Quantum Computing in High Energy Nuclear Physics

Xingyu Guo (QUNU Collaboration)

South China Normal University

Phys.Rev.D 105 (2022) 11, L11150
Sci. China Phys. Mech. Astron. 66(2023) 8, 281011
arXiv:2301.04179

1st USTC-PNP-NP MiniWorkshop

Contents

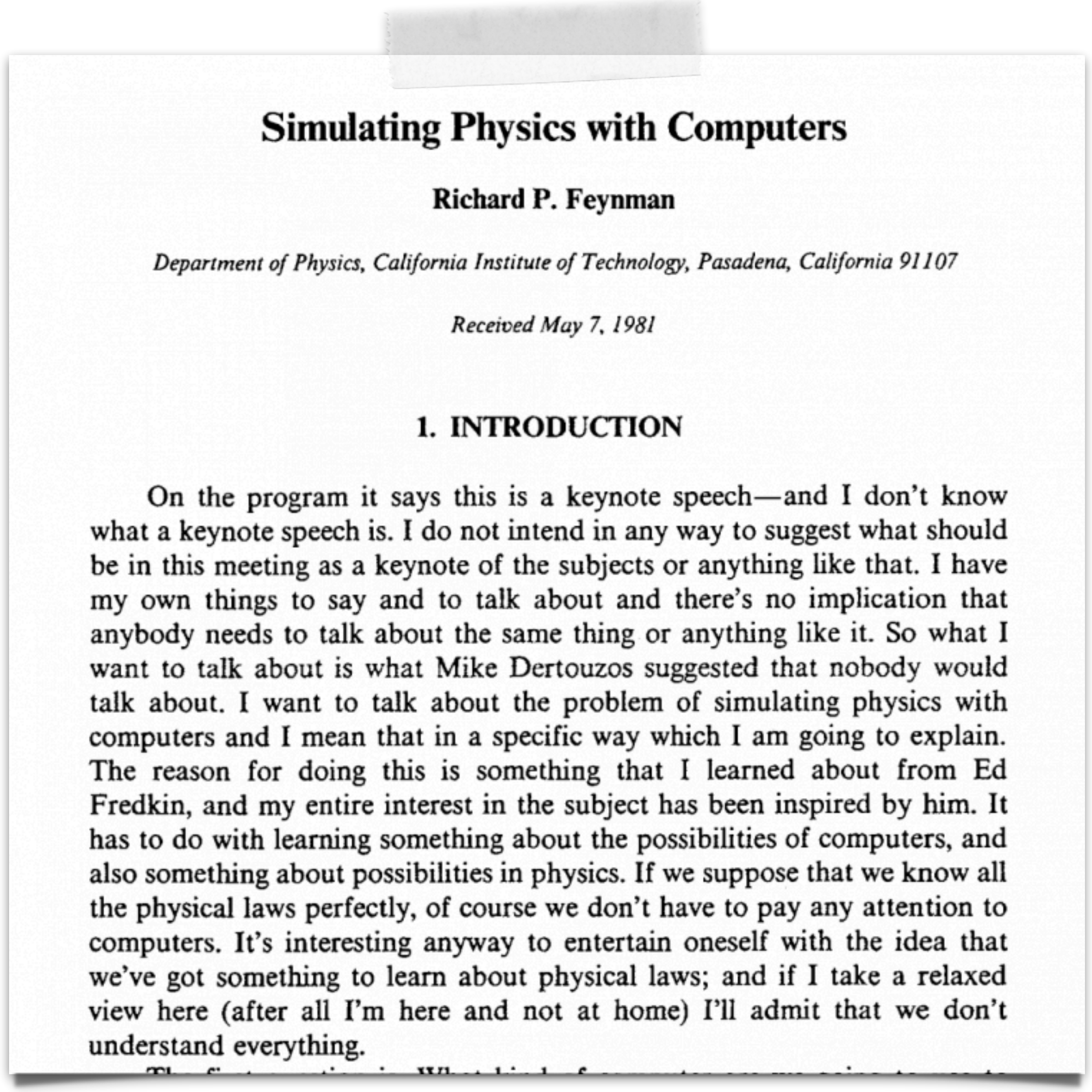
- Introduction
- Lattice NJL model in Hamiltonian formalism
- Quantum circuit for correlation functions
- Numerical results
- Summary

Quantum Computing

- Computing with quantum bits (“qubits”)
  - Hardware: How to build a quantum computer
  - Algorithm: How to “run” a quantum computer
- Exponential acceleration: quantum supremacy

Quantum Computing

- Computing with quantum bits ("qubits")
  - Hardware: How to build a quantum computer
  - Algorithm: How to "run" a quantum computer
- Exponential acceleration: quantum supremacy



Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that anybody needs to talk about the same thing or anything like it. So what I want to talk about is what Mike Dertouzos suggested that nobody would talk about. I want to talk about the problem of simulating physics with computers and I mean that in a specific way which I am going to explain. The reason for doing this is something that I learned about from Ed Fredkin, and my entire interest in the subject has been inspired by him. It has to do with learning something about the possibilities of computers, and also something about possibilities in physics. If we suppose that we know all the physical laws perfectly, of course we don't have to pay any attention to computers. It's interesting anyway to entertain oneself with the idea that we've got something to learn about physical laws; and if I take a relaxed view here (after all I'm here and not at home) I'll admit that we don't understand everything.

> "... and if you want to make a simulation of nature, you'd better make it quantum mechanical, ..."
>
> —Feynman

QC in High Energy Physics

- Parton structure
  - H. Lamm, S. Lawrence, Y. Yamauchi, Phys. Rev. Res. 2(2020), 013272
  - N. Mueller, A. Tarasov, R. Venugopalan, Phys. Rev. D 102(2020), 016007
- Jet production / fragmentation
  - A. Florio et al., arXiv:2301.11991
- Phase transition
  - A. M. Czajka, Z.-B. Kang, Y. Tee, F. Zhao, arXiv:2210.03062
  - A. Thompson, G. Siopsis, arXiv:2303.02425
  - K. Ikeda, D. E. Kharzeev, R. Meyer, S. Shi, arXiv:2305.00996
- Simulation of gauge theory
  - Z. Davoudi, A. F. Shaw, J. R. Stryker, arXiv:2212.14030
  - R. C. Farrell et al., Phys. Rev. D 107(2023), 054513
- And many more!

Quantum Computing in Physics

- Challenges
  - Space complexity
    - Limited number of qubits
    - Classical simulations: exponentially difficult
  - Time complexity
    - Noises
  - Gauge field
    - Analog simulation
    - Digital simulation
  - …

Quantum Computing in Physics

- Challenges
  - Space complexity
    - Limited number of qubits
    - Classical simulations: exponentially difficult
  - Time complexity
    - Noises
  - Gauge field
    - Analog simulation
    - Digital simulation
  - …

