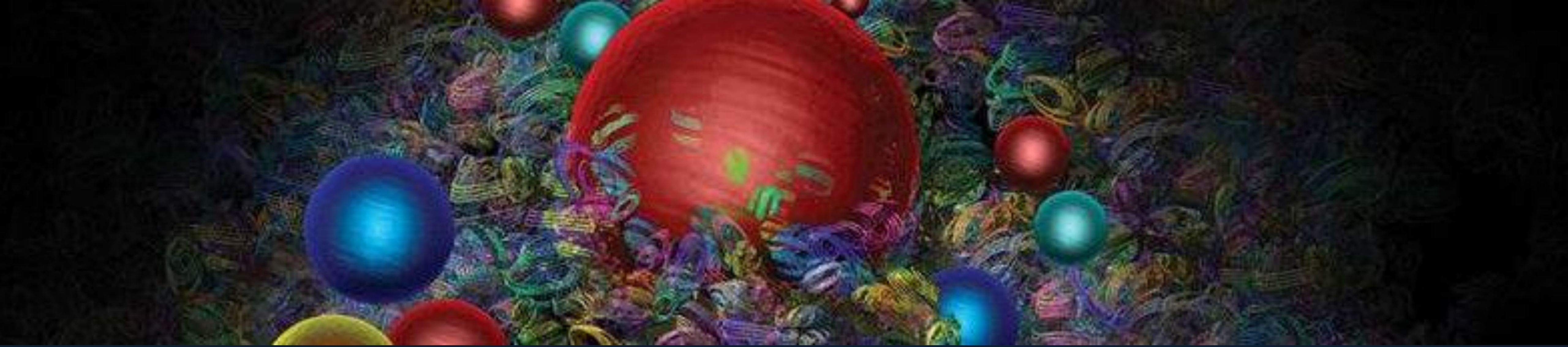

Global extraction of Generalized Parton Distributions (GPDs) with moment space parameterization

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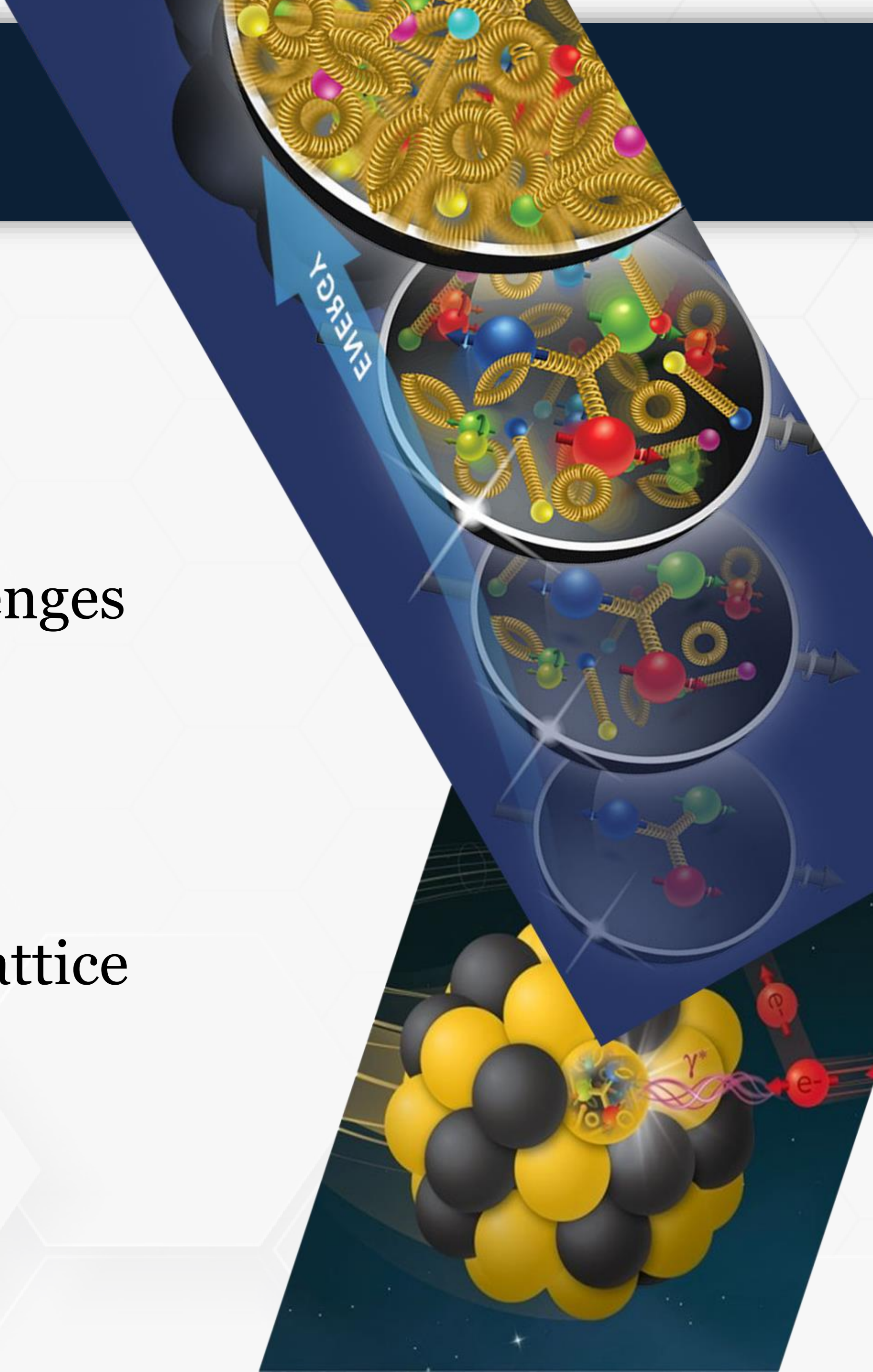
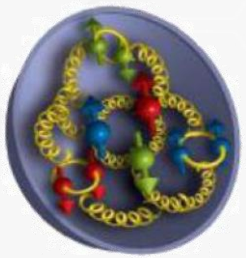
Outline

