Real-time non-perturbative dynamics in Schwinger model: jet production, chiral magnetic wave, and more

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References:

2301.11991: Florio, Frenklakh, Ikeda, Kharzeev, Korepin, SS, Yu

2305.00996: Ikeda, Kharzeev, Meyer, SS

2305.05685: Ikeda, Kharzeev, SS

outline

- motivation: real time dynamics in QFT
- model set up
- jet production
  - jet production
  - vector and axial charge transport
  - phase structure
- summary and outlook

motivation

“first principle” microscopic theory: quantum field theory $L \leftrightarrow H$

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perturbative calculation: scattering process, thermodynamics, transport

non-perturbative calculation(lattice QFT): thermodynamics, transport "first principle" microscopic theory: quantum field theory $L \leftrightarrow H$

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real time dynamics of non-perturbative theory?

"first principle" microscopic theory: quantum field theory $L \leftrightarrow H$

perturbative calculation: scattering process, thermodynamics, transport

non-perturbative calculation(lattice QFT): thermodynamics, transport

real time dynamics of non-perturbative theory?

time evolution of a quantum (field) state:

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$$

Ideally, ***quantum simulation*** for ***full QCD in 3+1 D***, but …

1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_z - g\gamma^1 A - m)\psi\right)\mathrm{d}x\,.$$

$$L(t) = \int \left(-\frac{F^{\mu\nu}F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^\mu\partial_\mu - g\gamma^\mu A_\mu - m)\psi\right)\mathrm{d}x\,.$$

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

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static energy

coupling b/w electric and fermion fields

kinetic energy

electric field energy

Hamiltonian

1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_z - g\gamma^1 A - m)\psi - j^1_{\text{ext}}(t) A \right) \mathrm{d}x \, .$$

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E: electric field

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coupling w/ external source (jets)

$$j^1_{\text{ext}}(x,t) = g\,[\delta(x-t) + \delta(x+t)]\,\theta(t)$$

Hamiltonian

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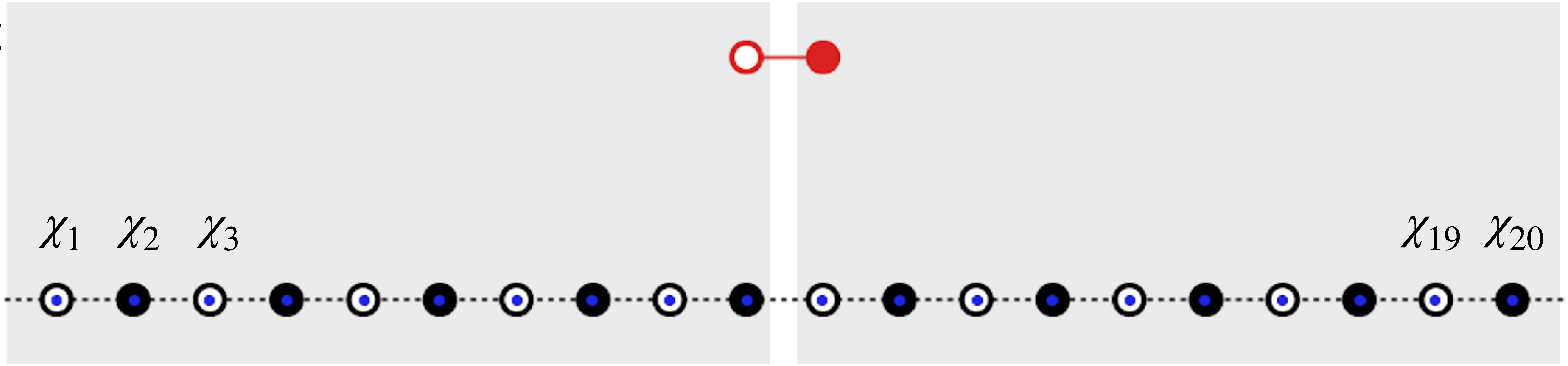
discretize and matrix(gate) representation:

Hamiltonian

1+1D Sc



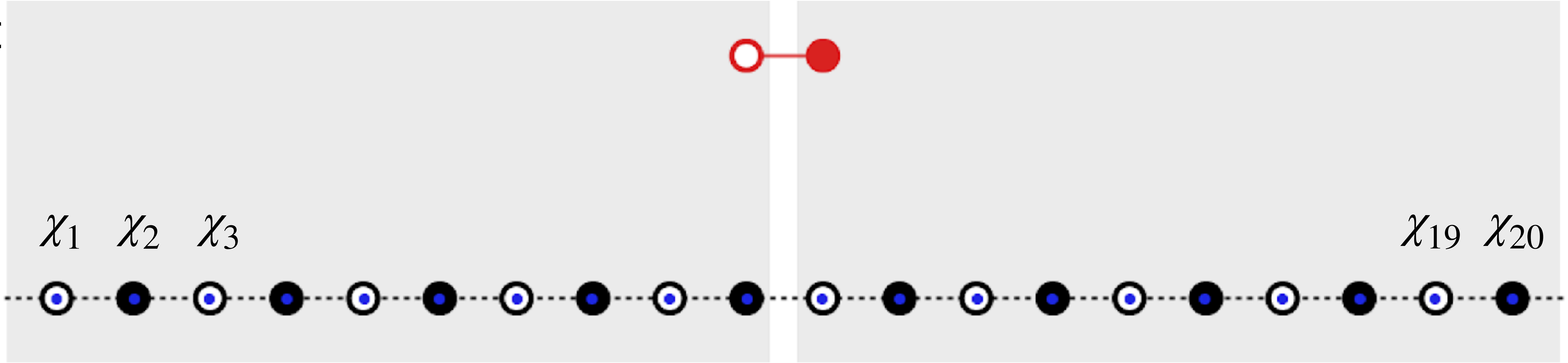
discretize and matrix(gate) representation:

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

$$\psi(x = a\ n) \quad \leftrightarrow \quad \frac{1}{\sqrt{a}} \begin{pmatrix} \chi_{2n} \\ \chi_{2n-1} \end{pmatrix}$$

Kogut-Susskind

1+1D Sc



Pauli matrices: X, Y, Z

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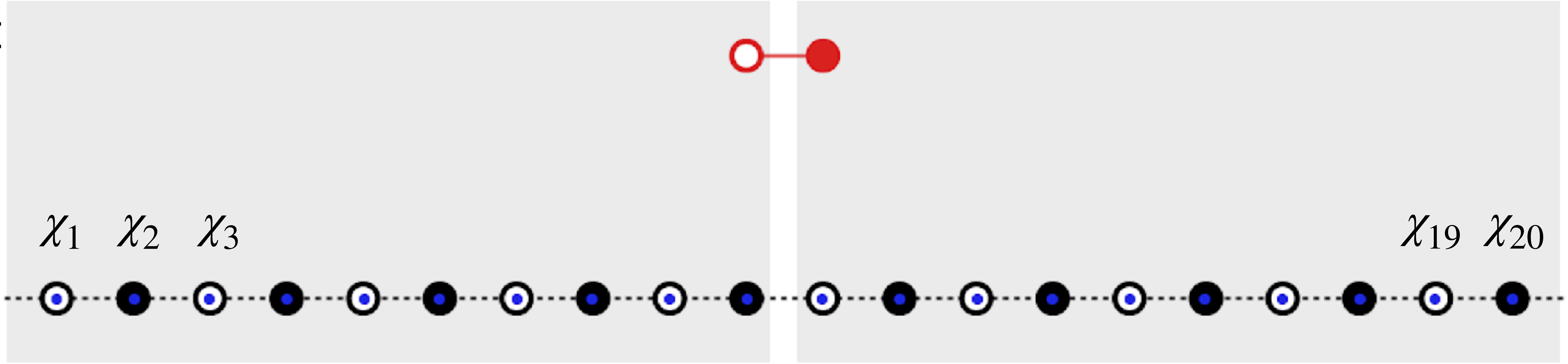
$$\chi_n = \frac{X_n - iY_n}{2}\prod_{m=1}^{n-1}(-iZ_m)$$

$$X_n \equiv \underbrace{I}_{1^{\rm st}} \otimes \cdots \otimes \underbrace{I}_{(n-1)^{\rm th}} \otimes \underbrace{X}_{n^{\rm th}} \otimes \underbrace{I}_{(n+1)^{\rm th}} \otimes \cdots \otimes I$$

Hamiltonian

1+1D Sc



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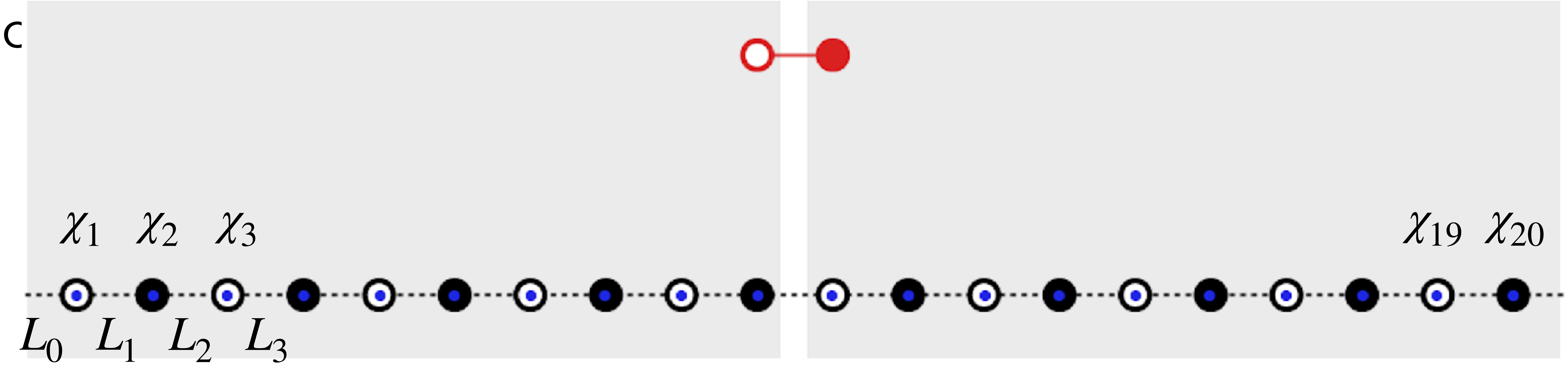
Jordan-Wigner

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm}, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0.$$

1+1D Sc



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gauge field fixed by Gauss' law: $\partial_1 E - g\,\bar{\psi}\gamma^0\psi = j^0_{\text{ext}}$

$E(x = an) \quad \leftrightarrow \quad L_n$

$$L_n - L_{n-1} - \frac{Z_n + (-1)^n}{2} = \frac{1}{g}\int_{(n-1/2)a}^{(n+1/2)a} \mathrm{d}x\, j^0_{\text{ext}}(x,t)\ ,$$

Hamiltonian

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$$H(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1} + Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{a\,g^2}{2}\sum_{n=1}^{N-1} L_n^2(t) \,.$$

why quantum computer?

dimension of state vector $= 2^N$

dimension of Hamiltonian $= 2^N \times 2^N$

N : number of lattice sides

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dimension of Hamiltonian $=$ ~~$2^N \times 2^N$~~ sparse $\sim 2N \times 2^N$

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N	dimension	memory of Hamiltonian	# of qubit (N)
8	256	~ 131 kB	8
12	4,096	~ 3.1 MB	12
16	65,536	~ 67 MB	16
20	1,048,576	~ 1.3 GB	20
24	16,777,216	~ 26 GB	24
28	268,435,456	~ 481 GB	28

unrealistic in a "classical" computer,
but plausible in the state-of-art quantum computer?

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classical hardware in this work

performance not satisfying…

Hamiltonian

(repeating 4)

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time-dependent Schroedinger equation:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\,i\,H|\psi(t)\rangle$$

$$q_{n,t} \equiv \langle \psi^\dagger(a\,n)\psi(a\,n)\rangle_t = \frac{\langle Z_n\rangle_t + (-1)^n}{2a},$$

$$Q_t \equiv \int \langle \psi^\dagger(x)\psi(x)\rangle_t\,\mathrm{d}x = a\sum_{n=1}^{N} q_{n,t},$$

$$\nu_{n,t} \equiv \langle \bar\psi(a\,n)\psi(a\,n)\rangle_t = \frac{(-1)^n\langle Z_n\rangle_t}{2a},$$

$$\nu_t \equiv \int \langle \bar\psi(x)\psi(x)\rangle_t\,\mathrm{d}x = a\sum_{n=1}^{N} \nu_{n,t},$$

$$\Pi_{n,t} \equiv \langle E(a\,n)\rangle_t = g\,\langle L_n\rangle_t,$$

$$E_{\mathrm{ele},t} \equiv \frac{1}{2}\int \langle E^2(x)\rangle_t\,\mathrm{d}x = \frac{a\,g^2}{2}\sum_{n=1}^{N-1}\langle L_n^2\rangle_t\,.$$

I. jet production

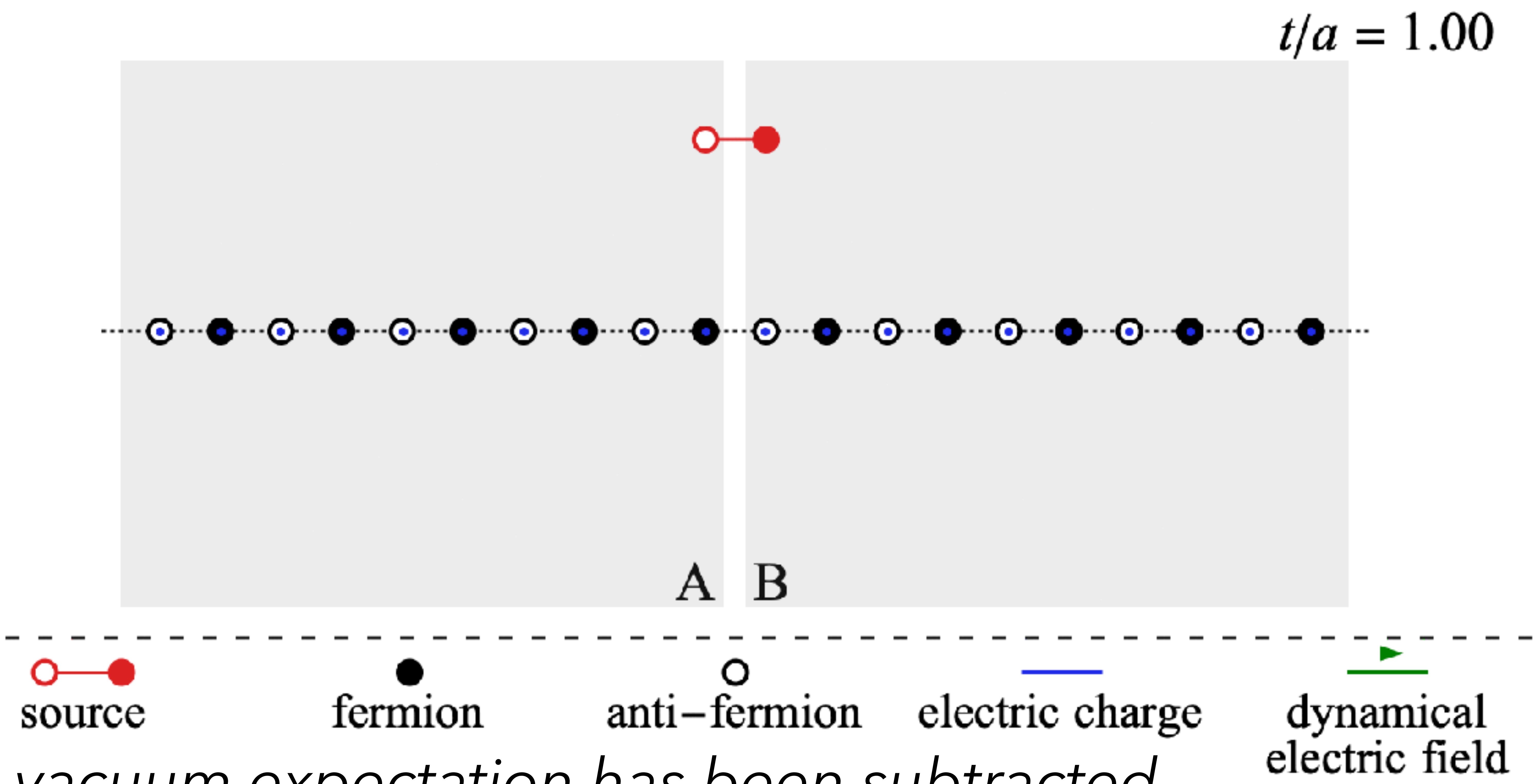
A. Florio, D. Frenklakh, K. Ikeda, D. Kharzeev, V. Korepin, *SS*, K. Yu

PhysRevLett.131.021902 (arXiv: 2305.05685)