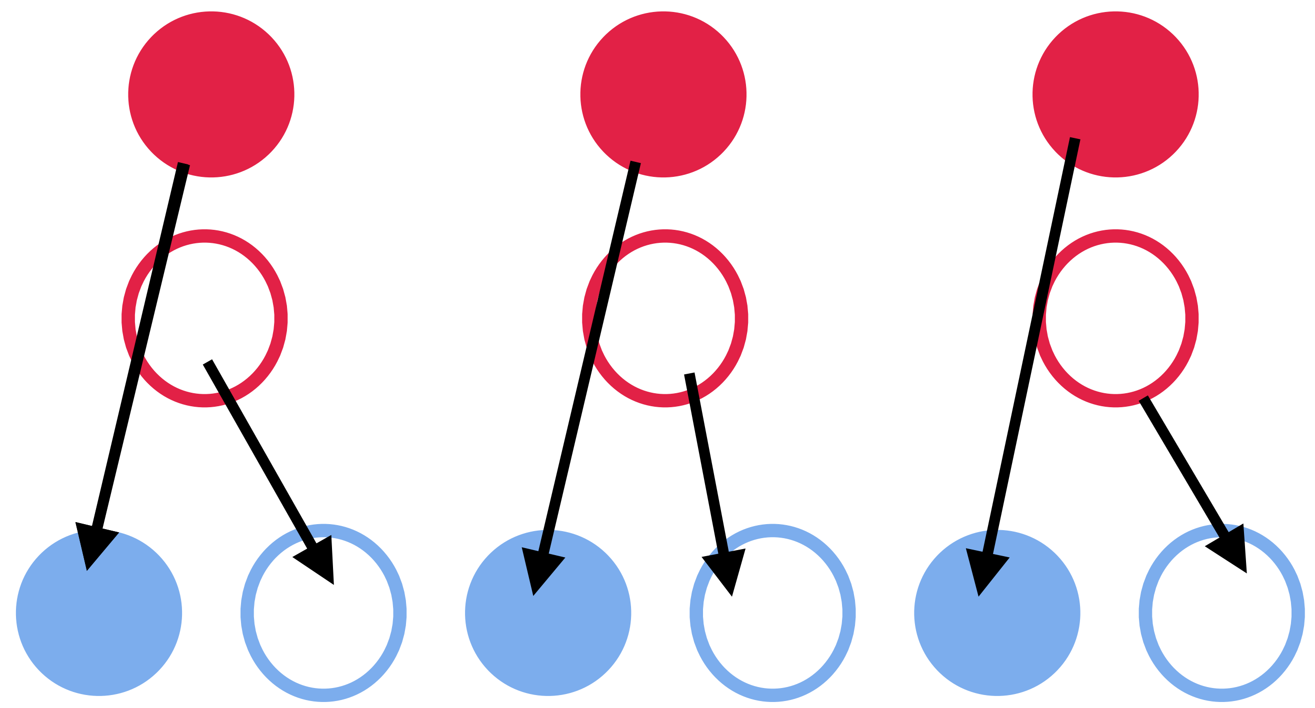
1+1D NJL Model

- Lagrangian:

$$\mathscr{L} = \bar{\psi}_\alpha(i\gamma^\mu\partial_\mu - m_\alpha)\psi_\alpha + g(\bar{\psi}_\alpha\psi_\alpha)^2$$

- Discretization: staggered fermion

$$\psi_\alpha(x) = \begin{pmatrix} \psi_{\alpha,1}(x) \\ \psi_{\alpha,2}(x) \end{pmatrix} \equiv \begin{pmatrix} \phi_{\alpha,2n} \\ \phi_{\alpha,2n+1} \end{pmatrix}$$



- Hamiltonian:

$$H = \sum_{\alpha,n} [-\frac{i}{2a}(\phi^\dagger_{\alpha,n}\phi_{\alpha,n+1} - h.c) + (-1)^n m_\alpha \phi^\dagger_{\alpha,n}\phi_{\alpha,n}]$$
$$-g \sum_{\alpha,n=even} [\phi^\dagger_{\alpha,n}\phi_{\alpha,n} + \phi^\dagger_{\alpha,n+1}\phi_{\alpha,n+1} - 2\phi^\dagger_{\alpha,n}\phi_{\alpha,n}\phi^\dagger_{\alpha,n+1}\phi_{\alpha,n+1}]$$

Parton Distribution Function

- Non-perturbative
- Light-cone: not equal-time
- Proton, Neutron, Pion, Nuclear, ...
- Study method:
  - Global fitting
  - Lattice QCD calculation: quasi-PDF, pseudo-PDF

Calculation of PDF

- Definition:

$$f_{q/h} = \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h | \bar{\psi}(z, -z) \gamma^+ \psi(0,0) | h \rangle$$

$$= \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \bar{\psi}(0, -z) e^{-iHz} \gamma^+ \psi(0,0) | h \rangle$$

- where $\gamma^+ = \gamma^0 + \gamma^1$.
- Relevant dimension: 1+1 D
- Prepare the hadronic state
- Calculate the (dynamical) correlation function

Calculation of PDF

- Definition:

$$f_{q/h} = \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h \,|\, \bar{\psi}(z, -z) \gamma^+ \psi(0k0) \,|\, h \rangle$$

$$= \int \frac{dz}{4\pi} e^{-ixM_h z} \langle \boxed{h} \,|\, e^{iHz} \bar{\psi}(0, -z) e^{-iHz} \gamma^+ \psi(0,0) \,|\, \boxed{h} \rangle$$

- where $\gamma^+ = \gamma^0 + \gamma^1$.
- Relevant dimension: 1+1 D
- Prepare the hadronic state
- Calculate the (dynamical) correlation function

Calculation of PDF

- Definition:

$$f_{q/h} = \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h | \bar{\psi}(z, -z) \gamma^+ \psi(0k0) | h \rangle$$

$$= \int \frac{dz}{4\pi} e^{-ixM_h z} \langle \boxed{h} | \boxed{e^{iHz} \bar{\psi}(0, -z) e^{-iHz} \gamma^+ \psi(0,0)} | \boxed{h} \rangle$$

- where $\gamma^+ = \gamma^0 + \gamma^1$.
- Relevant dimension: 1+1 D
- Prepare the hadronic state
- Calculate the (dynamical) correlation function

Calculation of PDF

- Definition:

$$f_{q/h} = \int \frac{dz}{4\pi} e^{-ixM_h z} \langle h | \bar{\psi}(z, -z)\gamma^+ \psi(0k0) | h \rangle$$

$$= \int \frac{dz}{4\pi} e^{-ixM_h z} \langle \boxed{h} | \boxed{e^{iHz}\bar{\psi}(0, -z) e^{-iHz}\gamma^+ \psi(0,0)} | \boxed{h} \rangle$$

- where $\gamma^+ = \gamma^0 + \gamma^1$.
- Relevant dimension: 1+1 D
- Prepare the hadronic state
- Calculate the (dynamical) correlation function

How to do this on a quantum computer?

Basic Elements of Quantum Computing

- Operator: unitary
  - Single qubit: X(σ_x), Y(σ_y), Z(σ_z), Rx(θ)($e^{i\theta\sigma_x}$), ...
  - $X_n = I \otimes I \otimes \cdots \otimes X \otimes \cdots$
  - Two(Multi) qubits: CNOT($\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$)
- Measurements: Hermitian
  - X, Y, Z
- The Hamiltonian should be expressed in XYZs!

Jordan-Wigner Transformation

$$\phi_{\alpha,n} = \prod_{\beta=1}^{\alpha-1} \tilde{\sigma}^z_{\beta,\frac{N}{2}} \tilde{\sigma}^z_{\alpha,n} \sigma^+_{\alpha,n} \equiv \Xi^z_{\alpha,n} \sigma^+_{\alpha,n} ,$$

$$\sigma^\pm_{\alpha,n} = \frac{1}{2}(\sigma^x_{\alpha,n} \pm i\sigma^y_{\alpha,n})$$

$$\tilde{\sigma}^z_{\alpha,n} \equiv \prod_{i<n} \sigma^z_{\alpha,i}$$

- Keeps the anti-commuting relation of ϕ.