- Intro – nucleon structure and GPDs
- How GPDs are constrained and what are the challenges
- Moment space parameterization of GPDs
- Global extraction of GPDs from experiments and lattice
- Summary and outlook



Nucleon spin and 3D structure

Naïve quark model	Gluonic contribution	Transverse motions	
quark spin	gluon spin	quark orbital AM	gluon orbital AM

$$\frac{\hbar}{2} =$$  $+$  $+$  $+$ 

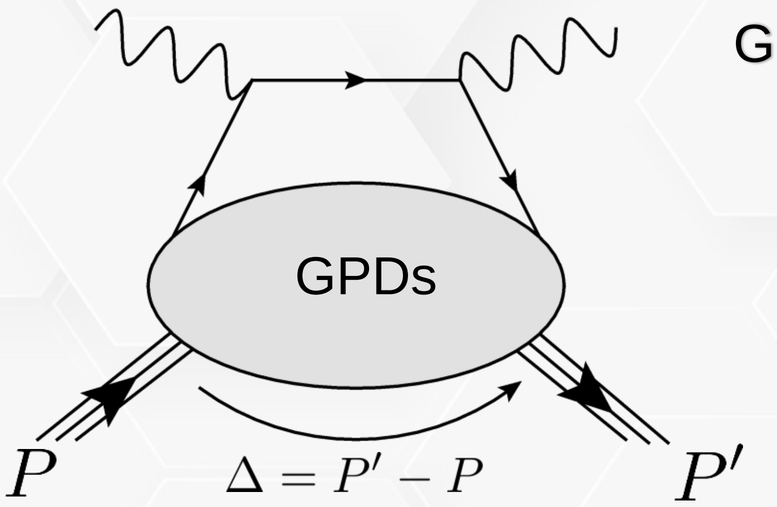
R. Jaffe and A. Manohar Nucl. Phys. B 337, 509 (1990)

Jaffe-Manohar sum rule

Nucleon spin can be explored with parton distributions localized in coordinate space.

Generalized parton distributions (GPDs)

To identify partons in the coordinate space, we consider the so-called generalized parton distributions (GPDs).



D. Muller et. al. Fortsch.Phys. 42 101 (1994)
X. Ji Phys. Rev. Lett. 78, 610 (1997)

GPDs are distributions unifying parton distributions and form factors

$$F(x, \Delta^\mu) = F\left(x, \xi, t\right)$$

x : average parton momentum fraction

ξ : skewness – longitudinal momentum transfer $\xi \equiv -n \cdot \Delta/2$

t : total momentum transfer squared $t \equiv \Delta^2$

GPDs are partonic form factors? – at zero skewness

3D mass & spin structures with GPDs

GPDs reduce to form factors when integrated over *x* X. Ji, J. Phys. *G* 24 1181-1205 (1998)

Charge FFs

$$\int \mathrm{d}x H(x,\xi,t) = F_1(t)$$

$$\int \mathrm{d}x E(x,\xi,t) = F_2(t)$$

Gravitational FFs

$$\int \mathrm{d}x\ xH(x,\xi,t) = A(t) + (2\xi)^2 C(t)$$

$$\int \mathrm{d}x\ xE(x,\xi,t) = B(t) - (2\xi)^2 C(t)$$

GPDs also provide an intuitive 3D image of nucleon:

M. Burkardt, Int. J. Mod. Phys. *A* 18 173-208 (2003)

$$\rho_q^{\mathrm{Unp}}(x,\boldsymbol{b}) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} H_q(x,-\boldsymbol{\Delta}^2) = \mathscr{H}_q(x,\boldsymbol{b})$$

which contains information of nucleon spin structure, e. g. transverse spin

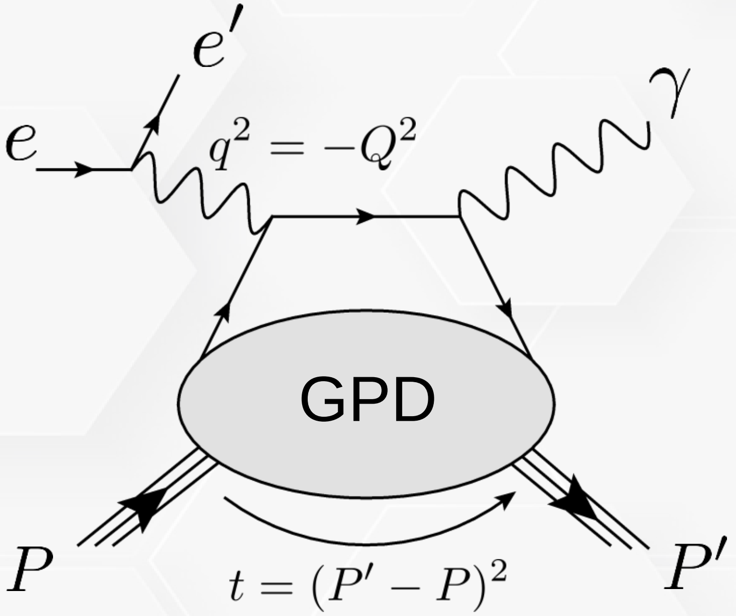
$$J_q^T(x) = \int \mathrm{d}^2\boldsymbol{b}(b^y \times xP^+)\rho_q^T(x,\boldsymbol{b})$$

Y. Guo et. al. Nucl. Phys. B 969 115440 (2021)

Experimental constraints on GPD

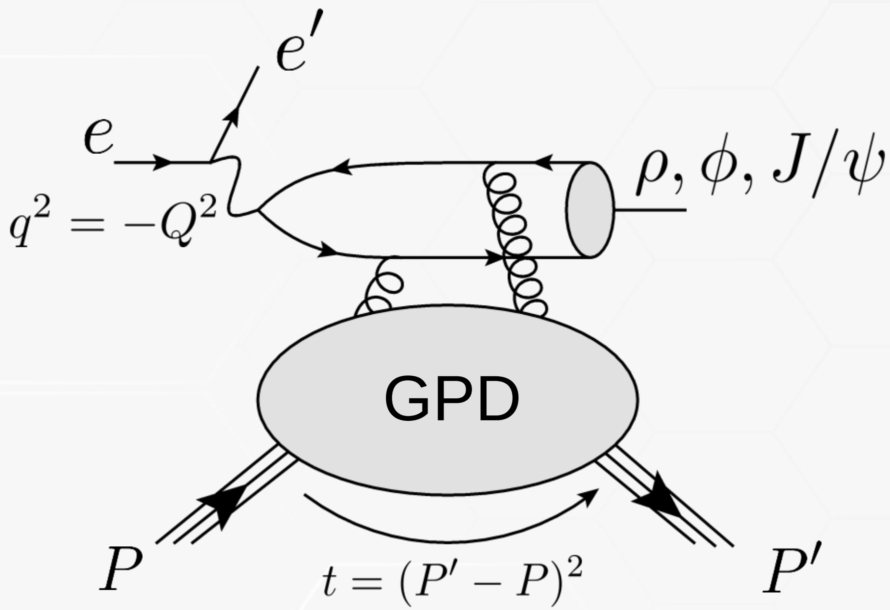
Deeply virtual processes

Diffractive processes can provide us access to the 3D structures.



