

# Phenomenology of exotic hadron and heavy quark polarization

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*USTC-PNP-Nuclear Physics Mini Workshop*  
**2023.9.16**

## Collaborators:

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X(3872): 2107.00969 Bc: 2111.08490

# Outline

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## 1. Introduction

heavy ion collisions & charmed hadrons

## 2. Heavy quark polarization

## 3. Production of charmed hadrons: $D$ , $X(3872)$ , $B_c$ , $J/\psi$

**D meson spectrum** ( $c - \bar{q}$ )

**$J/\psi$  spectrum** ( $c - \bar{c}$ )

**$B_c$  production** ( $c - \bar{b}$ )

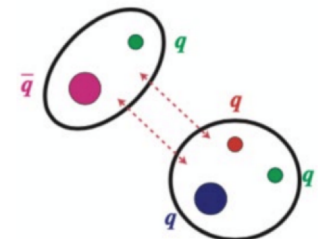
**$X(3872)$  as a tetraquark:**  $c + \bar{c} + q + \bar{q} \rightarrow X(3872)$  **in QGP**

**as a meson molecule:**  $c + \bar{q} \rightarrow D$ ,  $D^0 + \bar{D}^{*0} \rightarrow X(3872)$  **in hadronic gas**

**$T_{cc}$  as a meson molecule**  $D^0 D^{*0}$ ,  $D^0 D^{*+}$ ,  $D^+ D^{*0}$

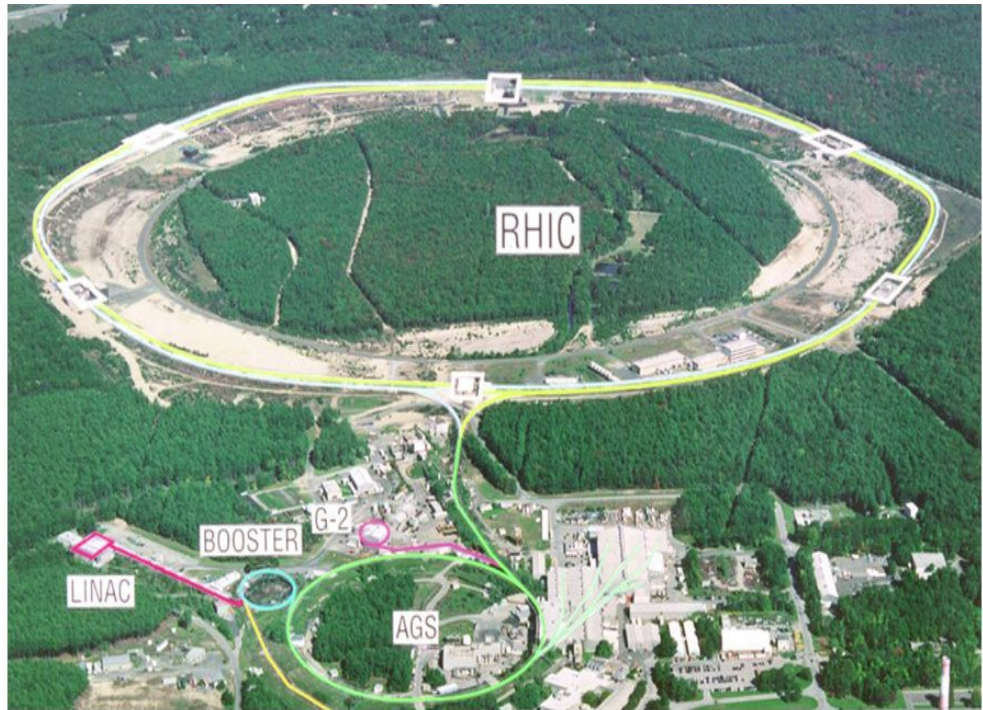
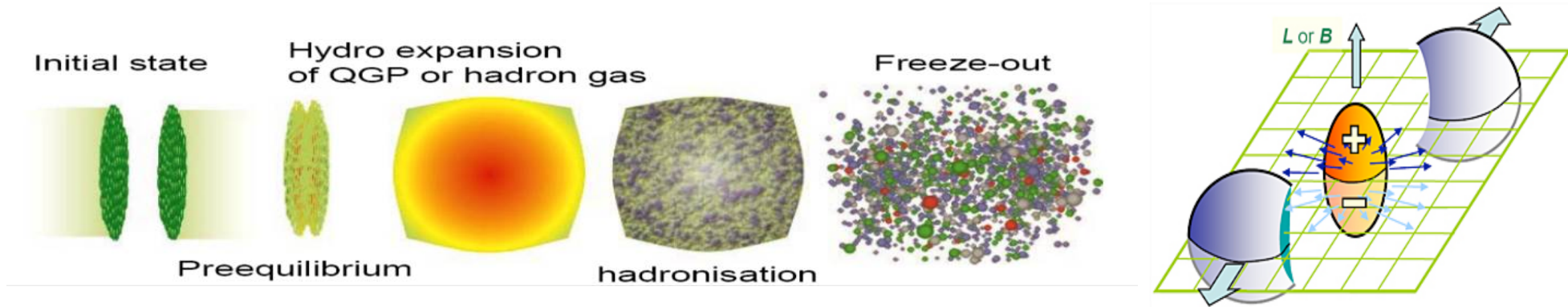


Tetraquark



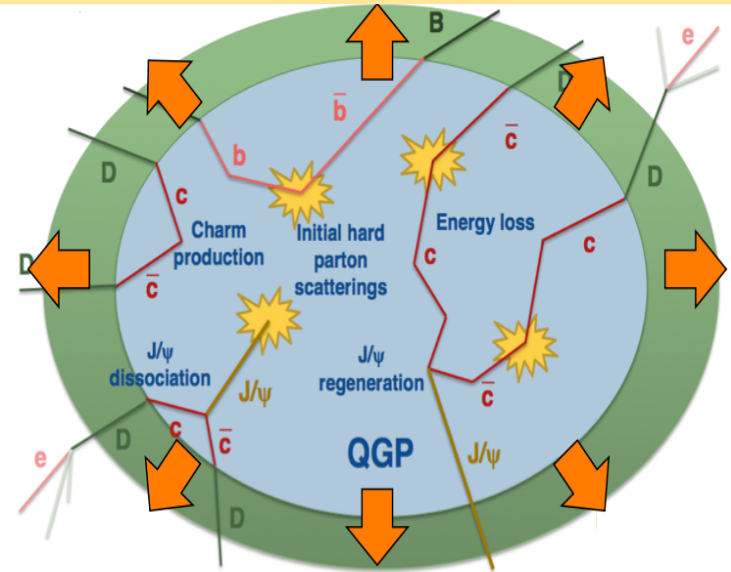
## 4. Summary

# 1. introduction



**RHIC energy:** nucleus velo.  
 $\sim 100 \text{ GeV}$   $v \sim 0.9999c$

## Expansion of Quark-Gluon Plasma



$$c + \bar{c} \leftrightarrow J/\psi + g$$

**Hot medium effects:**  
**screening** + inelastic coll.

# 1. introduction: Properties of charmed mesons

(1) For D mesons, produced at  $T = T_c$

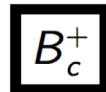
(2) For  $J/\psi$  or bottomonium,

they can be produced inside QGP with  $T > T_c$  due to larger binding energies

	$J/\psi$	$\chi_c$	$\psi'$	$D_s$	$D_s^*$	$D^0$	$D^{*0}$
$V = F$	1.42	-	-	1.14	1.10	1.10	1.08
$V = U$	3.09	1.30	1.24	2.50	1.98	2.35	1.80

Tsinghua Group, *Chin.Phys.C* 44 (2020) 8, 084101

(3) For  $B_c$

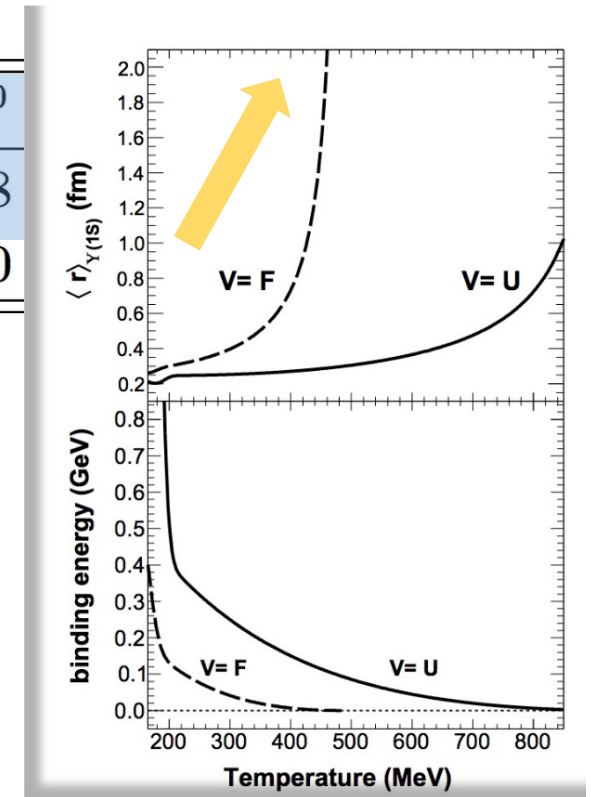


$I(J^P) = 0(0^-)$   
 $I, J, P$  need confirmation.

Quantum numbers shown are quark-model predictions.

States of $B_c$	1S	1P	2S
$T_d/T_c (V = U)$	3.27	1.59	1.41
$T_d/T_c (V = F)$	1.51	-	-

Liu, Carsten, et al, *Phys.Rev.C* 87 (2013) 1, 014910



BYC, Zhao,

*Phys.Lett.B* 772 (2017) 819-824

(4) For X(3872)

tightly bound tetraquark/charmonium-like(2P) states ? Molecular states?