Preparing the Hadronic State

- Hadronic states are the eigenstates of the Hamiltonian with certain quantum numbers.
- e.g.: How to find the (lowest few) eigenstates of a Hamiltonian?
- Variational method!
- A quantum computing version: Quantum alternating operator ansatz(QAOA)
  [E. Farhi, J. Goldstone, S. Gutmann, arXiv:1411.4028]

QAOA

- Divide the Hamiltonian

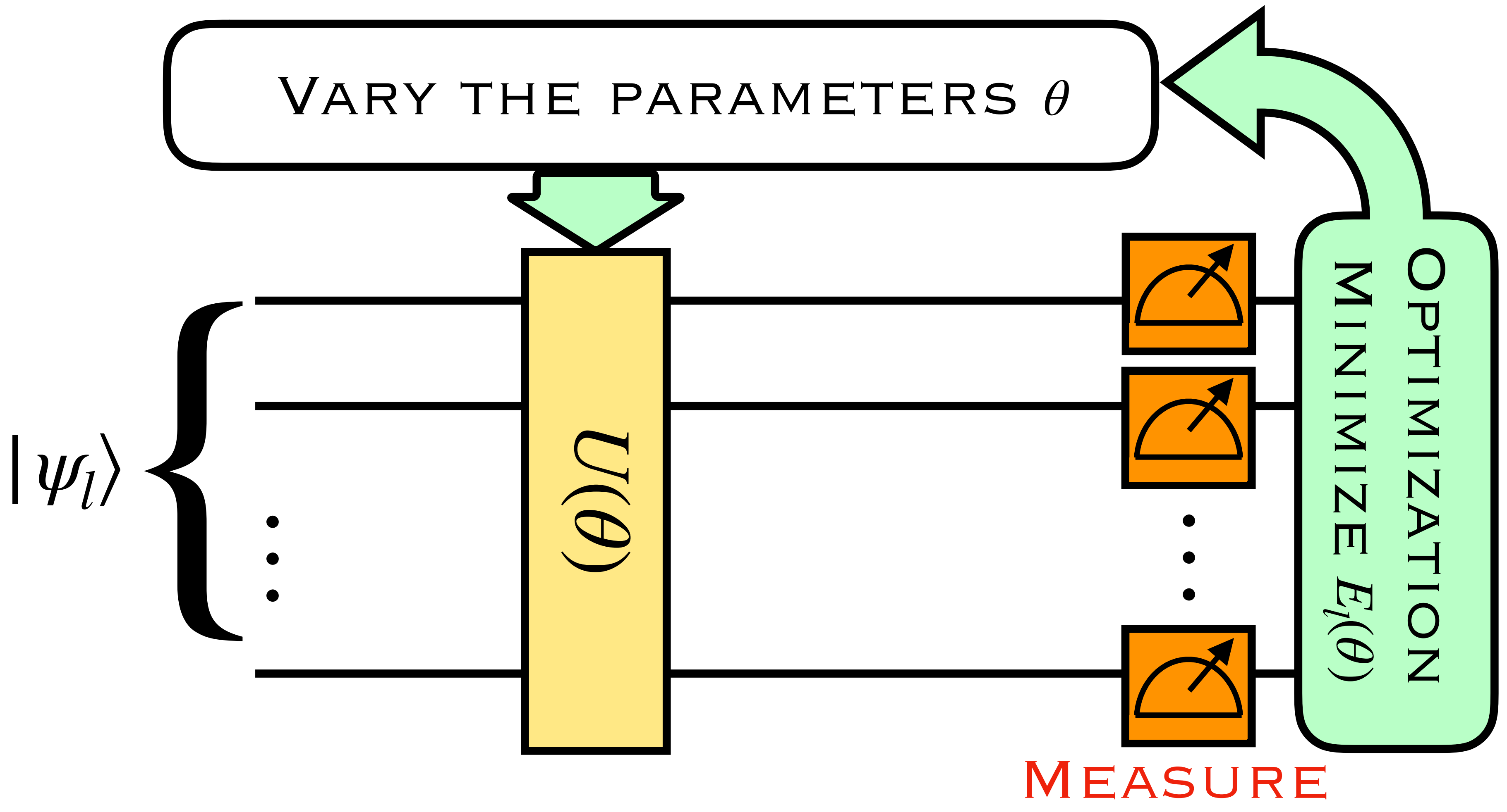
$$H = H_1 + H_2 + \cdots + H_k$$

  - $[H_i, H_{i+1}] \neq 0$
  - Each H_i preserves the same symmetry as H
- Parameterized symmetry-preserving operator

$$U(\theta) \equiv \prod_{i=1}^{p} \prod_{j=1}^{k} \exp(i\,\theta_{ij} H_j)$$

- The m-th state with quantum number l is

$|\psi_{lm}(\theta)\rangle = U(\theta)|\psi_{lm}\rangle$

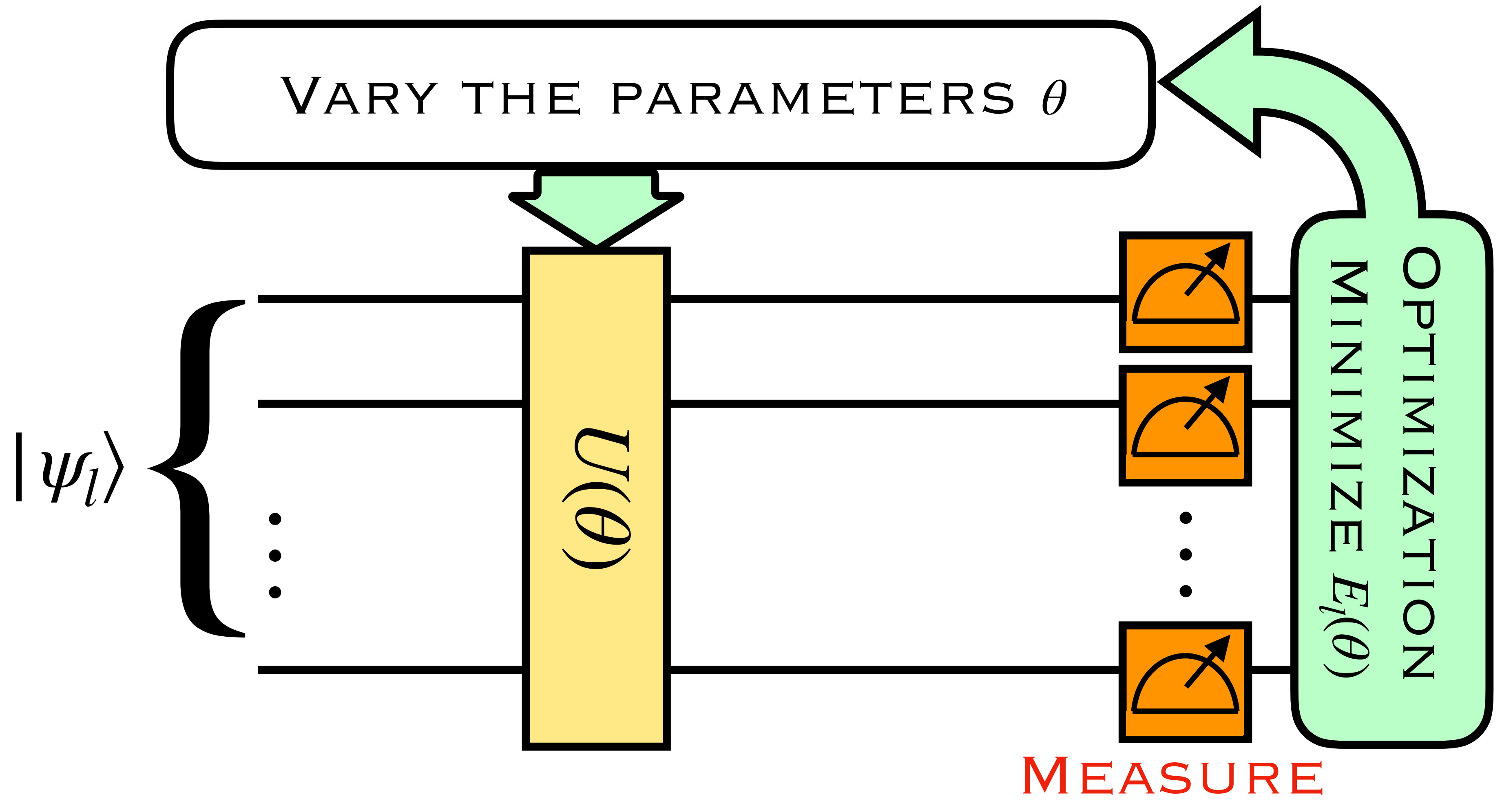


QAOA

- Cost function: weighted combination of the energy expectations [J. S. Pedernales, R. D. Candia, I. L. Egusquiza, J. Casanova, E. Solano, Phys. Rev. Lett. 113(2014), 020505]

$$E_l(\theta) = \sum_{i=1}^{k} w_{li} \langle \psi_{li}(\theta) | H | \psi_{li}(\theta) \rangle$$
$$\omega_{l1} > \omega_{l2} > \cdots > \omega_{lk}$$