Deeply virtual Compton scattering

X. Ji, Phys. Rev. D 55, 7114 (1997)

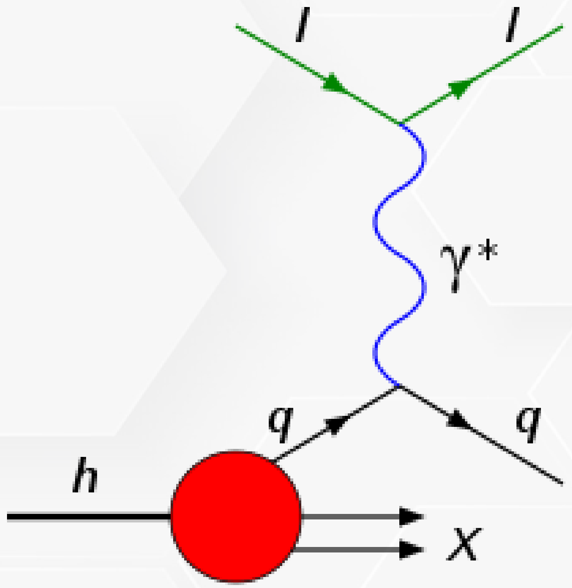


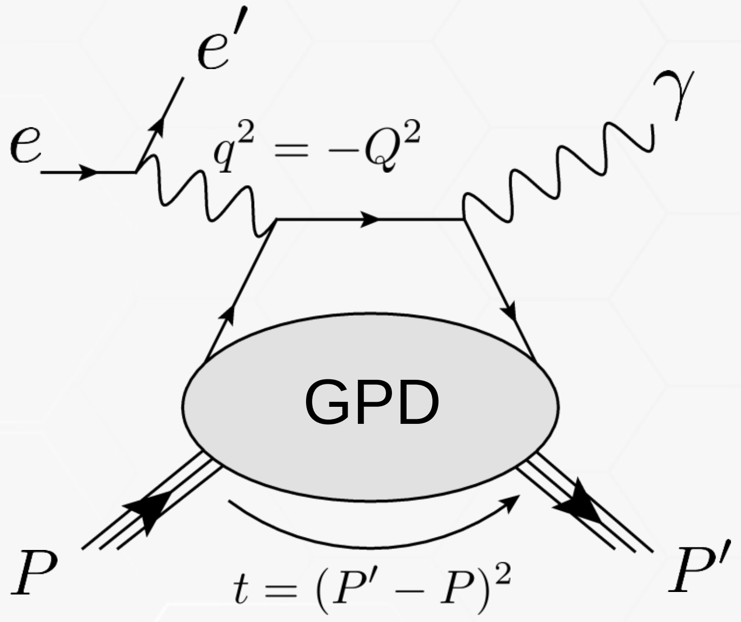
Deeply virtual meson production

A.V. Radyushkin Phys. Lett. B 385 333-342 (1996)
J. C. Collins et. al. Phys. Rev. D 56 2982-3006 (1997)

A huge difference in exclusive productions

The problem in exclusive productions: partons are not directly measured.





DIS: parton will hadronize into final state and almost on-shell.

DVCS: parton does not hadronize, it returns to the nucleon.

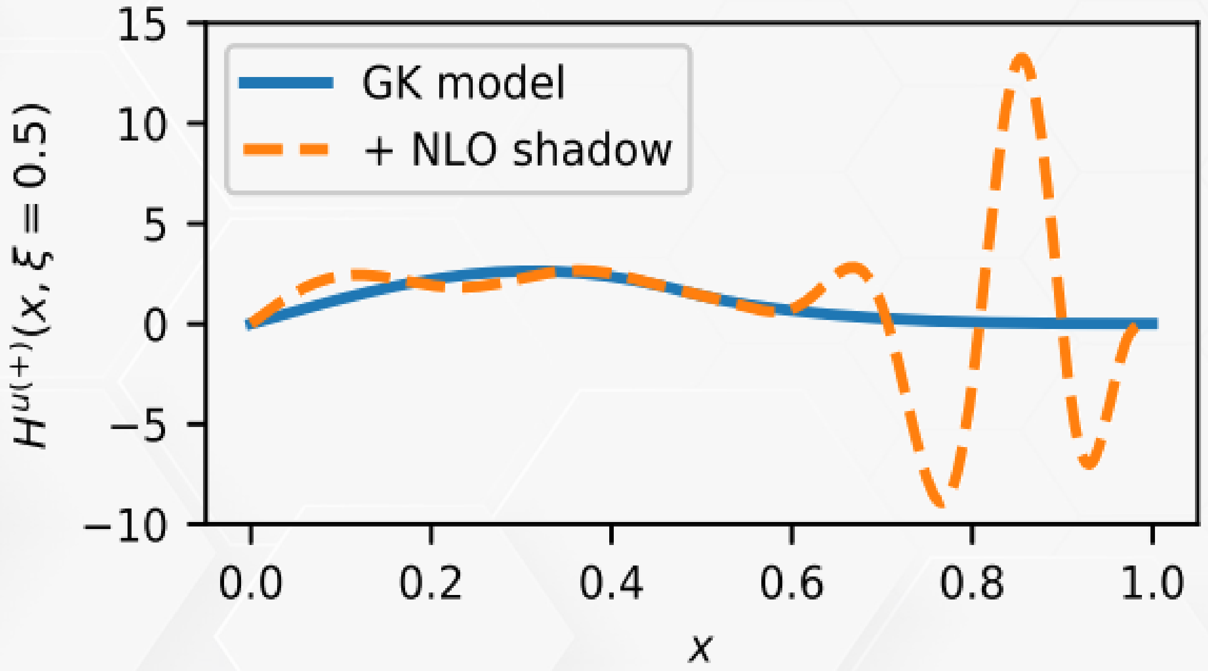
Inverse problem and shadow GPDs

The problem in exclusive productions: partons are not directly measure.

$$\mathcal{H}_{CFF}(\xi,t) = -\sum_q Q_q^2 \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x-\xi+i0} + \frac{1}{x+\xi-i0} \right) H_q(x,\xi,t) \ ,$$

One cannot obtain a unique solution of the GPD from the CFF measurements alone.

Global analysis is necessary!



