  $C_{B_j}$  are fine-tune parameters

$$C_{M_j} = \begin{cases} \frac{1}{1+R_{V/P}} & \text{for } J^P = 0^- \text{ mesons} \\ \frac{R_{V/P}}{1+R_{V/P}} & \text{for } J^P = 1^- \text{ mesons} \end{cases} \quad C_{B_j} = \begin{cases} \frac{R_{O/D}}{1+R_{O/D}} & \text{for } J^P = (1/2)^+ \text{ baryons} \\ \frac{1}{1+R_{O/D}} & \text{for } J^P = (3/2)^+ \text{ baryons} \end{cases} \quad \begin{array}{l} R_{V/P} \approx 0.5 \\ R_{O/D} \approx 2.0 \end{array}$$

Include decay contributions

$$f_{h_j}^{(final)}(p) = f_{h_j}(p) + \sum_{i \neq j} \int dp' f_{h_i}(p') D_{ij}(p', p)$$

decay function  $D_{ij}(p', p)$  is determined by the decay kinematics and decay branch ratios in PDG

**Model inputs:**  $f_{q_i}(p)$  which are fixed by experimental data

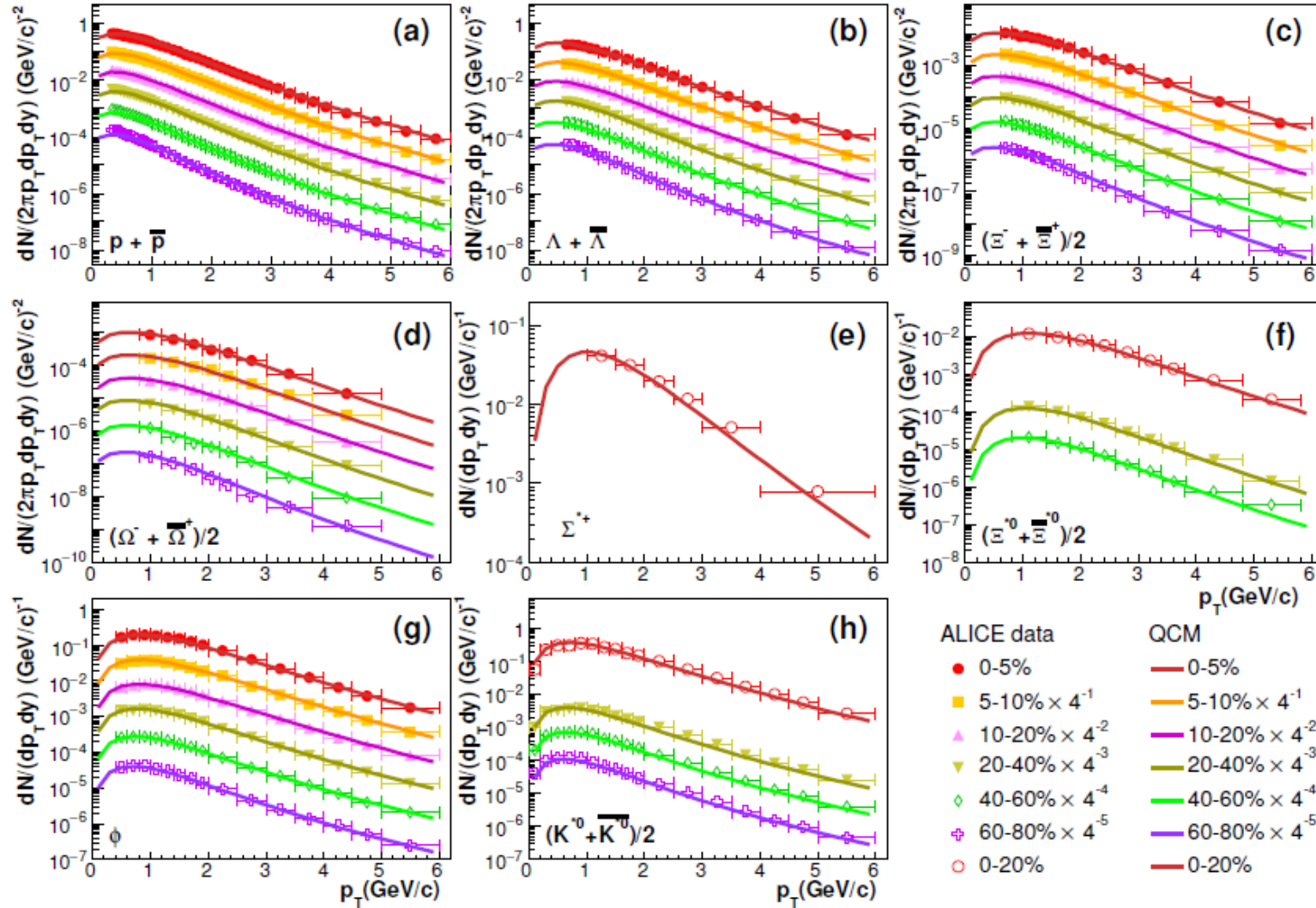
## 4.

# Systematic explanation of $p_T$ spectra of light-flavor hadrons

Refs:

- Jun Song, Hai-hong Li, and Feng-lan Shao, Phys. Rev. D105, 074027 (2022).
- Yan-ting Feng, Jun Song, and Feng-lan Shao, Phys. Rev. C106, 034910 (2022).
- Hai-hong Li, Feng-lan Shao, and Jun Song, Chin. Phys. C 45, 113105 (2021).
- Jian-wei Zhang, Hai-hong Li, Feng-lan Shao, and Jun Song, Chin.Phys. C44, 014101(2020).
- Jun Song, Xing-rui Gou, Feng-lan Shao, Zuo-tang Liang, Phys. Lett. B774, 516(2017).
- Xing-rui Gou, Feng-lan Shao, Rui-qin Wang, Hai-hong Li, Jun Song, Phys. Rev. D96,094010(2017).

## p-Pb collisions at 5.02 TeV



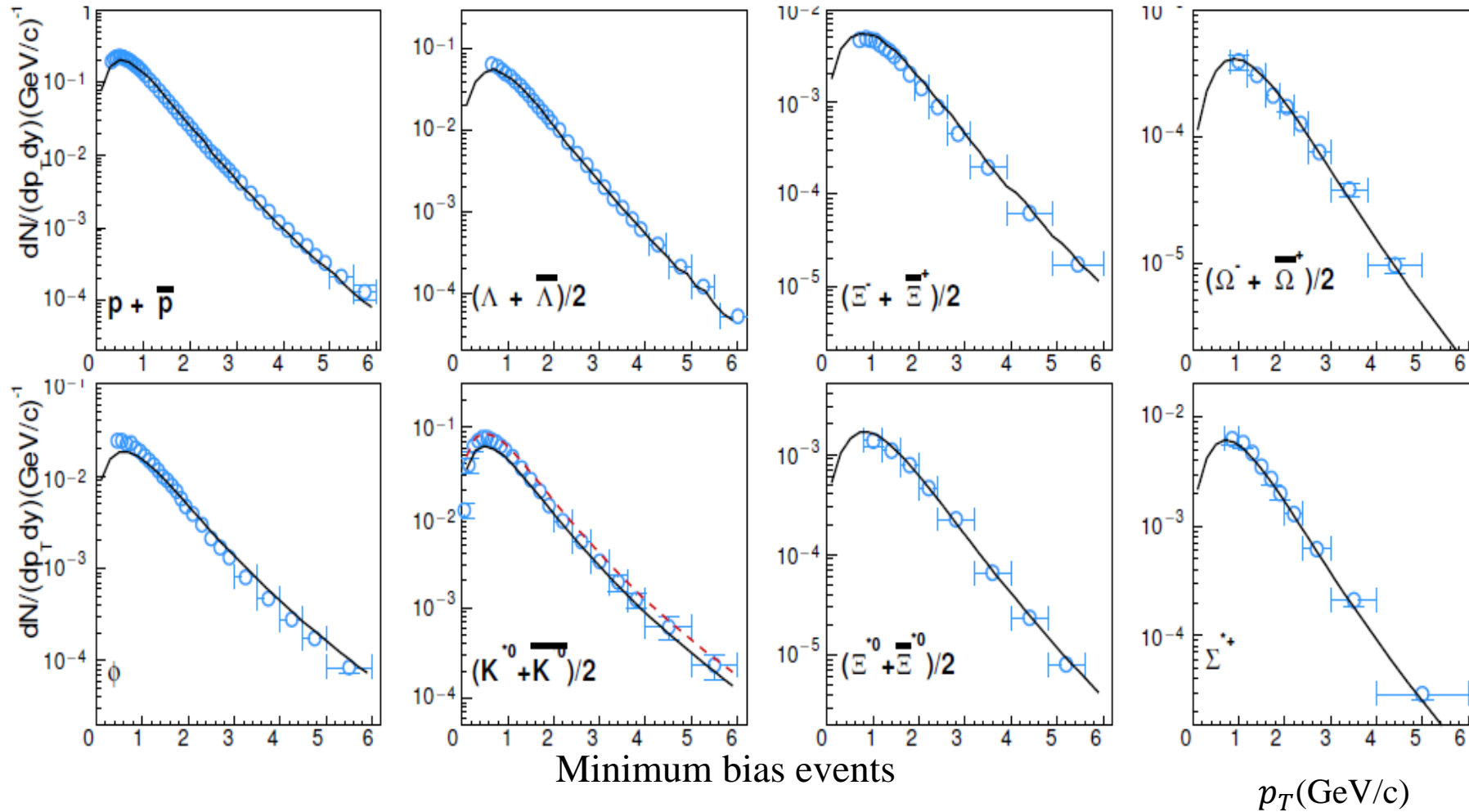
Two inputs:

$$f_u(p_T) = f_d(p_T) = f_{\bar{u}}(p_T) = f_{\bar{d}}(p_T)$$

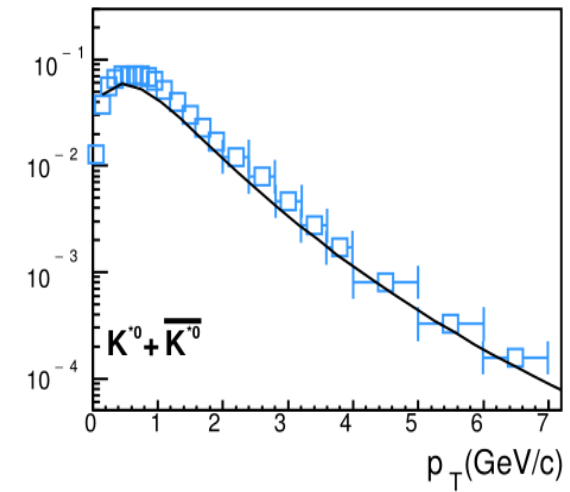
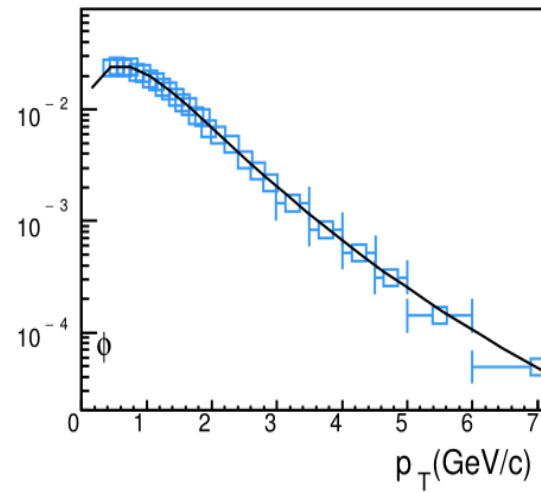
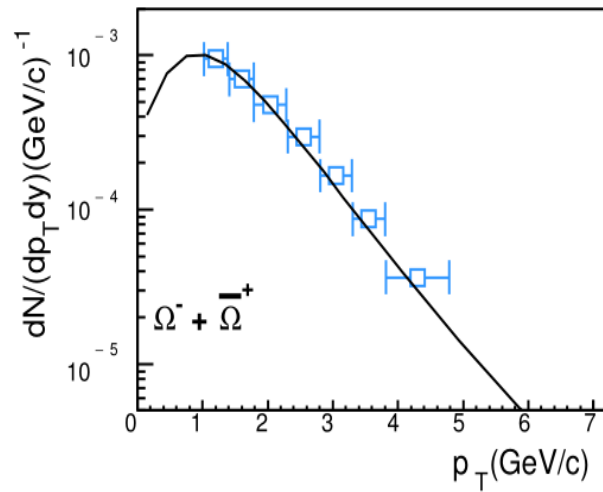
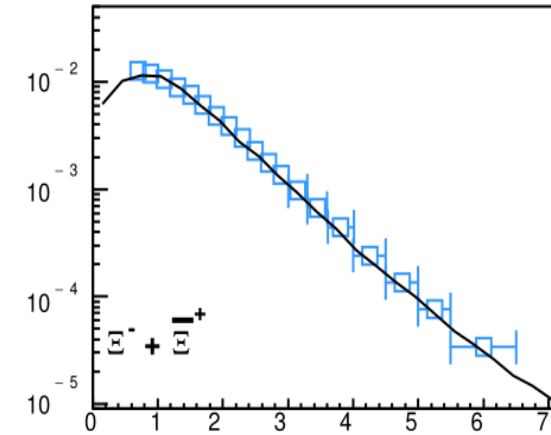
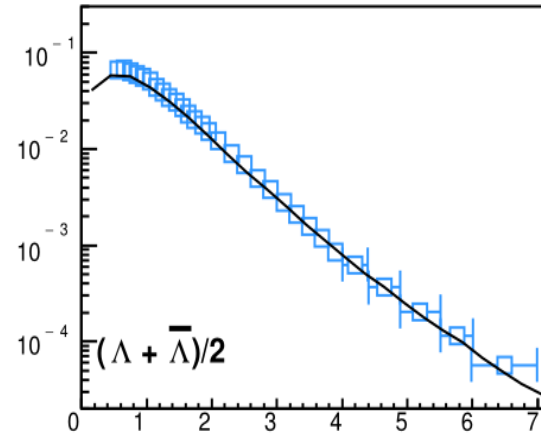
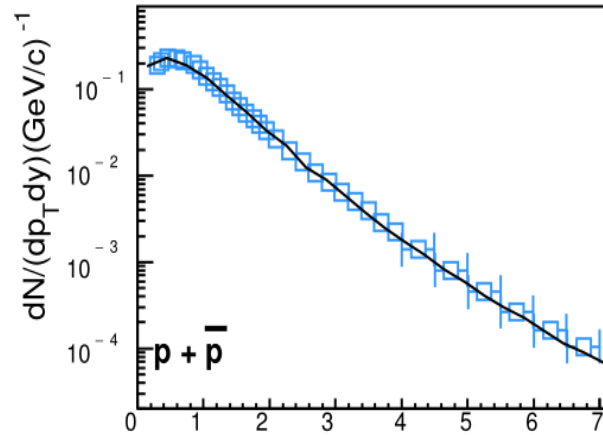
$$f_s(p_T) = f_{\bar{s}}(p_T)$$

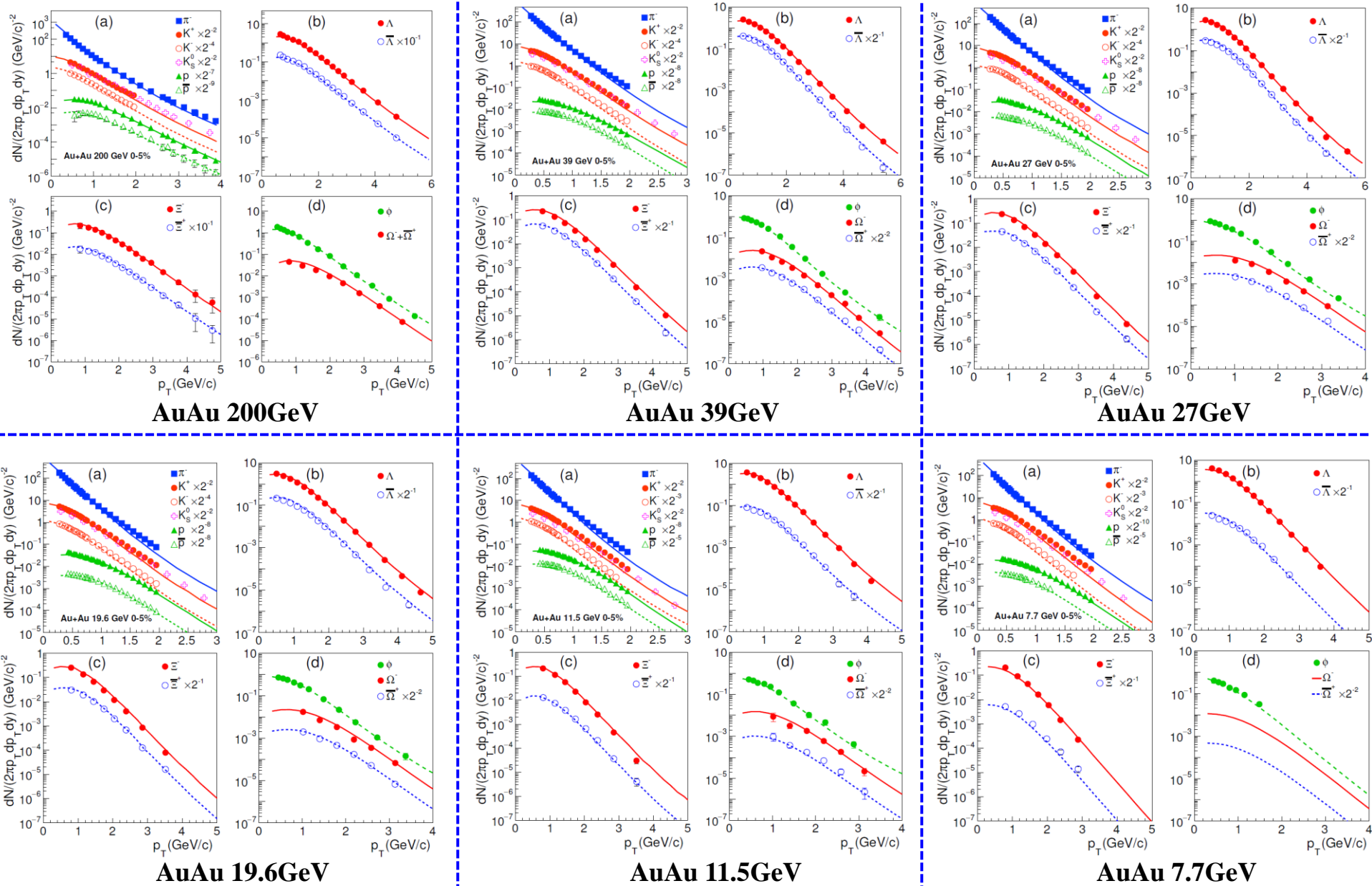


## pp collisions at 7 TeV



## pp collisions at 13 TeV

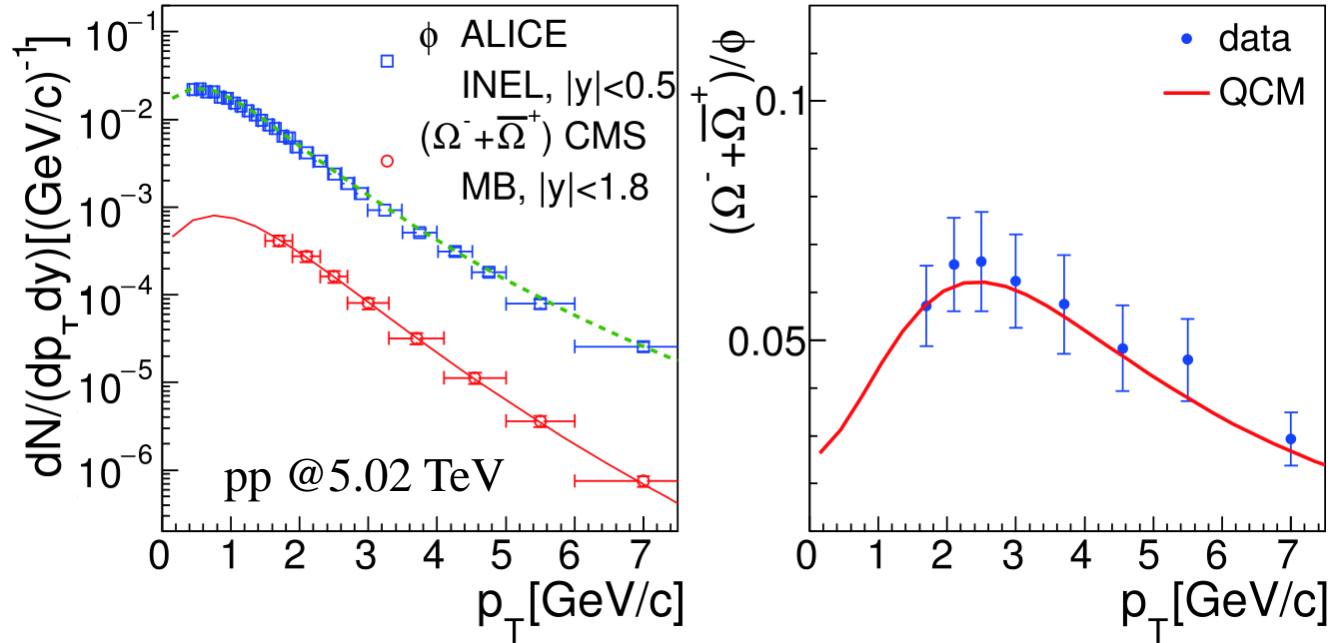




Three inputs  $f_u(p_T) \approx f_d(p_T)$ ,  $f_{\bar{u}}(p_T) \approx f_{\bar{d}}(p_T)$ ,  $f_s(p_T) \approx f_{\bar{s}}(p_T)$  at each energy.