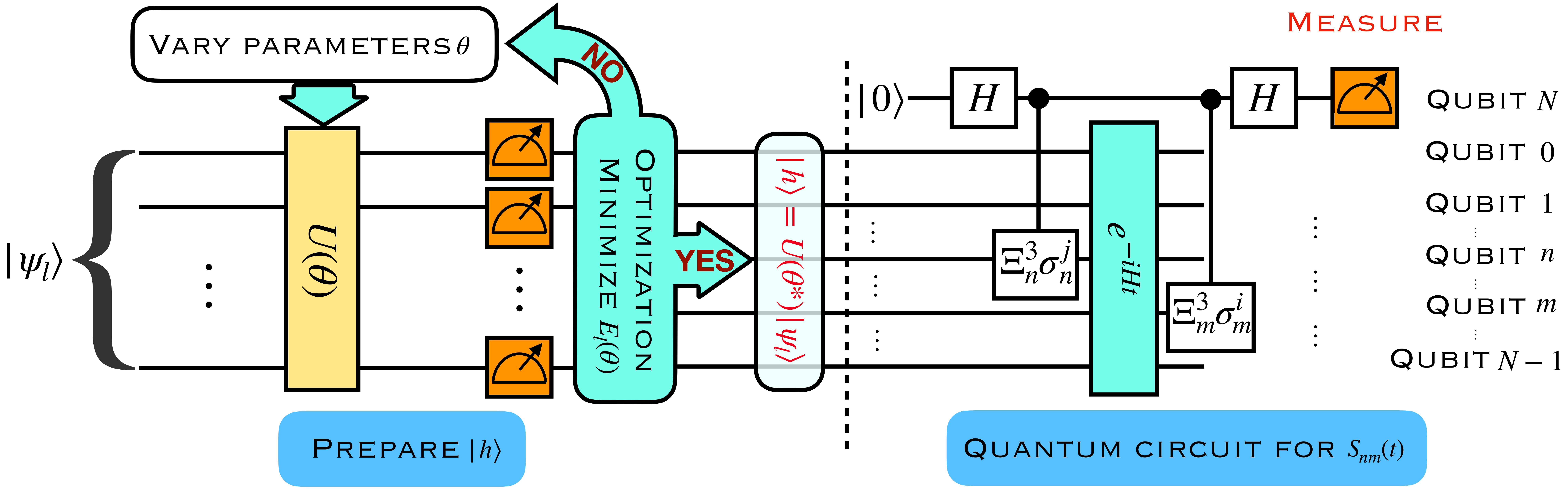
- The eigenstates are obtained by minimizing E_l
- The minimizing is done by a classical computer.
- Hybrid algorithm



Dynamical Correlation Function

- The measurement on the quantum computer is achieved by one auxiliary qubit and the controlled gates. [A. Francis, J. K. Fredrick's A. F. Kemper, Phys. Rev. B 101(2020), 014411]
- $|\alpha\rangle_a|0\rangle_b \rightarrow \frac{\sqrt{2}}{2}|\alpha\rangle_a(|0\rangle_b + |1\rangle_b) \rightarrow |\phi\rangle \equiv \frac{\sqrt{2}}{2}(|\alpha\rangle_a|0\rangle_b + \hat{O}|\alpha\rangle_a|1\rangle_b)$
- $\langle\phi|I_a \otimes X_b|\phi\rangle = \frac{1}{2} + Re(\langle\alpha|\hat{O}|\alpha\rangle)$
- $\langle\phi|I_a \otimes Y_b|\phi\rangle = \frac{1}{2} - Im(\langle\alpha|\hat{O}|\alpha\rangle)$
- What we measure: $S_{mn}(t) = \langle h|\hat{O}_{mn}(t)|h\rangle$, $\hat{O}_{mn}(t) = e^{iHt}\Xi_m^3\sigma_m^i e^{-iHt}\Xi_n^3\sigma_n^j$

Full Quantum Circuit



Meson Mass

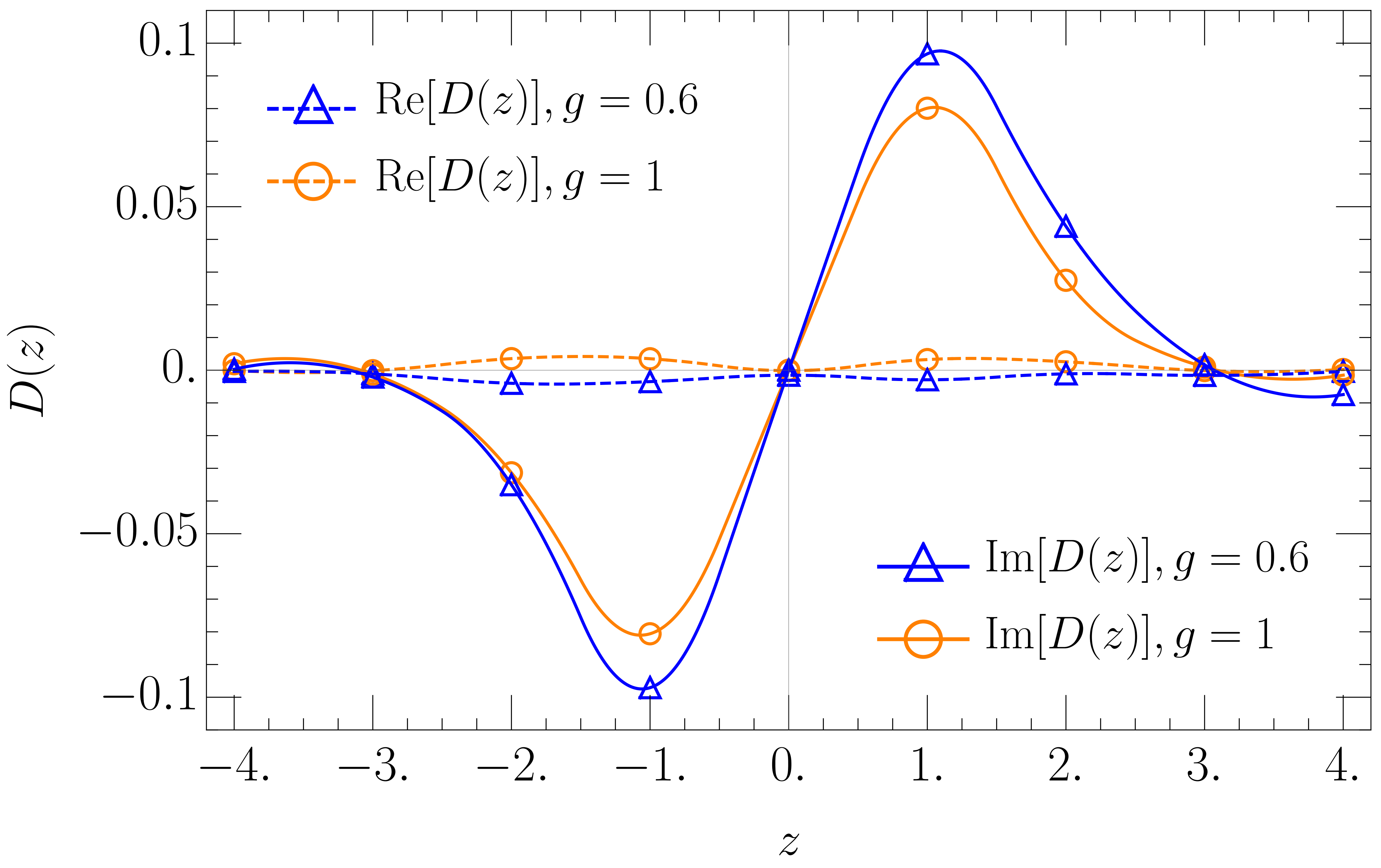
- $ma = 0.2$, 12 qubits

g	0.2	0.4	0.6	0.8	1.0
$M_{h,QC}a$	1.002	1.810	2.674	3.534	4.352
$M_{h,NUM}a$	1.001	1.801	2.659	3.509	4.342