## 2. Heavy quark dynamical evolution

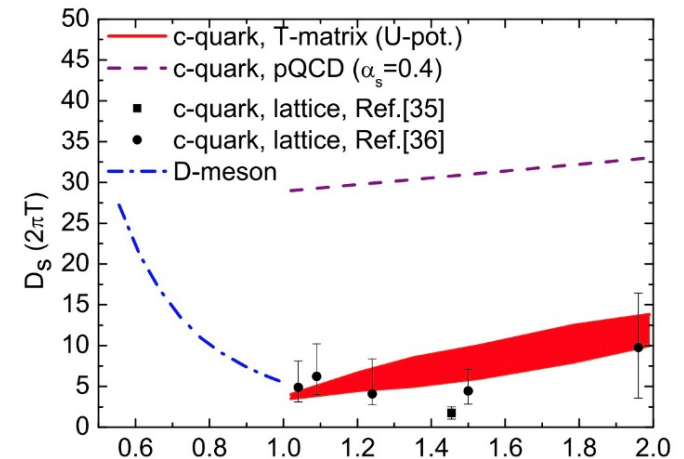
$$\frac{d\vec{p}}{dt} = -\eta\vec{p} + \vec{\xi} + f_g$$

$$\eta = \kappa/(2TE) \quad \kappa D_s = 2T^2$$

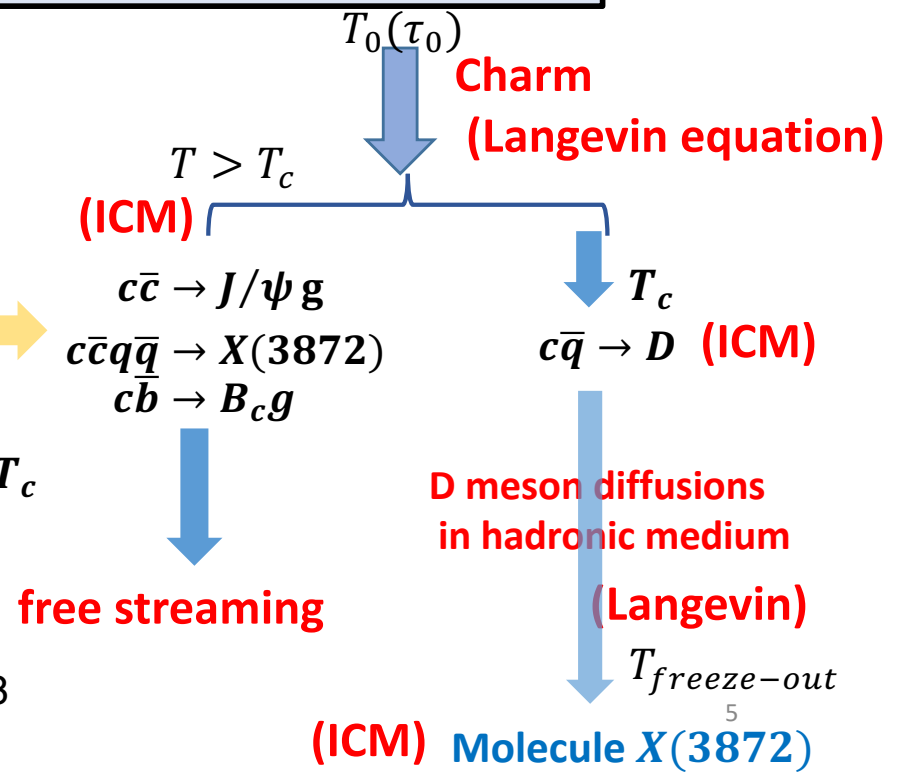
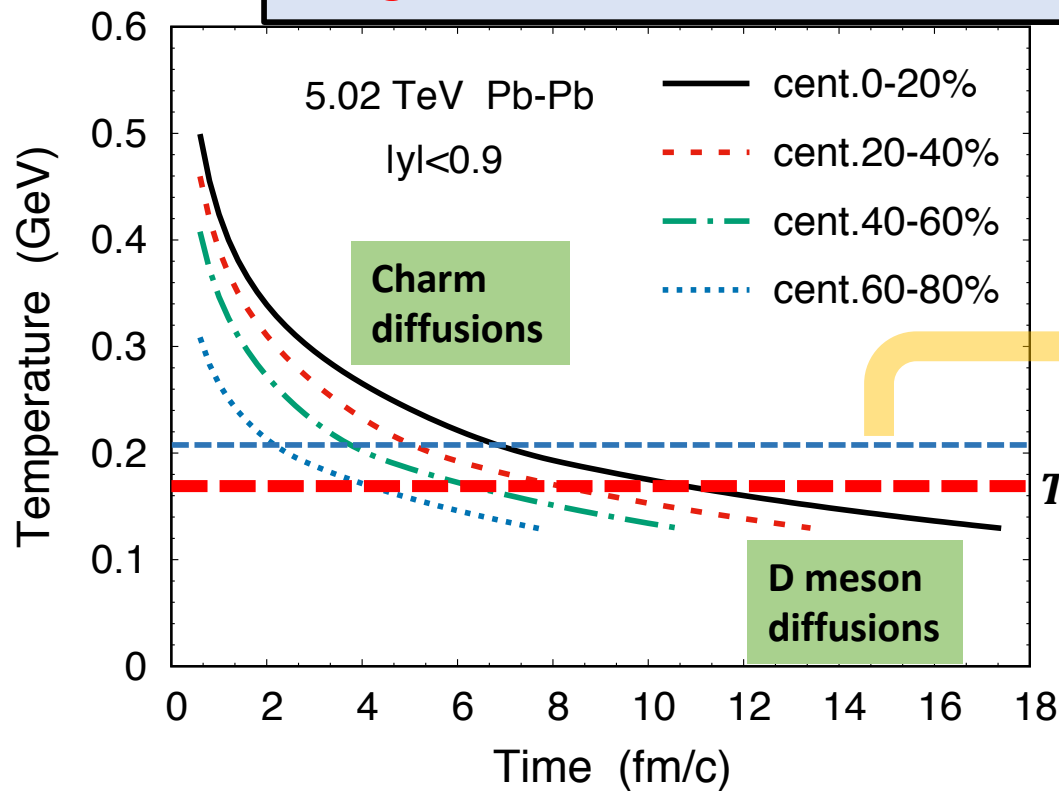
$$D_s(2\pi T) = 5$$

$D_s, \kappa$ :

Spatial and Momentum Diffusion coefficients

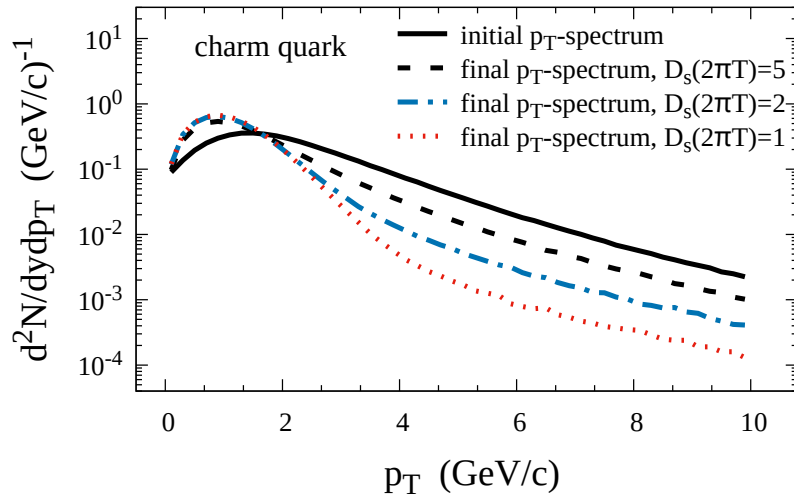


**Langevin + Instantaneous coalescence model (LICM)** [et al PRL 2012](#)

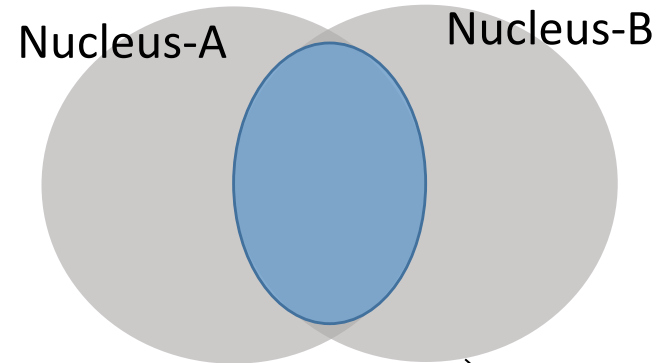


# 2. Heavy quark dynamical evolution

## (1) initial distribution



charm initial spectrum: FONLL model



$$\frac{dN^{test}}{d\vec{x}_T} \propto T_A(\vec{x}_T - \frac{\vec{b}}{2}) T_B(\vec{x}_T + \frac{\vec{b}}{2})$$

Charm initial positions:

Proportional to the  $N_{coll}(\vec{x}_T)$ ,

Corrected by shadowing effect (EPS09)

## (2) Charmonium coalescence at the hadronization temperature

$$P_{c+\bar{c} \rightarrow \psi}(\vec{x}_M, \vec{p}_M) = g_M \int d\vec{x}_1 d\vec{x}_2 \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3} \frac{d^2 N_1}{d\vec{x}_1 d\vec{p}_1} \frac{d^2 N_2}{d\vec{x}_2 d\vec{p}_2} f_M^W(\vec{x}_r, \vec{q}_r) \\ \times \delta^{(3)}(\vec{p}_M - \vec{p}_1 - \vec{p}_2) \delta^{(3)}(\vec{x}_M - \frac{\vec{x}_1 + \vec{x}_2}{2})$$

➤  $g_M = 1/12$  Vector meson degeneracy factor from color and spin

➤  $f_M^W(\vec{x}_r, \vec{q}_r)$ : Wigner function.  $(\vec{x}_r, \vec{q}_r)$  in the center of mass frame of  $c - \bar{c}$

### 3. charmed hadron production

**Wigner function: encodes the information of formed states**

$$f_{J/\psi}^W(\vec{x}_r, \vec{q}_r) = 8 \exp\left[-\frac{x_r^2}{\sigma^2} - \sigma^2 q_r^2\right]$$

$$W(\mathbf{r}, \mathbf{p}) = \int d^3\mathbf{y} e^{-i\mathbf{p}\cdot\mathbf{y}} \psi\left(\mathbf{r} + \frac{\mathbf{y}}{2}\right) \psi^*\left(\mathbf{r} - \frac{\mathbf{y}}{2}\right)$$

$$\sigma^2 = \frac{4(m_1 + m_2)^2}{3(m_1^2 + m_2^2)} \langle r^2 \rangle_M$$

$$\sqrt{\langle r^2 \rangle_{J/\psi}} = 0.54 \text{ fm}$$

Give **consistent formation conditions** on the relative distance and relative momentum of two particles.