V. Bertone et. al. SciPost Phys.Proc. 8 (2022) 107

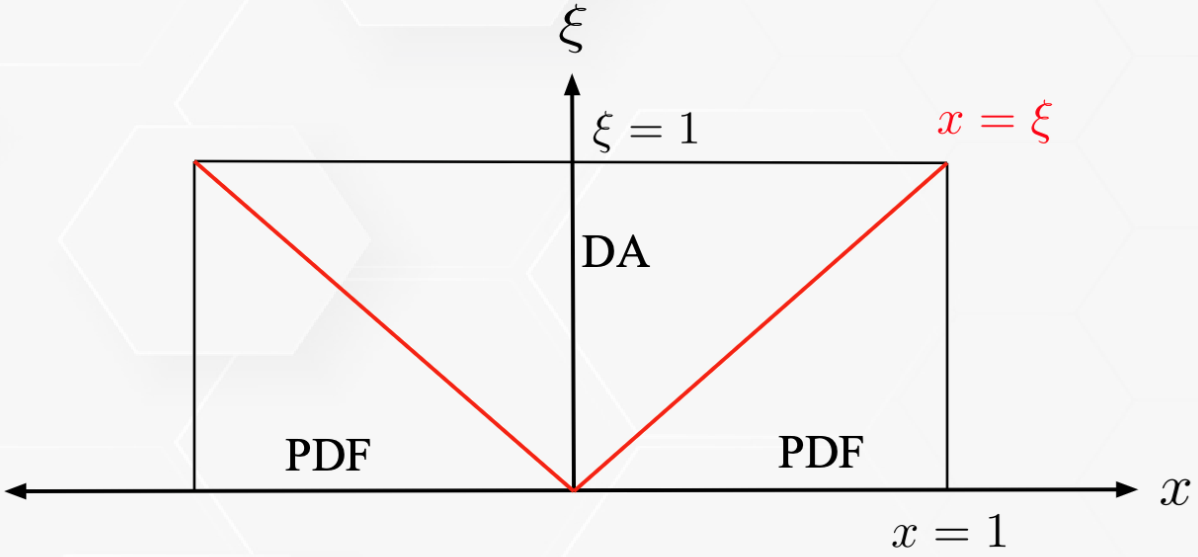
Scheme for GPD global analysis

- Parameterization of GPDs
  - Various GPD species and flavors
- Compute GPD observables
  - QCD scale evolution
- Inputs (Constraints) on GPDs
  - Both lattice and experiments
  - Constraints in x- and moment space
- Compare and iterate
  - Computation efficiency required!

GPDs from moment space

Non-analyticity in modeling GPDs

GPDs are not analytical function on the whole domain of definition



GPDs are continuous but not analytical on the cross-over line.

$$\mathcal{H}_{CFF}(\xi, t) = -\sum_q Q_q^2 \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x - \xi + i0} + \frac{1}{x + \xi - i0} \right) H_q(x, \xi, t) \ ,$$

Physical observables are most sensitive to the non-analytical region.

Polynomiality condition and more

The polynomiality condition is another important constraints on the GPDs:

$$\int_{-1}^{+1} dx x^{n-1} H(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A_{ni}(t) + (-2\xi)^n C_{n0}(t)\big|_{n\ \text{even}} \quad ,$$

$$\int_{-1}^{+1} dx x^{n-1} E(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}(t) - (-2\xi)^n C_{n0}(t)\big|_{n\ \text{even}} \quad ,$$

X. Ji, J. Phys. G 24 1181-1205 (1998)

It's the result of Lorentz symmetry and has strong physical significance.

However, implementing this is not trivial – You have a set of infinite integral equations

Besides, solving evolution of GPDs is another hard task in the x-space.

GPDs in terms of moments

GPDs can be formally expanded in the conformal moment space:

D. Mueller and A. Schafer 2006

$$F(x,\xi,t)=\sum_{j=0}^{\infty}(-1)^j p_j(x,\xi)\mathcal{F}_j(\xi,t)$$

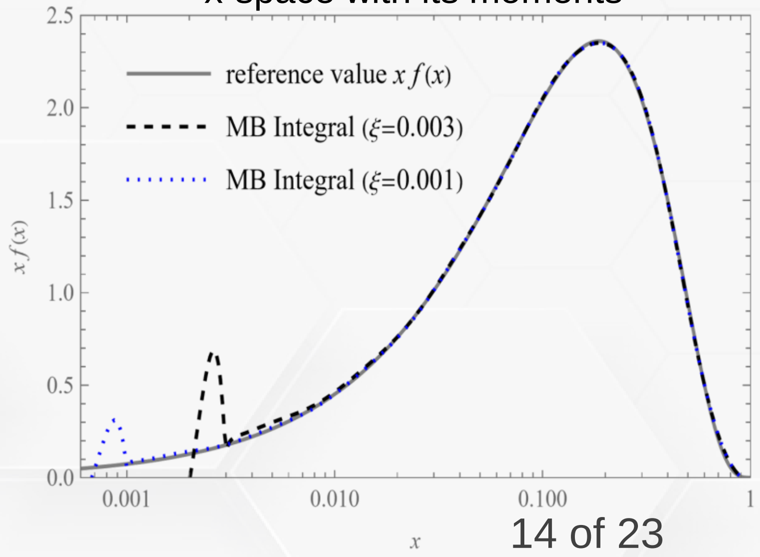
$p_j(x,\xi)$: Orthogonal basis in terms of Gegenbauer polynomials

$\mathcal{F}_j(\xi,t)$: Moments of GPDs to be parameterized

Whereas GPDs in x-space can be reconstructed by resumming all the moments through a complex integral in the moment space.

$$F(x,\xi,t)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\,\frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)\ ,$$

Example of the reconstruction of GPD in x-space with its moments



Pros and cons

Advantages:

- Polynomiality condition get naturally imposed.
- Avoid the non-analyticity at the cross-over line.
- LO evolution is simple and fast (within second).
- NLO evolution is slower but still possible and practical (order of 10 seconds).