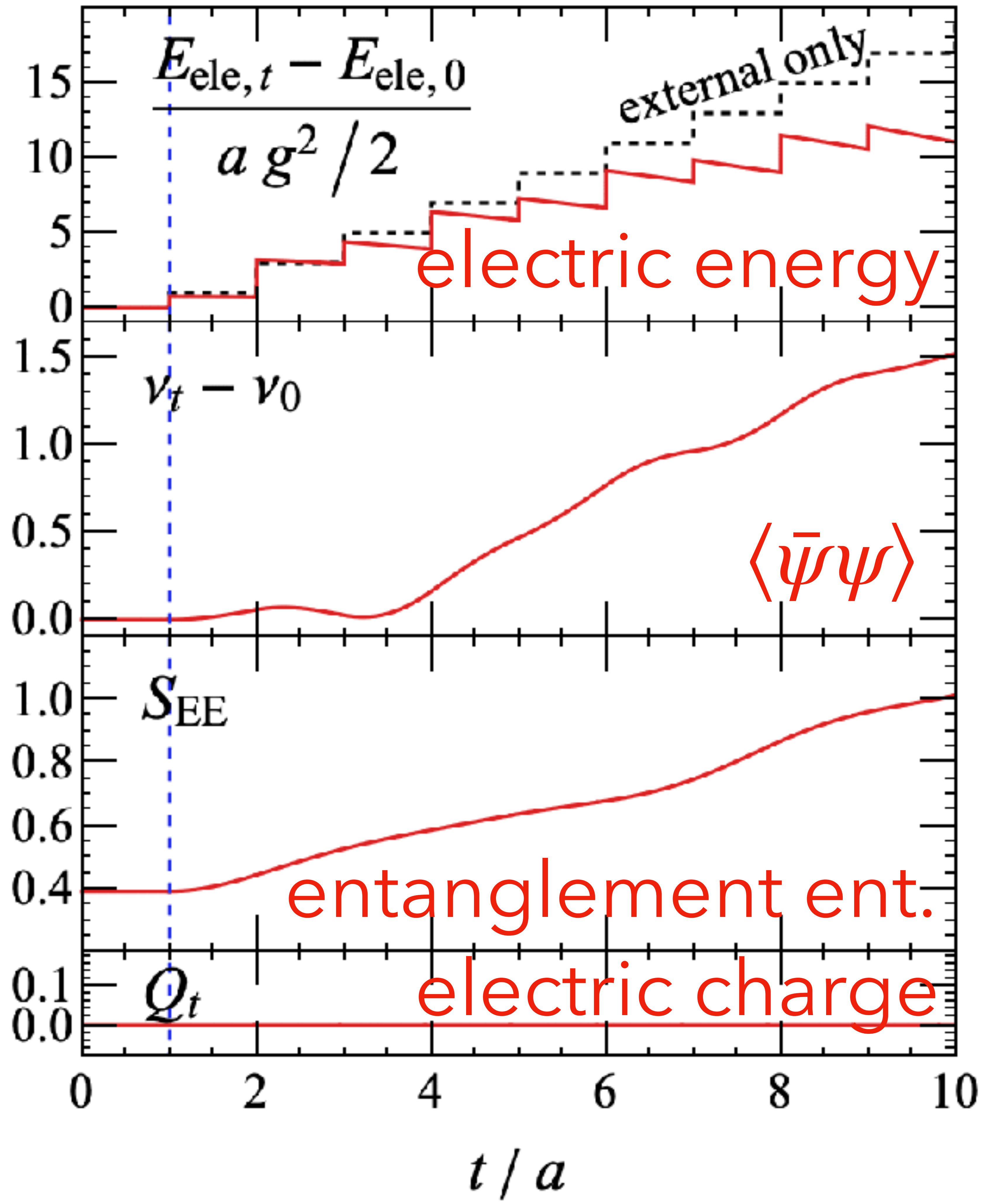
real-time evolution

initial state: vacuum $H(t=0)\,|\,\psi(t=0)\rangle = E_0\,|\,\psi(t=0)\rangle$

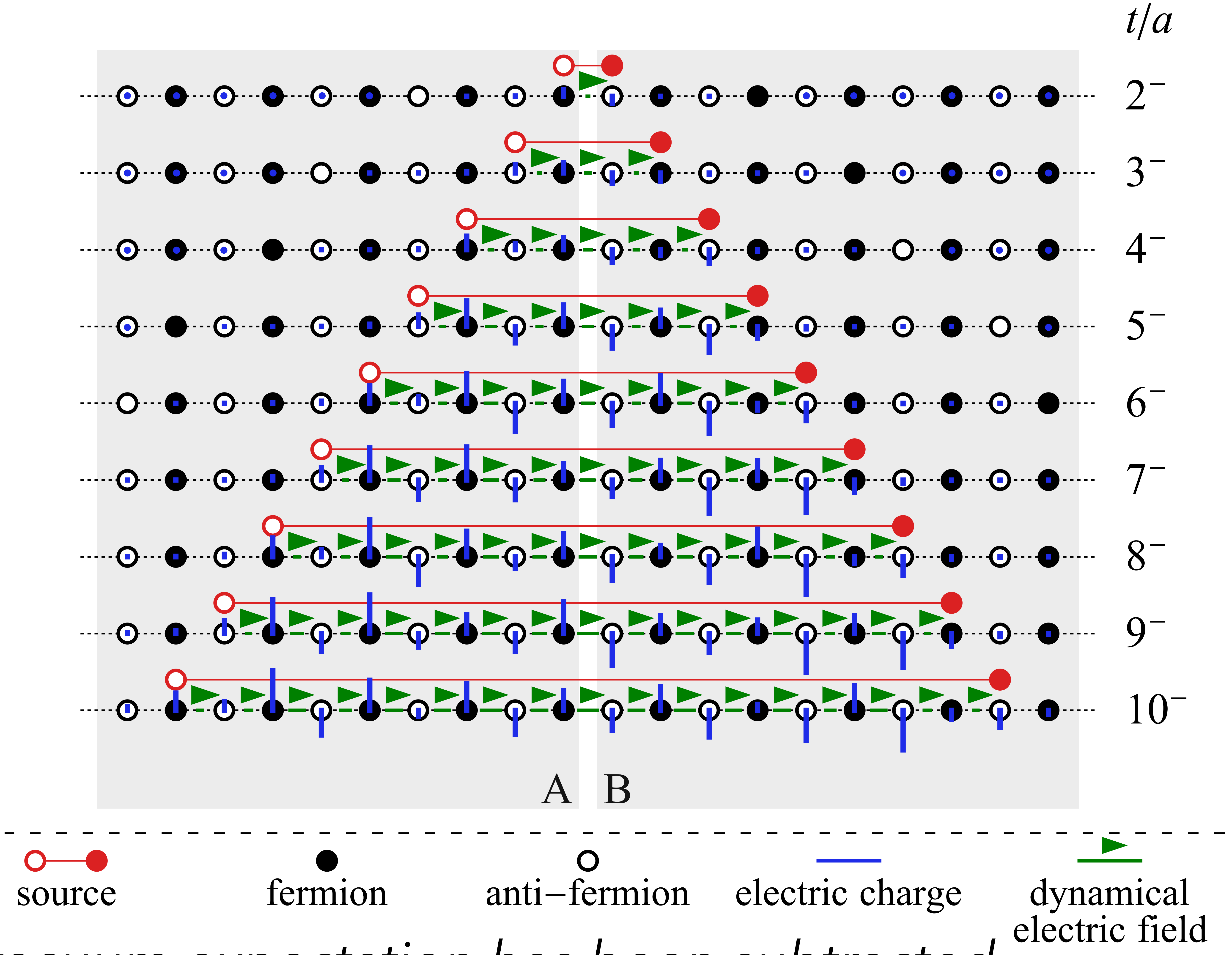
$$\frac{\partial}{\partial t}\,|\,\psi(t)\rangle = -\,i\,H(t)\,|\,\psi(t)\rangle$$

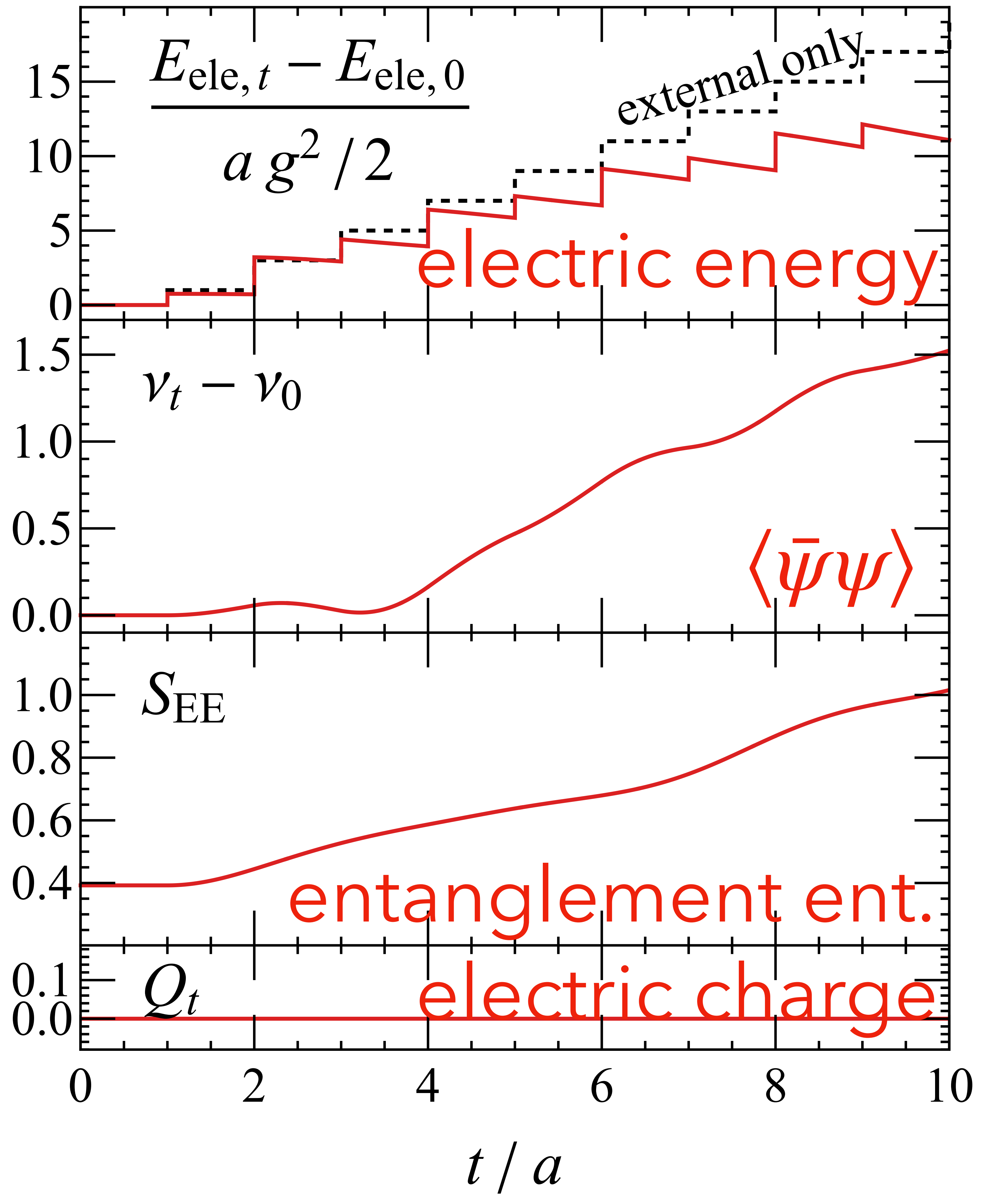


vacuum expectation has been subtracted



vacuum modification



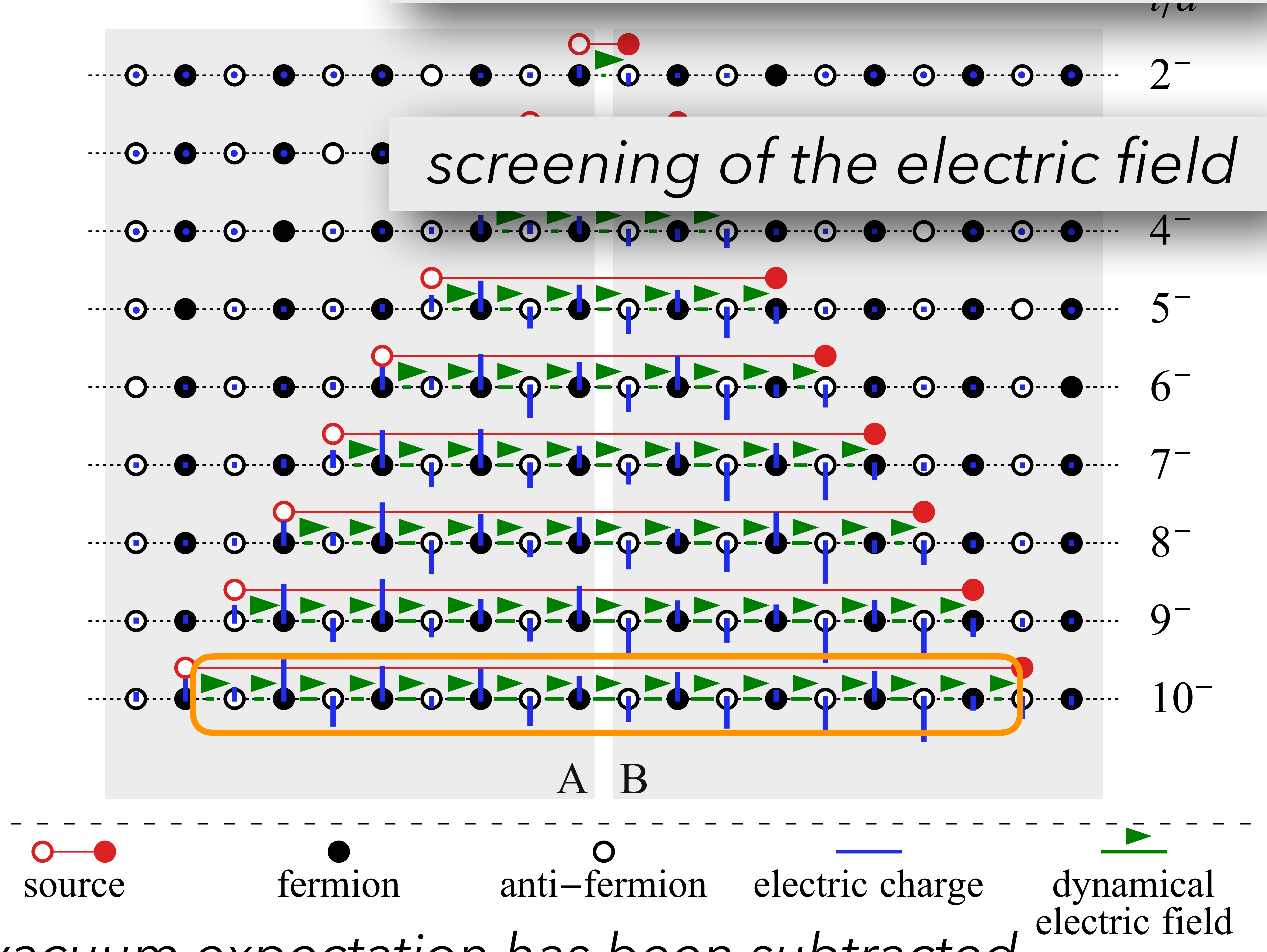


vacuum expectation has been subtracted

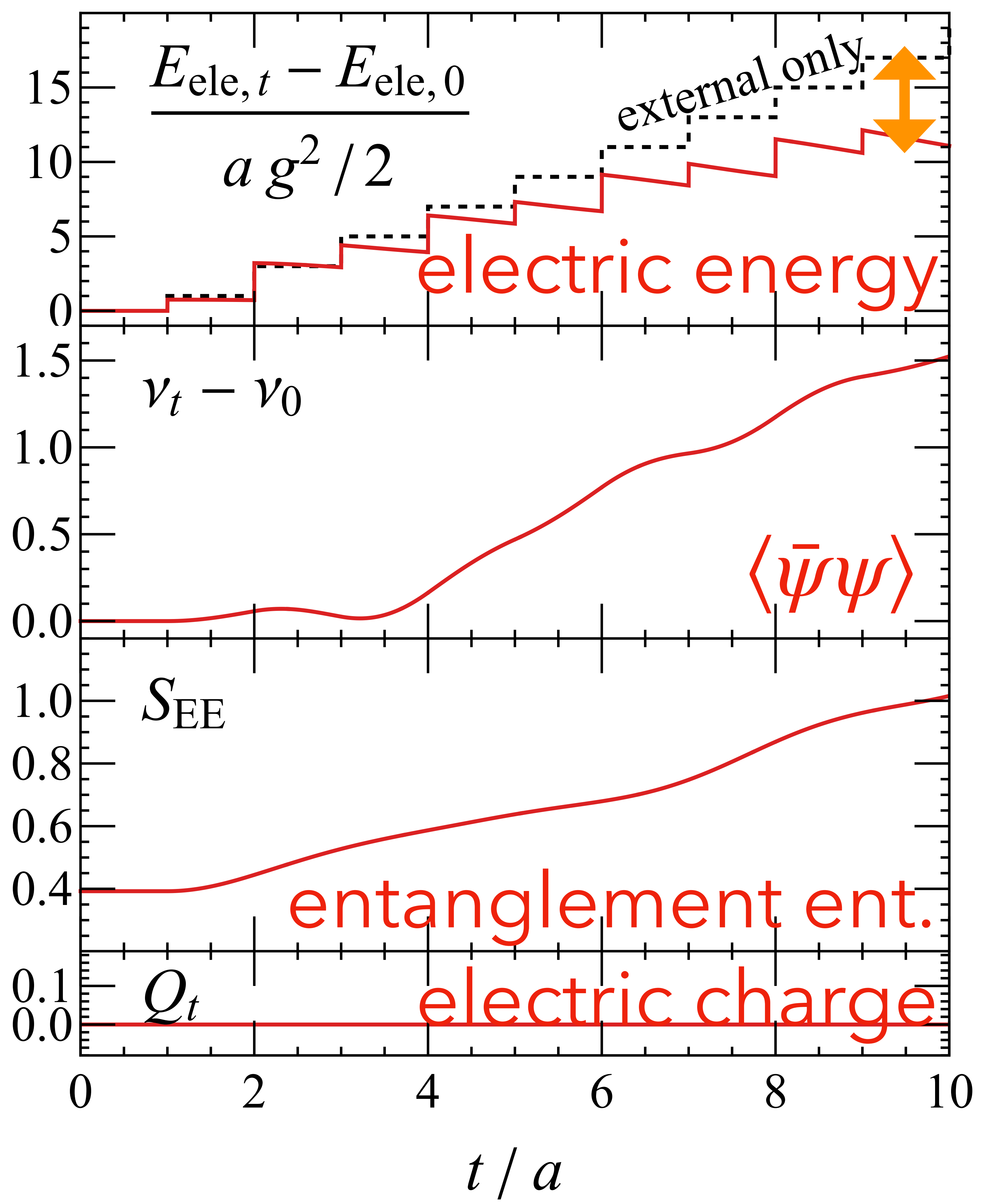
vacuum modification

effects of pair production:

screening of the electric field



vacuum expectation has been subtracted



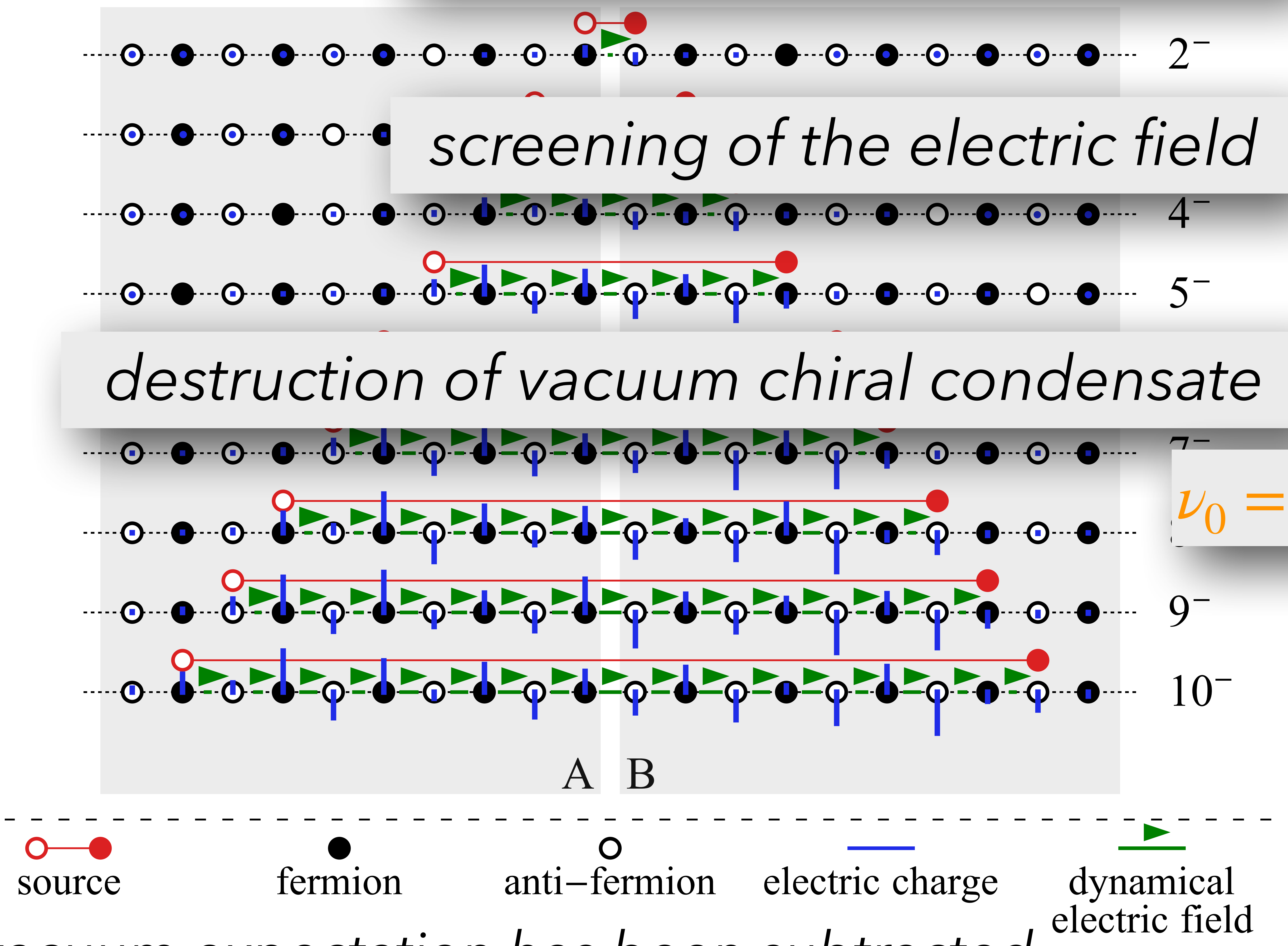
vacuum modification

effects of pair production:

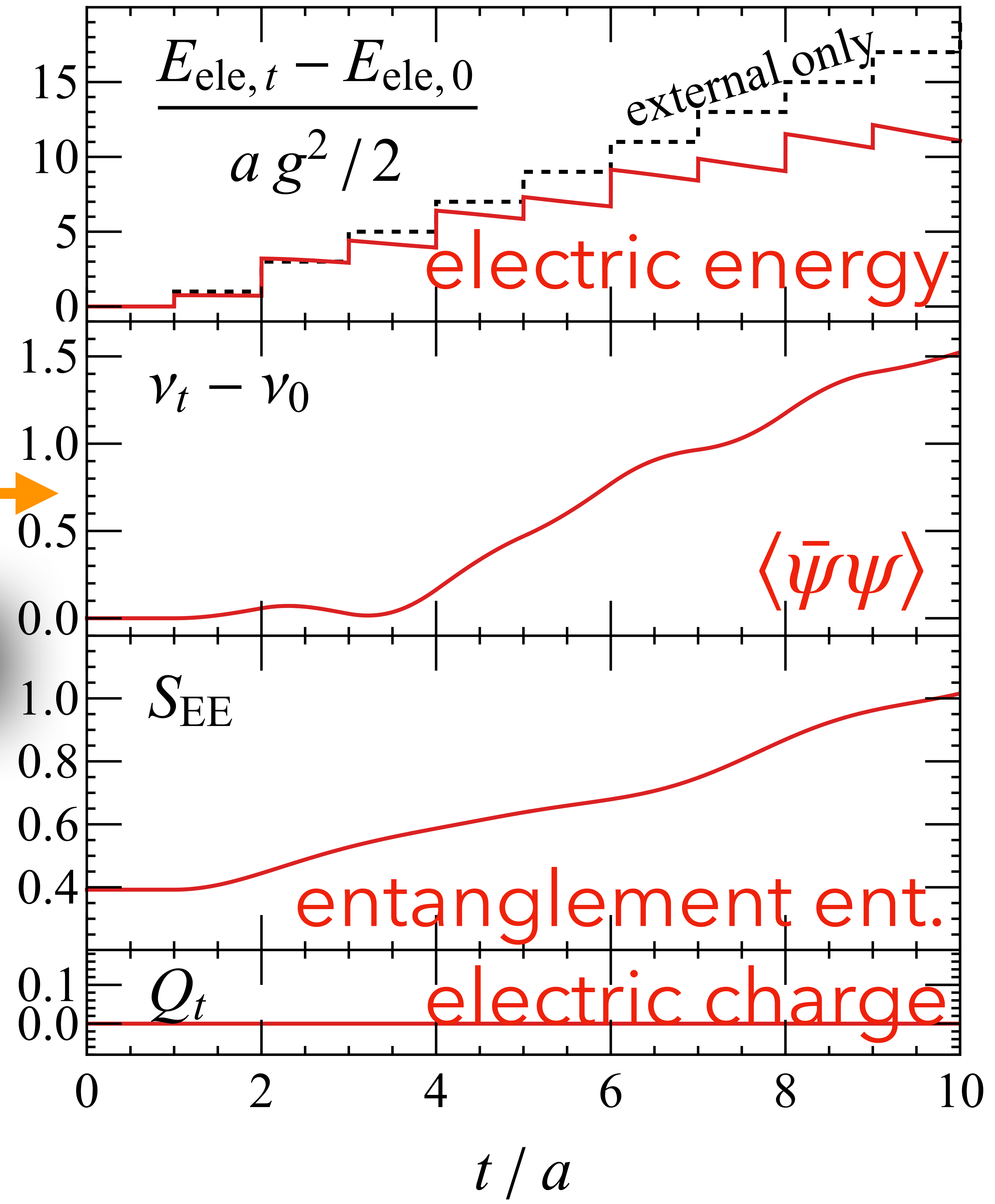
screening of the electric field

destruction of vacuum chiral condensate

$\nu_0 = -5.16$



vacuum expectation has been subtracted



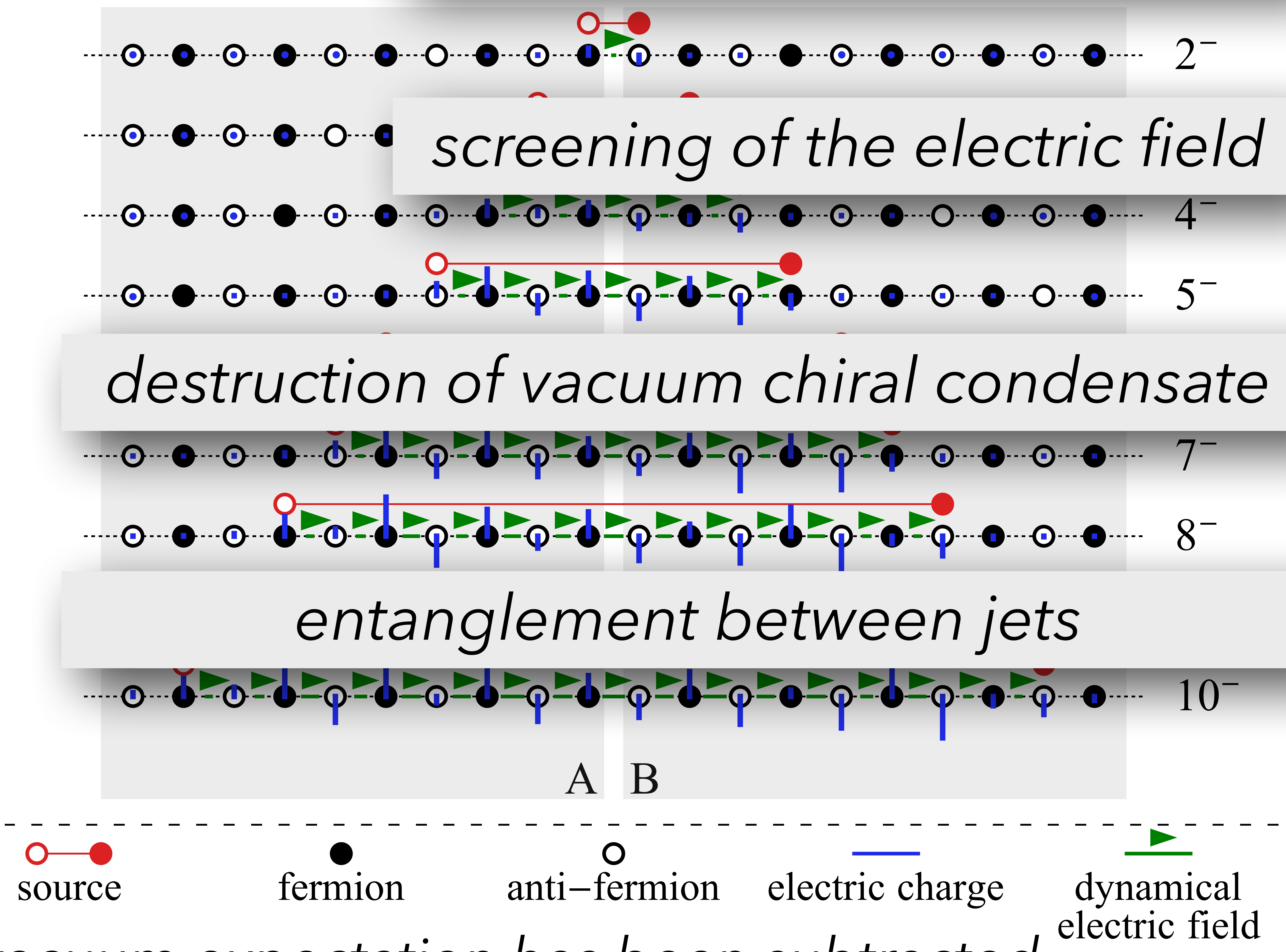
vacuum modification

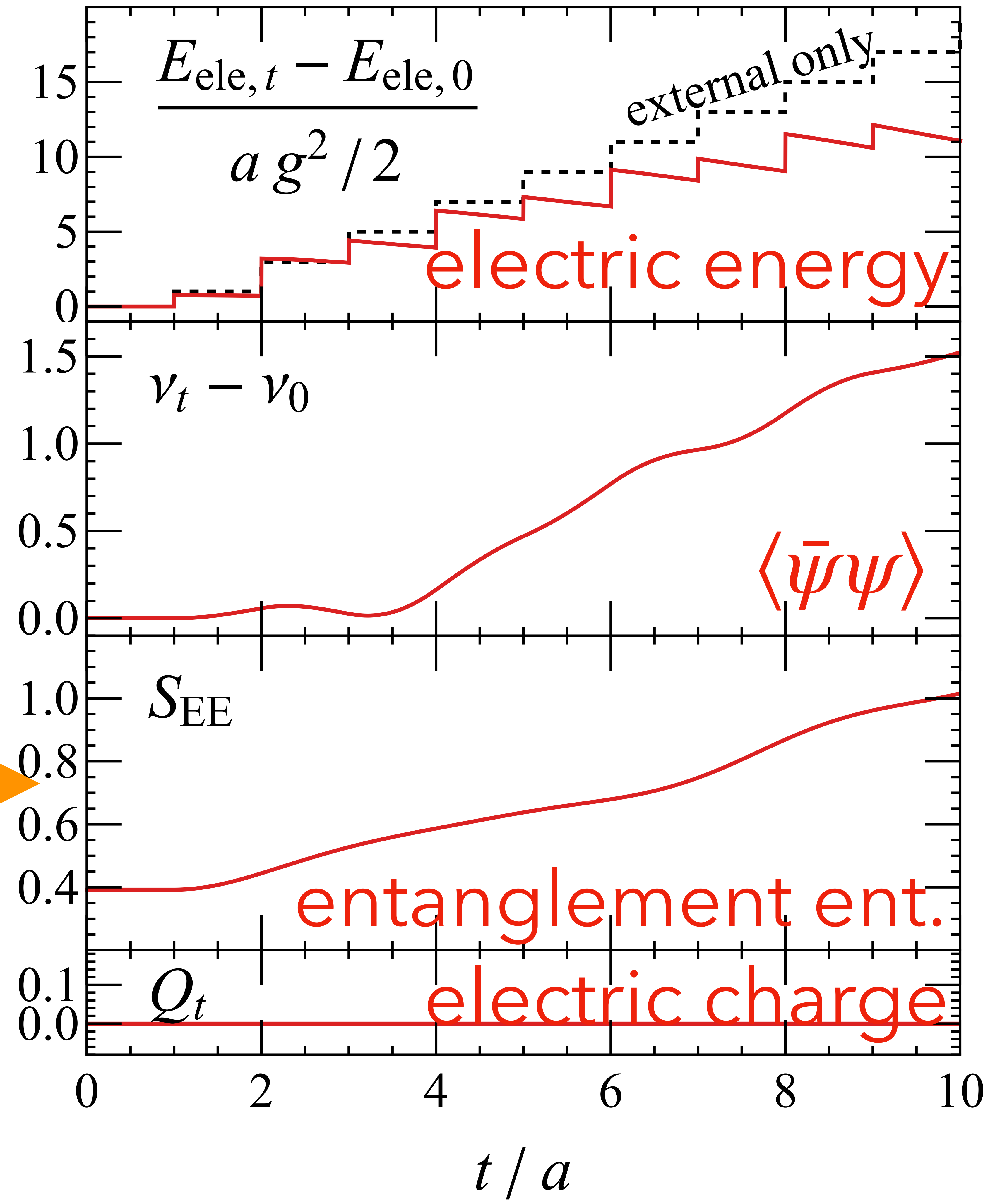
effects of pair production:

screening of the electric field

destruction of vacuum chiral condensate

entanglement between jets



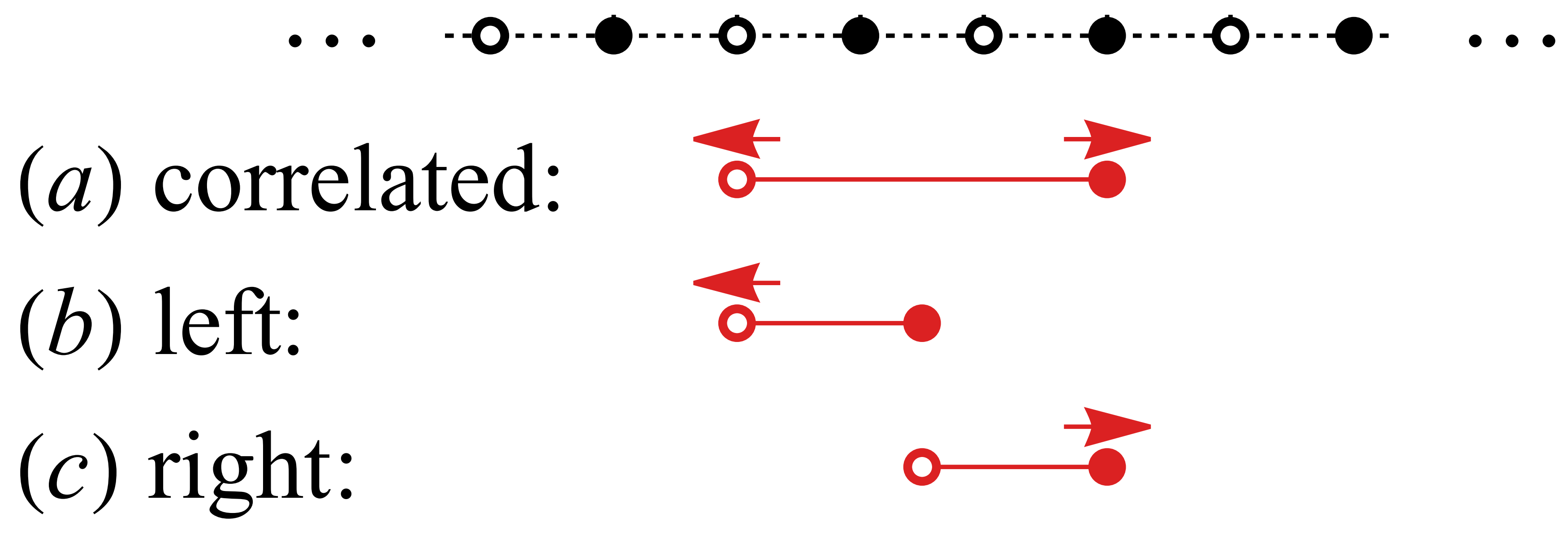


vacuum expectation has been subtracted

Background estimation: uncorrelated sources

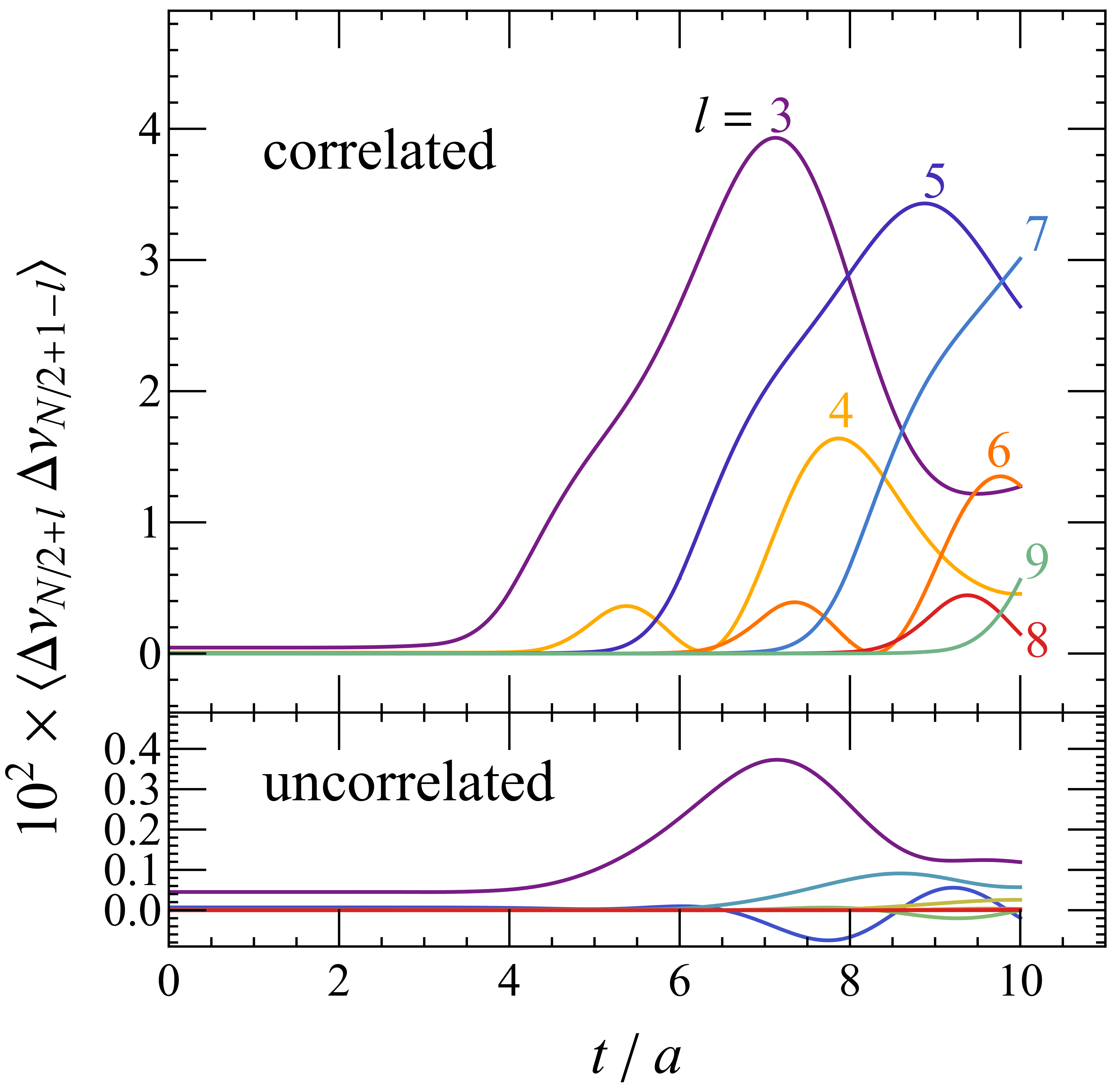
$$|\psi_{\text{uncorr}}\rangle = \frac{1}{\sqrt{2}}|\psi_{\text{left}}\rangle + \frac{e^{i\varphi}}{\sqrt{2}}|\psi_{\text{right}}\rangle$$