**The width  $\sigma$  in the Wigner function**

is connected with the internal structure of the formed state

### Hadron Spectrum in heavy-ion collisions

$$\frac{d^2 N_\psi}{dy_M d\vec{p}_T} = \int d\vec{x}_M \frac{dp_z}{2\pi} \langle P_{c+\bar{c} \rightarrow \psi}(\vec{x}_M, \vec{p}_M) \rangle_{events} \times \frac{(\Delta N_{c\bar{c}}^{AA})^2}{\Delta y_M}$$

$$\Delta N_{c\bar{c}}^{AA} = \int d\vec{x}_T T_A(\vec{x}_T - \frac{\vec{b}}{2}) T_B(\vec{x}_T + \frac{\vec{b}}{2}) \frac{d\sigma_{pp}^{c\bar{c}}}{dy} R_S(\vec{b}, \vec{x}_T) \Delta y_{c\bar{c}}$$

Shadowing factor

# 3. Heavy quark polarization in magnetic field

- Landau-Lifshitz-Gilbert (LLG) equation

arXiv:1805.01776 (2018)

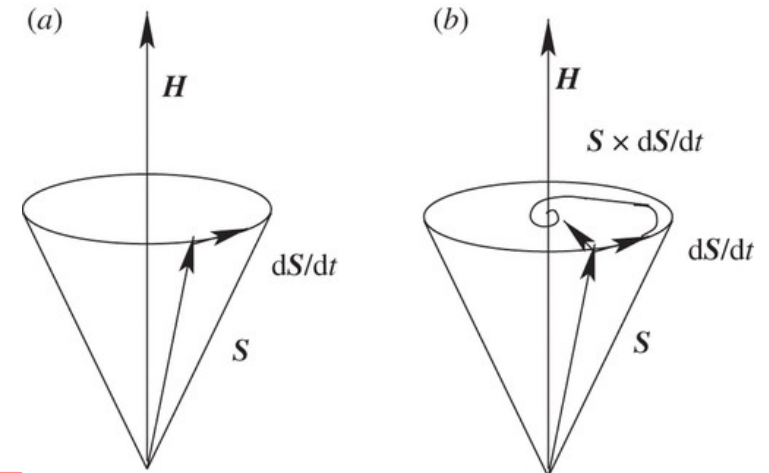
$$\frac{d\vec{S}}{dt} = -\frac{\gamma}{1 + \alpha^2} [\vec{S} \times (\vec{H} + \vec{H}_{th})] - \frac{\alpha\gamma}{1 + \alpha^2} \vec{S} \times [\vec{S} \times (\vec{H} + \vec{H}_{th})]$$

$$\vec{S} = \vec{s}/|\vec{s}|$$

Unit vector

stochastic dynamics of a spin in the medium with magnetic field

Polarization of heavy quark is induced by:  
 spin-magnetic field interaction  
 + particle-particle interaction



$$\gamma = \frac{g|Q|}{2m_Q}$$

Electric charge  
 gyromagnetic ratio

$\alpha = 0.1$  Damping factor  
 (to be determined later)

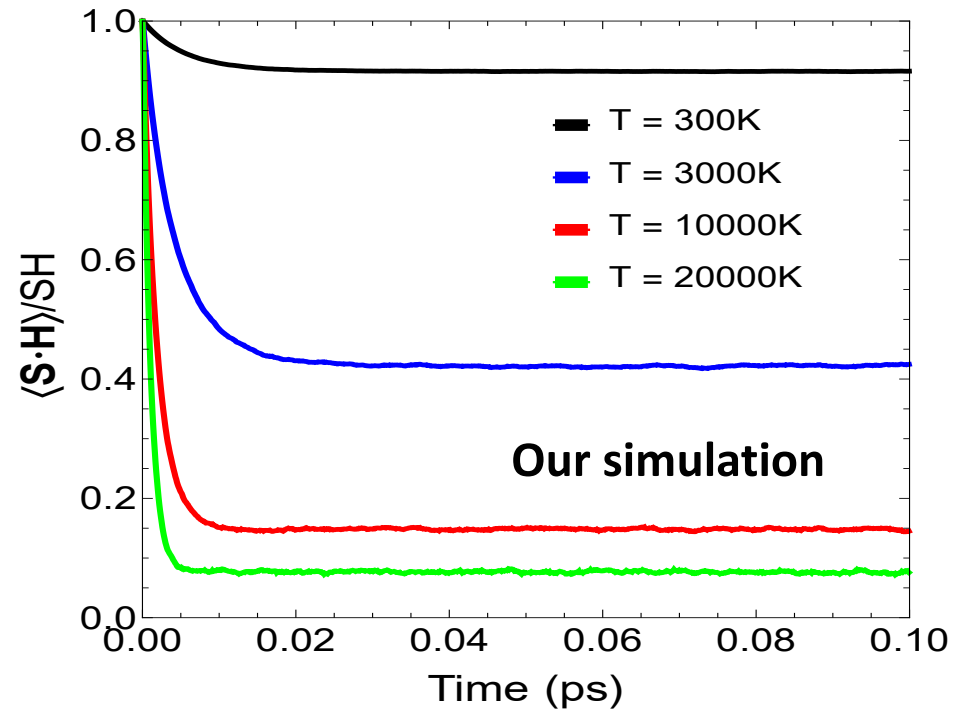
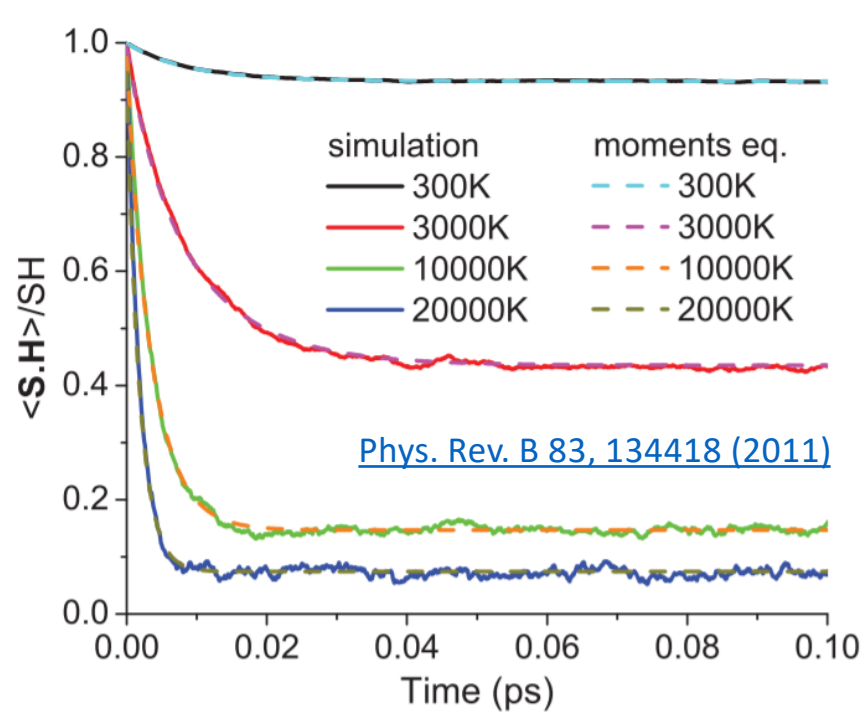
noise term

$$\langle \vec{H}_{th}(t) \rangle = 0$$

$$\langle H_{th,i}(t) H_{th,j}(t') \rangle = \frac{4\alpha T}{\gamma^2} \delta_{i,j} \delta(t - t')$$



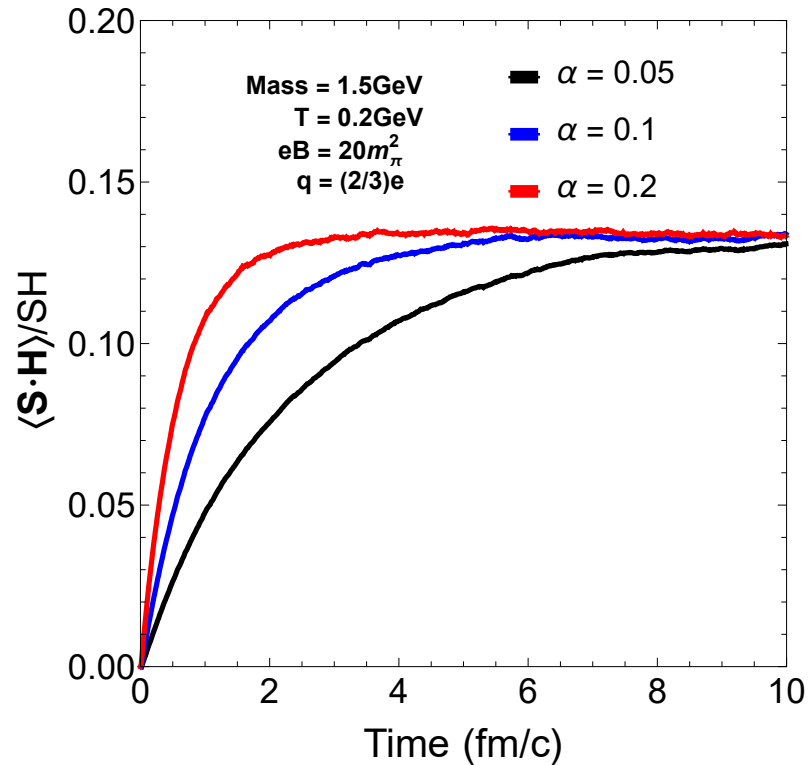
# 3. Heavy quark polarization in magnetic field



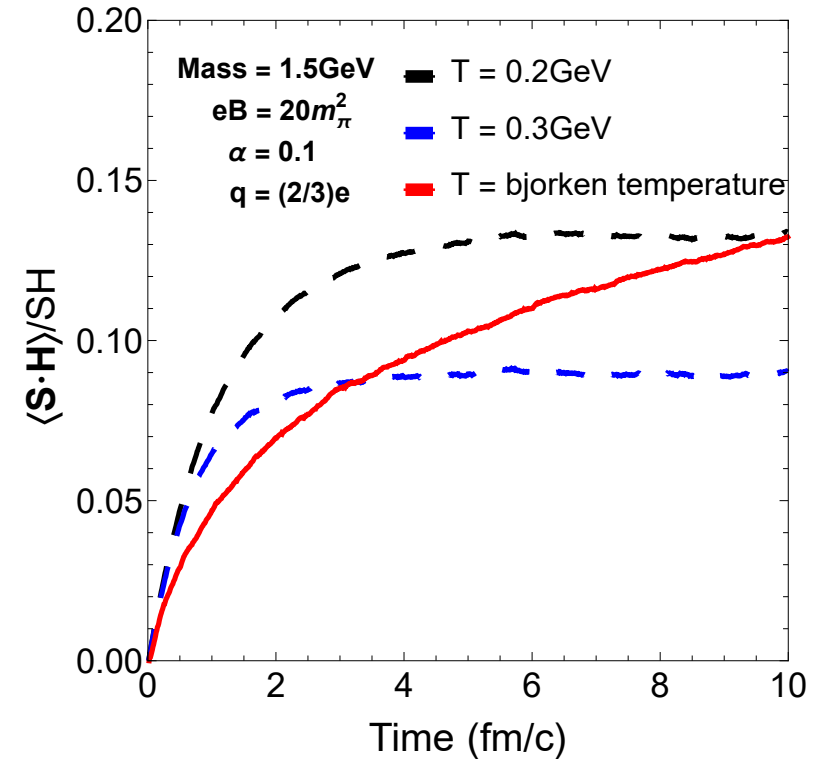
Compare with the reference (PRB 83, 134418, 2011)