- QAOA has good accuracy
- The majority of meson mass comes from interaction rather than quark mass.
- For $ma = 0.8$, the quark masses are dominant.

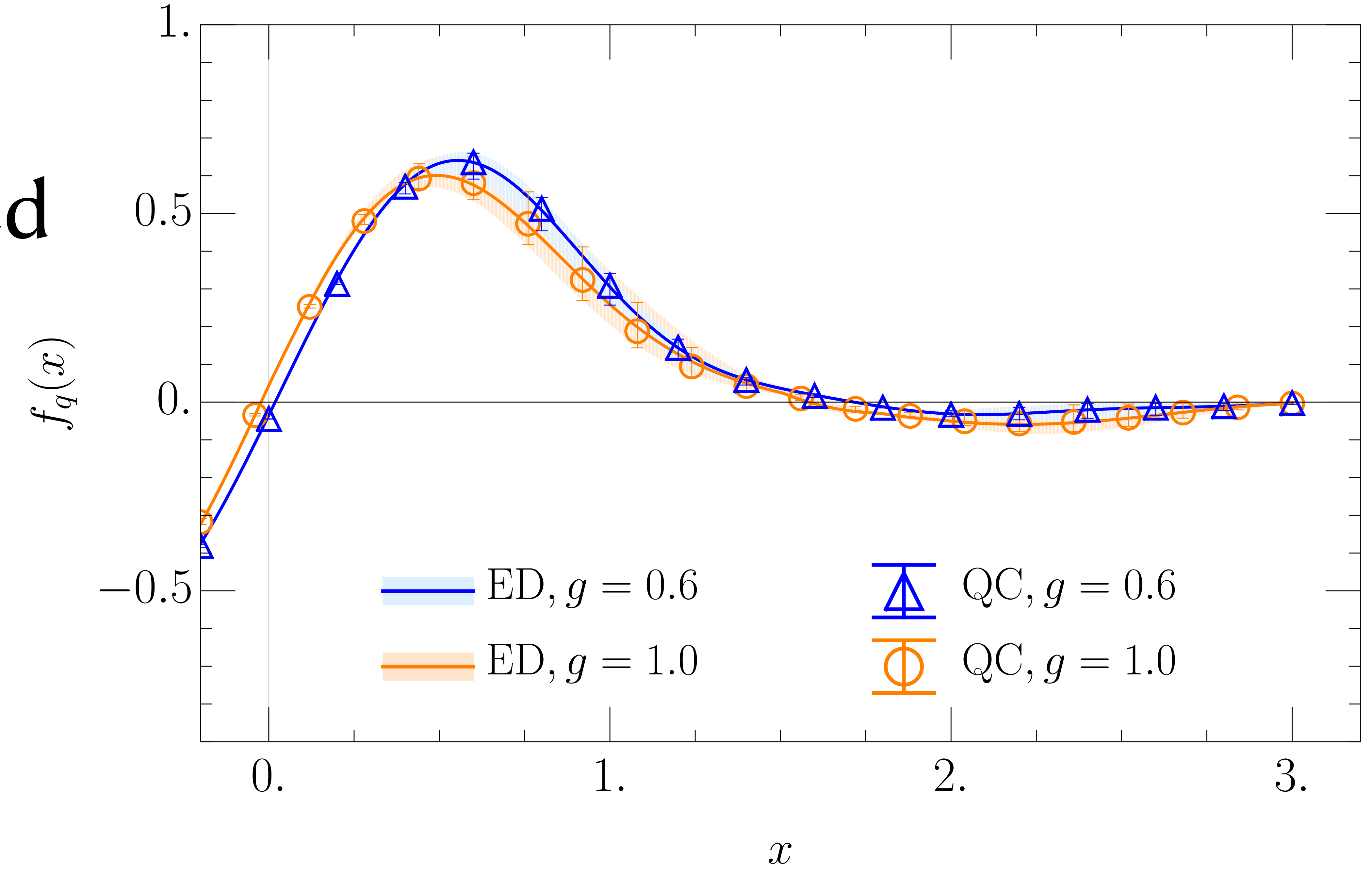
Correlation Function

- Real part consistent with 0:
  $f_q(x) = -f_q(-x) = f_{\bar{q}}(x)$
- Bounded state behavior.



Meson PDF

- Error bars/bands from interpolation methods.
- QC results matches well with exact diagonalization ones.
- Qualitative in agreement with pion PDFs in 2d QCD[Y. Jia, S. Liang, X. Xiong, R. Yu, Phys. Rev. D 98(2018), 054011].