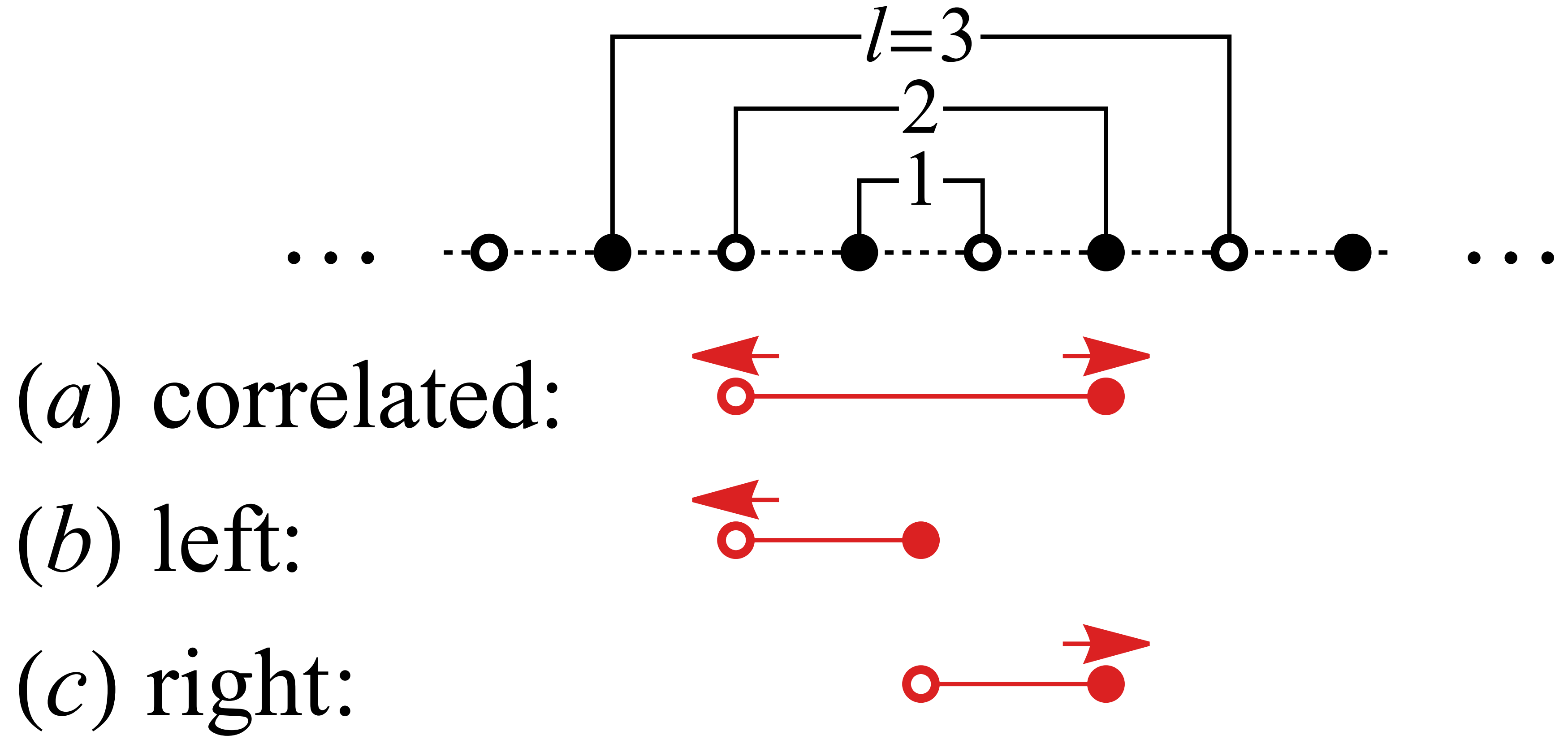
$$\begin{aligned}&\langle\langle\psi_{\text{uncorr}}|O|\psi_{\text{uncorr}}\rangle\rangle\\ &\equiv \int \langle\psi_{\text{uncorr}}|O|\psi_{\text{uncorr}}\rangle\frac{\mathrm{d}\varphi}{2\pi}\\ &= \frac{\langle\psi_{\text{left}}|O|\psi_{\text{left}}\rangle}{2} + \frac{\langle\psi_{\text{right}}|O|\psi_{\text{right}}\rangle}{2}\end{aligned}$$



observables of entanglement

$\nu_n \equiv \bar{\psi}_n \psi_n \qquad \Delta\nu_n \equiv \nu_n - \langle \nu_n \rangle_{\text{vac}}$





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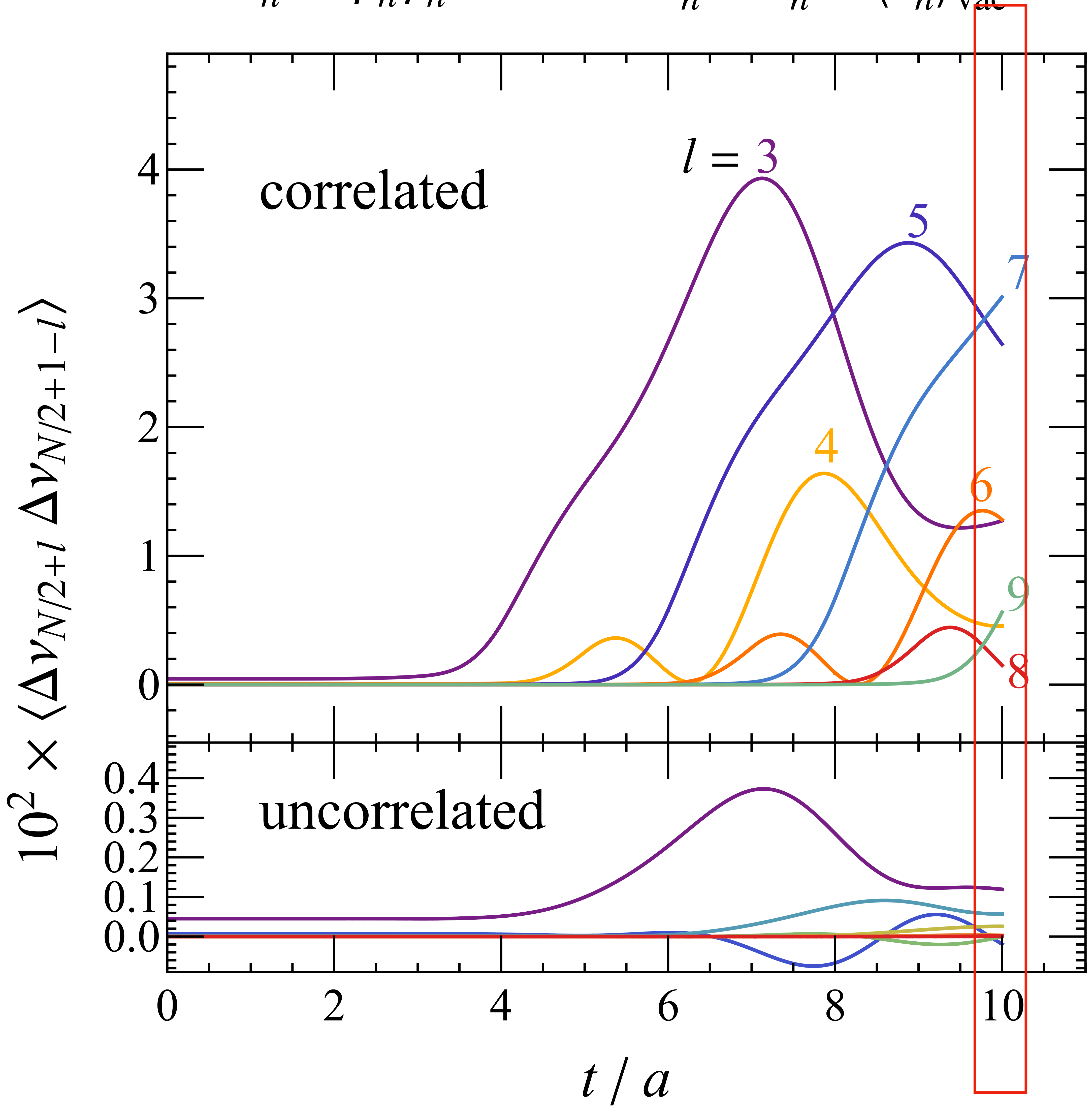

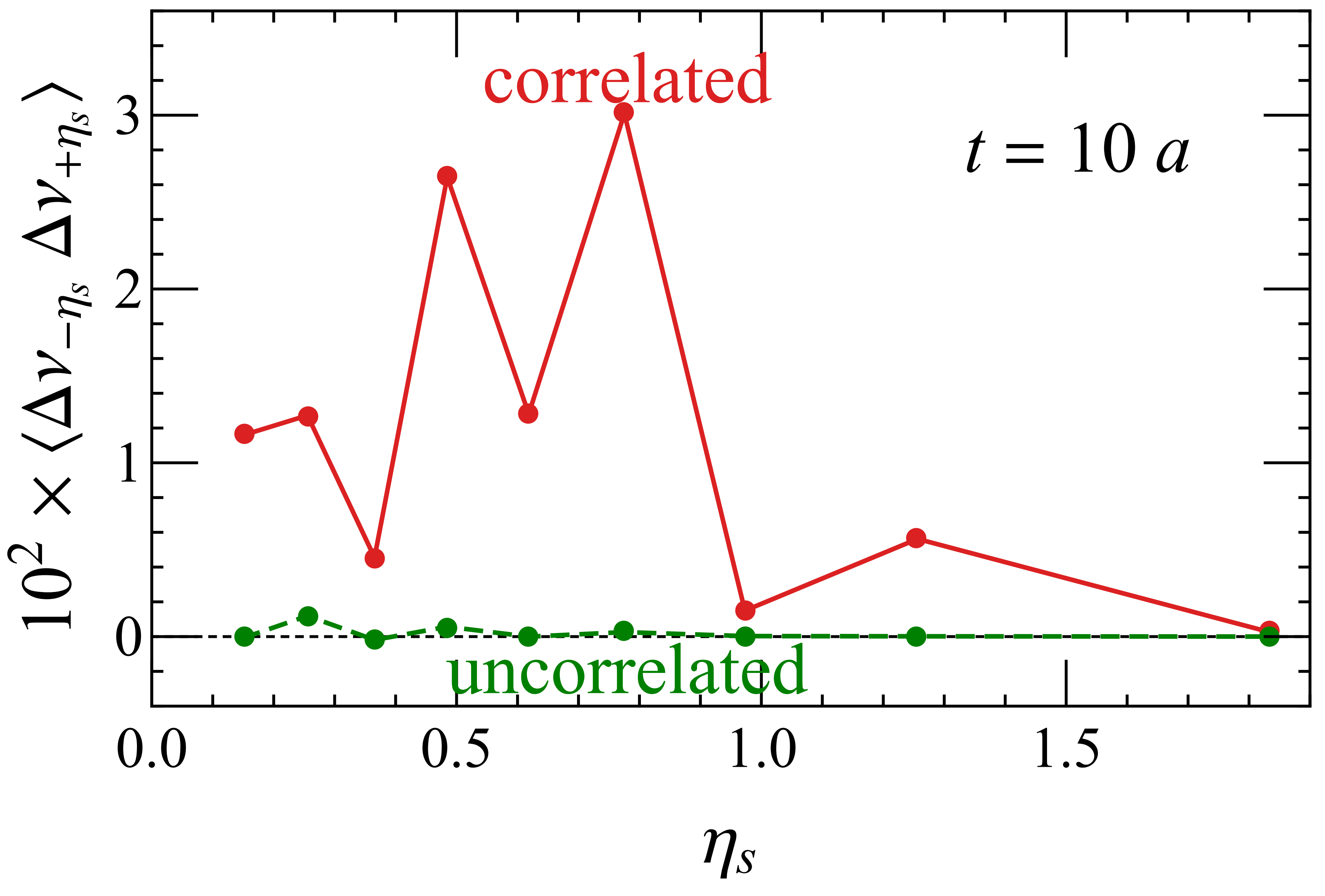

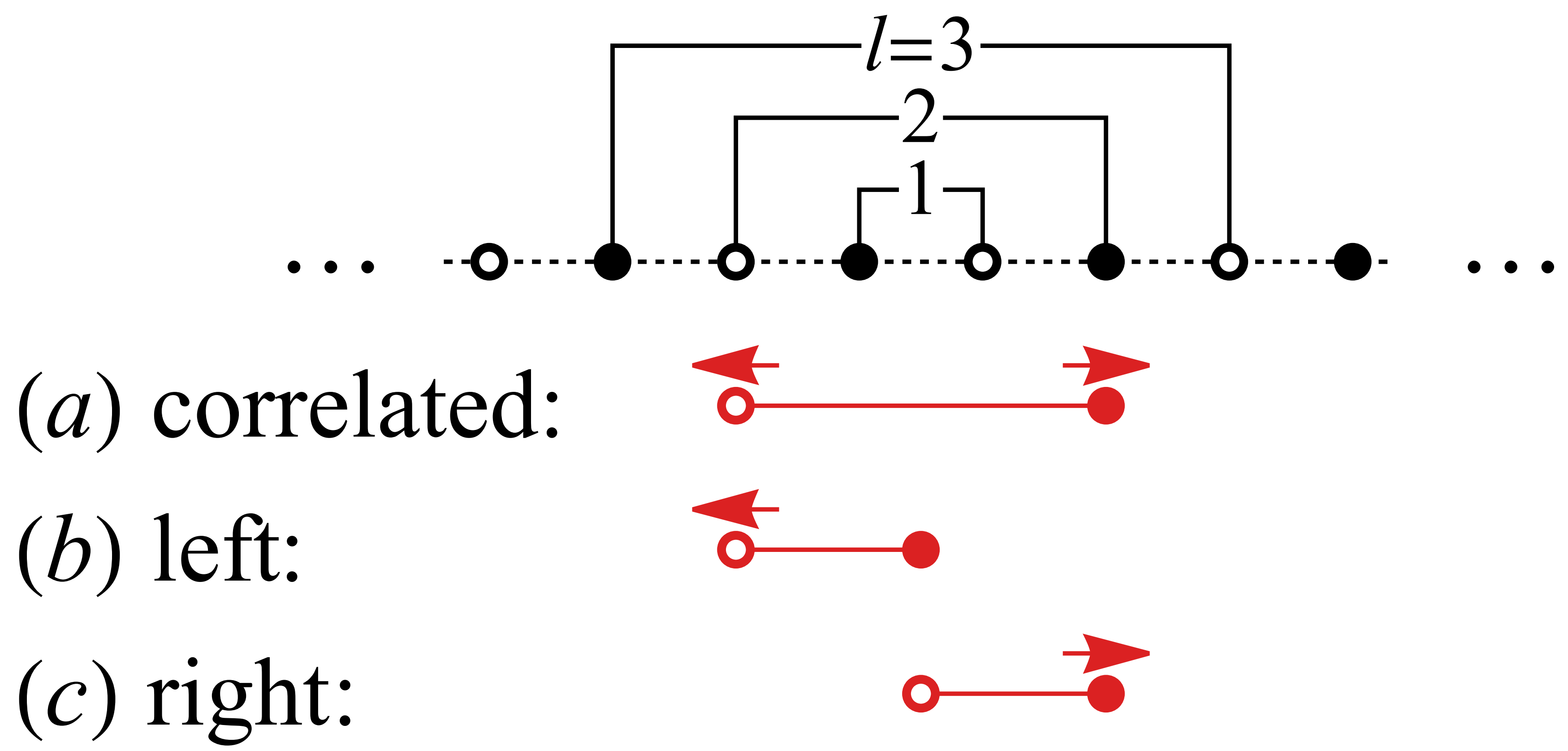

II. Vector & Axial Charge Transport

axial and vector transport in Schwinger model

in Schwinger model: $$Q = J_5, \quad J = -Q_5$$

axial and vector transport in Schwinger model

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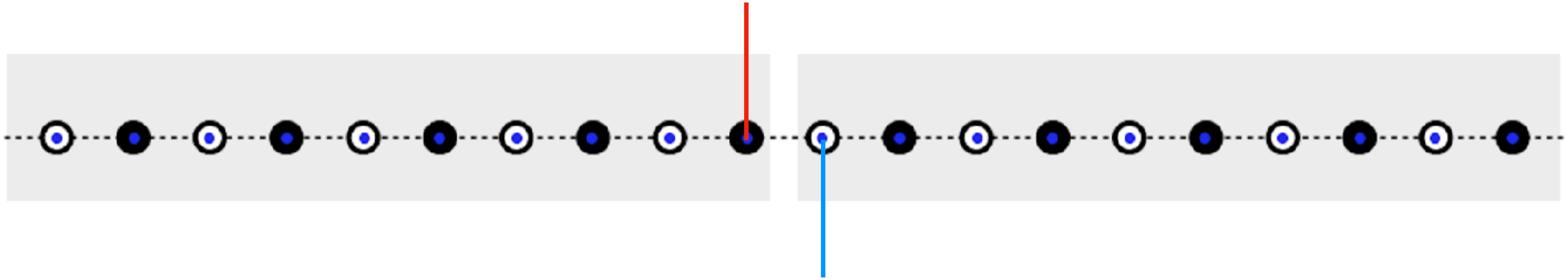
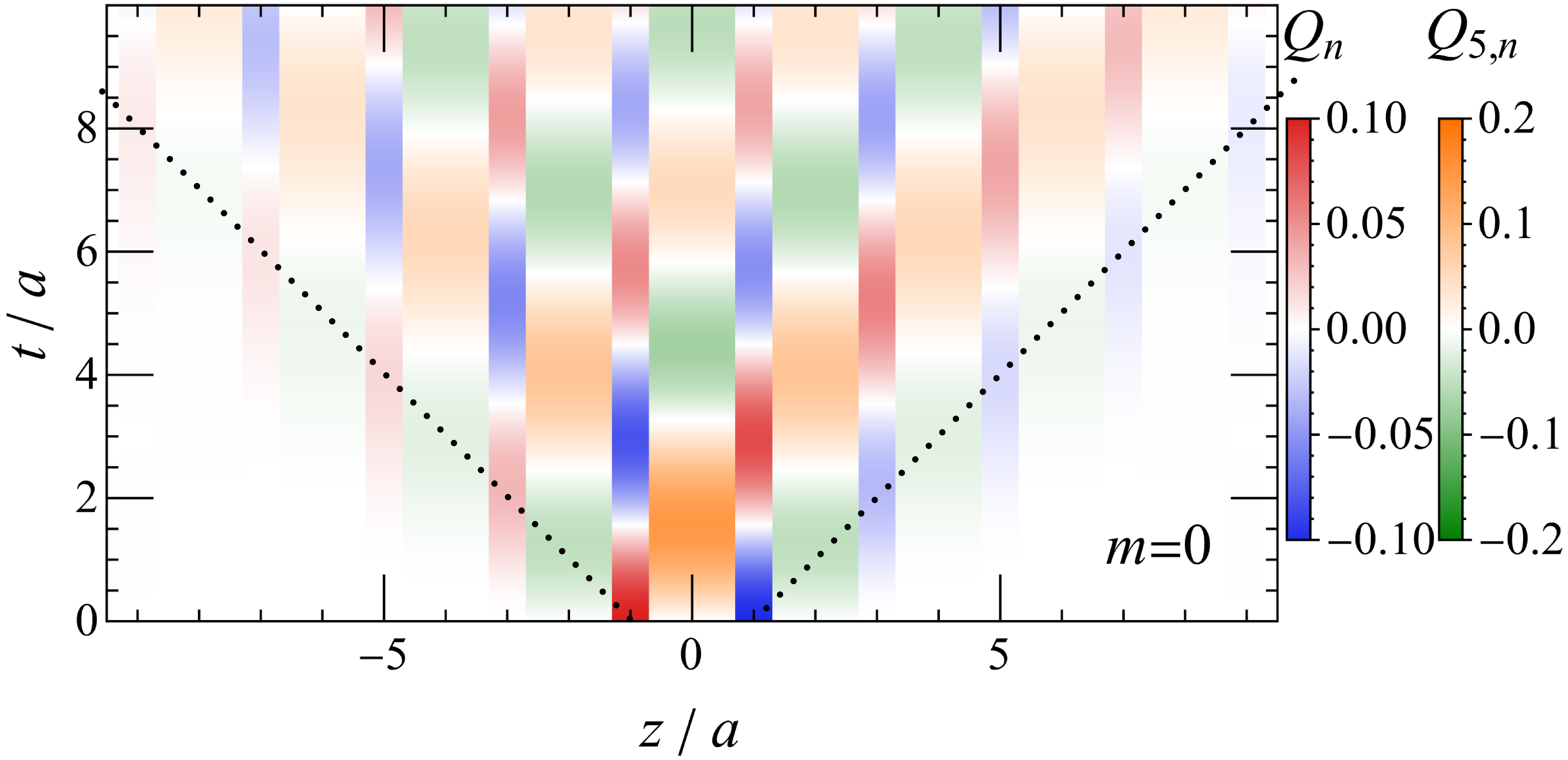