## Analytical aspect of EVC :

$\Omega/\phi$  ratio as the function of  $p_T$



$$f_s(p_{T_s}) \sim \begin{cases} p_T \exp\left[-\frac{\sqrt{p_T^2 + m^2}}{T}\right] & p_{T_s} \lesssim 1.0 \frac{GeV}{c} \\ \left(1 + \frac{p_T}{p_{T_0}}\right)^{-n} & p_{T_s} \gtrsim 1.0 \frac{GeV}{c} \end{cases}$$

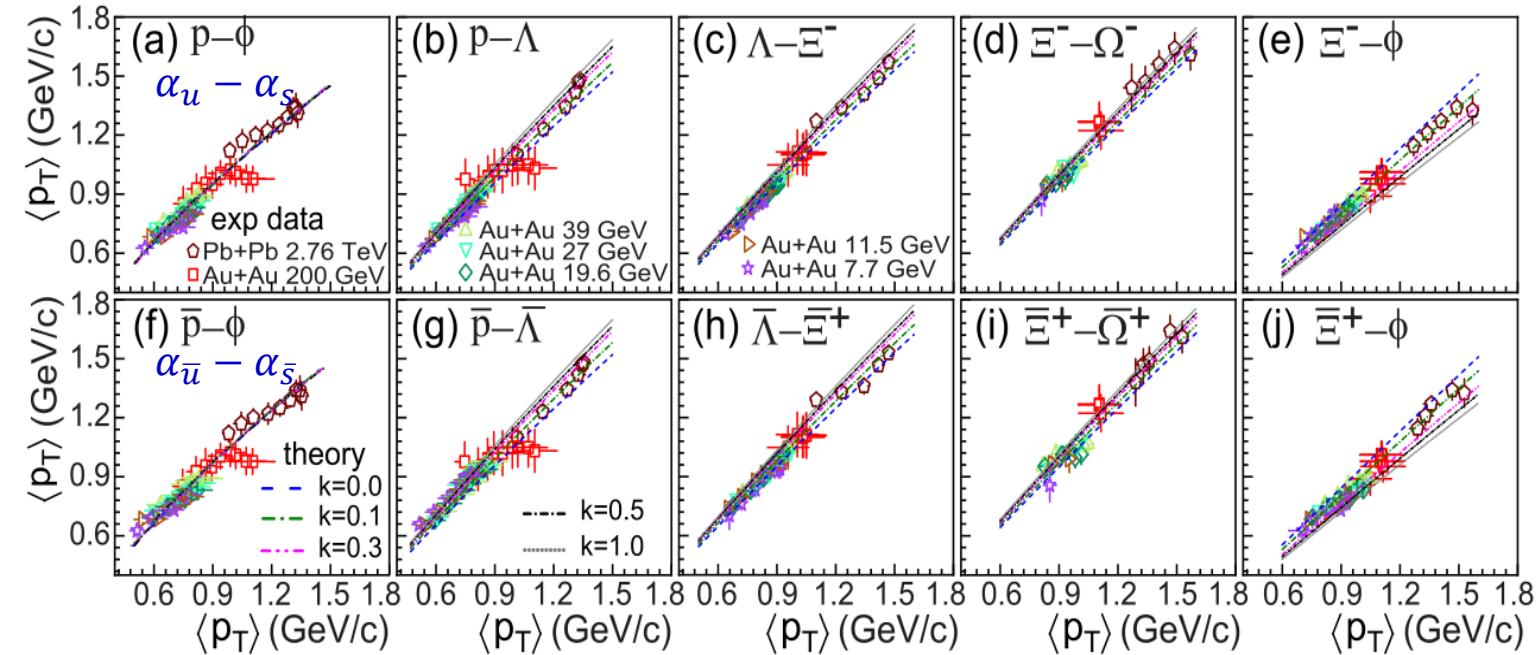
$$[\ln f_s(\xi)]'' \sim \begin{cases} < 0 & p_{T_s} \lesssim 1.0 GeV/c \\ > 0 & p_{T_s} \gtrsim 1.0 GeV/c \end{cases}$$

$$\left[\ln \frac{\Omega}{\phi}(p_T)\right]' \sim \begin{cases} > 0 & p_T \lesssim 2.5 GeV/c \\ < 0 & p_T \gtrsim 2.5 GeV/c \end{cases}$$

$$\left[\ln \frac{\Omega}{\phi}(p_T)\right]' = -\frac{1}{6} [\ln f_s(\xi)]'' \quad \frac{p_T}{3} < \xi < \frac{p_T}{2}$$

## Analytical aspect of EVC :

$\langle p_T \rangle$  correlation between two hadrons



Quark thermal distribution :

$$f_{q_i}(p_T) = N p_T^k e^{-\sqrt{p_T^2 + m_i^2}/T_i}$$

$$\langle p_T \rangle_M = (m_{q_1} + m_{\bar{q}_2}) \sqrt{\frac{2}{\alpha_M} \frac{\Gamma(k+1)}{\Gamma(k+\frac{1}{2})} \frac{K_{k+\frac{3}{2}}(\alpha_M)}{K_{k+1}(\alpha_M)}}$$

$$\langle p_T \rangle_B = (m_{q_1} + m_{q_2} + m_{q_3}) \sqrt{\frac{2}{\alpha_B} \frac{\Gamma(\frac{3k}{2}+1)}{\Gamma(\frac{3k}{2}+\frac{1}{2})} \frac{K_{\frac{3k}{2}+3}(\alpha_B)}{K_{\frac{3k}{2}+1}(\alpha_B)}}$$

$$\alpha_M = \frac{m_{q_1}}{T_1} + \frac{m_{\bar{q}_2}}{T_2} = \alpha_{q_1} + \alpha_{\bar{q}_2}$$

$$\alpha_B = \alpha_{q_1} + \alpha_{q_2} + \alpha_{q_3}$$

# 5.

## **EVC explanation of $v_2$ of hadrons**

Refs:

- Jun Song, Hai-hong Li, and Feng-lan Shao, Eur. Phys. J. C (2021) 81:5

## Hadron $v_2$ as functions of quark $v_2$

$$v_{2,M_i}(p_T) = v_{2,q_1}(x_1 p_T) + v_{2,\bar{q}_2}(x_2 p_T),$$

$$v_{2,B_i}(p_T) = v_{2,q_1}(x_1 p_T) + v_{2,q_2}(x_2 p_T) + v_{2,q_3}(x_3 p_T).$$

$$v_{2,\Omega}(p_T) = 3v_{2,s}(p_T/3),$$

$$v_{2,p}(p_T) = 3v_{2,u}(p_T/3),$$

$$v_{2,\phi}(p_T) = v_{2,s}(p_T/2) + v_{2,\bar{s}}(p_T/2),$$

$$v_{2,\Lambda}(p_T) = 2v_{2,u}\left(\frac{1}{2+r}p_T\right) + v_{2,s}\left(\frac{r}{2+r}p_T\right),$$

$$v_{2,\Xi}(p_T) = v_{2,u}\left(\frac{1}{1+2r}p_T\right) + 2v_{2,s}\left(\frac{r}{1+2r}p_T\right)$$

$$r = \frac{x_s}{x_u} = \frac{m_s}{m_u}$$

## quark $v_2$ in terms of hadron $v_2$

$$v_{2,u}(p_T) = \frac{1}{3}v_{2,p}(3p_T),$$

$$v_{2,u}(p_T) = \frac{1}{3} [2v_{2,\Lambda}((2+r)p_T) - v_{2,\Xi}((1+2r)p_T)],$$

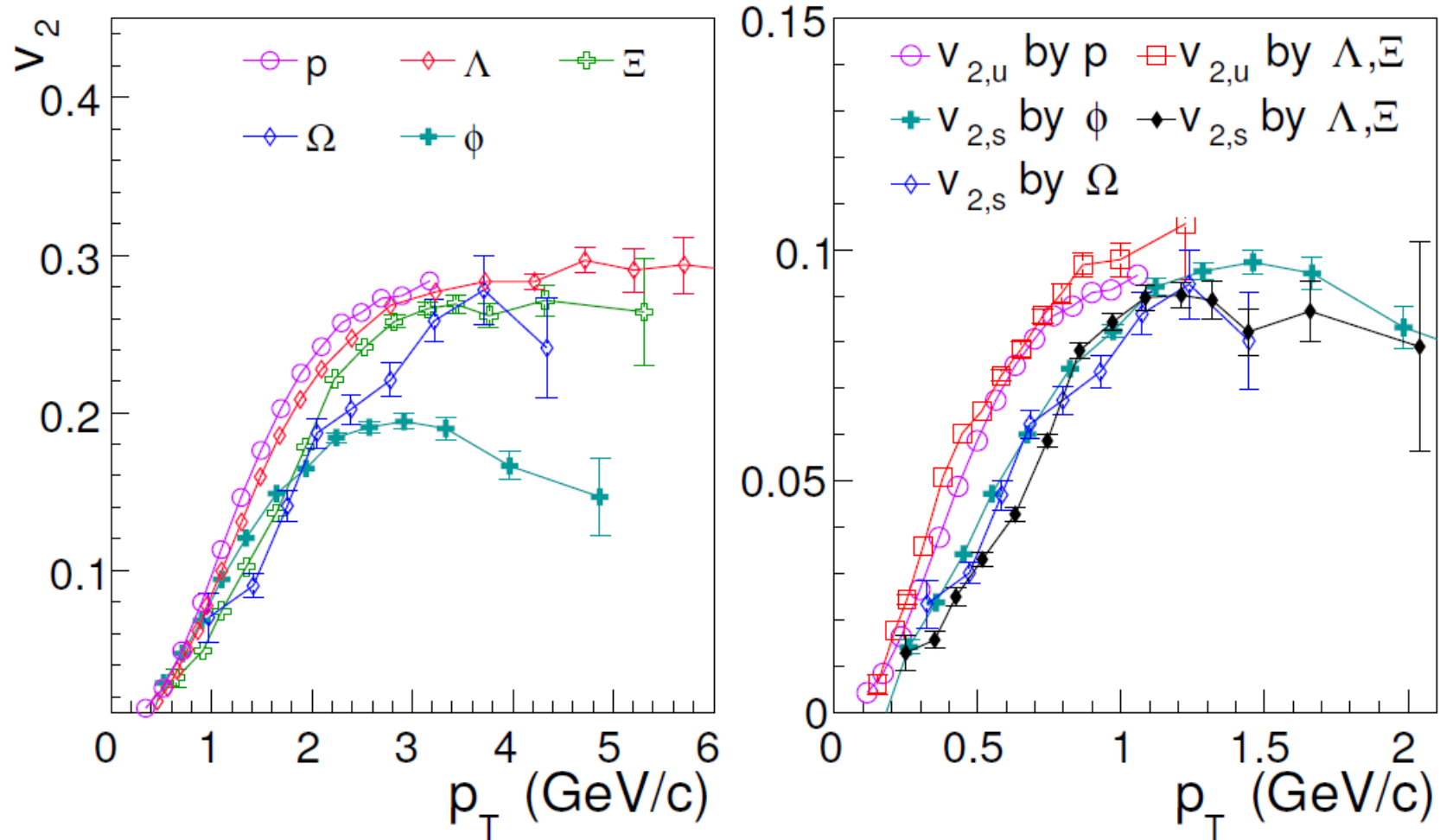
$$v_{2,s}(p_T) = \frac{1}{3}v_{2,\Omega}(3p_T),$$