# 3. Heavy quark polarization in magnetic field

## Polarization rate:



## T dependence:

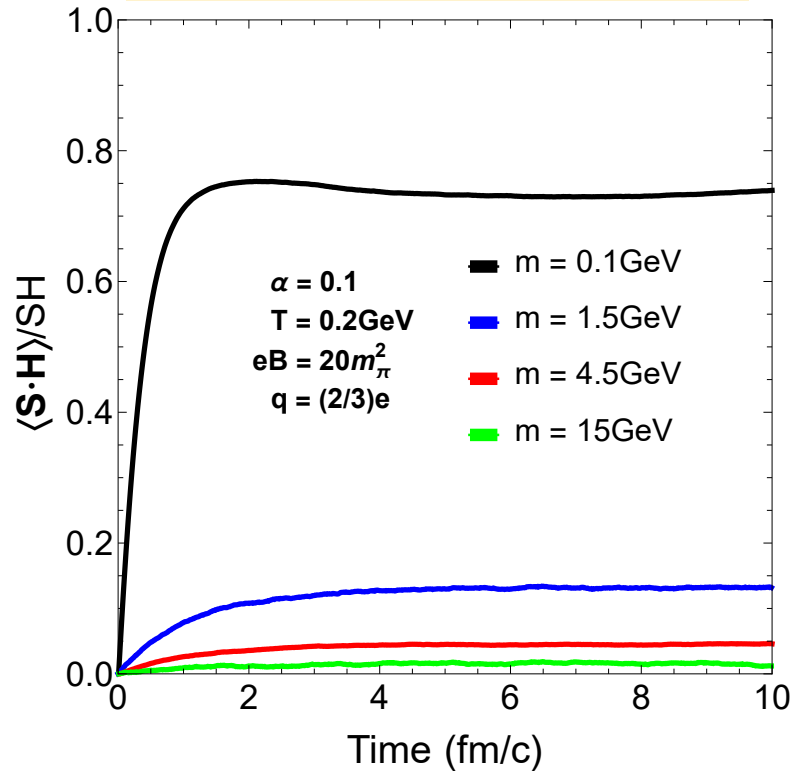


Zhiwei, Anping, Baoyi, in progress

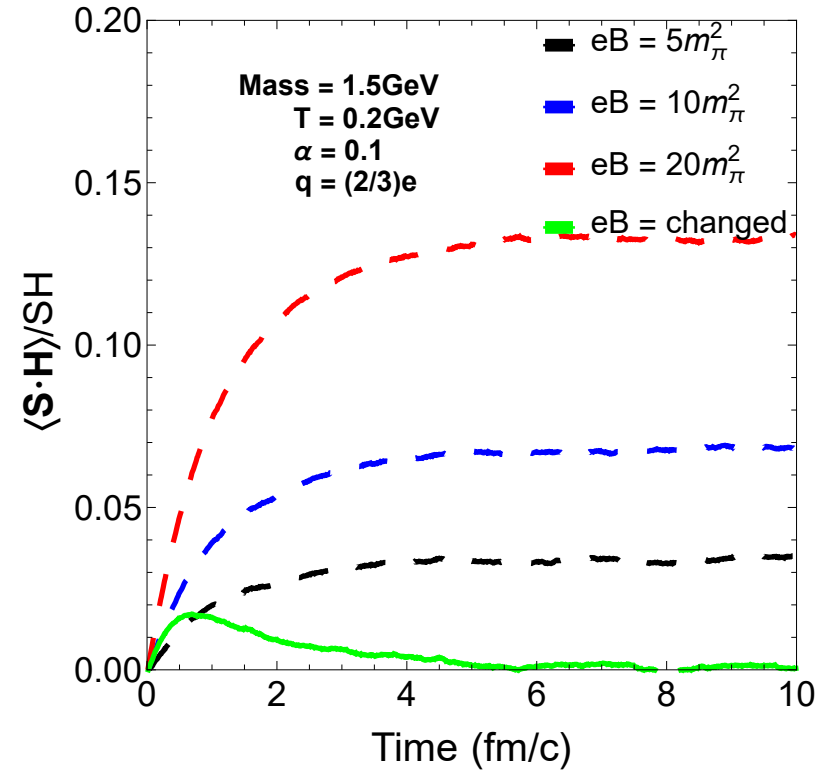
- Damping factor alpha do not affect the equilibrium value, only affect the rate.
- Higher temperature → smaller polarization

# 3. Heavy quark polarization in magnetic field

## Mass dependence:



## B dependence:



Zhiwei, Anping, Baoyi, in progress

$$eB = \frac{eB_0}{1+(t/t_B)^2}, \text{ where } t_B = 0.6 \text{ fm/c}, B_0 = \frac{2\gamma Ze}{\pi b^2}$$

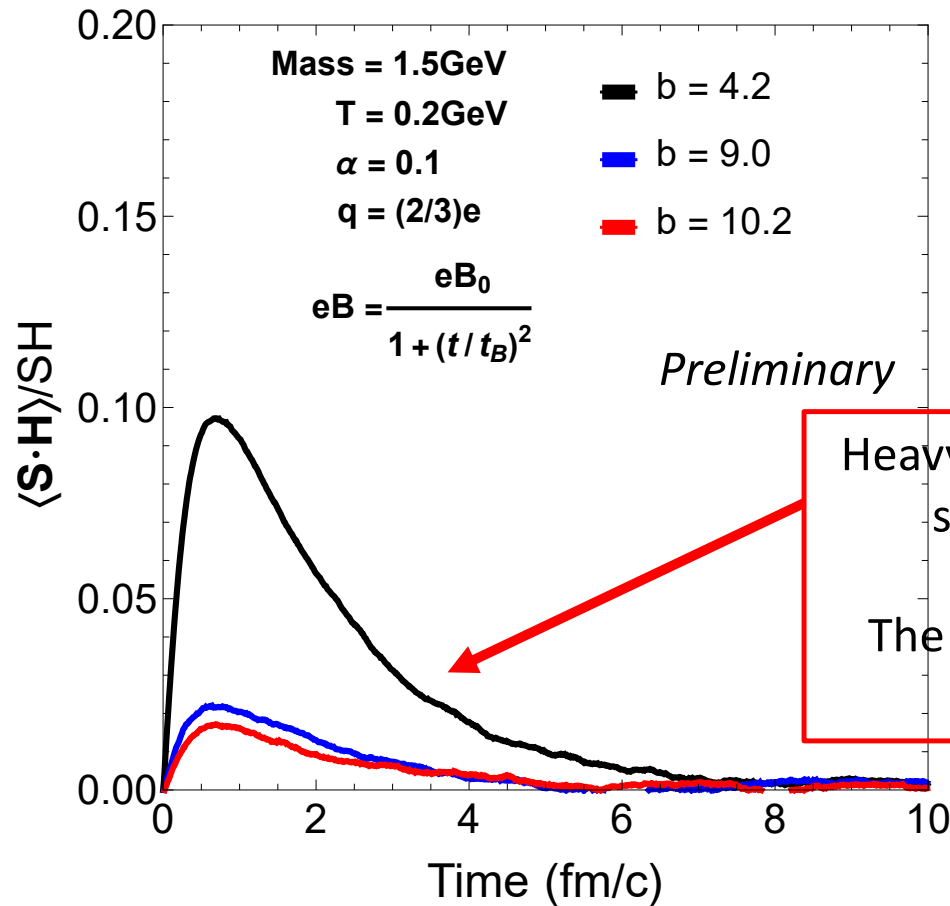
- Larger Quark mass → smaller polarization
- Higher B field → larger polarization

PRB Volume 737, 7 October 2014, Pages 262-266

# 3. Heavy quark polarization in magnetic field

In the changing systems:

$$B_0 = \frac{2\gamma Ze}{\pi b^2}$$



Further improvement:  
 LLG + Spin-coalescence-model

Heavy quarks may hadronized in this time scale (where LLG equation stops)  
 The polarization of HQ may be partially preserved in charmed hadron.

Magnetic field induced polarization & thermal particle random collisions

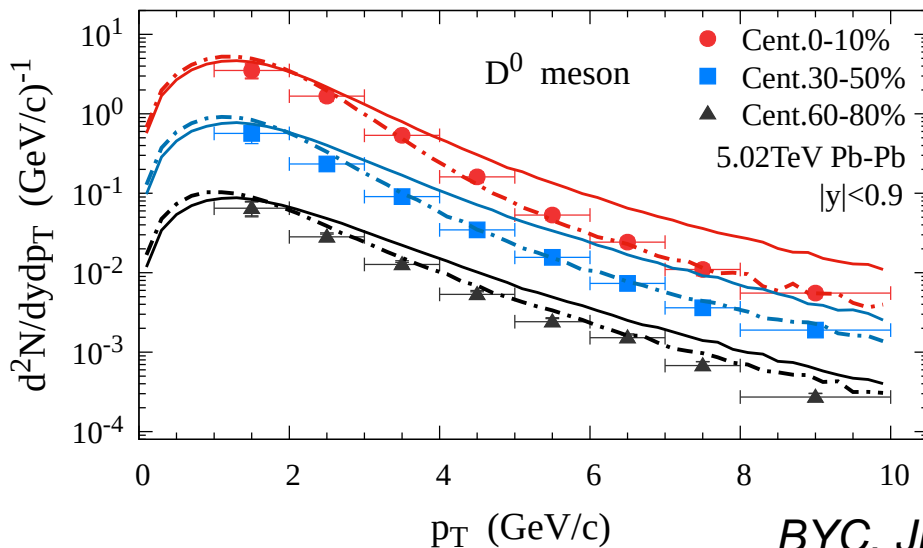
# 4. charmed hadron production: D meson

## ● D meson coalescence

$$P_{c\bar{q}\rightarrow D^0}(\vec{p}_M) = H_{c\rightarrow D^0} \int \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3} \frac{dN_1}{d\vec{p}_1} \frac{dN_2}{d\vec{p}_2} f_D^W(\vec{q}_r) \times \delta^{(3)}(\vec{p}_M - \vec{p}_1 - \vec{p}_2)$$