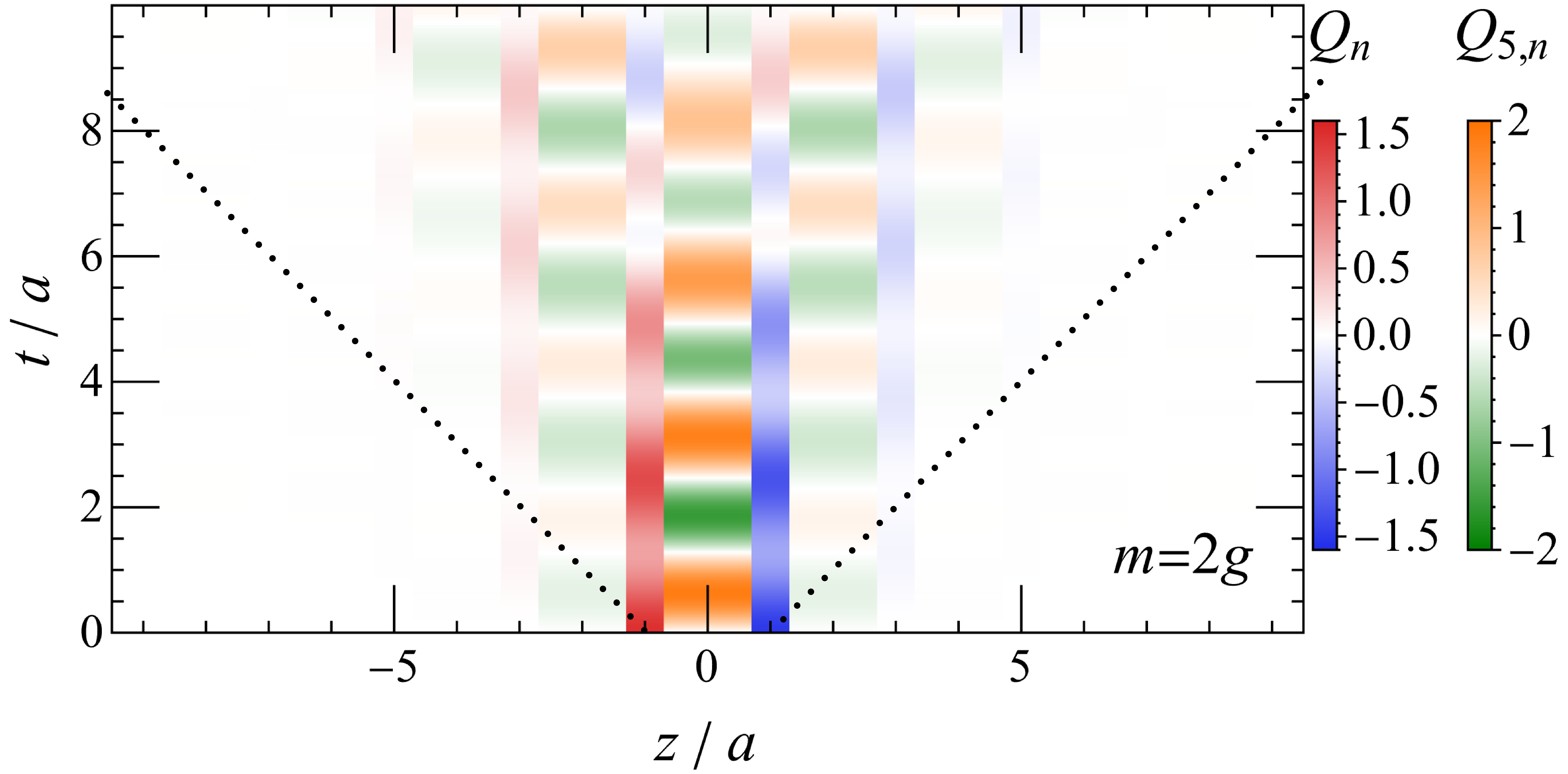
$$Q_n \equiv \langle \bar{\psi}(a\,n)\gamma^0\psi(a\,n)\rangle = \frac{\langle Z_n\rangle + (-1)^n}{2a},$$

$$Q_{5,n} \equiv \langle \bar{\psi}(a\,n)\gamma^5\gamma^0\psi(a\,n)\rangle = \frac{\langle X_n Y_{n+1} - Y_n X_{n+1}\rangle}{4a}$$

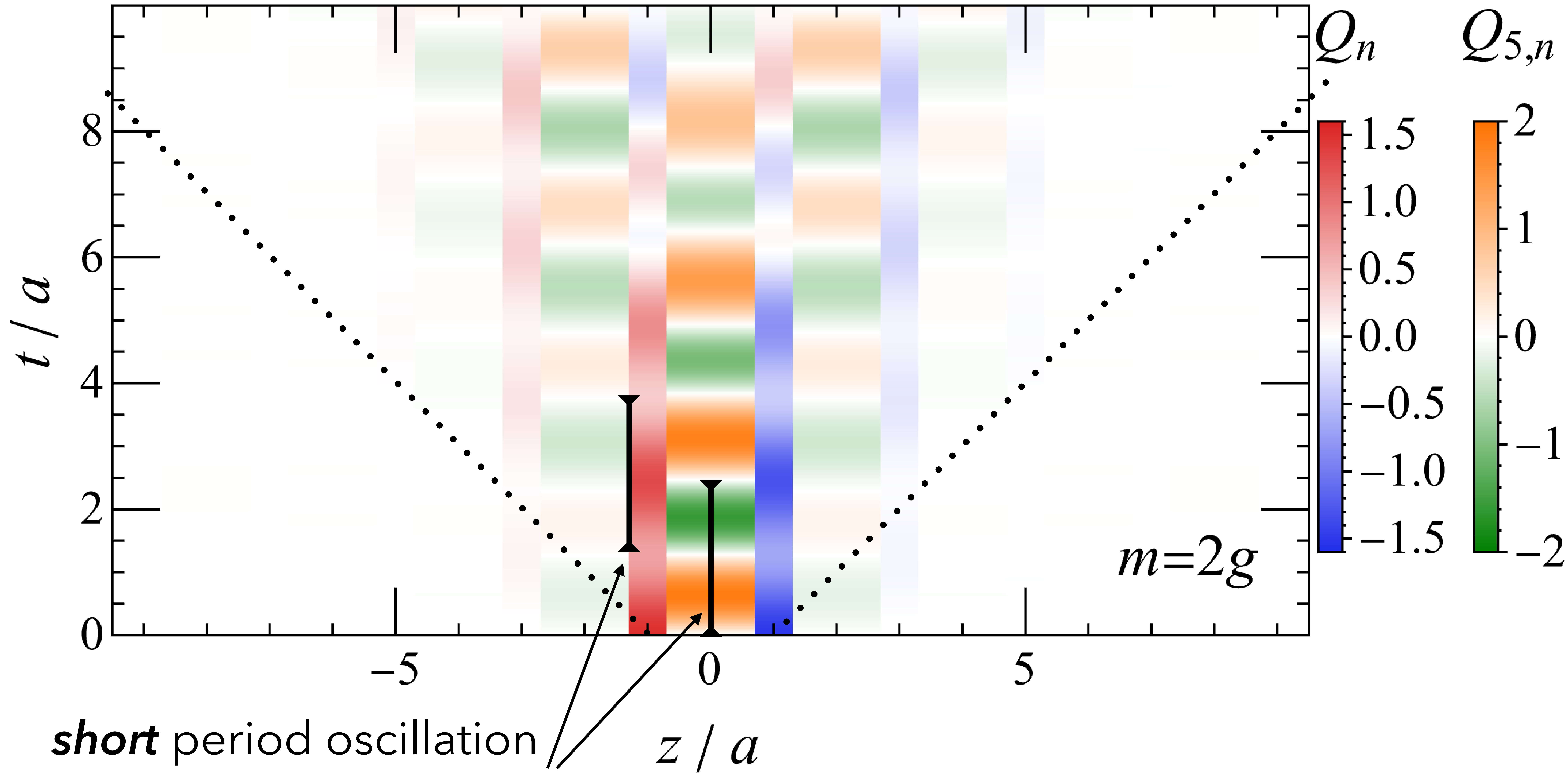


Q_n
$Q_{5,n}$
0.10
0.05
0.00
−0.05
−0.10
0.2
0.1
0.0
−0.1
−0.2
$m=0$
$t\,/\,a$
0
2
4
6
8
−5
0
5
$z\,/\,a$

massive case

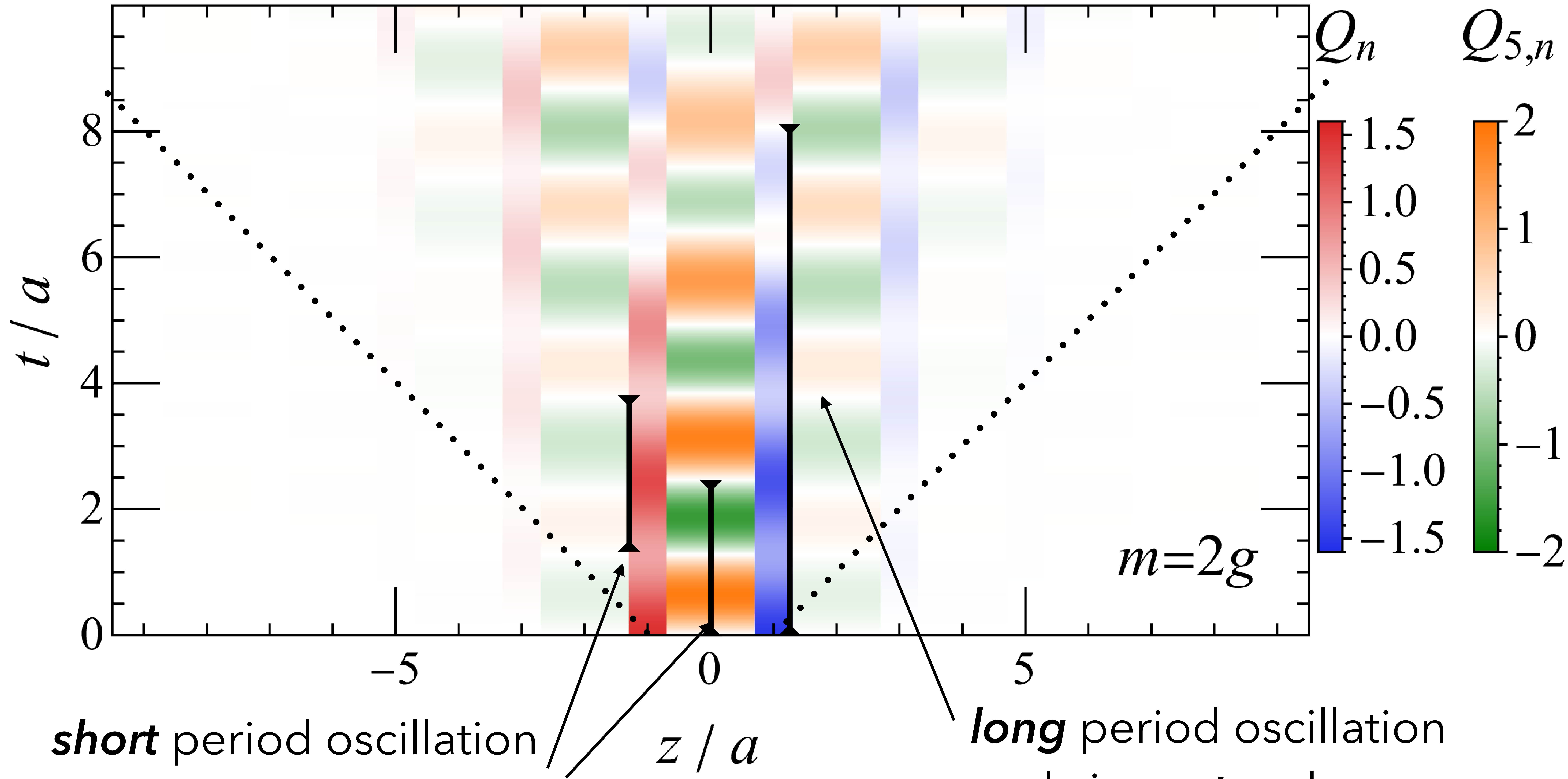


massive case



short period oscillation
in both ***vector*** and ***axial*** charge

massive case



short period oscillation
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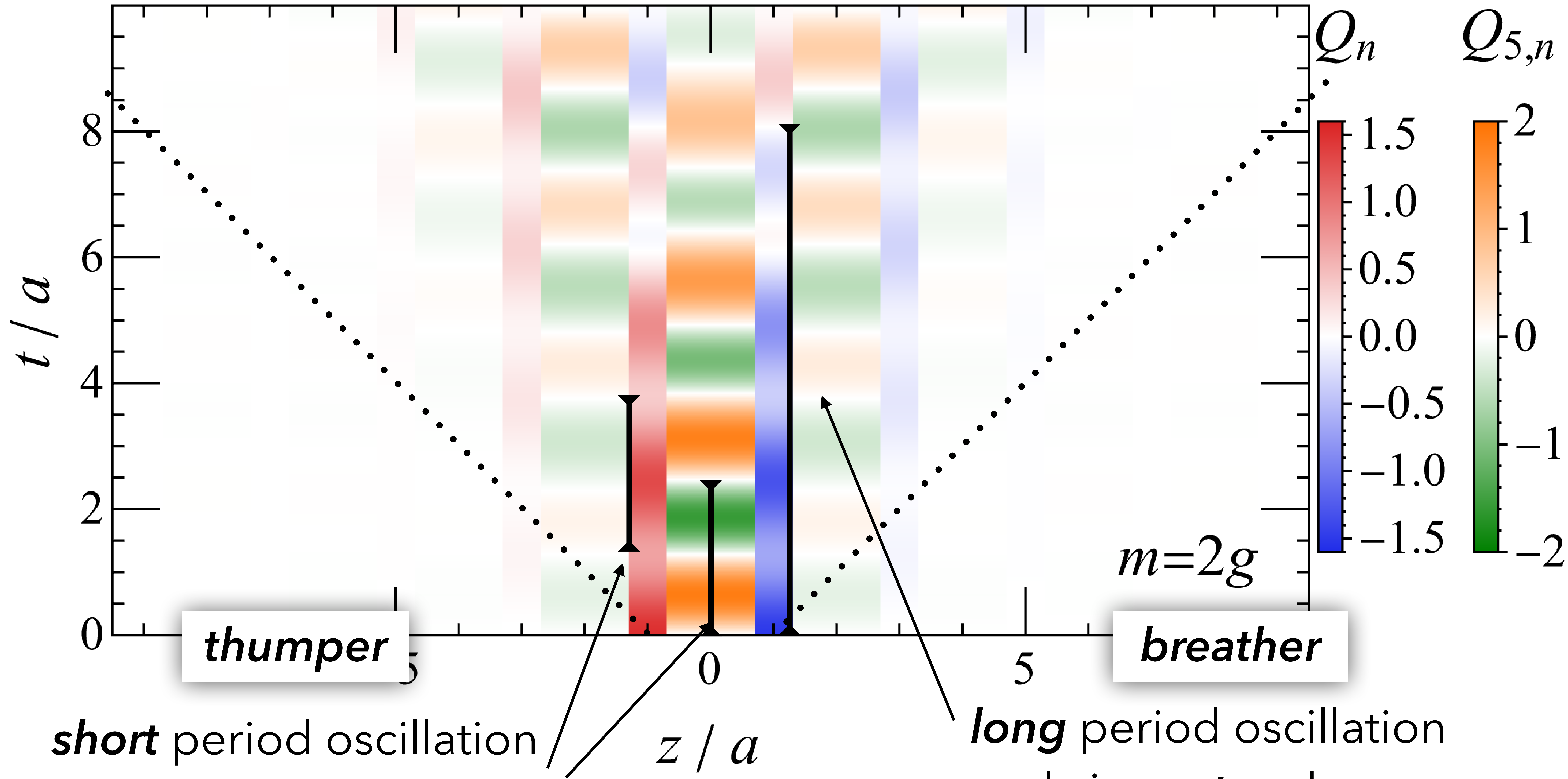
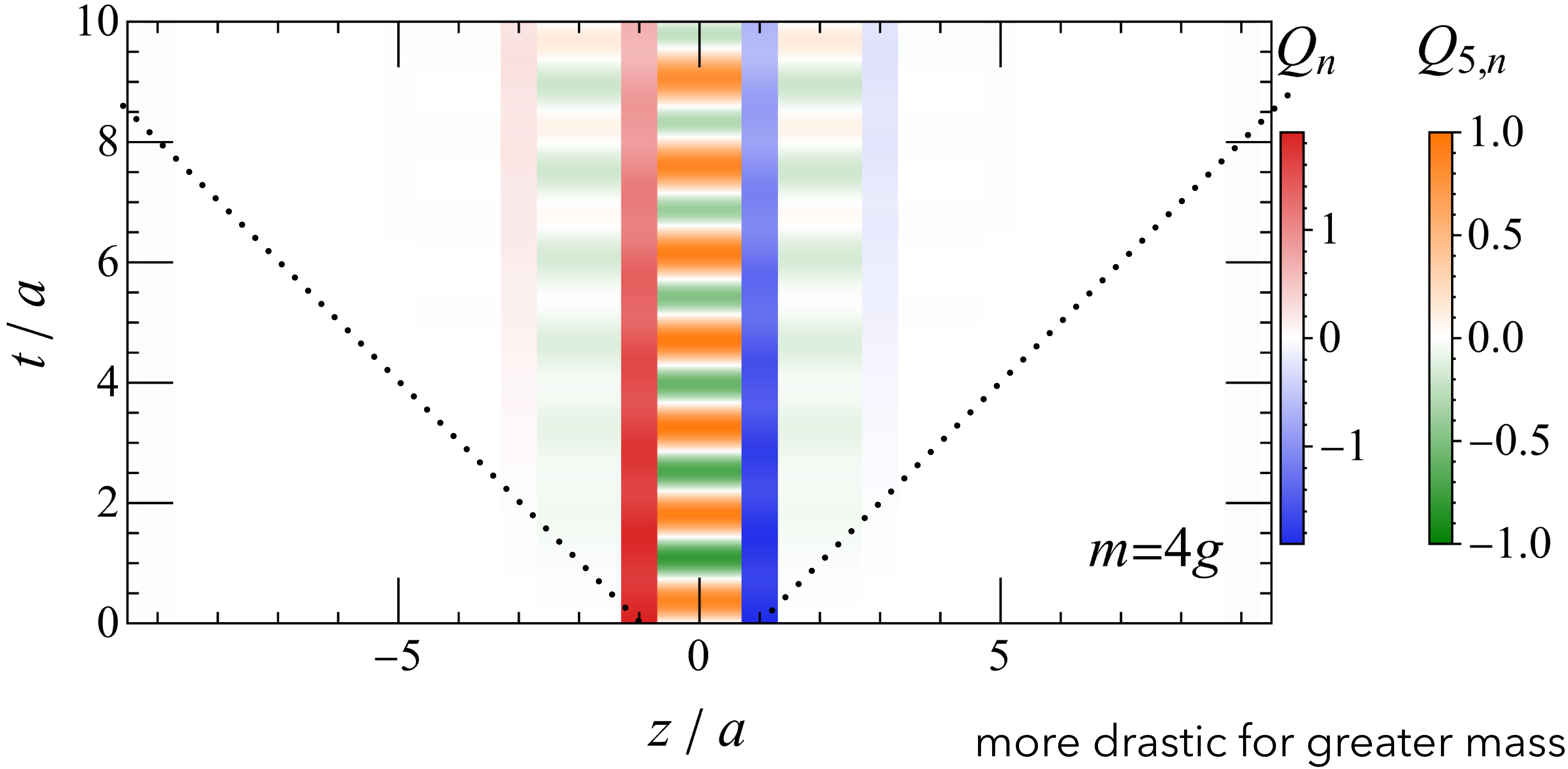
long period oscillation
only in ***vector*** charge