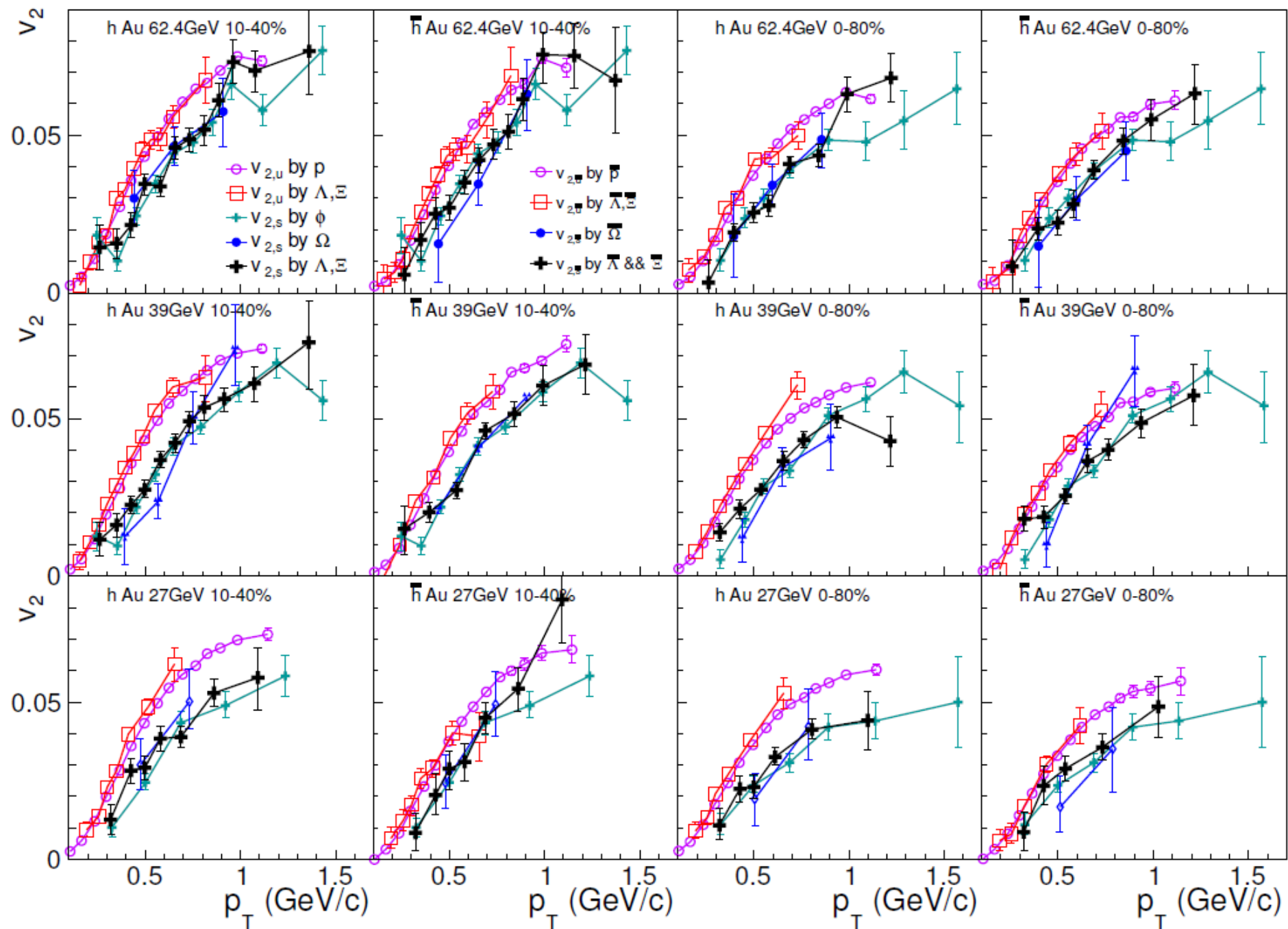
$$v_{2,s}(p_T) = \frac{1}{2}v_{2,\phi}(2p_T),$$

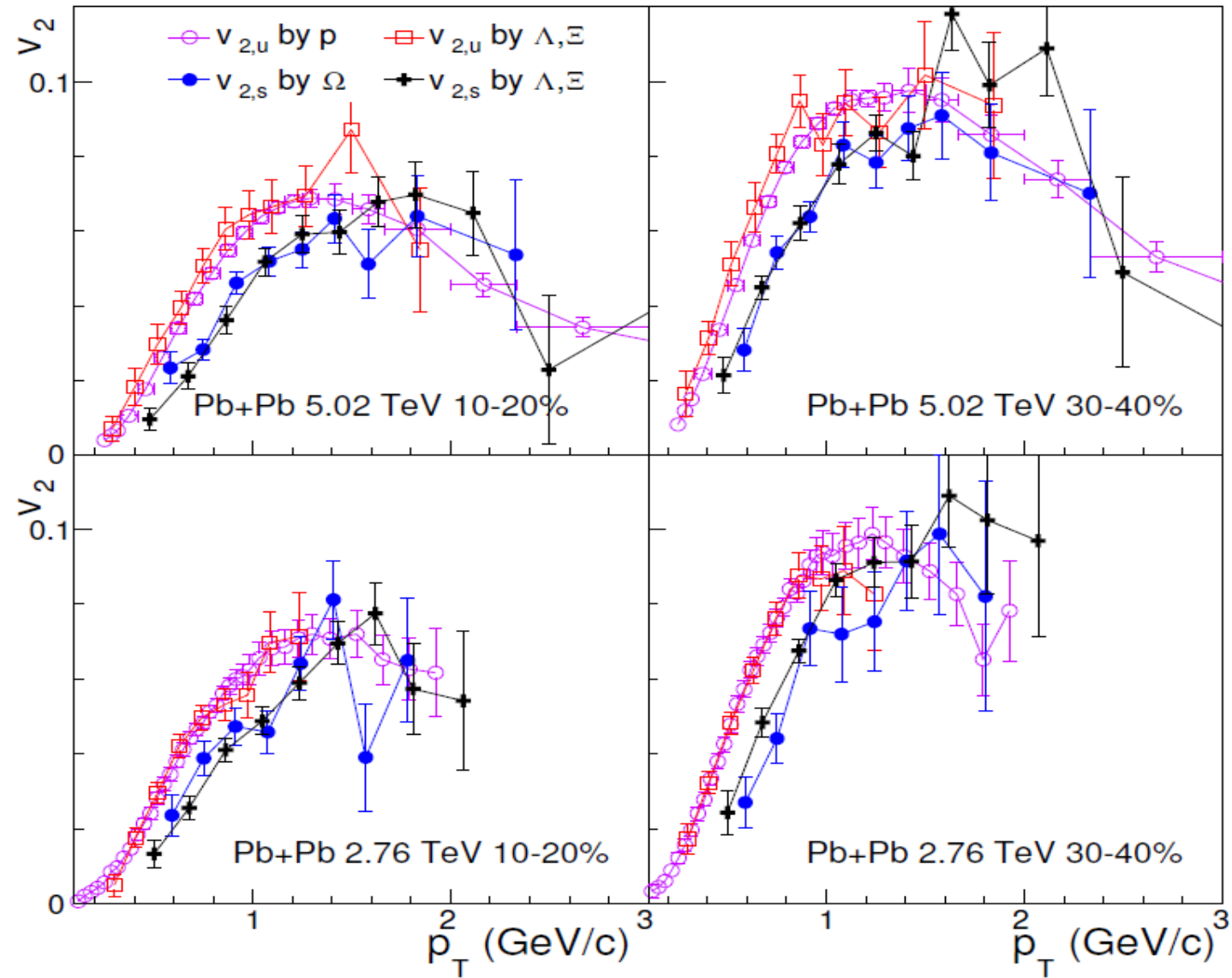
$$v_{2,s}(p_T) = \frac{1}{3} \left[ 2v_{2,\Xi} \left( \frac{1+2r}{r} p_T \right) - v_{2,\Lambda} \left( \frac{2+r}{r} p_T \right) \right].$$



### AuAu 200 GeV for 30-80% centrality







## 6.

# EVC mechanism of charm hadron production

Refs:

- Jun Song, Hai-hong Li, and Feng-lan Shao, *Eur. Phys. J. C* 83:852(2023).
- Jun Song, Hai-hong Li, and Feng-lan Shao, *Phys. Rev. D* 105, 074027 (2022).
- Hai-hong Li, Feng-lan Shao, and Jun Song, *Chin. Phys. C* 45, 113105 (2021).
- Jun Song, Hai-hong Li, and Feng-lan Shao, *Eur. Phys. J. C* 78:344 (2018).
- Hai-hong Li, Feng-lan Shao, and Jun Song, *Phys. Rev. C* 97, 064915 (2018).