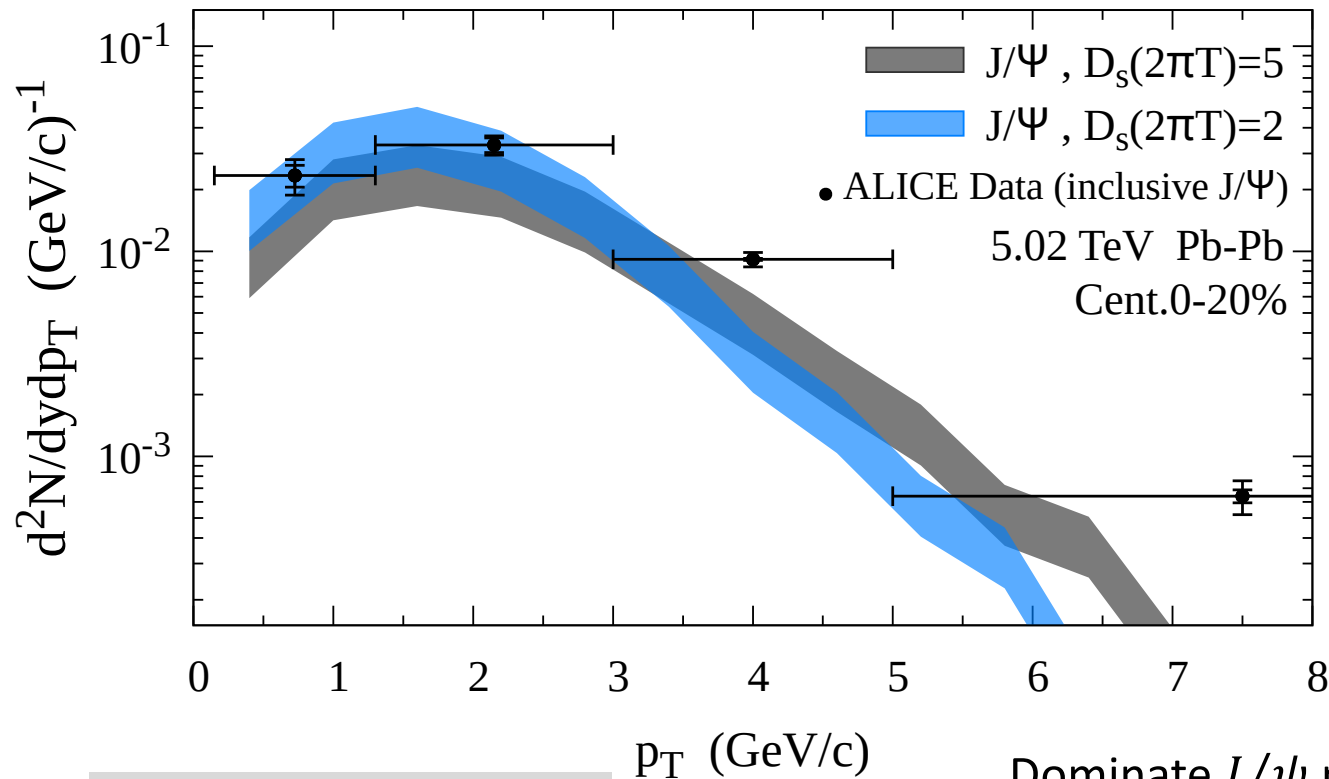
$$\frac{d^2 N_D}{dy_M d\vec{p}_T} = \int \frac{dp_z}{2\pi} \langle P_{c\bar{q}\rightarrow D^0}(\vec{p}_M) \rangle_{events} \times \frac{\Delta N_{c\bar{c}}^{AA}}{\Delta y_M}$$

- $H_{c\rightarrow D^0} = 9.5\%$  (20%): Charm turning into **direct**  $D^0$  ( $D^{*0}$ ) at Tc
- $\frac{dN_1}{d\vec{p}_1}$ : **charm** momentum distribution
- $\frac{dN_2}{d\vec{p}_2}$ : **light quark** momentum distribution: Fermi.



- We take the ratio of prompt  $D^0$  over charm:  
 $N(D^0)/N_{c\bar{c}} = 39\%$   
**ALICE pp, arXiv:2105.06335**
- Different thermalization:  $D_s(2\pi T) = 5$  (solid line) and  $D_s(2\pi T) = 2$  (dotted-dashed line)

# 4. charmed hadron production: $J/\psi$



**Theoretical bands:**  
With/without  
the shadowing effect.

**Experimental data:**

inclusive production = primordial + B-decay +  $c - \bar{c}$  coalescence

Dominate  $J/\psi$  production at high  $p_T$

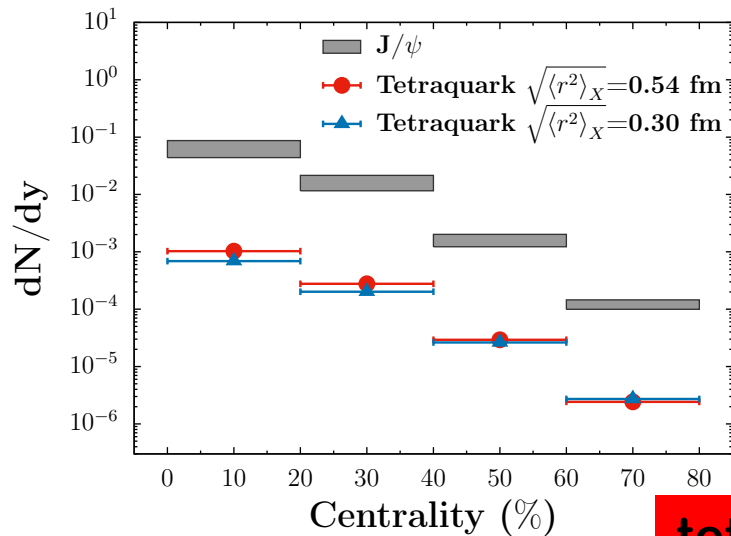
Dominate at low  $p_T$  and **total yield**

**Theoretical calculation:**

$c - \bar{c}$  coalescence

# 4. charmed hadron production: X(3872)

- $g_{X(3872)} = 1/432$  with X(3872) spin  $J=1$
- Root-mean-square radius of tetraquark:  $\langle r^2 \rangle_X = 0.30 - 0.54 \text{ fm}^2$
- diquark ( $cq$ ) is formed firstly, then two diquarks form a tetraquark state.



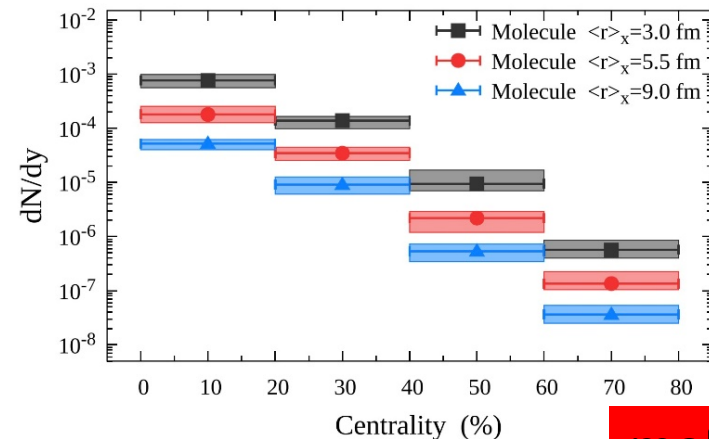
**tetraquark**

- Tetraquark yield is around **40 times** smaller than  $J/\psi$
- Tetraquark yield is controlled by both **spatial** and **momentum** part of the Wigner function

## Molecule state with potential model

$$V_{mole} = V_{\pi} + V_{\omega} + V_{\eta} + V_{\rho}$$

$\Lambda$	0.55	0.555	0.56	0.565	0.57	0.575	0.579
BE. (keV)	1600.3	1098.5	698.4	394.4	180.6	51.2	3.3
$\langle r \rangle$ (fm)	2.47	2.85	3.41	4.31	6.01	10.52	22.60
$\sqrt{\langle r^2 \rangle}$ (fm)	3.08	3.59	4.36	5.61	8.00	14.33	28.94

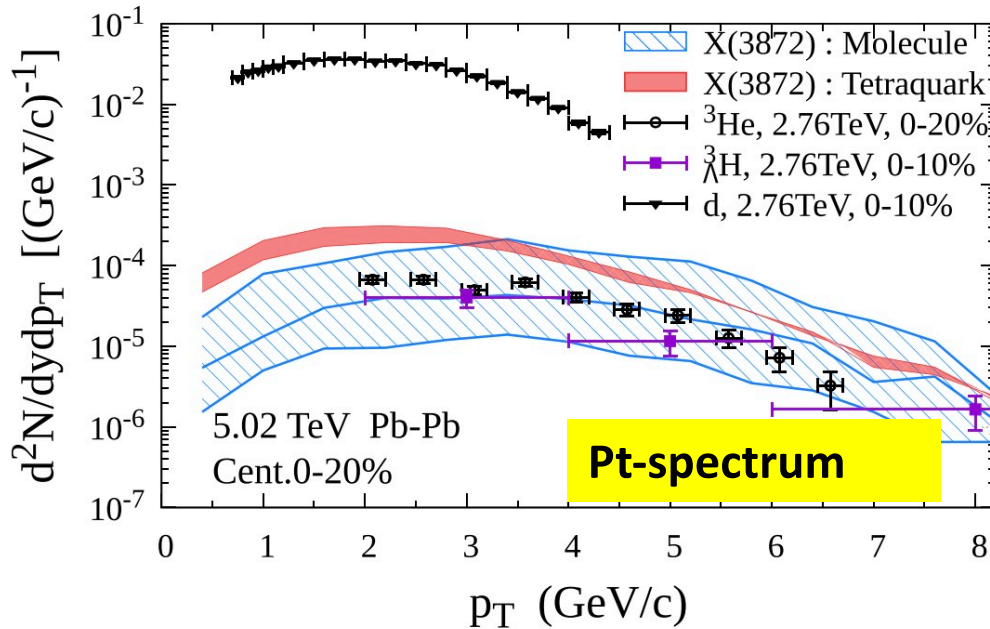
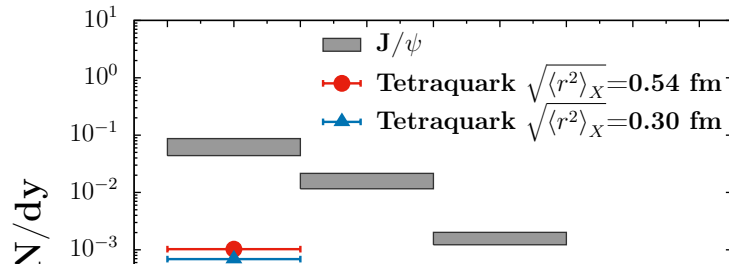