Disadvantages:

- Calculation of x-space observables takes extra time
- Not ideal for numerical x-space convolution.

  Precalculated x-space convolution /Precalculated x-space GPDs
- There are also constraints easier to impose in x space – positivity bound of GPDs

Examples of global extractions

Universal parameterization of GPD moments

Moments of GPDs are expandable in ξ due to the polynomiality condition. For small $\xi \lesssim 0.3$ which covers most of the current data, we consider the expansion of moments

$$\mathcal{F}_j(\xi,t) = \mathcal{F}_{j,0}(t) + \xi^2 \mathcal{F}_{j,2}(t) + \xi^4 \mathcal{F}_{j,4}(t) + \cdots$$

The first term describes GPDs at $\xi = 0$, and is parameterized with a 5-parameter ansatz ($N, \alpha, \beta, \alpha', b$):

$$\mathcal{F}_{j,0}(t) = N \underbrace{B(j+1-\alpha, 1+\beta)}_{\text{Euler Beta Function}} \underbrace{\frac{j+1-\alpha}{j+1-\alpha(t)}}_{\text{Regge trajectory } \alpha(t)=\alpha+\alpha' t} \beta(t) \qquad \beta(t) = e^{-b|t|}$$

K. Kumericki et.al. Nucl.Phys.B 794 244-323 (2008)

The ξ-dependence of GPD can in principle be independently parameterized. Here we instead parameterize them with simple ratios: $\mathcal{F}_{j,2}(t) = R_{\xi^2} \mathcal{F}_{j,0}(t)$ $\mathcal{F}_{j,4}(t) = R_{\xi^4} \mathcal{F}_{j,0}(t)$

to avoid unconstrained parameters due to the lack of input.

GPD global analysis with DVCS

Experimental data and constraints

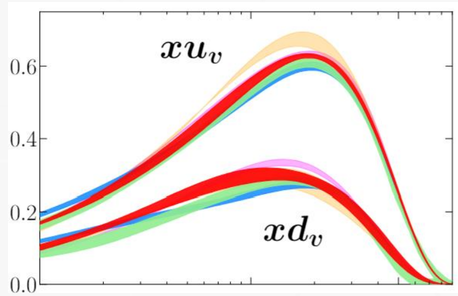
- Polarized and unpolarized PDFs from global analysis
  - Alternatively, one can fit to (polarized) DIS directly
- Neutron/ Proton charge form factors from global analysis
- Deeply virtual Compton scattering data at JLab/HERA
- Deeply virtual meson productions data at HERA

Lattice QCD simulations

- Lattice simulations of nucleon generalized form factors
- Lattice simulations of unpolarized and helicity GPDs at zero and non-zero ξ (skewness)

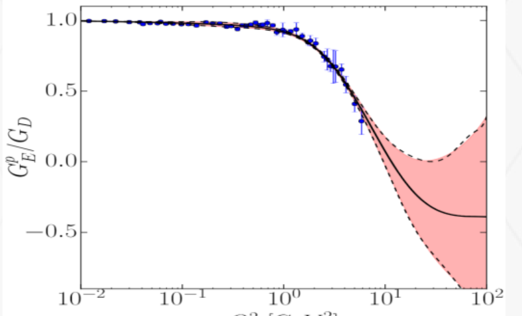
Sequential fit as first step to accelerate the convergence

Example of global PDF fit



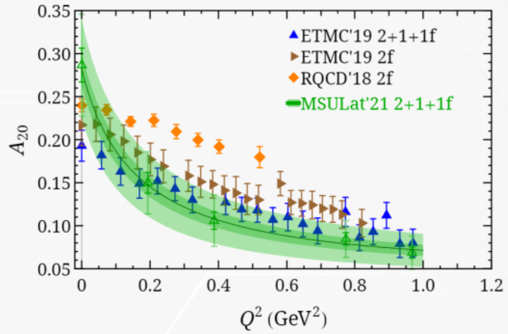
JAM 2022

Example of charge form factor fit



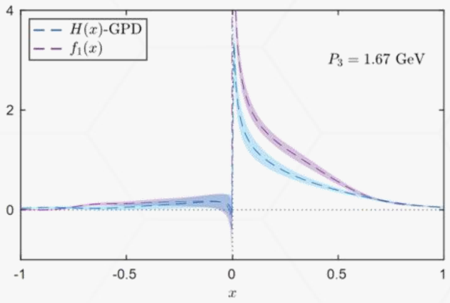
Z. Ye et al 2018

Example of lattice form factors



Huey-Wen Lin 2022

Example of lattice GPD



C. Alexandrou et. al. 2020

- JAM (2022) PDF global analysis results
- Globally extracted electromagnetic form factors (*Z. Ye et al* 2018)
- Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)

DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)

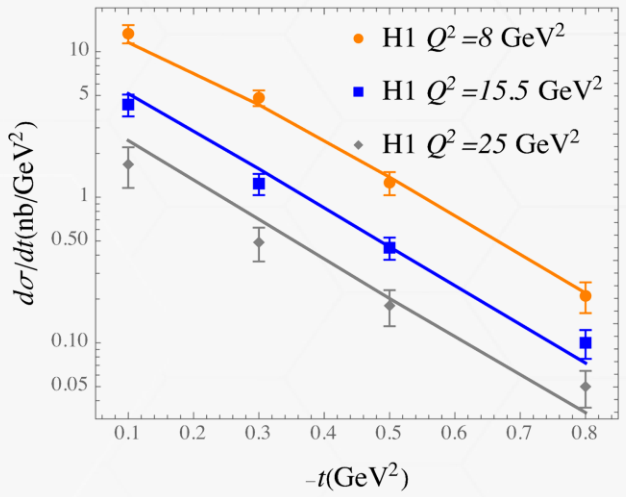
Examples of GUMP fits

The setup is rather basic:

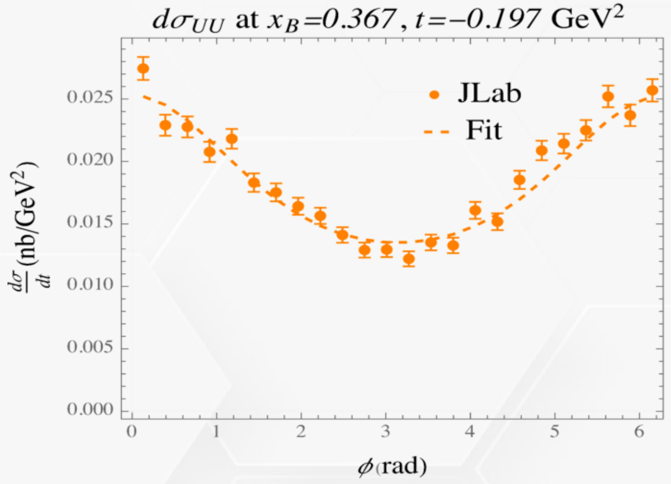
- Around 30 parameters
- Minimization of chi^2 with *iminuit*
- No uncertainty quantification at this point

Sub-fits	χ^2	N_{data}	$\chi^2_\nu \equiv \chi^2/\nu$
Semi-forward			
tPDF H	281.7	217	1.41
tPDF E	59.7	50	1.36
tPDF $\widetilde{H}$	159.3	206	0.84
tPDF $\widetilde{E}$	63.8	58	1.23
Off-forward			
JLab DVCS	1413.7	926	~ 1.53
H1 DVCS	19.7	24	~ 0.82
Off-forward total	1433	950	**1.53**
Total	2042	1481	**1.40**