## $p_T$ spectra of single-charmed hadrons in EVC

$$f_D(p_T) = \kappa_D f_c \left( \frac{r_{cu}}{1 + r_{cu}} p_T \right) f_{\bar{u}} \left( \frac{1}{1 + r_{cu}} p_T \right)$$

$$f_{D_s}(p_T) = \kappa_{D_s} f_c \left( \frac{r_{cs}}{1 + r_{cs}} p_T \right) f_{\bar{s}} \left( \frac{1}{1 + r_{cs}} p_T \right)$$

$$f_{\Lambda_c}(p_T) = \kappa_{\Lambda_c} f_c \left( \frac{r_{cu}}{2 + r_{cu}} p_T \right) f_u^2 \left( \frac{1}{1 + r_{cu}} p_T \right)$$

0.75-0.85

0.15-0.25

$$r_{cu} = \frac{m_c}{m_u}$$

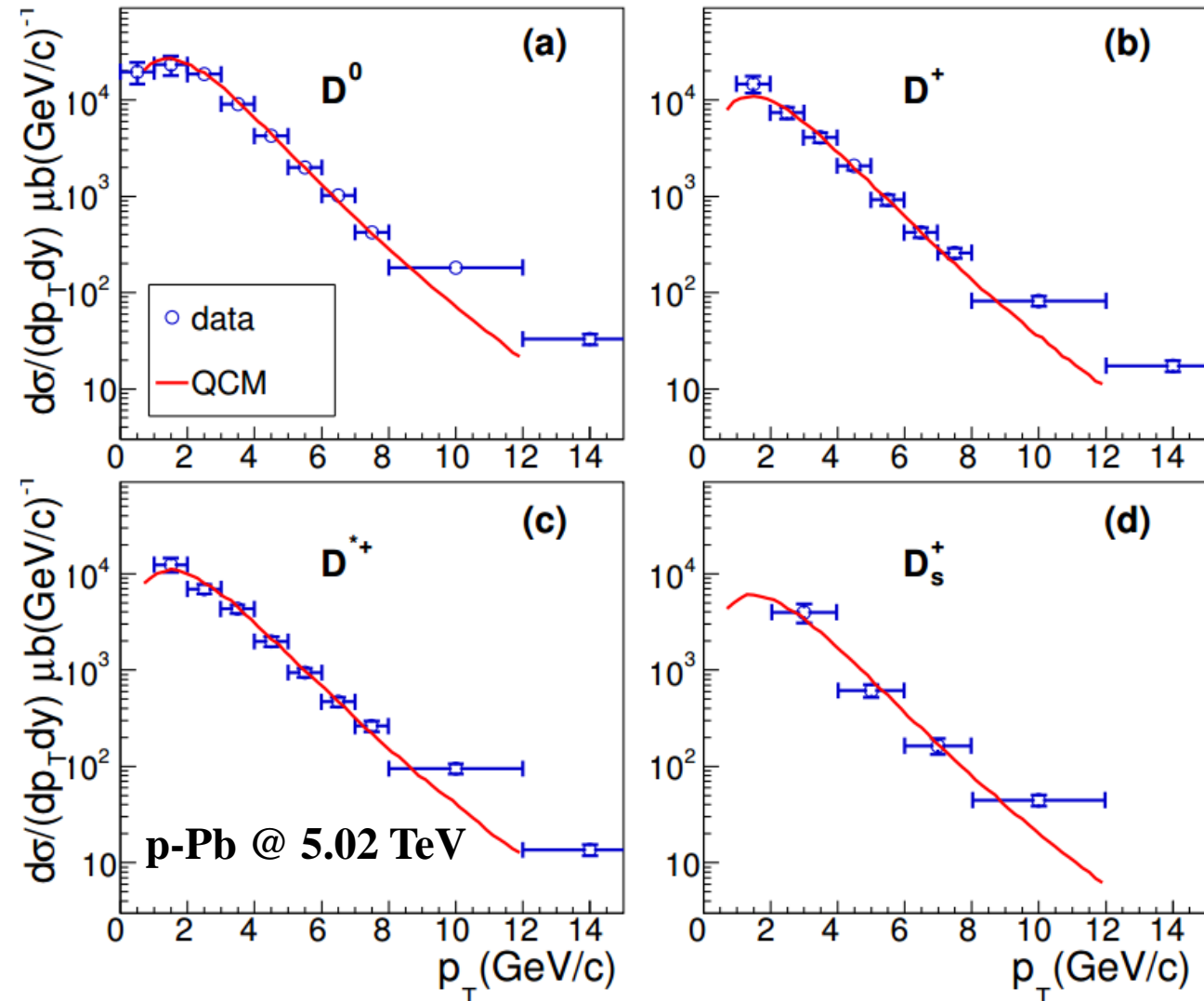
$$r_{cs} = \frac{m_c}{m_s}$$

$$m_c = 1.5 \text{ GeV}$$

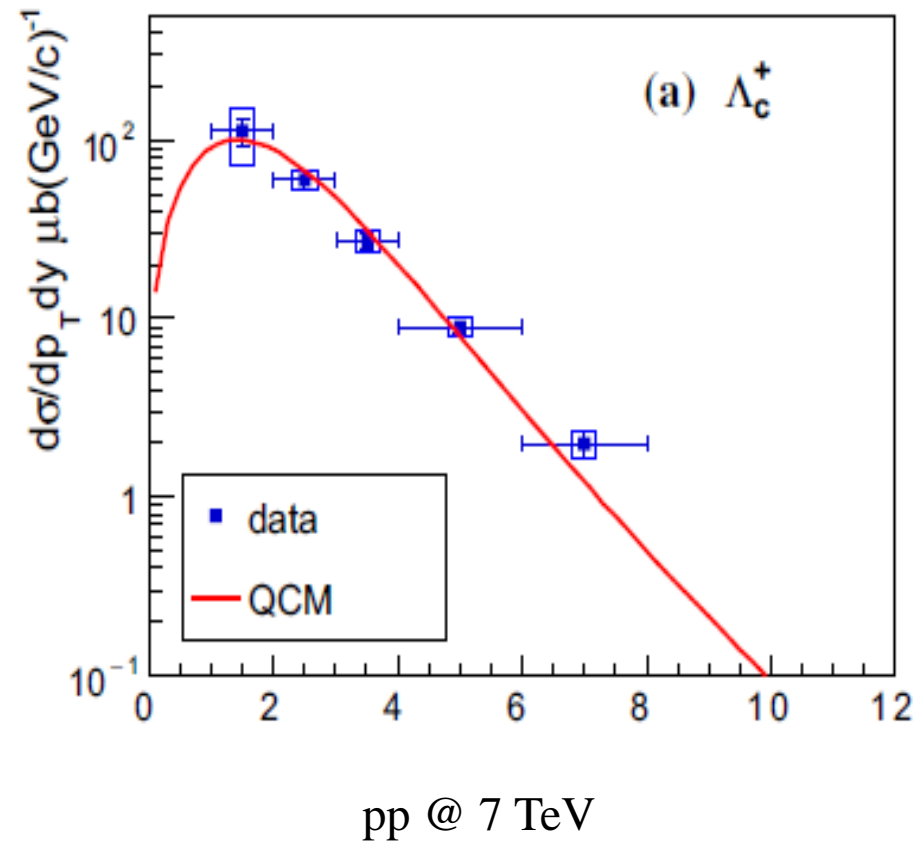
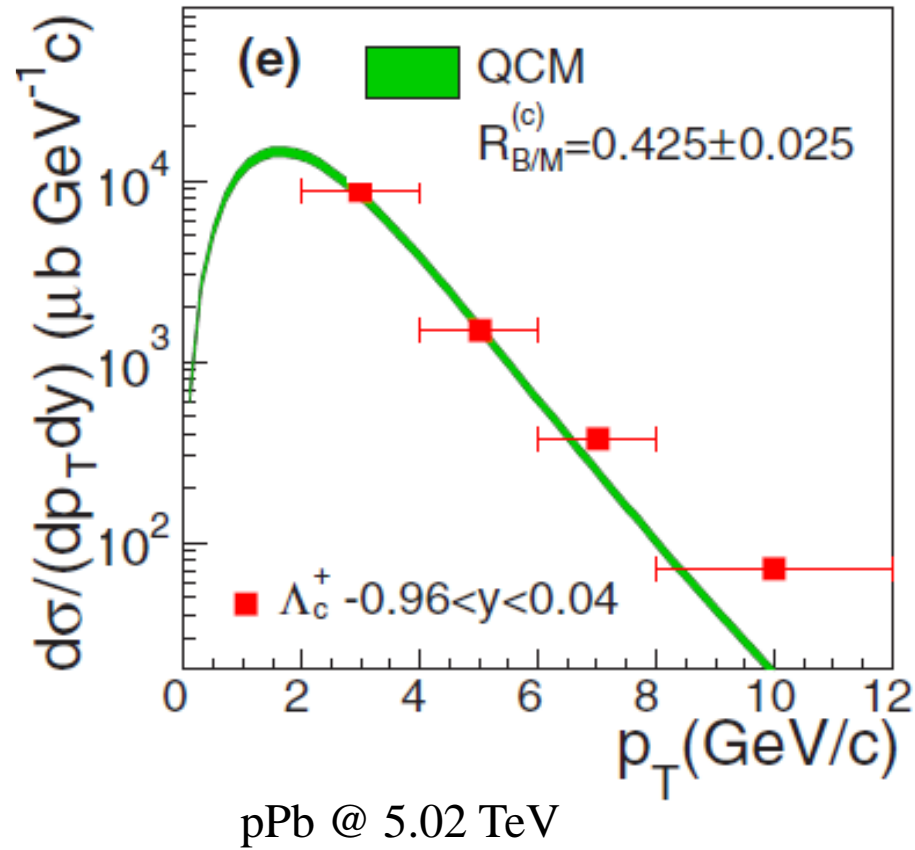
charm hadrons of  $p_T \lesssim 10 \text{ GeV}/c$  :