**molecule**

Bands:  
Volume dependence in freeze-out

# 4. charmed hadron production: X(3872)

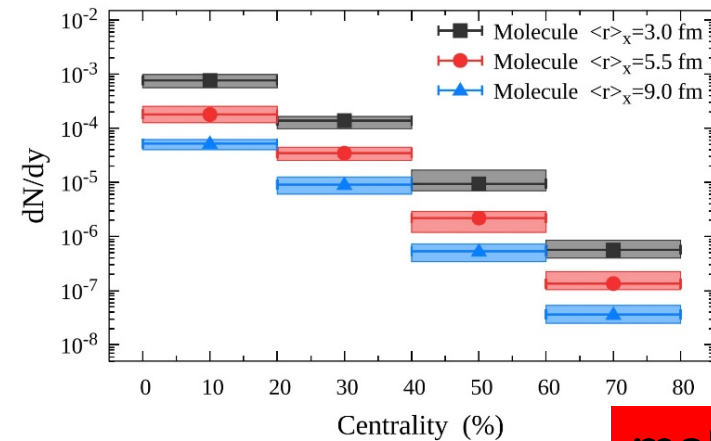
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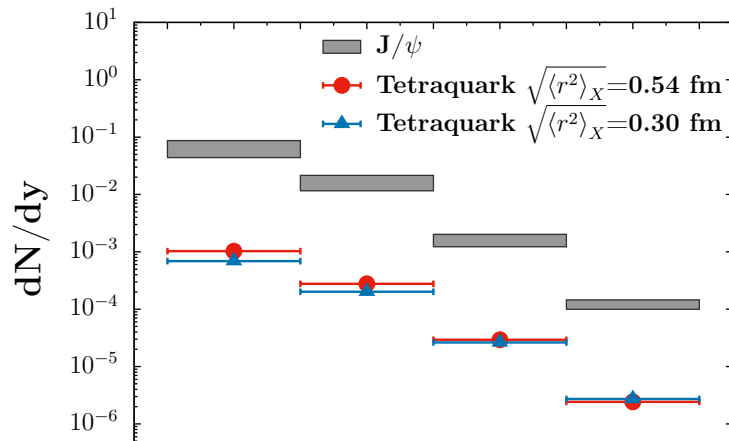
Bands:

**Volume dependence in freeze-out**



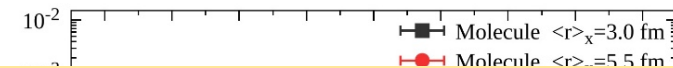
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● Our tetraquark yield  $\sim 10^{-3}$  is consistent with Cho. Prog.Part.Nucl.Phys. 95,279-322 (2017); when taking same coalescence temperature

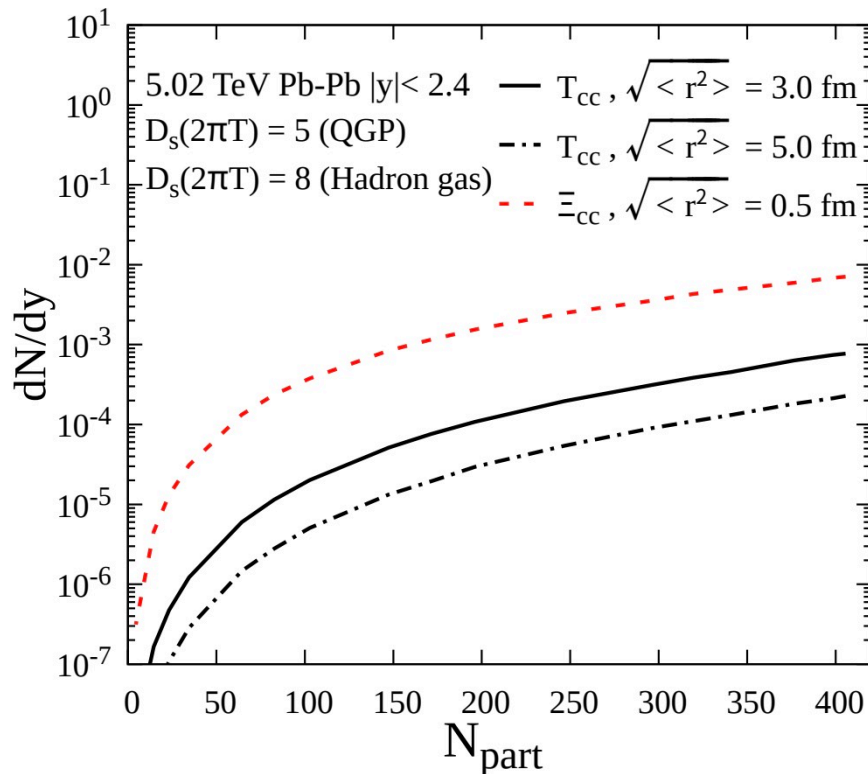
● **Relations** between tetraquark and molecule production: ours is consistent with rate equation model Rapp EPJA 57, 122 (2021);

different from AMPT model: Zhang PRL 126, 012301 (2021);

→ maybe due to its different formation conditions.  $\left\{ \begin{array}{l} 5\text{fm} < \text{relative distance} < 7\text{fm} \\ 2M_D < \text{pair mass} < 2M_{D^*} \end{array} \right.$

## 4. Mesonic molecule production: $T_{cc}$

- $T_{cc}$  includes  $T_{cc}^0, T_{cc}^+, T_{cc}^{++}$      $D^0 - D^{*0}, D^{0/+} - D^{*+ /0}, D^+ - D^{*+}$
- Wigner function is determined by the  $\langle r^2 \rangle_T$



in preparation...

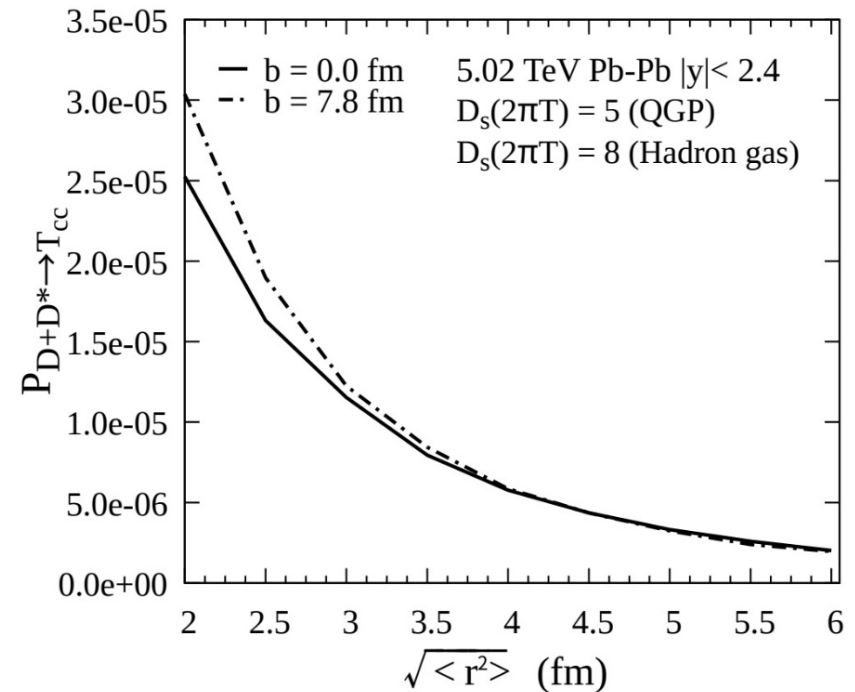
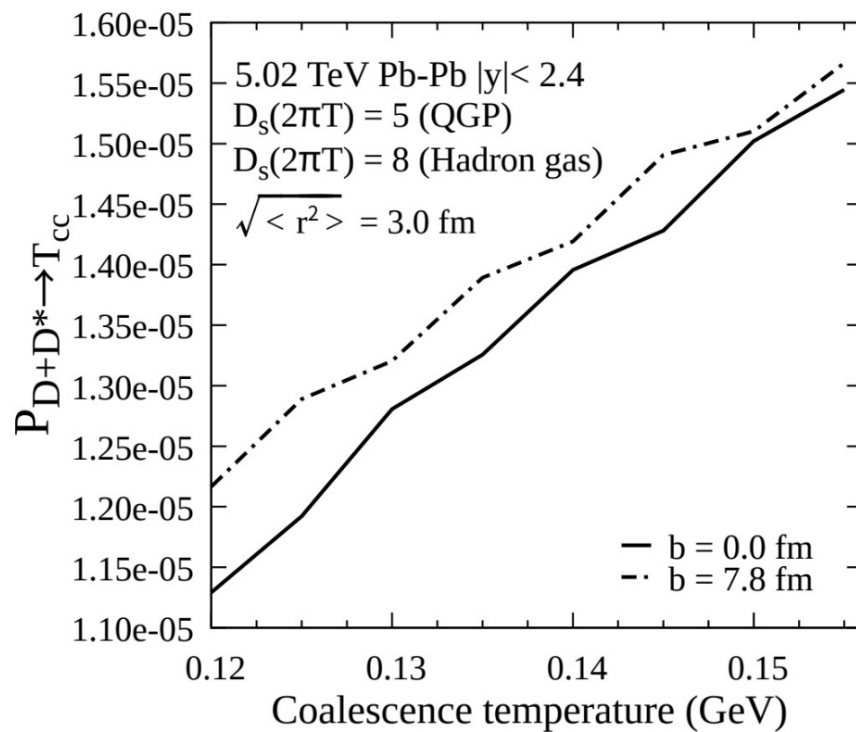
- $c \rightarrow (D^0, D^{*0})$
- Shadow factor = 0.65
- Molecules are produced at  $T=0.12$  GeV

Three states are included in the results.