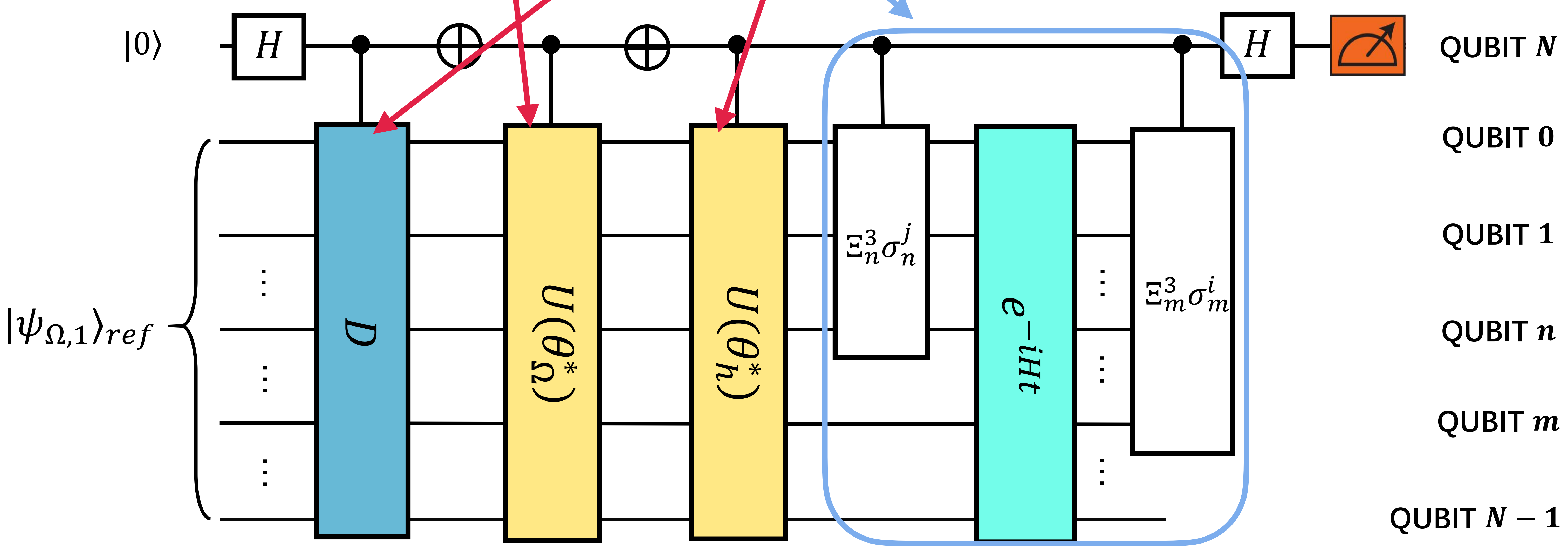


Light-Cone Distribution Amplitudes

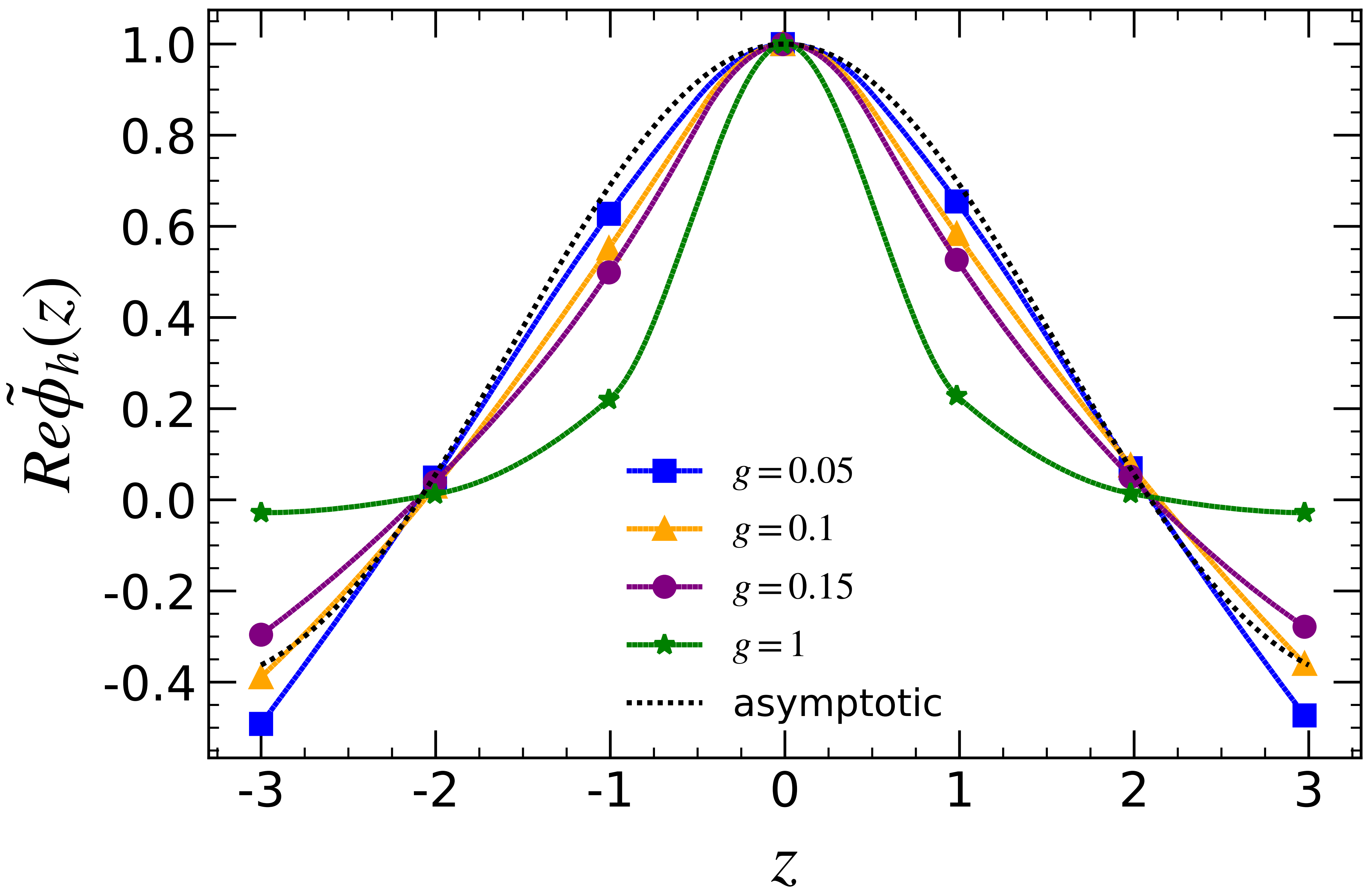
- The projection amplitude of a hadron onto multiple-parton states
- Describes the formation / decay of a hadron.
- $\phi_h(x) = \dfrac{1}{f_h} \int dz e^{-i(x-1)n\cdot Pz} \langle \Omega | \bar{\psi}(zn) \gamma^+ \psi(0) | h(P) \rangle$
- Now we want to measure $\langle \alpha | \hat{O} | \beta \rangle$
- We just need to prepare $| \phi' \rangle = \dfrac{1}{\sqrt{2}} (| \Omega \rangle | 0 \rangle + \hat{O} | h \rangle | 1 \rangle)$