**c** quark of  $p_{T,c} \lesssim 8$  + **l** quark of  $p_{T,l} \lesssim 2 \text{ GeV}/c$

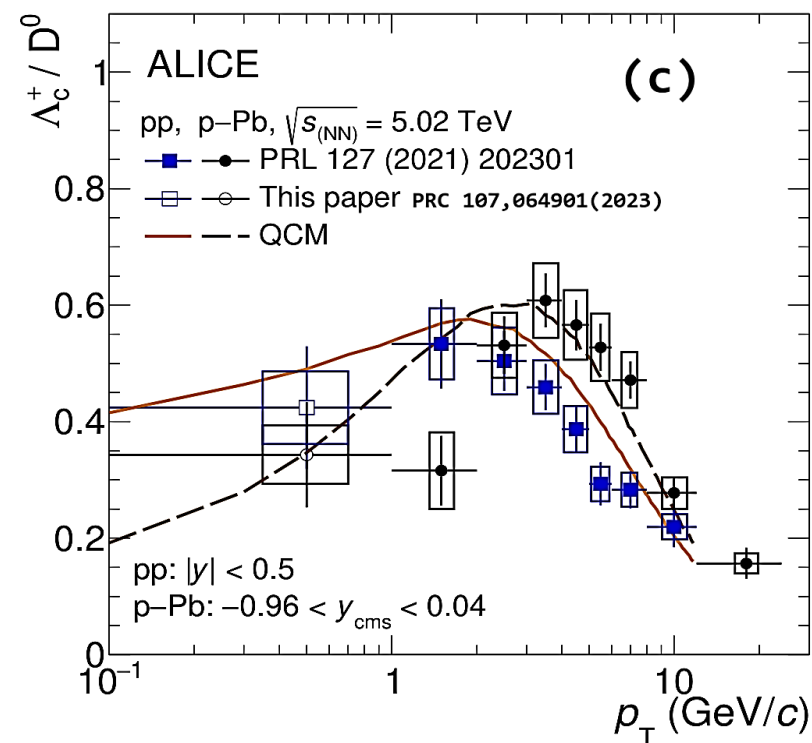
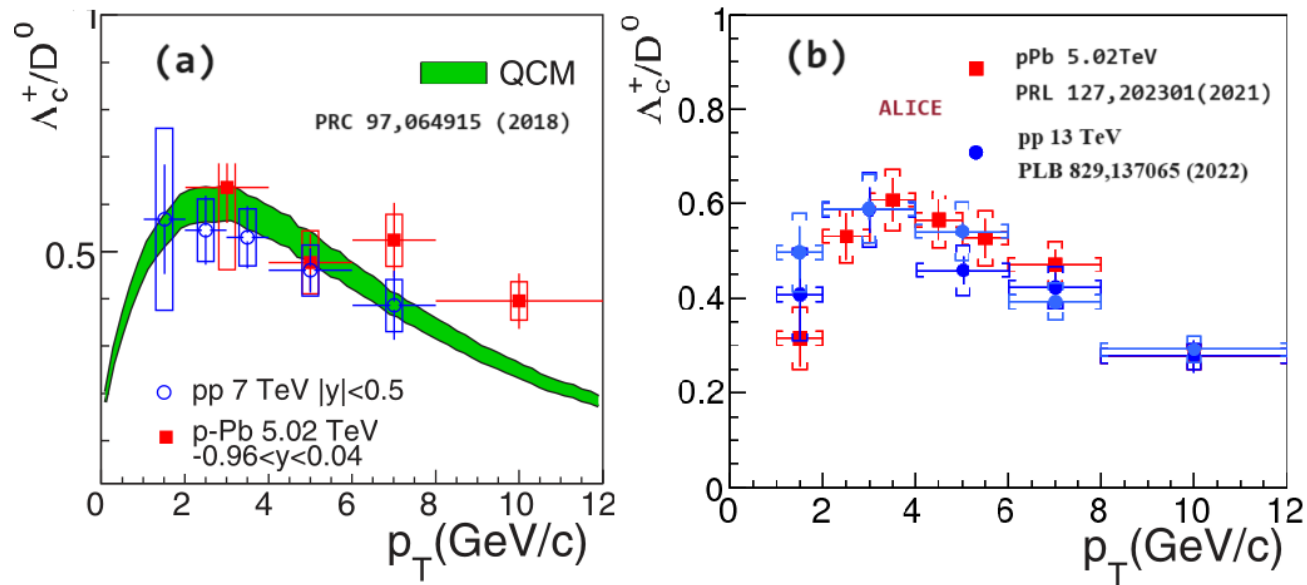
# $p_T$ spectra of $D$ mesons



# $p_T$ spectrum of $\Lambda_c^+$



# Non-monotonical $p_T$ dependence of charmed Baryon/Meson ratio



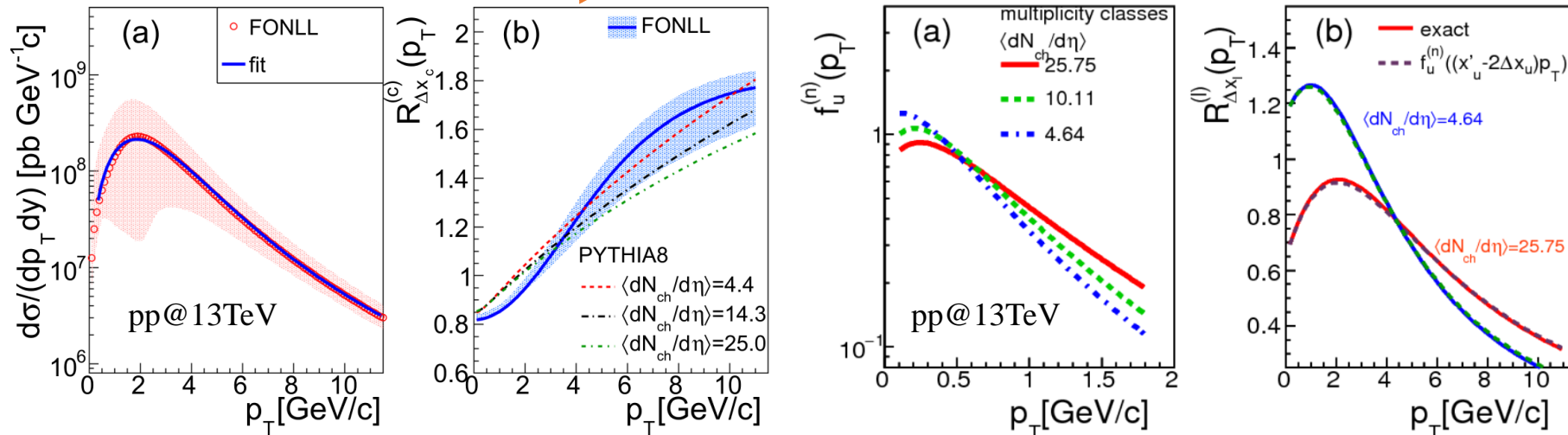


# Origin of $p_T$ dependence for $\Lambda_c^+ / D^0$ ratio

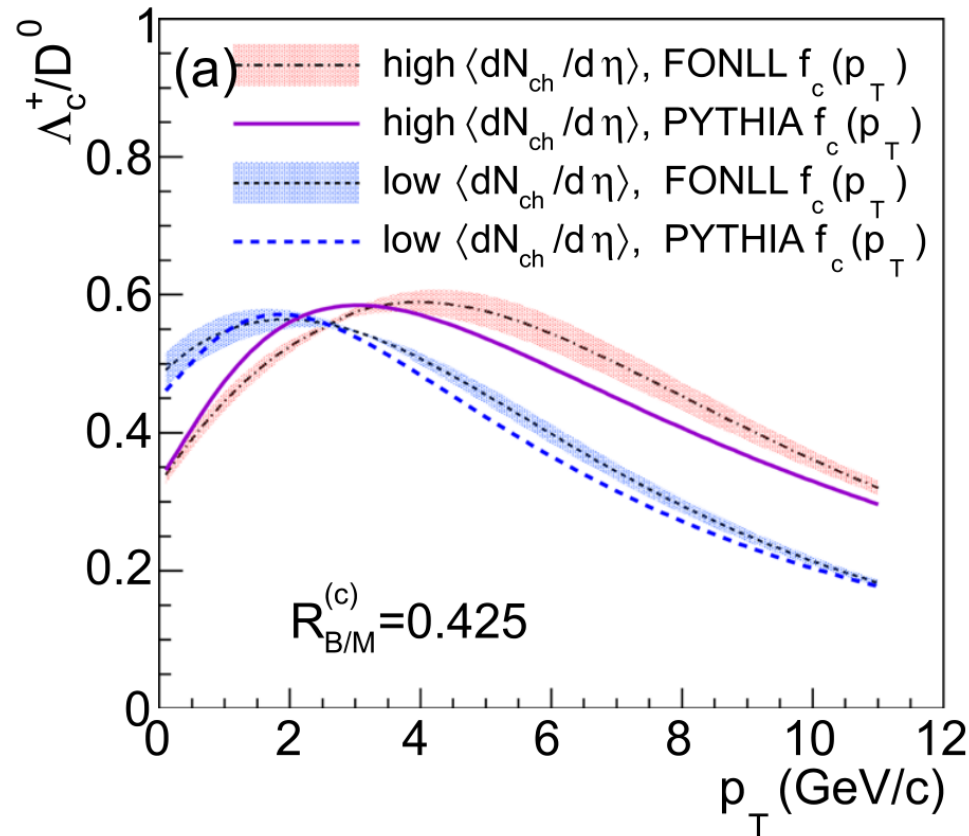
$$\frac{f_{\Lambda_c}(p_T)}{f_{D^0}(p_T)} = R^{(N_{qi})} \left[ \frac{A_{\Lambda_c}}{A_{D^0}} R_{\Delta x_c}^{(c)}(p_T) R_{\Delta x_l}^{(l)}(p_T) R_{\text{corr}}^{(cl)}(p_T) \right]$$