- $N_{cc}$  large:  $N_{cc}(\mathbf{b} = \mathbf{0}) \sim 35$  per rapidity at 5.02 TeV Pb-Pb,  $(N_{cc\bar{c}})^2$  &  $N_{cc\bar{c}}(N_{cc\bar{c}}-1)/2$
- $N_{cc}$  small:  $T_{cc}$  is more suppressed than X(3872), like at peripheral collisions.

## 4. Mesonic molecule production: $T_{cc}$

- $T_{cc}$ :  $D^0 - D^{*0}$ ,  $D^0 - D^{*+}$ ,  $D^+ - D^{*0}$
- Wigner function is determined by the  $\langle r^2 \rangle_T$



[2309.02987](#)

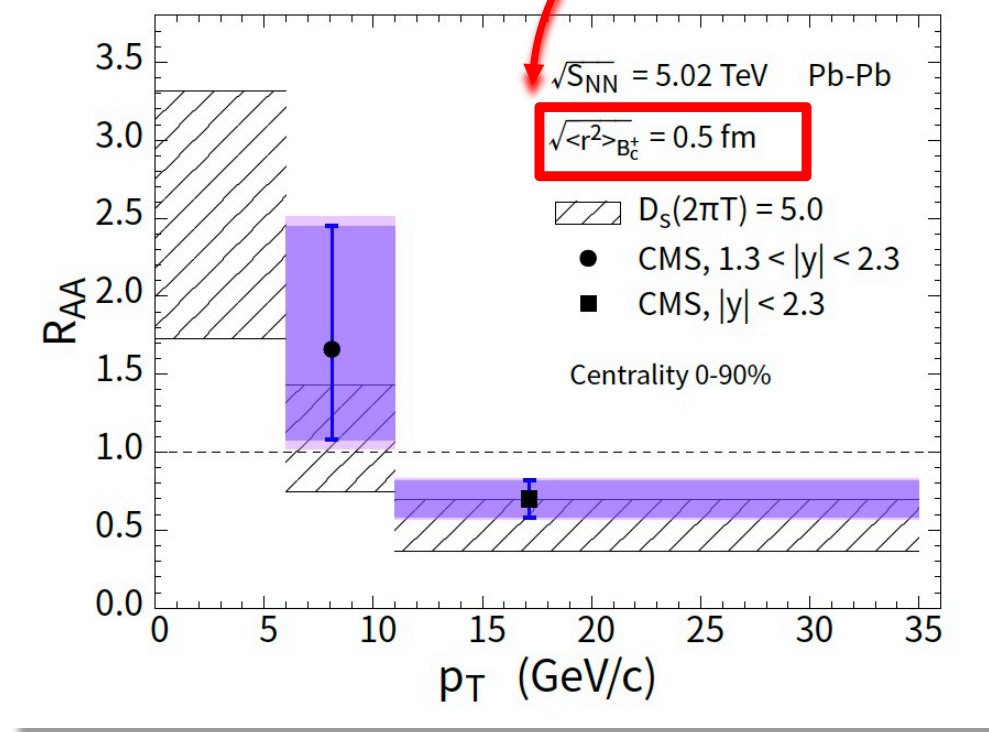
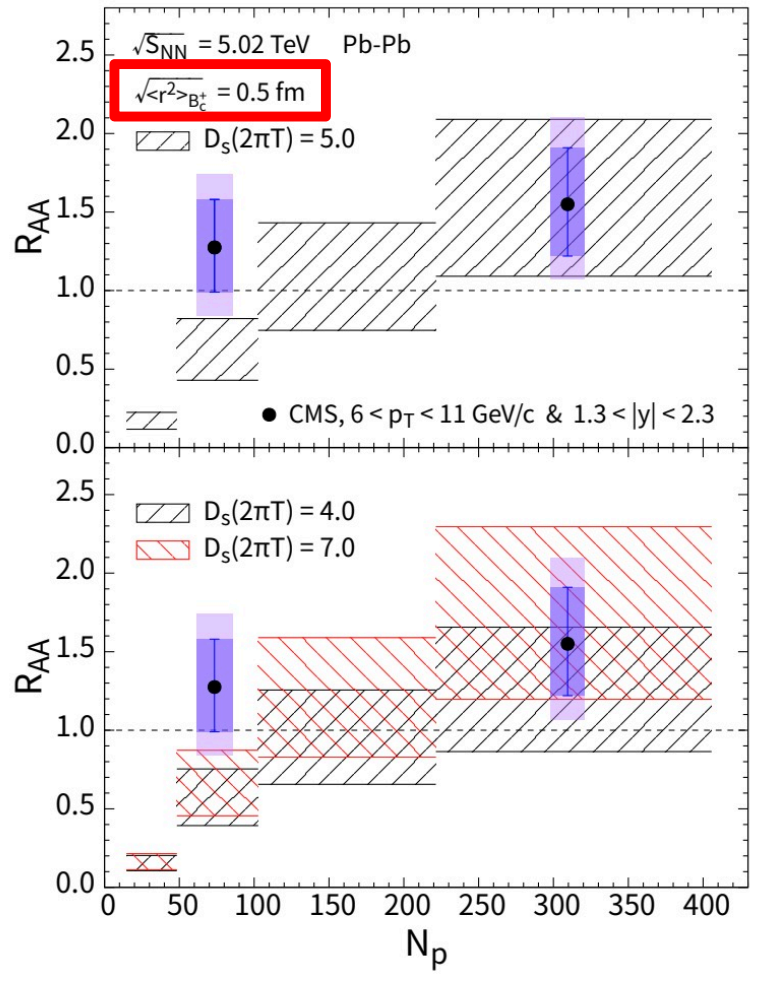
Coalescence probability per one  $D$  and  $D^*$

$$f^W(\vec{x}_r, \vec{q}_r) = 8 \exp\left[-\frac{x_r^2}{\sigma^2} - \sigma^2 q_r^2\right]$$

$$\sigma^2 = \frac{4}{3} \frac{(m_1 + m_2)^2}{m_1^2 + m_2^2} \langle r^2 \rangle_{19M}$$

# 4. charmed hadron production: $B_c$

Geometry size



BYC, Wen, Liu, [arXiv: 2111.08490](https://arxiv.org/abs/2111.08490)

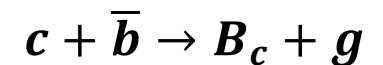
$$\frac{d\sigma_{pp}^{cc}}{dy} = 1.165 \text{ mb}$$

$B_c$ : spin 0  
fig:  $B_c(1s) + B_c(2s \rightarrow 1s)$

$$\frac{d\sigma_{pp}^{bb}}{dy} = 47.5 \mu\text{b}$$

$$\frac{d\sigma_{pp}^{Bc}}{dy} = (151.9 - 79.3) \text{ nb}$$

1)  $B_c$  final production is evidently enhanced, due to a large number of c and b quarks in QGP.



2)  $R_{AA} > 1$  at central collisions:

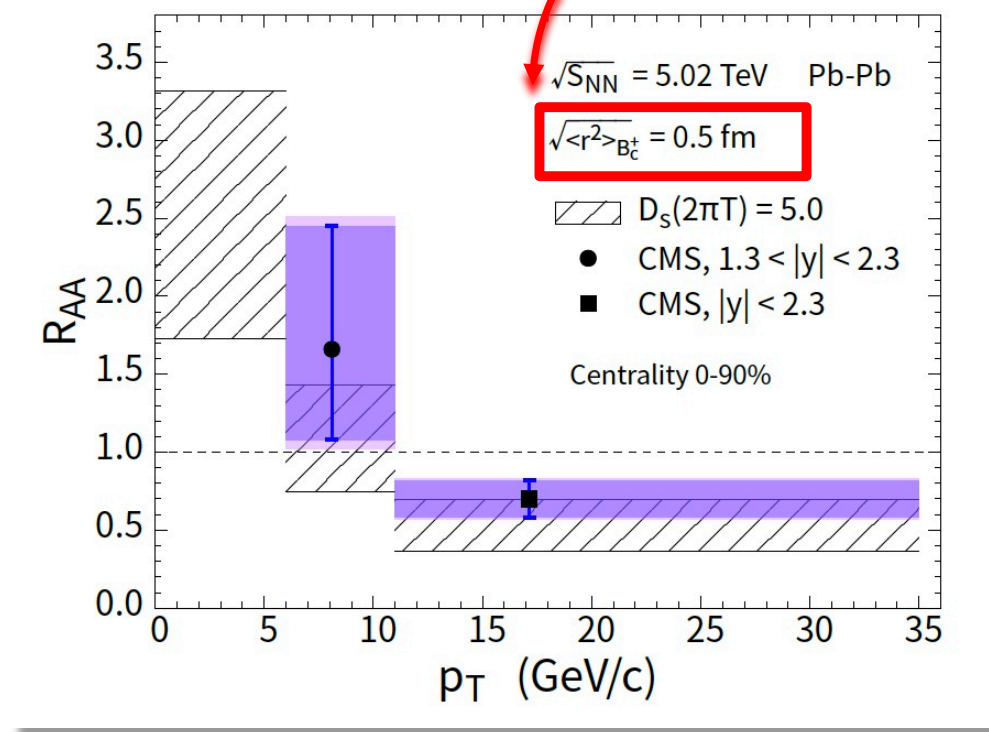
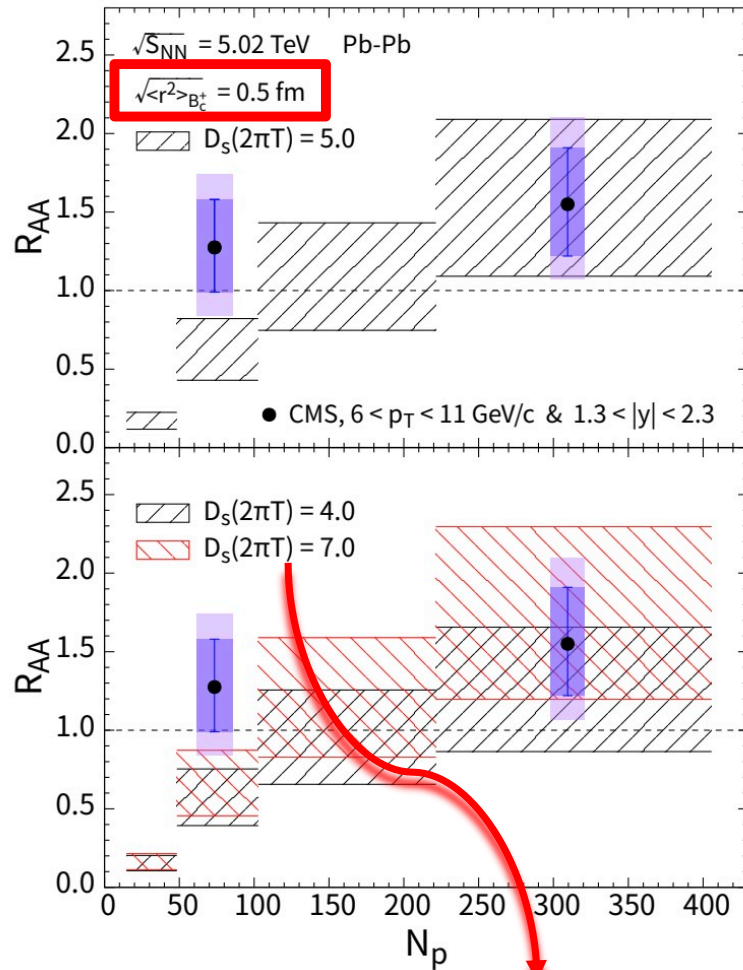
**QGP signal**