short period oscillation
in both ***vector*** and ***axial*** charge

long period oscillation
only in ***vector*** charge

even more massive



more drastic for greater mass

origin of fast and slow oscillating modes

$$H|k\rangle = E_k|k\rangle$$

$$|\Psi(t=0)\rangle = \sum_k c_k|k\rangle$$

$$O(t) \equiv \langle\Psi(t)|O|\Psi(t)\rangle = \sum_{k,l} c_k c_l^* e^{i(E_l - E_k)t}\langle l|O|k\rangle$$

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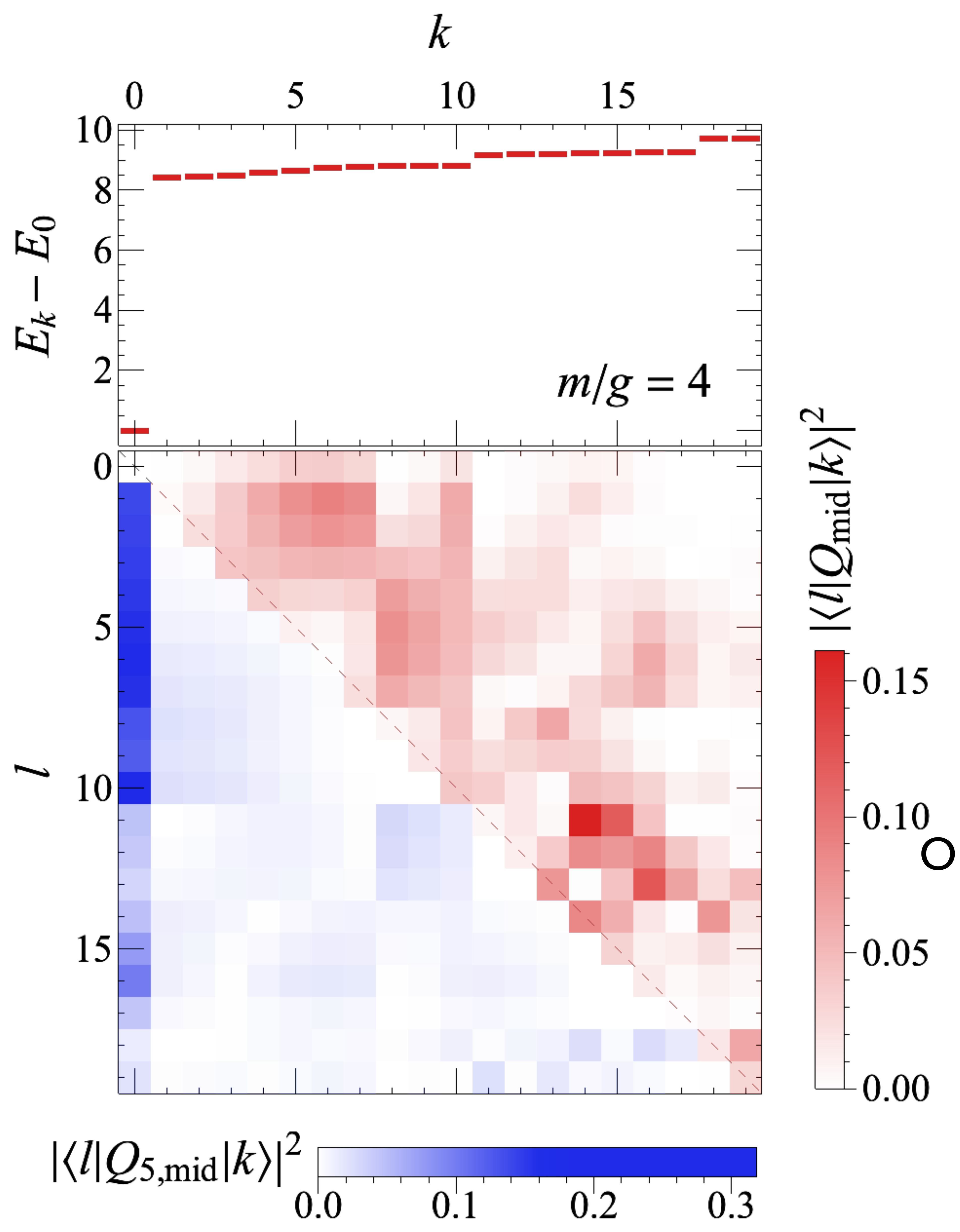
oscillation *frequency* ← energy difference

oscillation *strength* ← matrix element

origin of fast and slow oscillating modes



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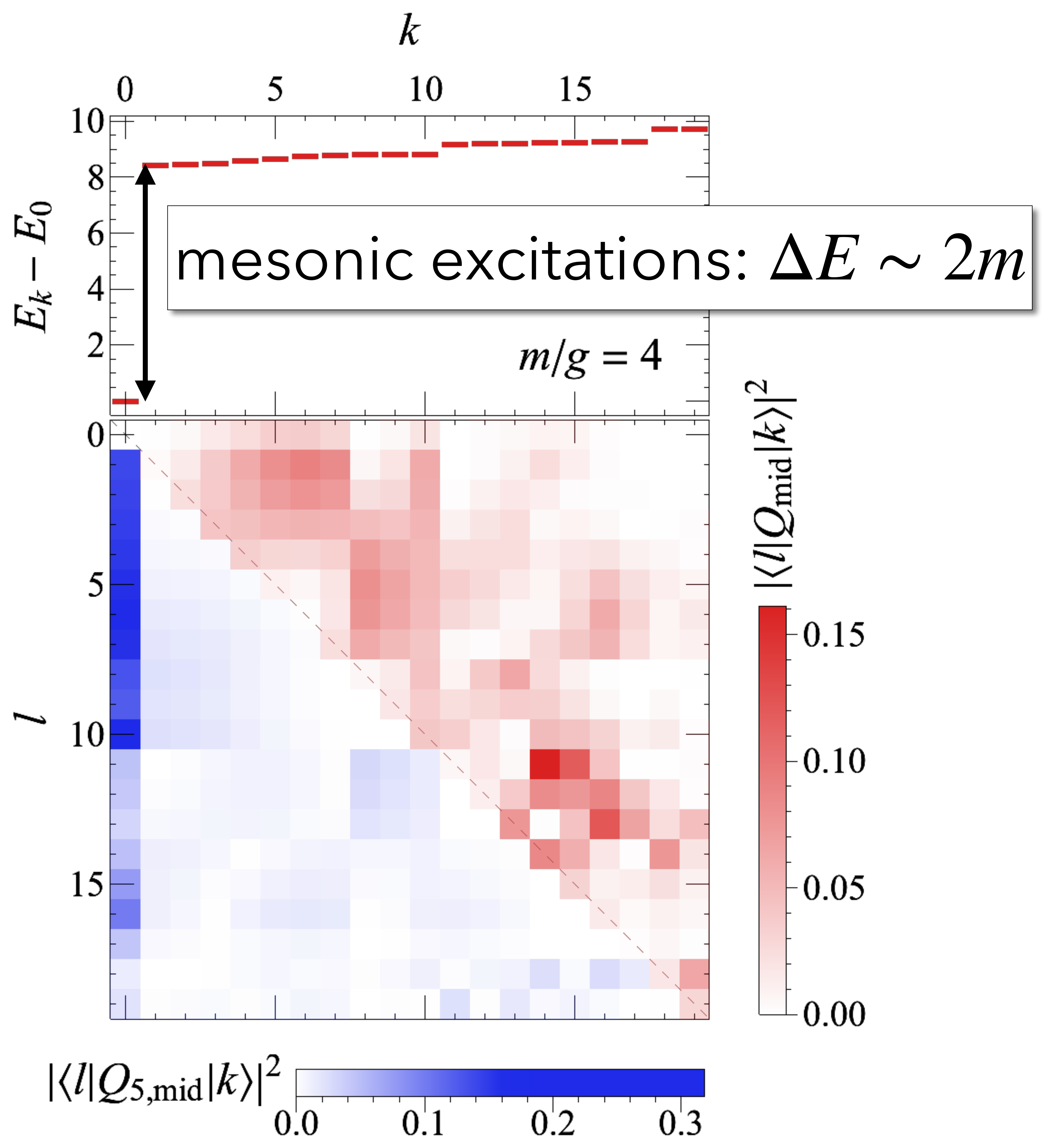

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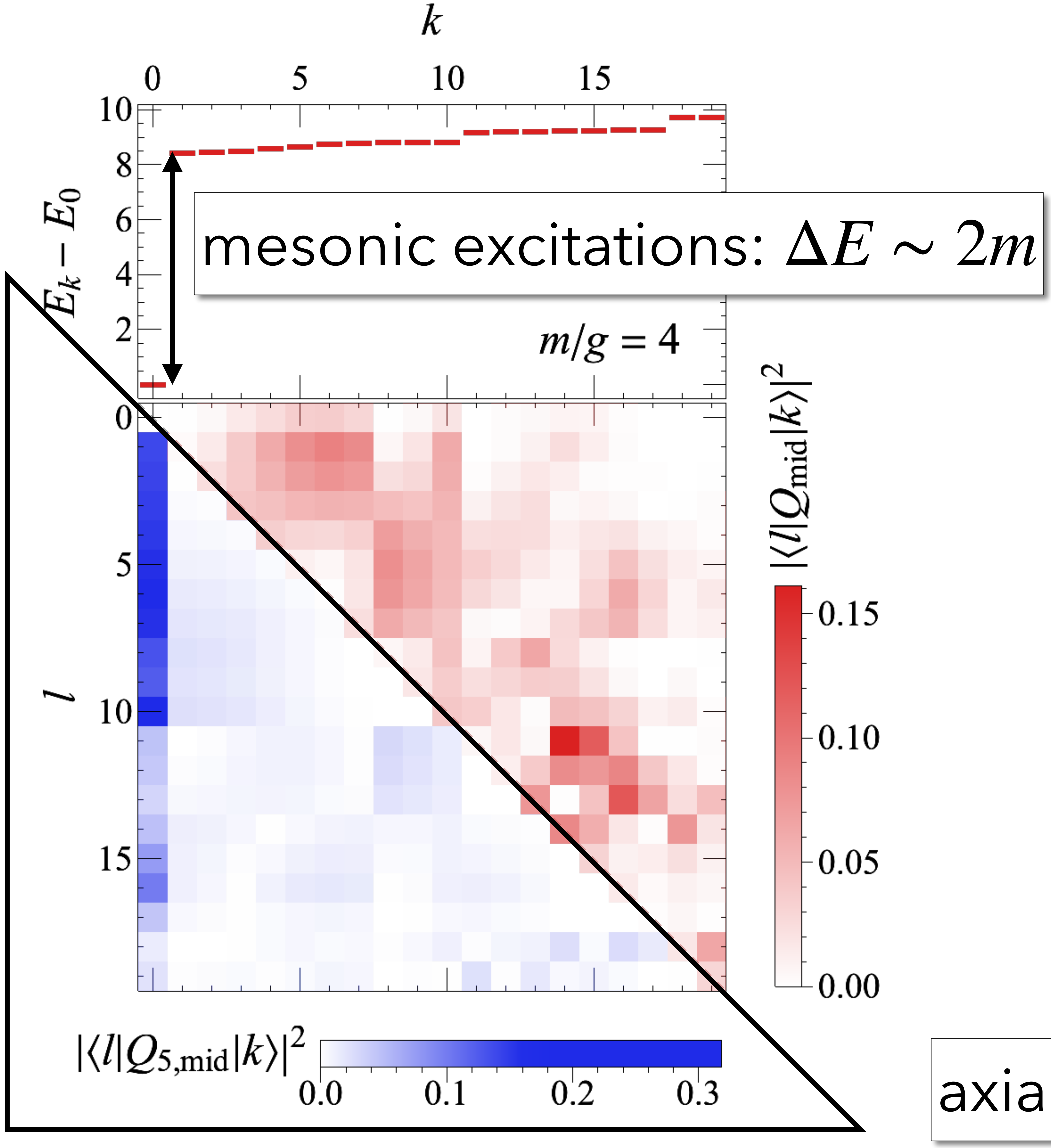

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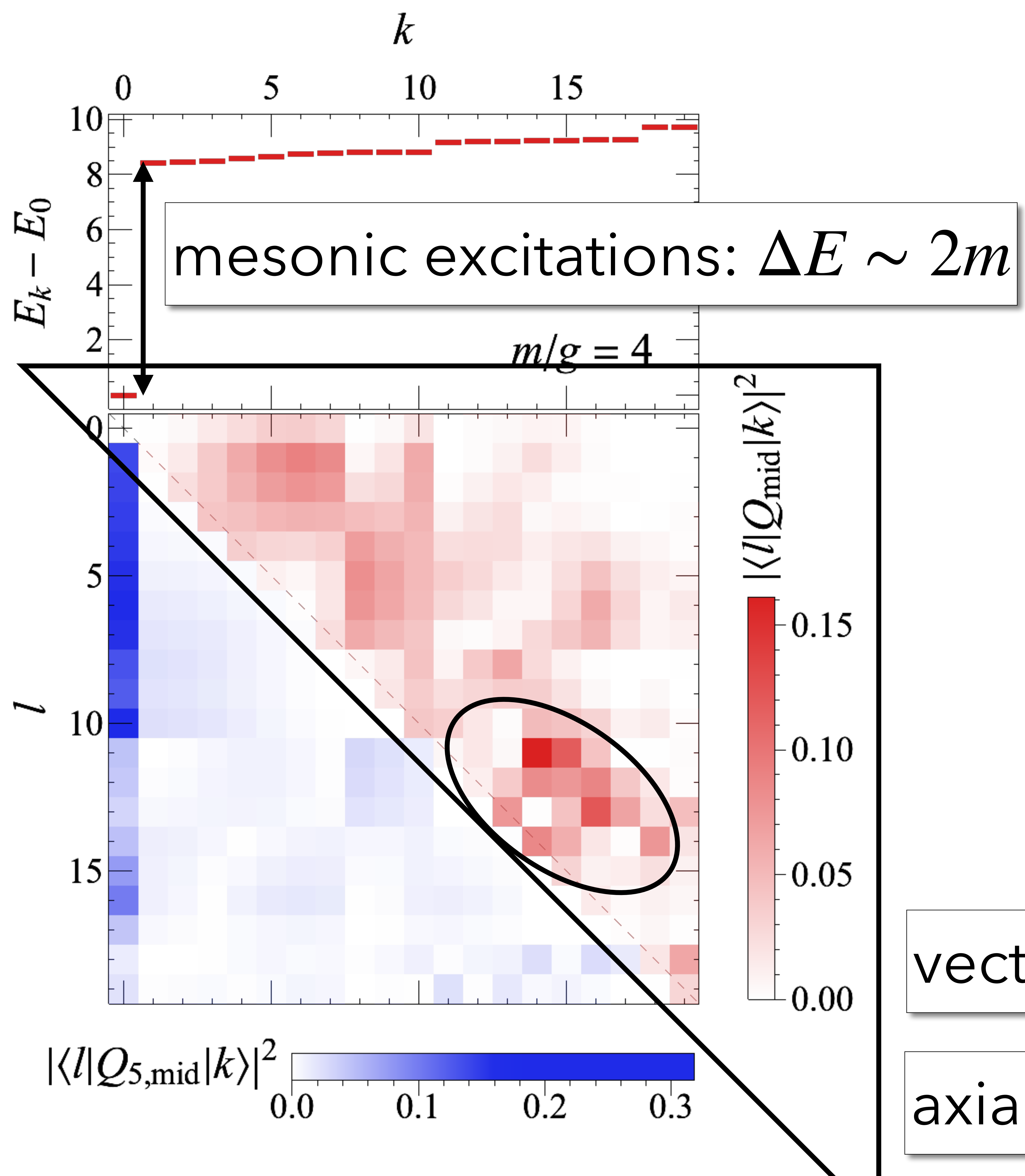
axial charge: ground state ↔ excitation