charm

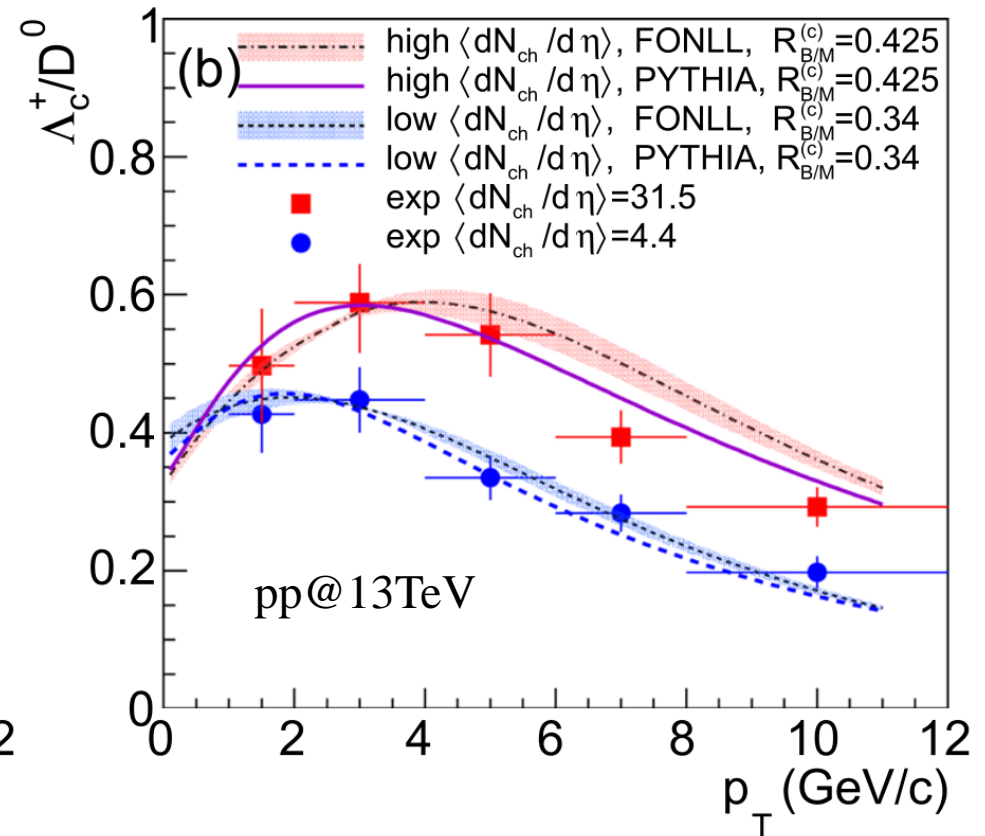
light-flavor



**Influence of  
the shape of quark distribution**



**Influence of  
the shape of quark distribution  
&  
baryon/meson production competition**

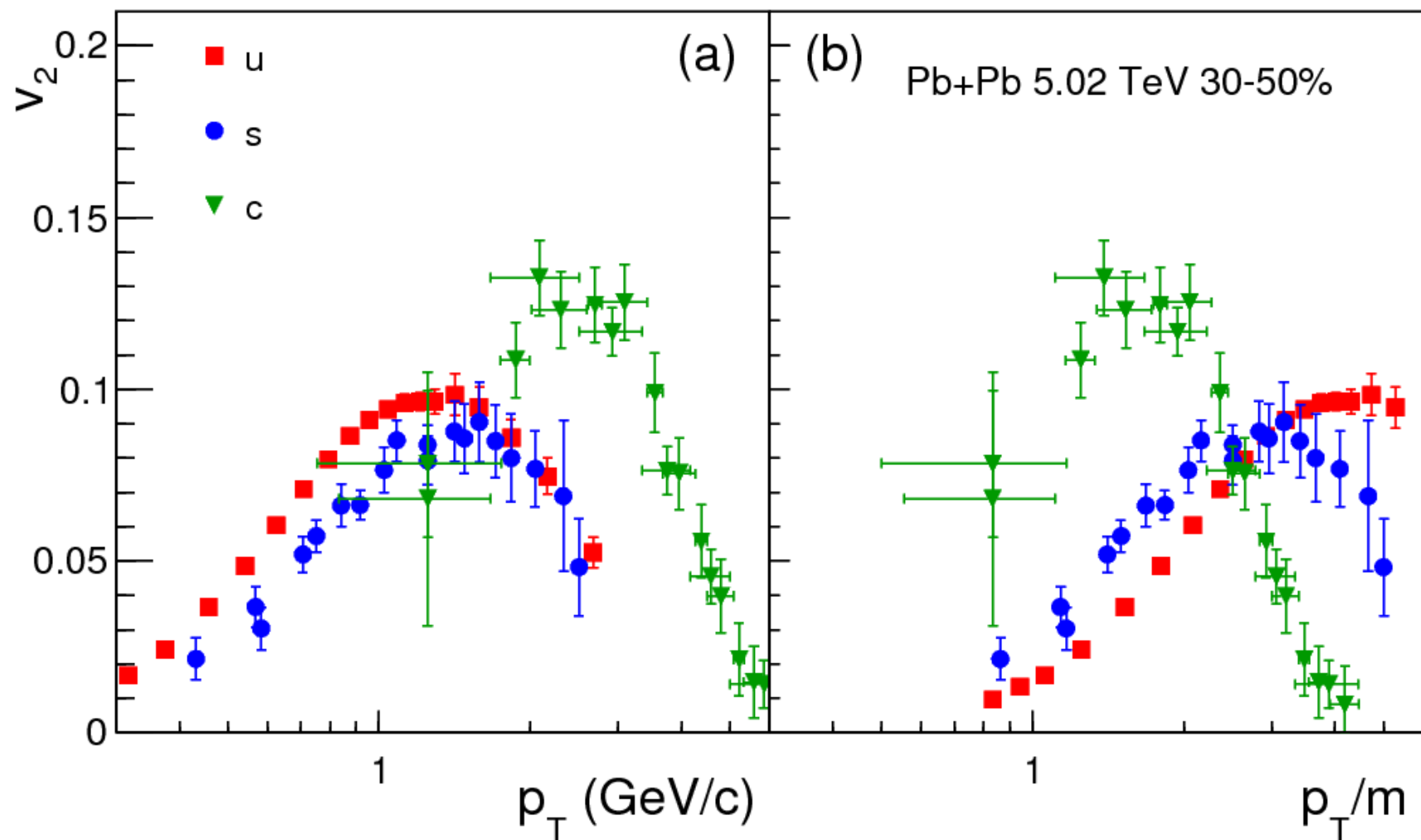


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**7.**

**understand parton system before  
hadronization**

## Compare $v_2$ of up, strange and charm quarks



# Effects of hadronization

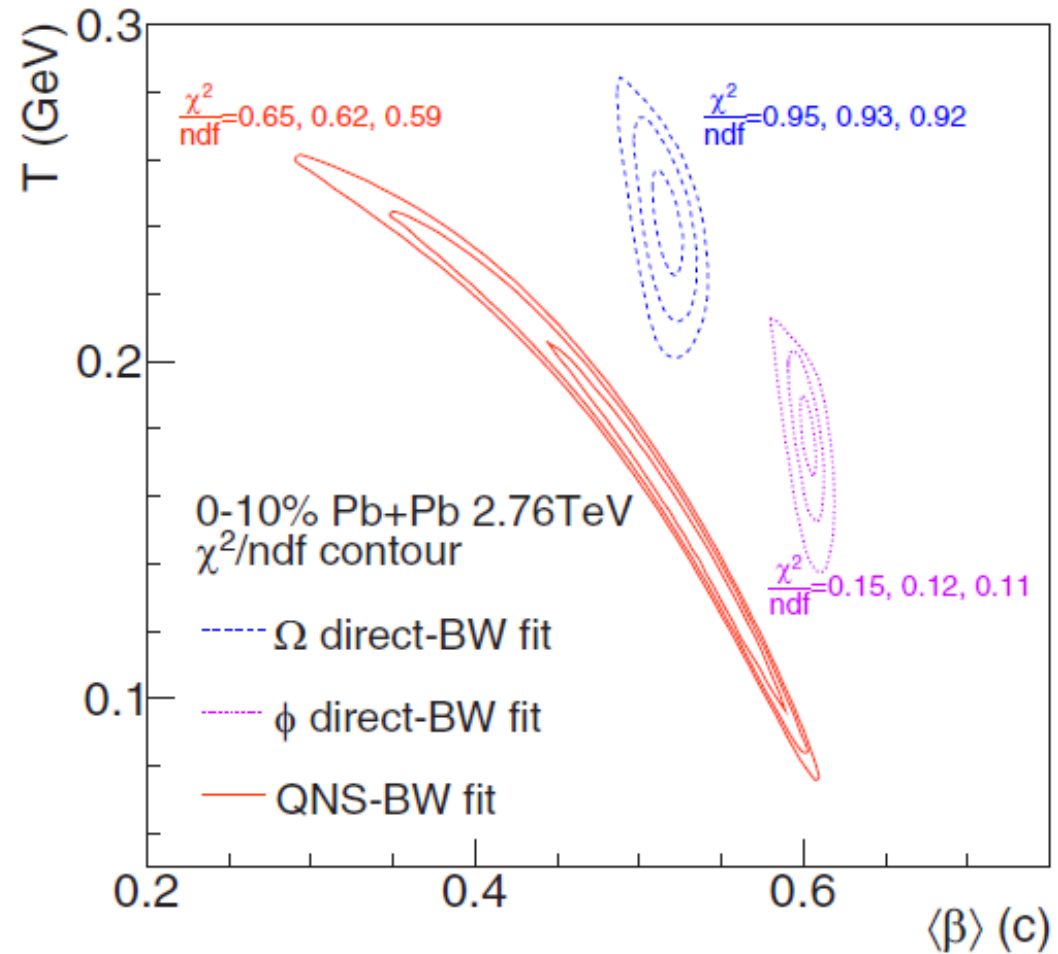


FIG. 4. Contour plot of blast-wave model fit of data of the  $p_T$  spectra of  $\Omega$  and  $\phi$  in central (0–10%) Pb + Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV [3,4].

# Summary

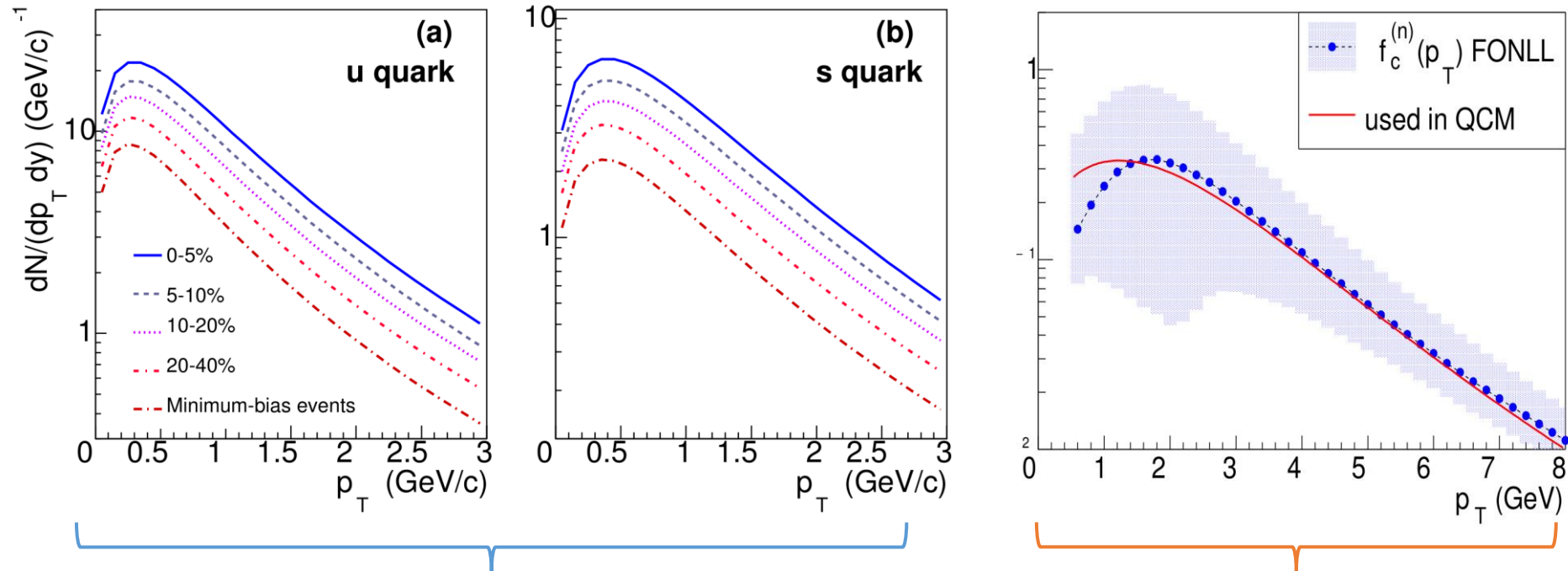
EVC is an effective mechanism

- ✓ quark number scaling for hadronic  $p_T$  spectra.
- ✓ self-consistent description for  $p_T$  spectra of light-flavor hadrons as well as single-charmed hadrons in pp and pPb collisions at LHC energies.
- ✓  $v_2$  of hadrons in heavy-ion collisions.
- ✓ energy-scan test of hadronic  $p_T$  spectra in Au+Au collisions at  $\sqrt{s_{NN}} = 7.7 - 200$  GeV.