$R_{AA} < 1$  at peripheral collisions:

**absence of initial production**

# 4. charmed hadron production: $B_c$

Geometry size



BYC, Wen, Liu, [arXiv: 2111.08490](https://arxiv.org/abs/2111.08490)

1)  $B_c$  final production is evidently enhanced, due to a large number of  $c$  and  $b$  quarks in QGP.

$$\frac{d\sigma_{p1}^c}{dy} + \frac{d\sigma_{p1}^b}{dy}$$

Different thermalization

of charm and bottom quarks on  $B_c$  production,

By taking spatial diffusion coefficient  $D_s(2\pi T) = 4$  and  $7$

+  $g$

## 5. Summary

- We study the heavy quark polarization in magnetic field.
- X(3872) as a **tightly bound tetraquark** and **a hadronic molecule**, is formed via different processes.  
Their production depends on the wave function of X(3872).  
**Therefore, heavy-ion collisions provide a new opportunity to study the nature of X(3872), and  $T_{cc}$**
- **$B_c$  meson is firstly observed in AA collisions,**  
**evident enhancement of  $R_{AA}$ : a very clear signal of QGP**

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Thank you'very much for your  
attention!

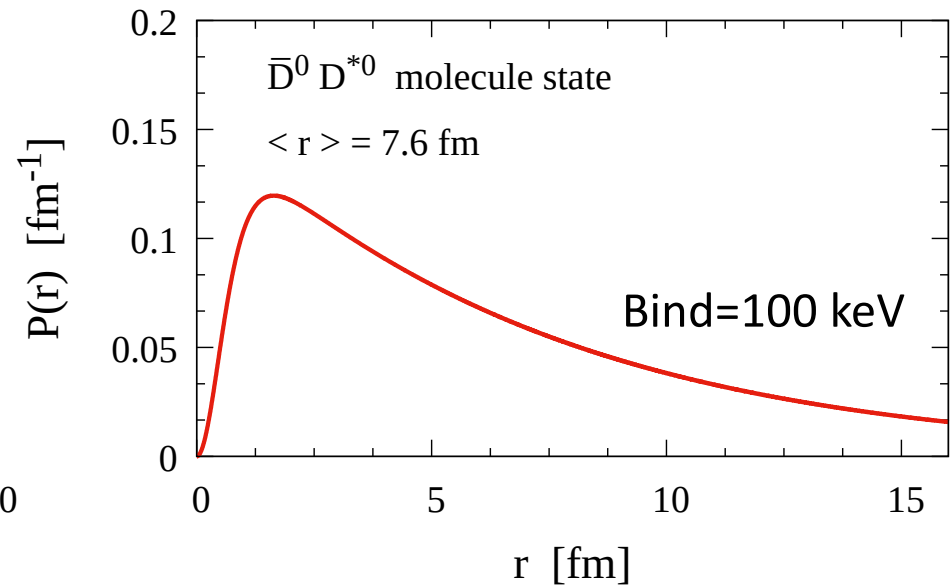
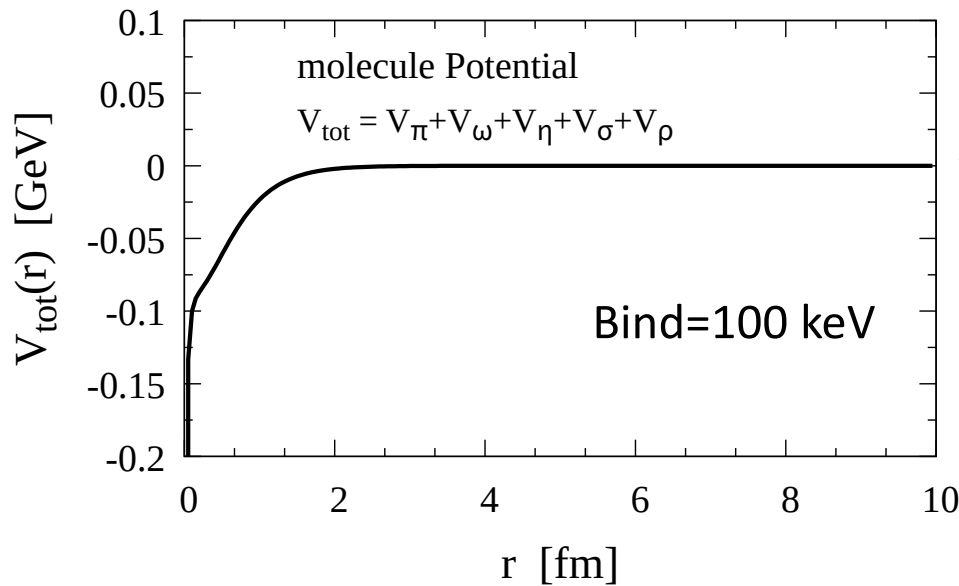
# Binding energy of X(3872)

## ➤ Molecule state based on potential model

$$V_{mole} = V_{\pi} + V_{\omega} + V_{\eta} + V_{\rho}$$

arXiv: 2107.00969

$\Lambda$	0.55	0.555	0.56	0.565	0.57	0.575	0.579
BE.(keV)	1600.3	1098.5	698.4	394.4	180.6	51.2	3.3
$\langle r \rangle$ (fm)	2.47	2.85	3.41	4.31	6.01	10.52	22.60
$\sqrt{\langle r^2 \rangle}$ (fm)	3.08	3.59	4.36	5.61	8.00	14.33	28.94





# 1. Properties of charmed mesons

$$B(B_c^+ \rightarrow J/\psi \mu^+ \nu) = (2.37 - 4.54)\%$$

# 3. Heavy quark polarization in magnetic field

- Landau-Lifshitz-Gilbert (LLG) equation

PRB 83, 134418 (2011)

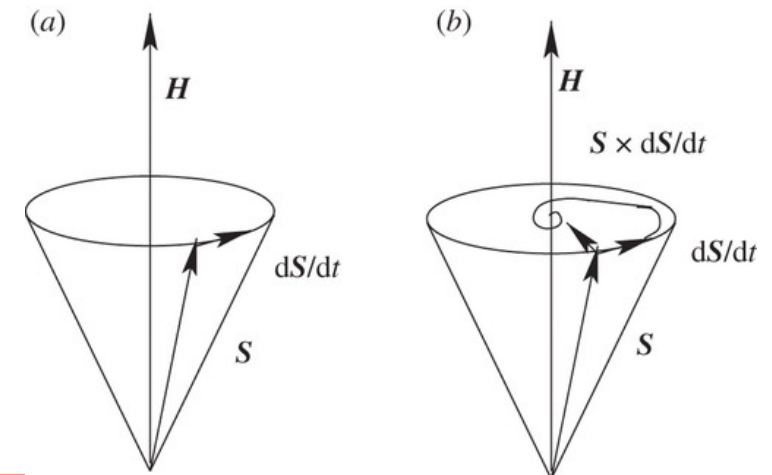
$$\frac{d\vec{S}}{dt} = -\frac{\gamma}{1 + \alpha^2} [\vec{S} \times (\vec{H} + \vec{H}_{th})] - \frac{\alpha\gamma}{1 + \alpha^2} \vec{S} \times [\vec{S} \times (\vec{H} + \vec{H}_{th})]$$

$$\vec{S} = \vec{s}/|\vec{s}|$$

Unit vector

stochastic dynamics of a spin in the medium with magnetic field

Polarization of heavy quark is induced by:  
**spin-magnetic field interaction**  
**+ particle-particle interaction**



$$\gamma = \frac{Q}{2m} \dots$$

Electric charge

$\alpha = 0.1$  Damping factor  
(to be determined later)

noise term

$$\langle \vec{H}_{th}(t) \rangle = 0$$

$$\langle H_{th,i}(t) H_{th,j}(t') \rangle = 2\gamma T \delta_{i,j} \delta(t - t')$$

-----

● **D meson coalescence**

$$P_{c\bar{q}\rightarrow D^0}(\vec{p}_M) = H_{c\rightarrow D^0} \int \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3} \frac{dN_1}{d\vec{p}_1} \frac{dN_2}{d\vec{p}_2} f_D^W(\vec{q}_r) \times \delta^{(3)}(\vec{p}_M - \vec{p}_1 - \vec{p}_2)$$

$$\frac{d^2 N_D}{dy_M d\vec{p}_T} = \int \frac{dp_z}{2\pi} \langle P_{c\bar{q}\rightarrow D^0}(\vec{p}_M) \rangle_{events} \times \frac{\Delta N_{c\bar{c}}^{AA}}{\Delta y_M}$$

- $H_{c\rightarrow D^0} = 9.5\%$  : Charm quarks turning into **direct**  $D^0$  at the phase transition
- $\frac{dN_1}{d\vec{p}_1}$ : **charm** momentum distribution
- $\frac{dN_2}{d\vec{p}_2}$ : **light quark** momentum distribution. See below.
- Assume all  $c \rightarrow D^0$  via the **coalescence process**, neglect the fragmentation. This simplification works well in low or moderate  $p_T$  region.

Light quark momentum

(local

$m_l =$

After coalescence at  $T_c$ ,

**D meson continues diffusion in hadronic medium via Langevin,**

**(with  $D_s(2\pi T) = 8$ )**

Until kinetic freeze-out  $T=0.14$  GeV