origin of fast and slow oscillating modes



$$H|k\rangle = E_k|k\rangle$$

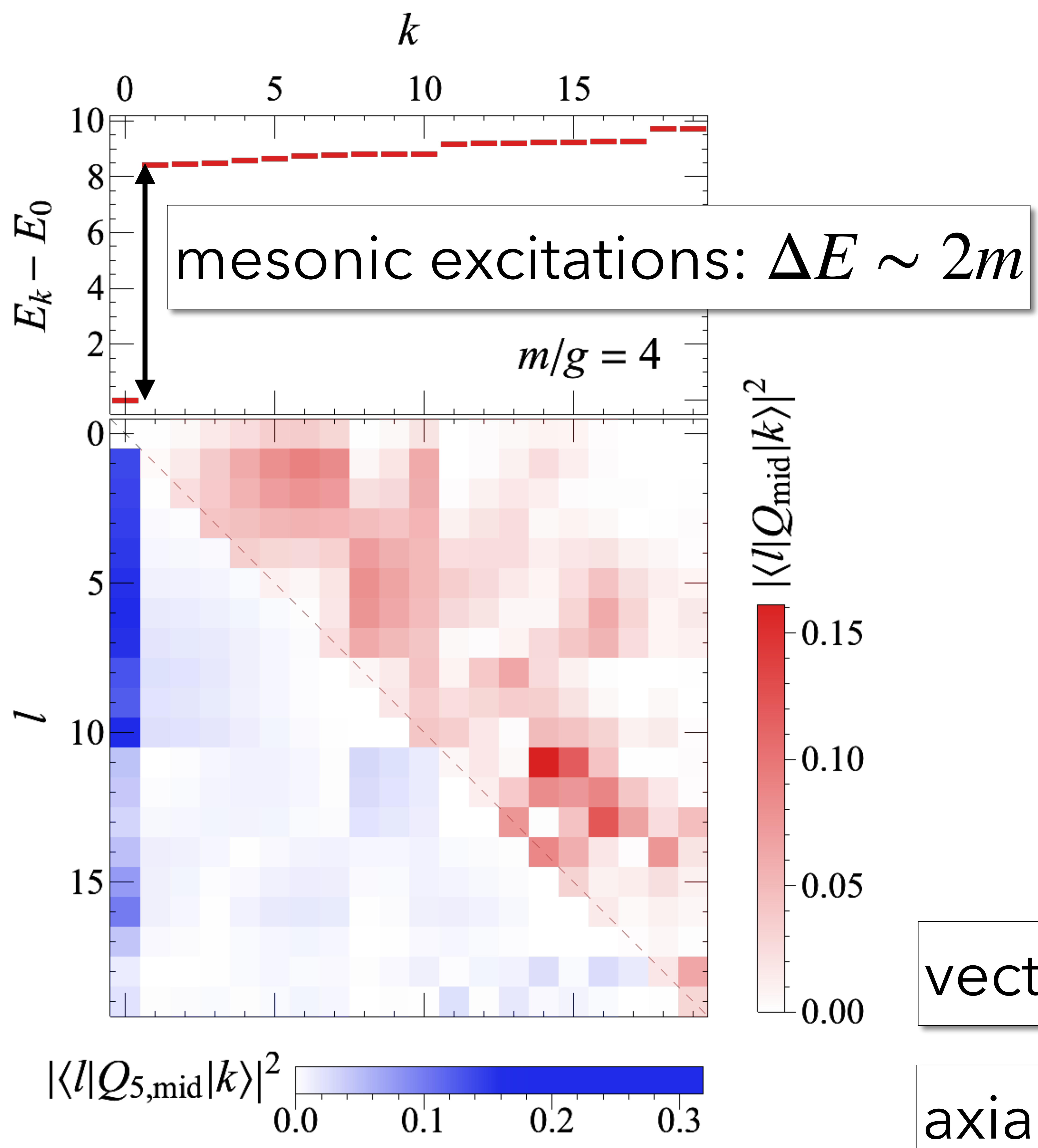
$$|\Psi(t=0)\rangle = \sum_k c_k|k\rangle$$

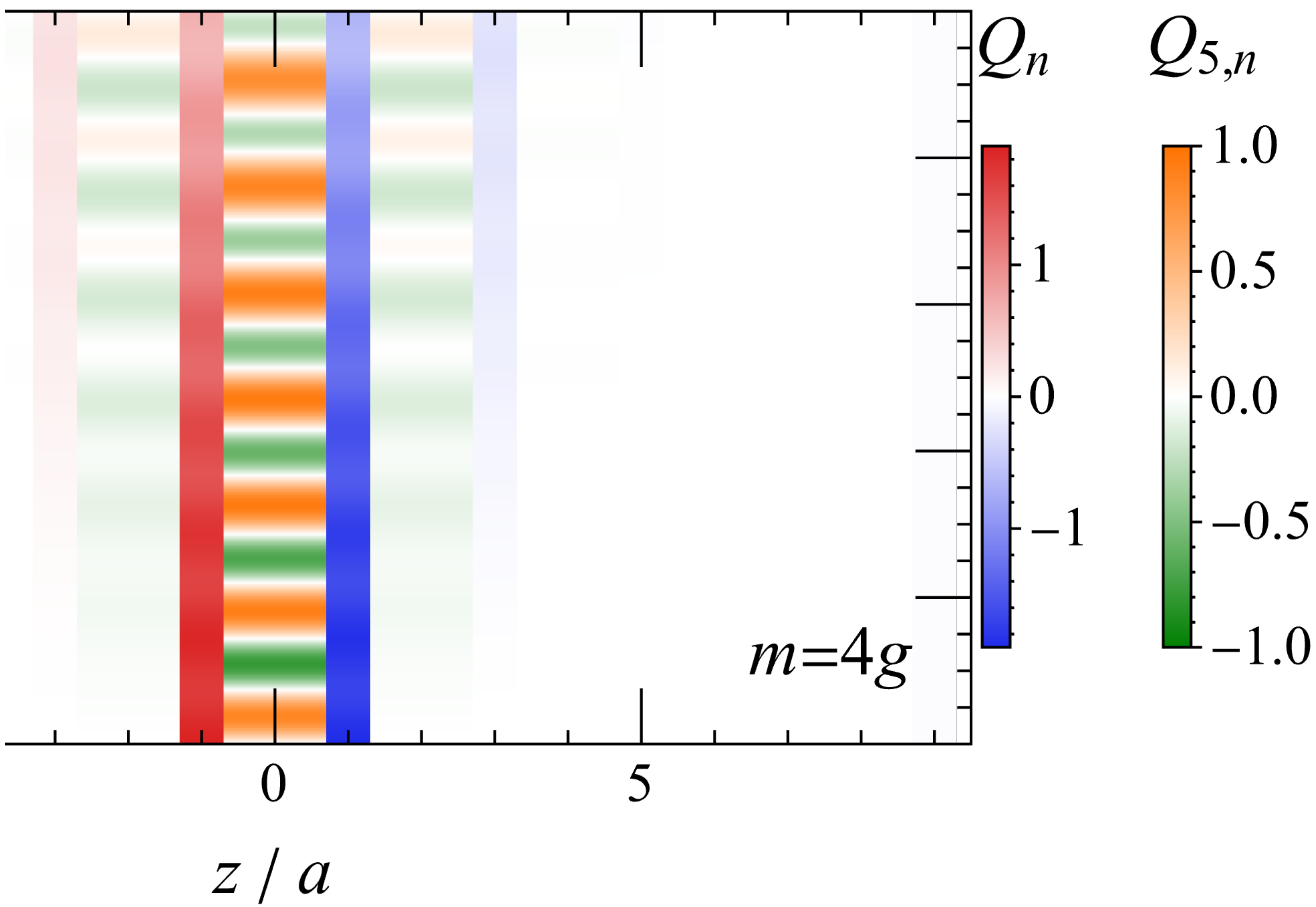
$$O(t) \equiv \langle\Psi(t)|O|\Psi(t)\rangle = \sum_{k,l} c_k c_l^* e^{i(E_l - E_k)t}\langle l|O|k\rangle$$

vector charge: excitation ↔ excitation

axial charge: ground state ↔ excitation

origin of fast and slow oscillating modes

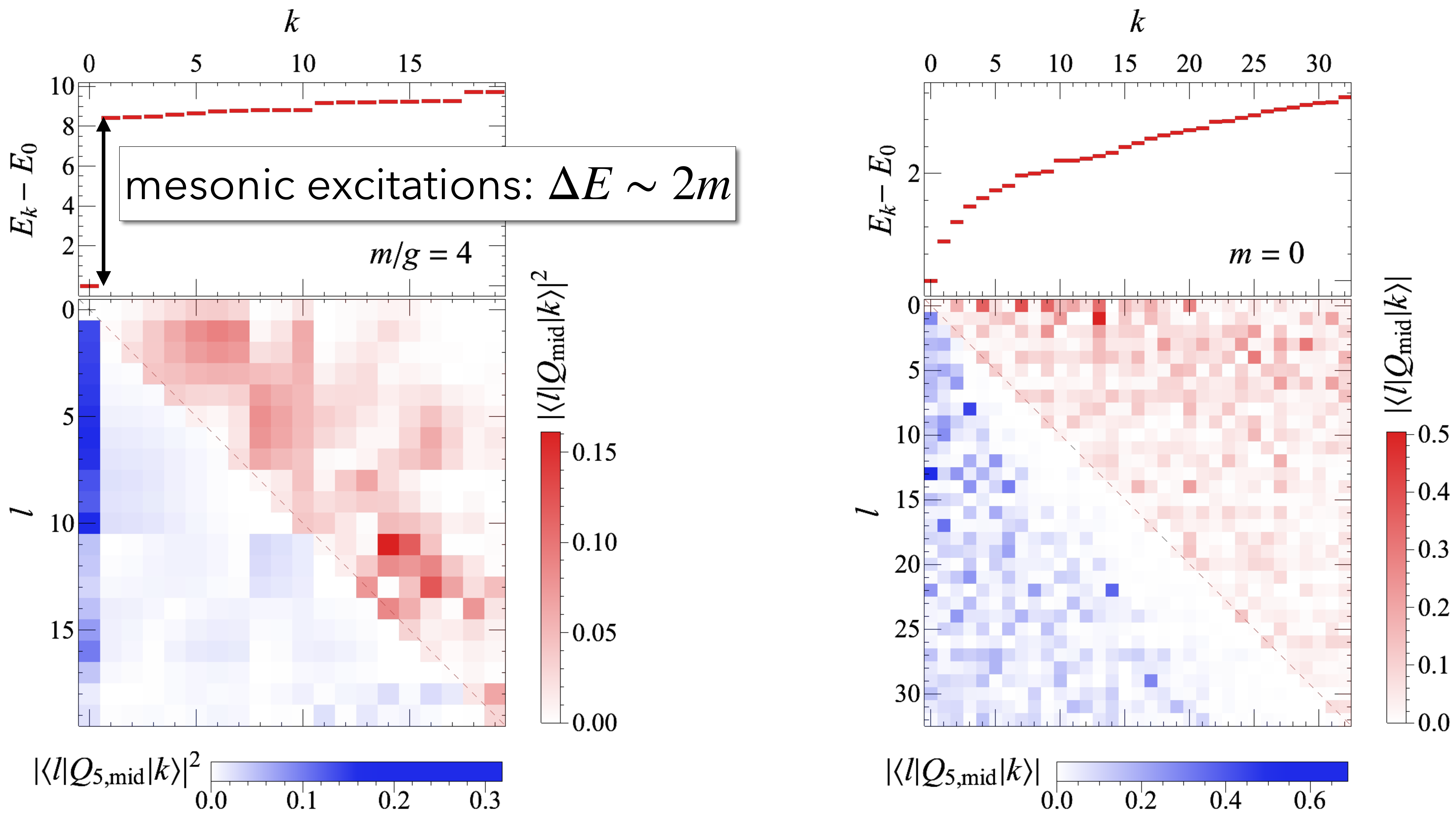




vector charge: excitation ↔ excitation

axial charge: ground state ↔ excitation

real-time evolution



III. Phase Structure

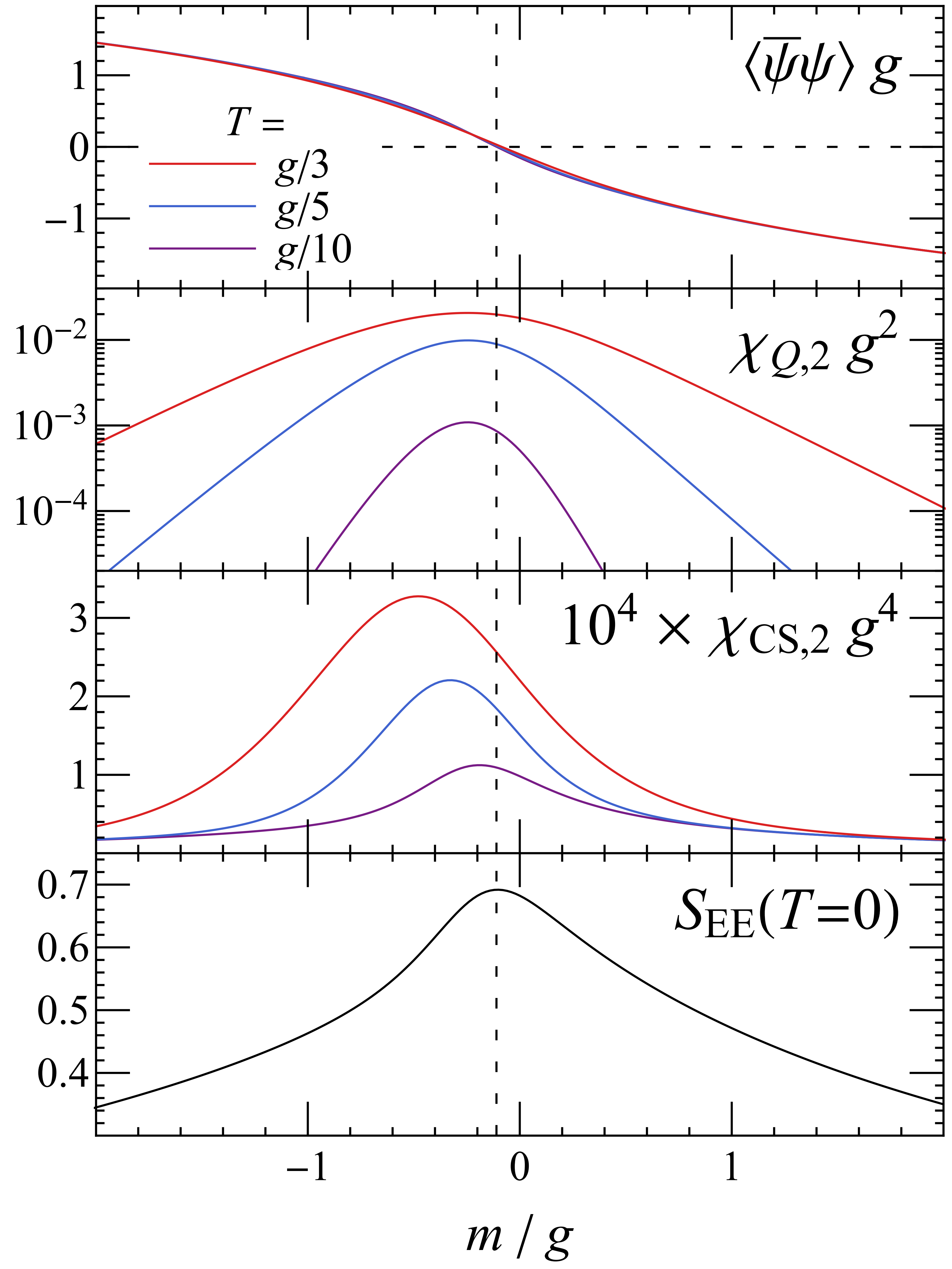
K. Ikeda, D. Kharzeev, R. Meyer, S. Shi, arXiv: 2305.00996

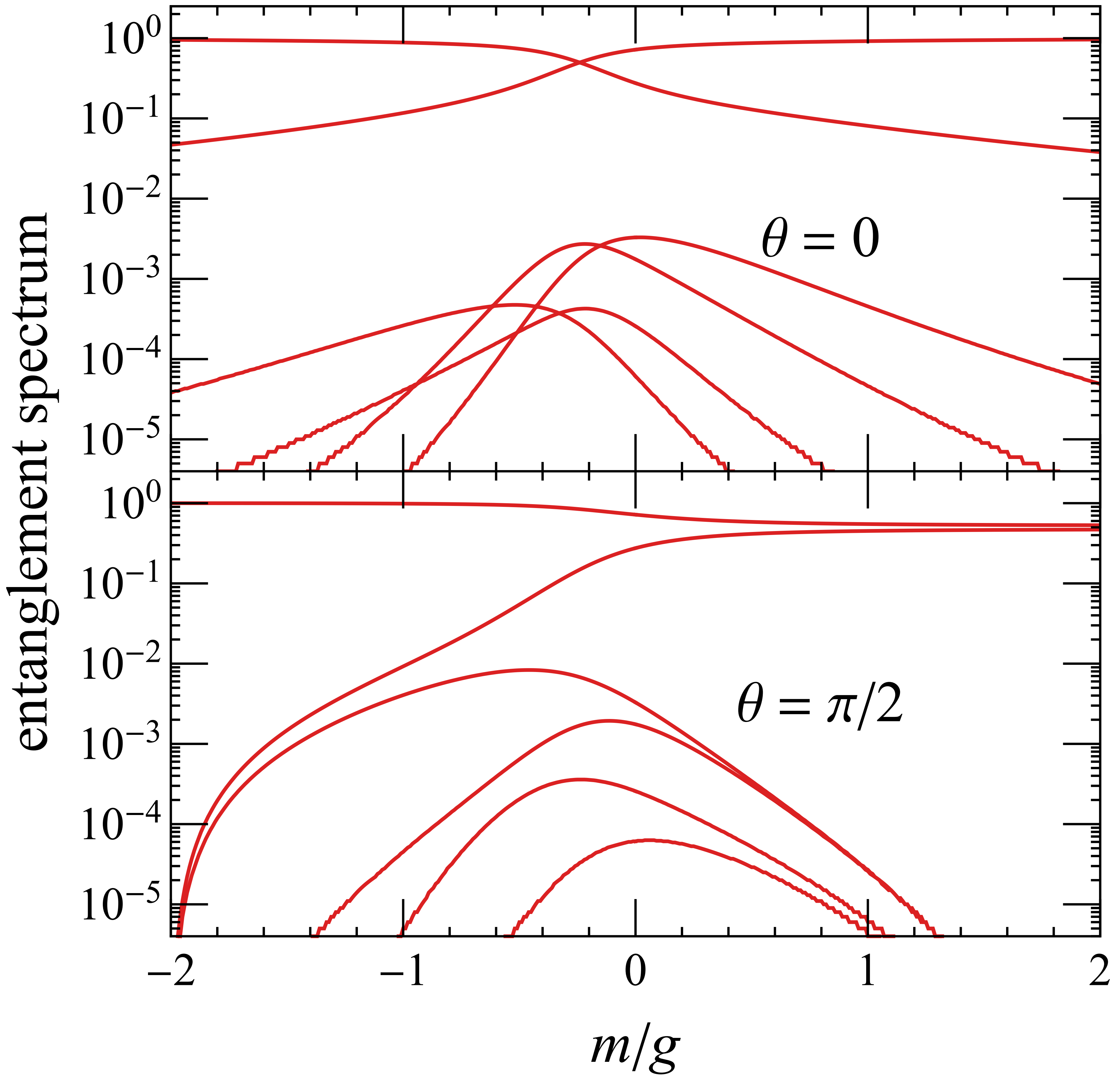
thermodynamic properties

- finite temperature, finite chemical potential:

$$\langle O \rangle_{\rm th} \equiv {\rm Tr}(\rho_{\rm th}\, O) \qquad \rho_{\rm th} \equiv \frac{e^{-(H-\mu Q)/T}}{{\rm Tr}(e^{-(H-\mu Q)/T})}$$