Thanks for your attention!

## Quark spectra at hadronization are known.

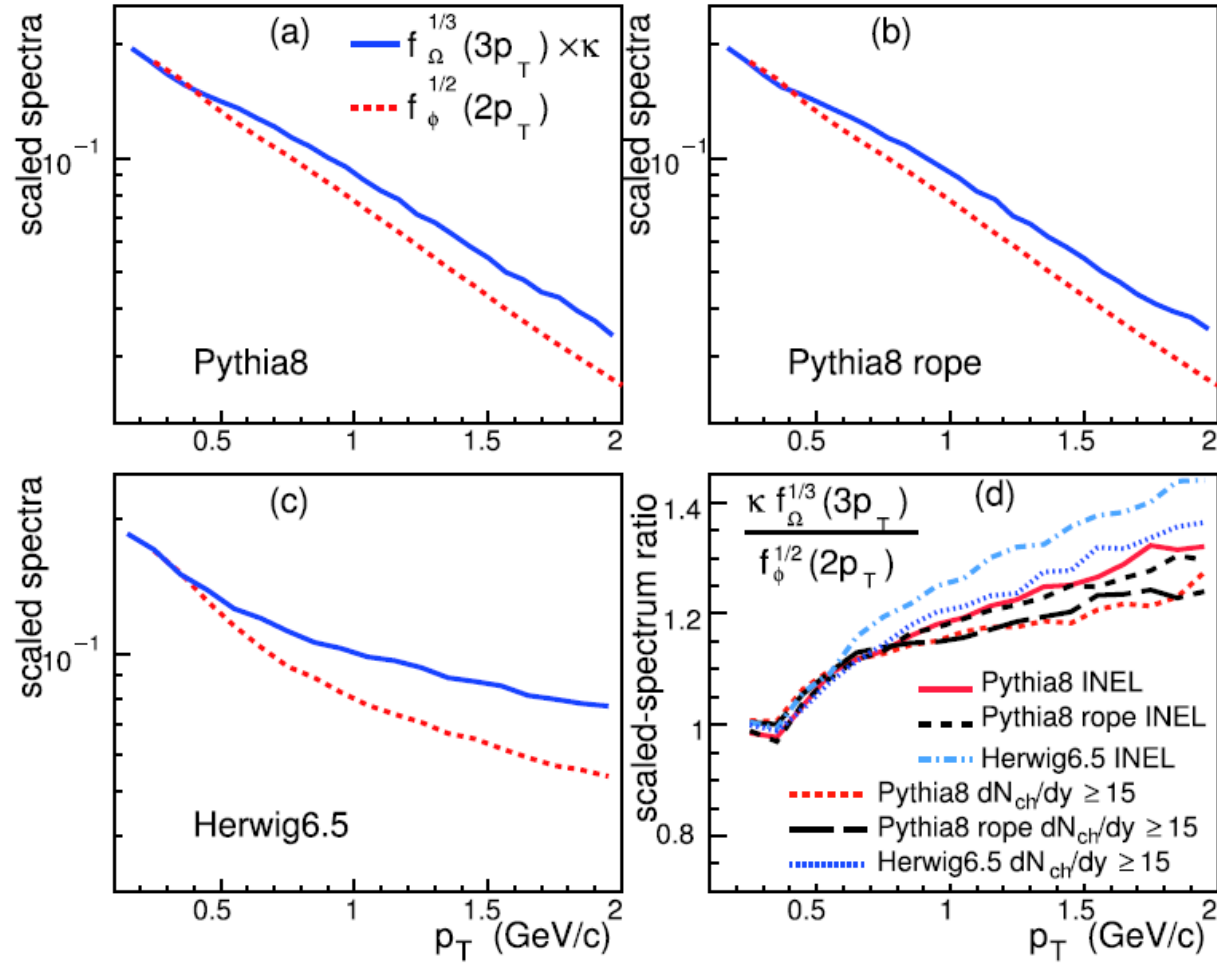
e.g., in p-Pb collisions at 5.02 TeV



obtained from light-flavor hadrons

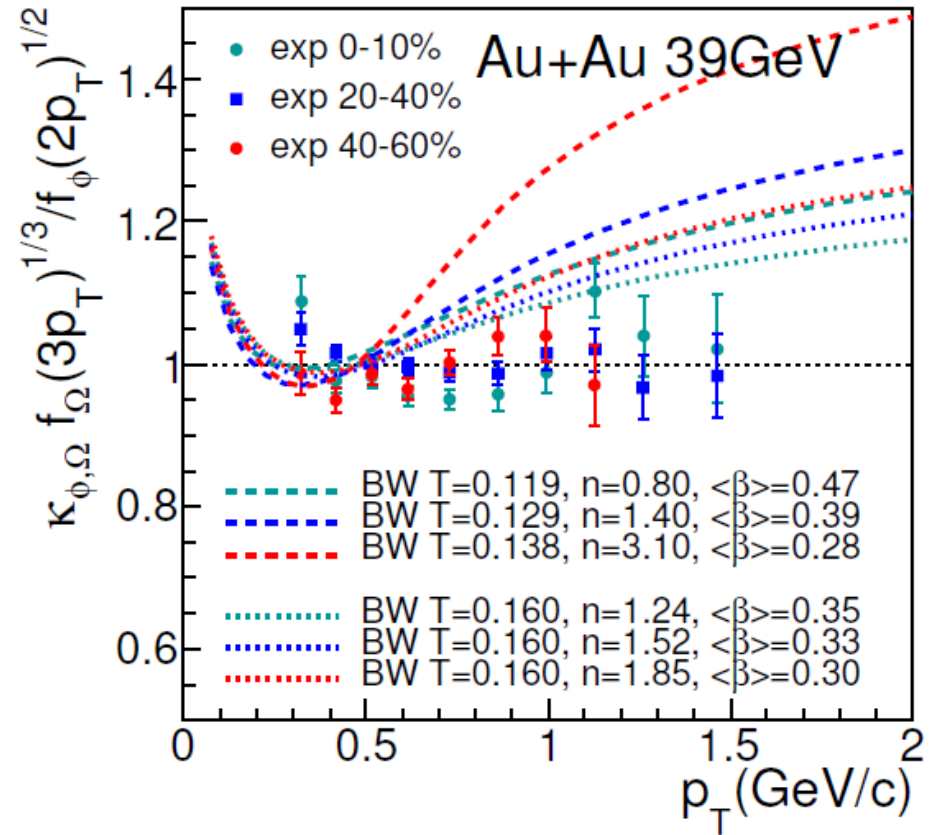
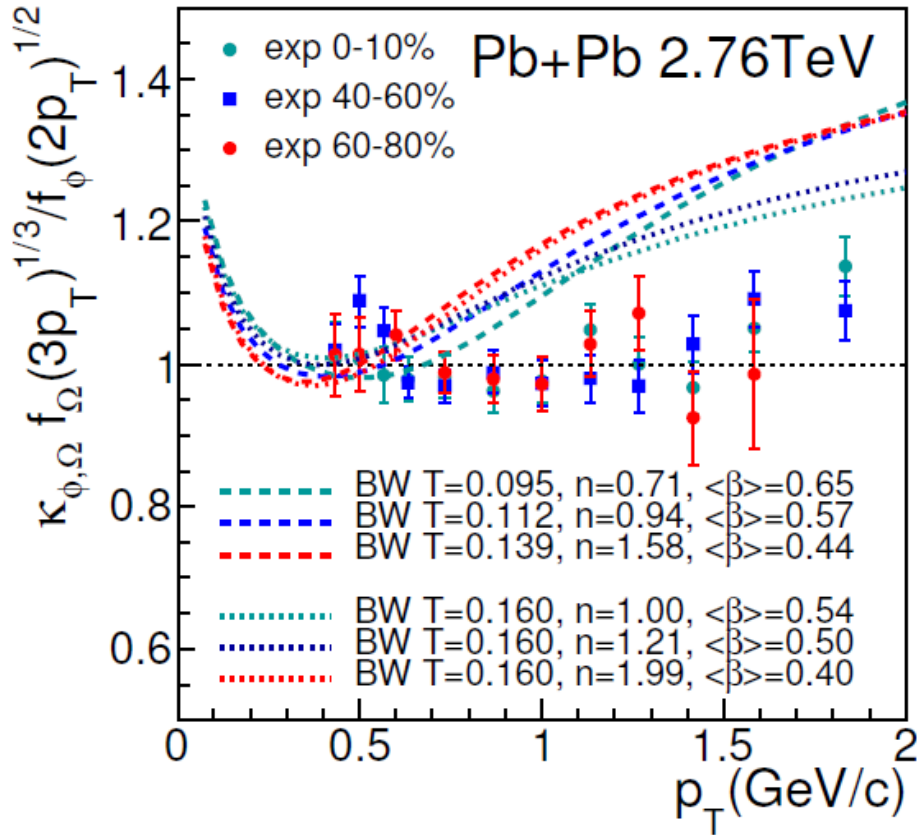
consistent with pQCD calculations

## QNS in PYTHIA & HERWIG





## QNS in Blast-wave model



## QNS in other (re-)combination/coalescence models