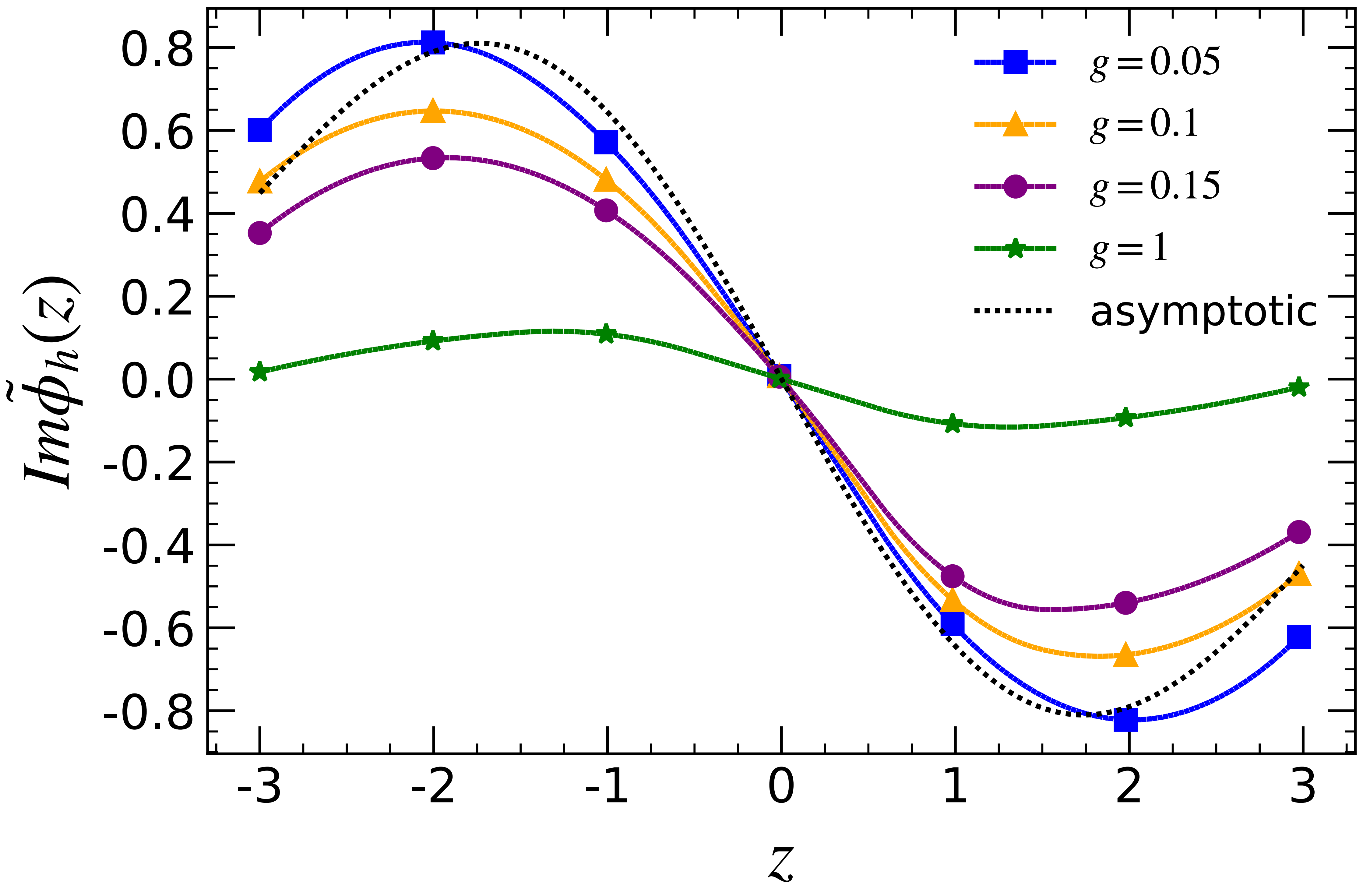
Quantum Circuit for LCDA

$$|\phi'\rangle = \frac{1}{\sqrt{2}}(|\Omega\rangle|0\rangle + \boxed{\hat{O}}|h\rangle|1\rangle)$$