different signals of the critical point





summary

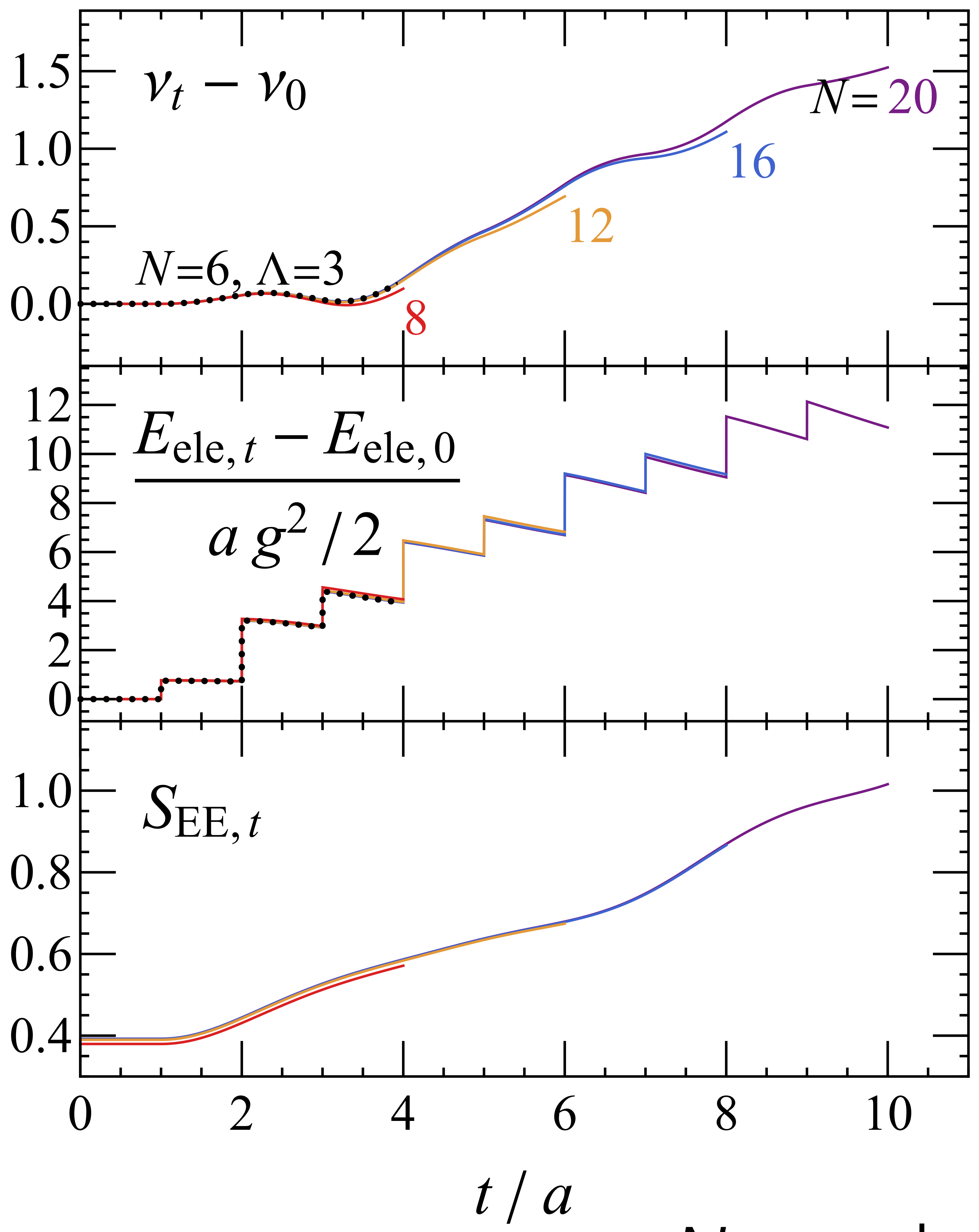
- real-time dynamics in Schwinger model
  - jet production: spread out of light cone, creation of fermion-antifermion pairs
  - charge transport: thumper and breather modes
  - critical point detected by different signals
- Need ***quantum computers*** to approach the continuum limit.

Students and Postdocs are welcome!

(so are questions/comments)

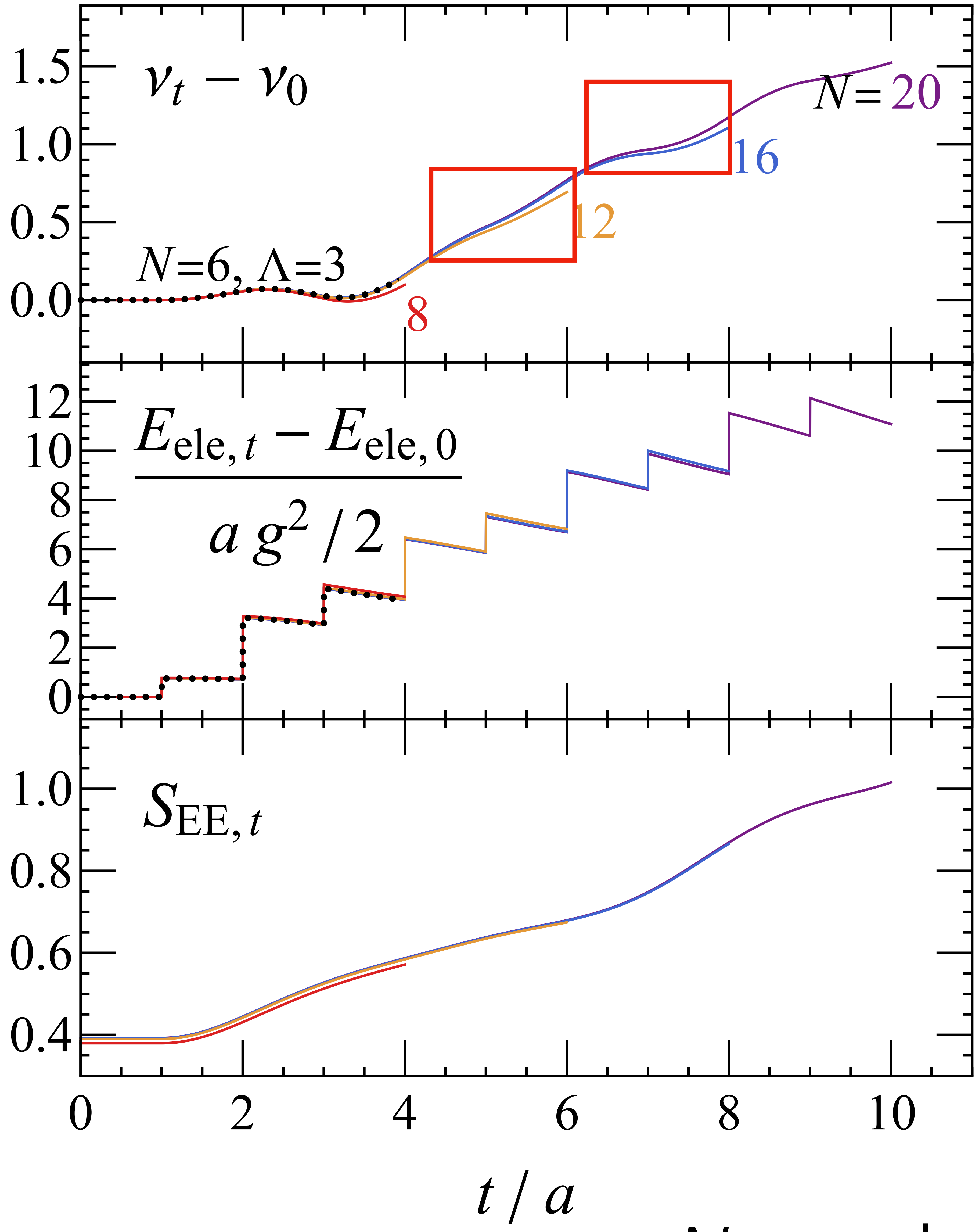
Backup Slides

size dependence and boundary effect



N: number of lattice sites

size dependence and boundary effect



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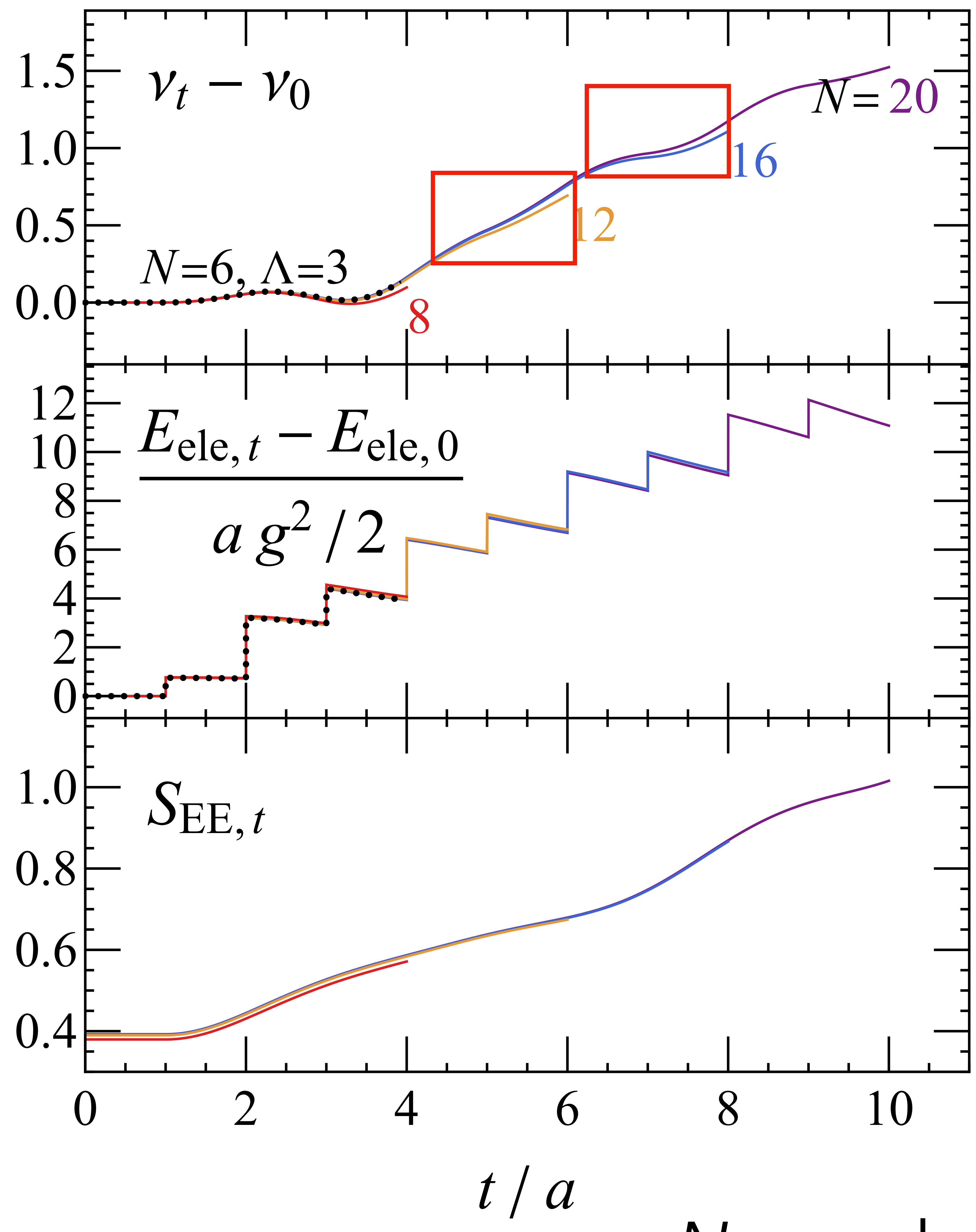
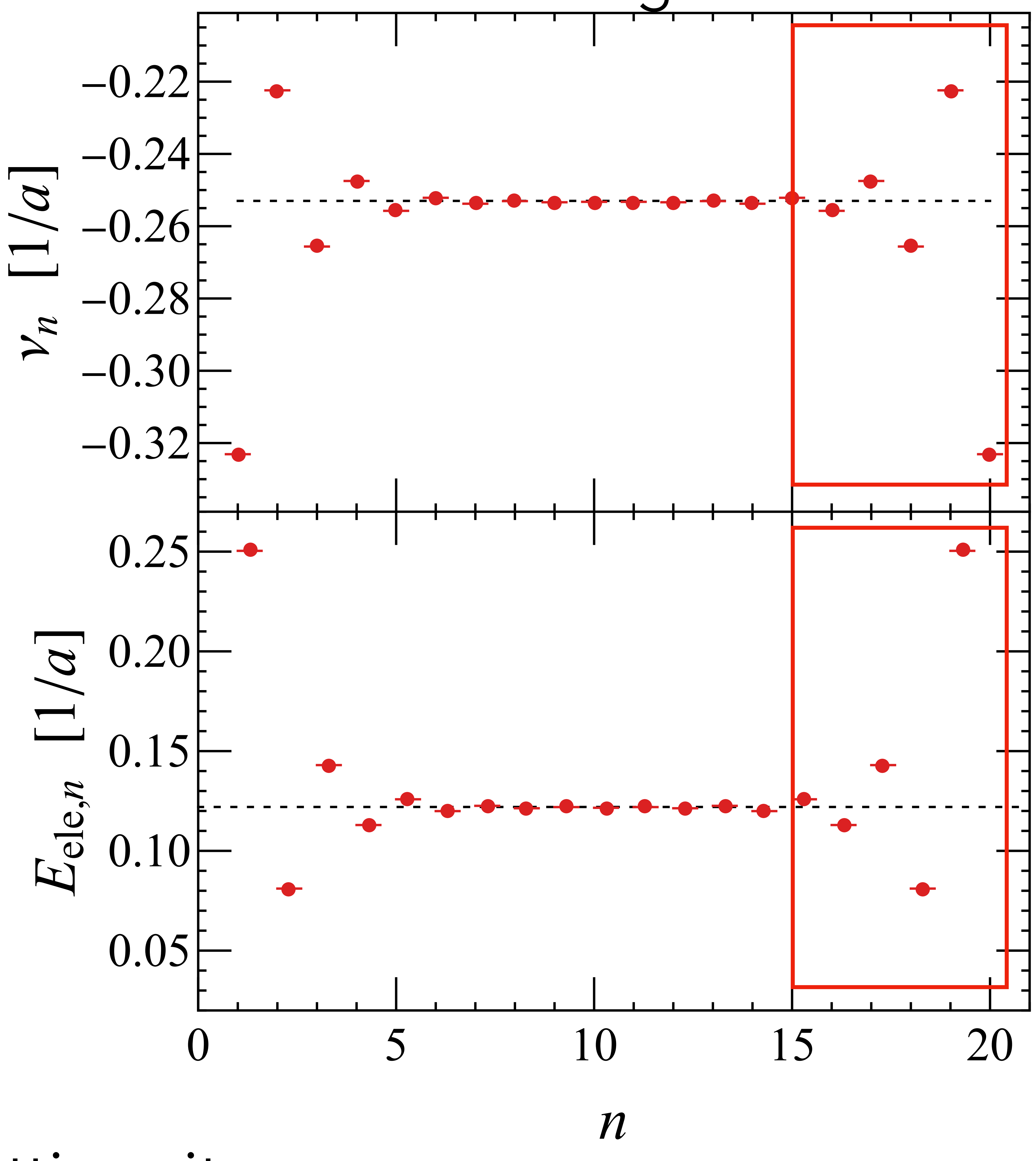
size dependence and boundary effect





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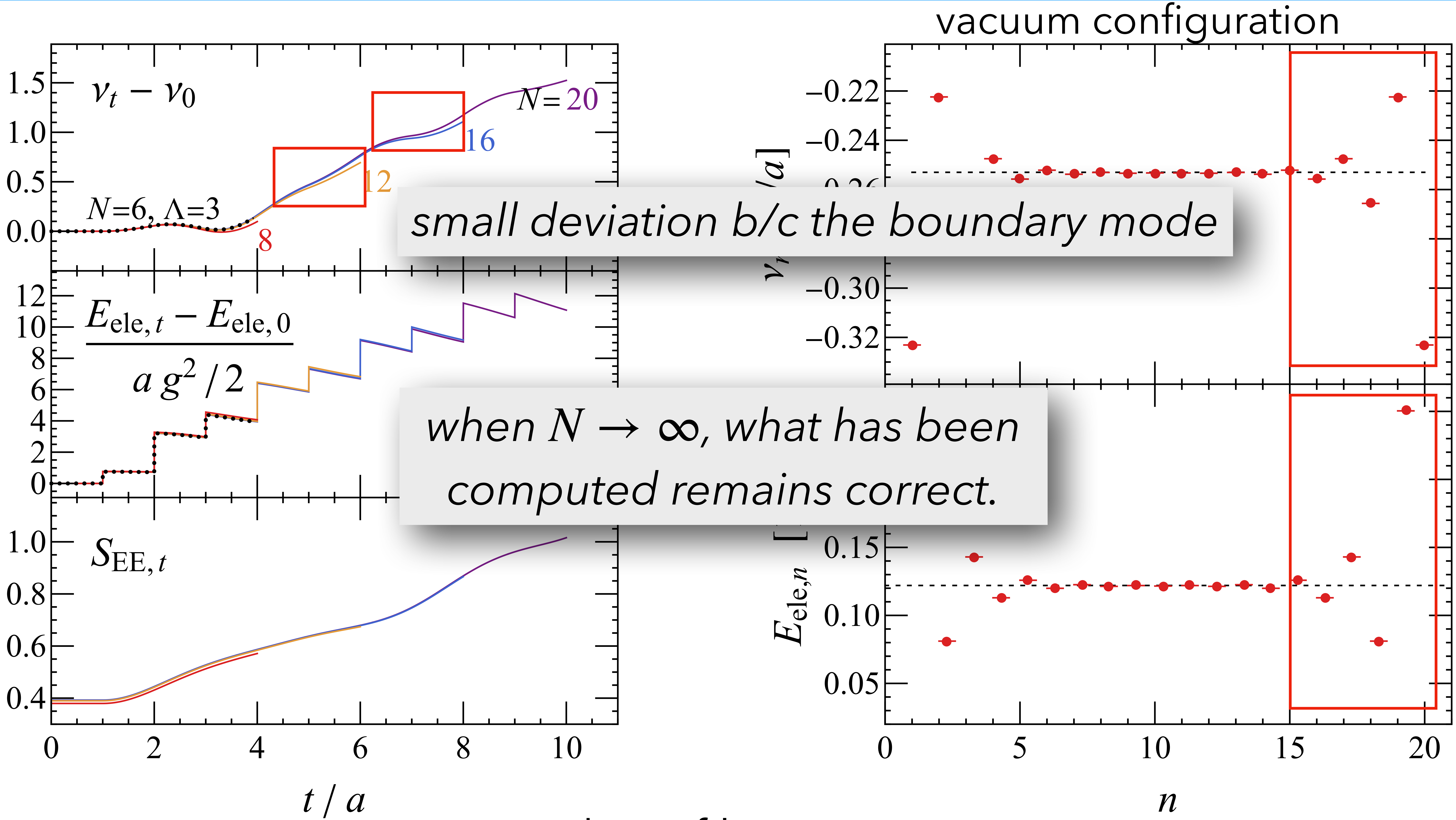
size dependence and boundary effect



small deviation b/c the boundary mode

when $N \to \infty$, what has been computed remains correct.

N: number of lattice sites

vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

near light-front:

uncorrelated states reproduce one-point observables

$\Longrightarrow$ *reliable baseline for two-point correlations*