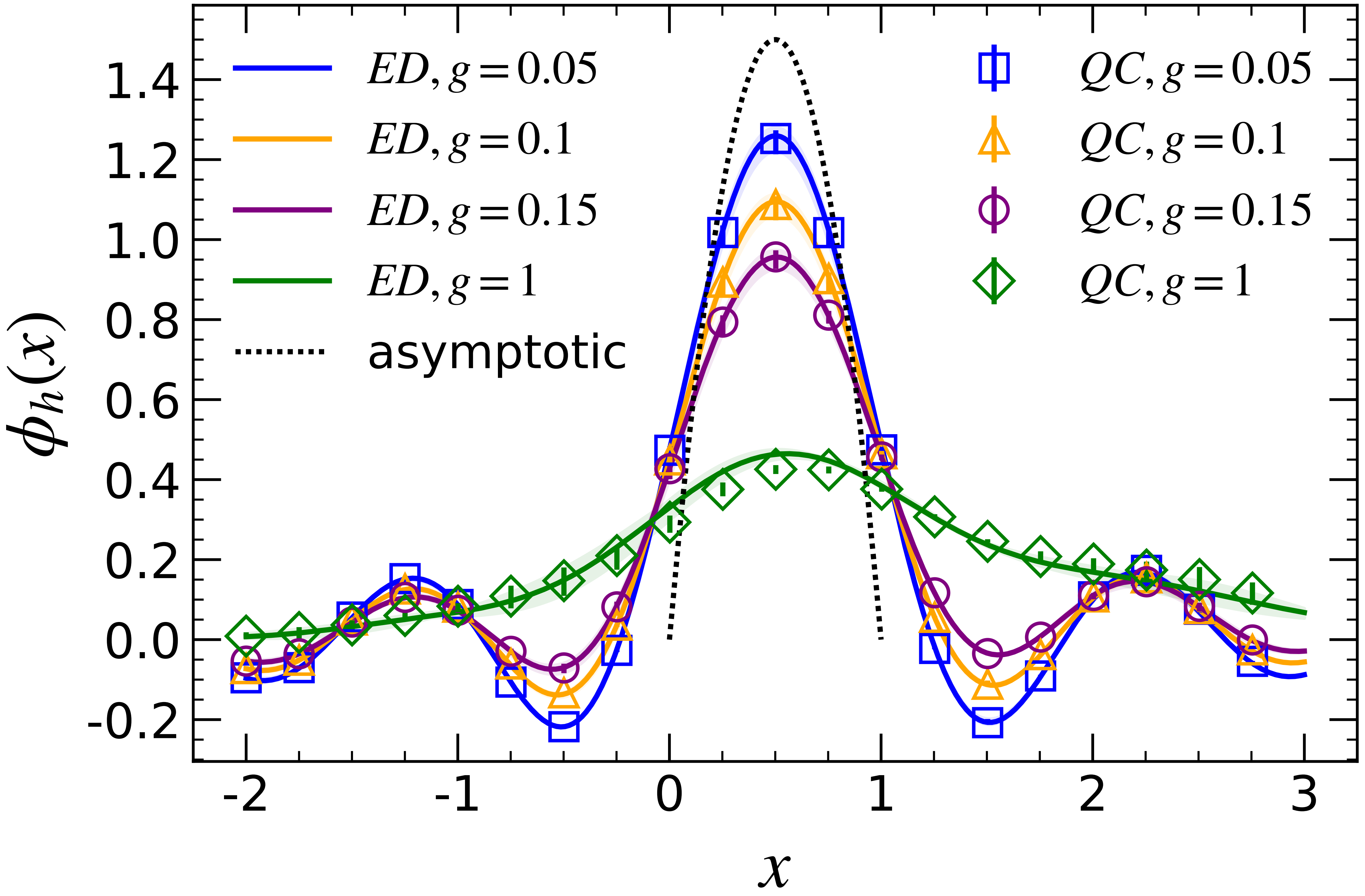
Results

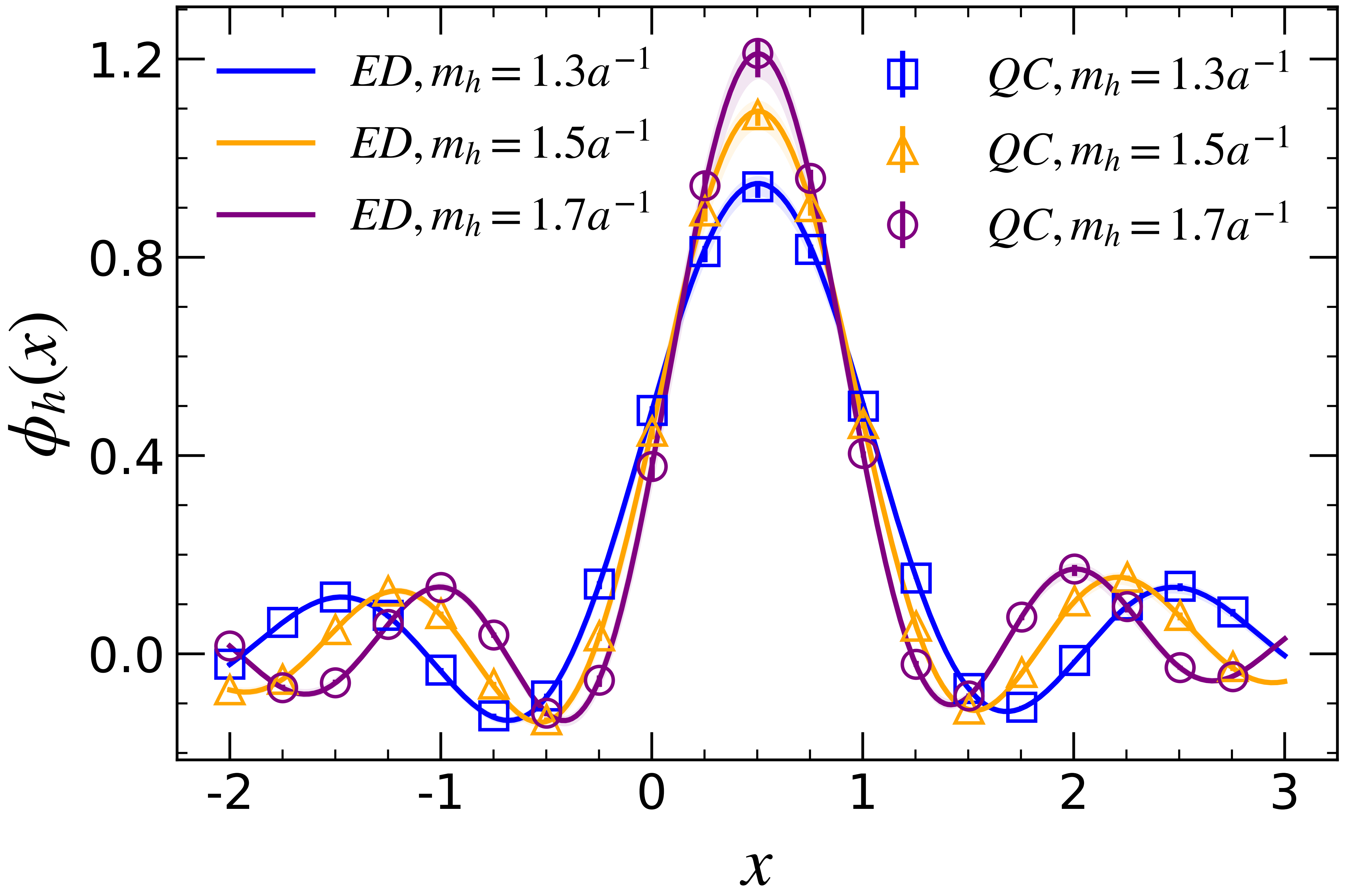




- m_h is fixed to be $1.5a^{-1}$
- Asymptotic corresponds to $\phi_h(x) = 6x(1-x)$

Results





- Peak gets narrower with decreasing coupling constant or increasing hadron mass
- Converges to asymptotic result.

Scattering Amplitude

- We have done simulating correlation functions.
- How about scattering amplitudes?
  - Especially nonperturbative ones!
- Current framework (Science 336, 1130; Quant. Inf, Compute. 14, 1014)
  - General
  - High lattice size requirement
  - Difficulty with bounded states
- But correlation functions and scattering amplitudes are connected!

LSZ Reduction Formula

$$i\mathcal{M} = R^{n/2} \lim_{p_i^2 \to m^2, k_j^2 \to m^2} G(\{p_i\},\{k_i\}) \times \left(\sum_{r=1}^{n_{out}} K^{-1}(p_r) \right) \left(\sum_{s=1}^{n_{in}-1} K^{-1}(k_s) \right)$$

$$G(\{p_i\},\{k_j\}) = \left(\sum_{i=1}^{n_{out}} \int d^4 x_i e^{ip_i \cdot x_i} \right) \left(\sum_{j=1}^{n_{in}} \int d^4 x_j e^{ik_j \cdot y_j} \right) \langle \Omega | \phi(x_1) \cdots \phi(x_{n_{out}}) \phi^\dagger(y_1) \cdots \phi^\dagger(y_{n_{in}-1}) \phi^\dagger(0) | \Omega \rangle$$

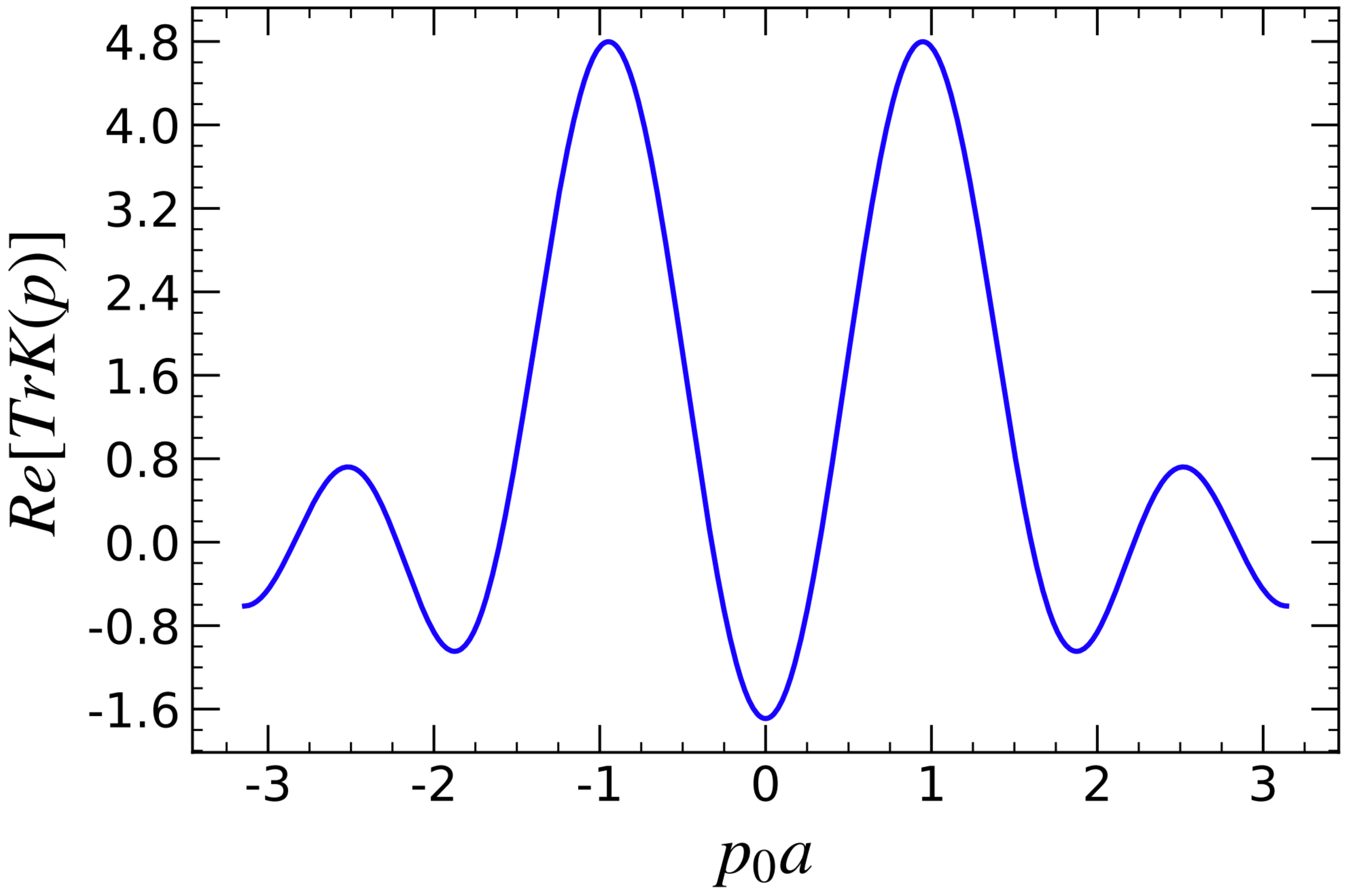
$$K(p) = \int d^4 x e^{p \cdot x} \langle \Omega | T\{\phi(x)\phi^\dagger(0) | \Omega \rangle$$

$$R = |\langle \Omega | \phi(0) | h(p=0) \rangle|^2$$

- Scattering function is a n-point correlation function divided by propagators of incoming and outgoing particles.

Quark Proporgator

- p_1 is fixed to be zero.
- Poles at $p_0 = \pm m$.
- Additional poles may be bound states.



Summary

- First direct simulation of the parton structure on a quantum computer
  - Meson PDF
  - LCDA
  - Scattering Amplitudes
- Quantum algorithm agrees well with exact diagonalization
- Results qualitatively reasonable
- Methods generally applicable

Thank you!