Example of fit to DVCS at H1



Example of fit to DVCS at JLab

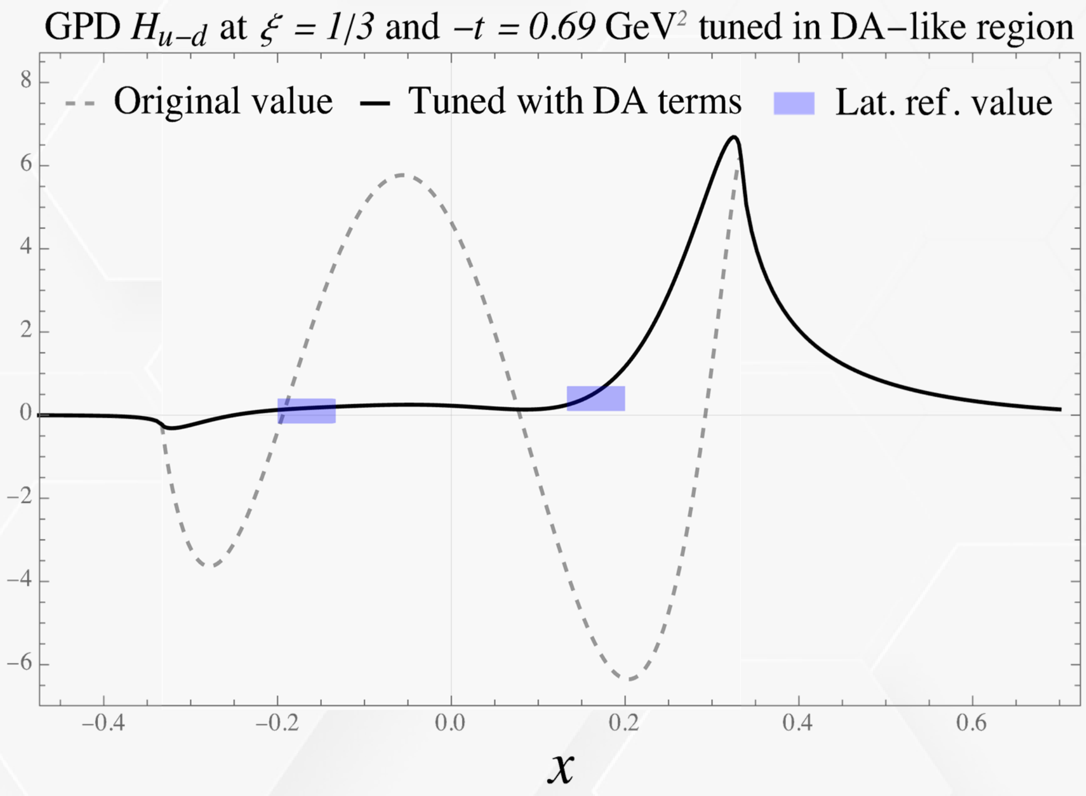


Y. Guo et. al. JHEP 05 150 (2023)

Overcoming the shadow GPDs with lattice

The extracted GPDs has wiggling in the DA region, could be caused by the shadow GPDs

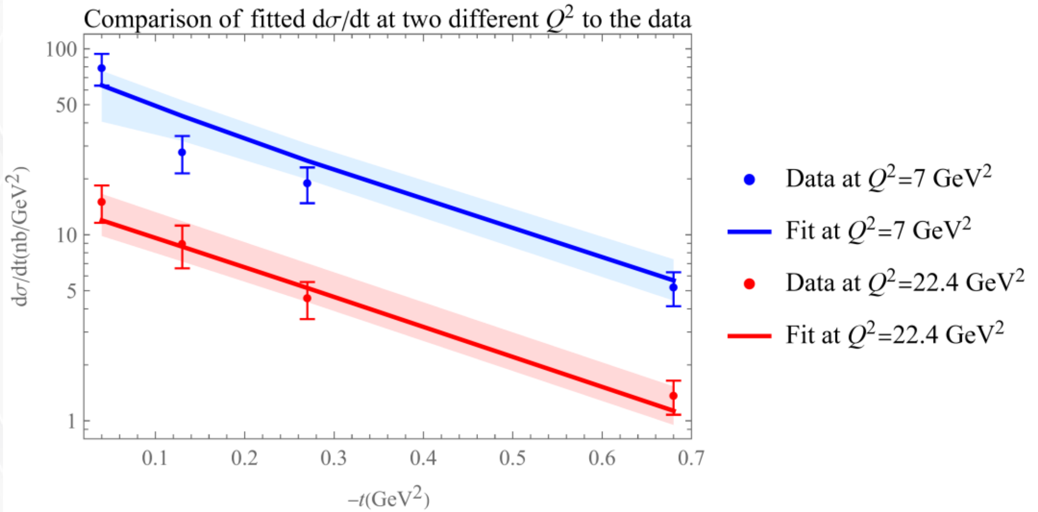
Example of GPDs tuned to lattice input

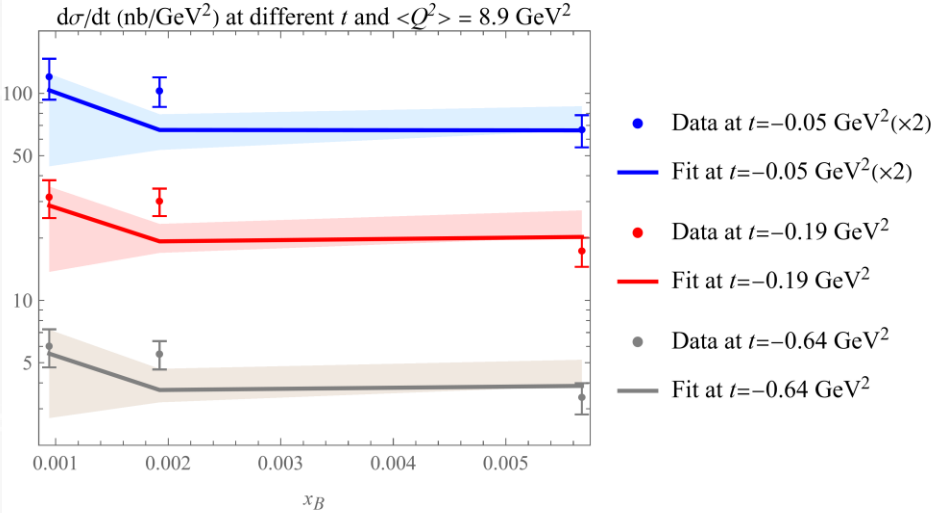


The wiggling can be constrained by the lattice input which gives information on the x-dependence

Moving on to the gluon GPDs

Fitting to DV J/ψP and small-x gluon PDFs with gluon GPDs at NLO





Y. Guo et. al. arxiv: 2409.17231

- ❑ Good agreements with the DV$J/\psi$P cross-sections;
- ❑ Cannot fit the data without next-to-leading corrections or the J/ψ mass corrections;
- ❑ Bands are NOT statistical uncertainties. They are obtained by a scale variation

– large scale dependence also indicates strong higher-order corrections beyond NLO;

Global analysis to the next-to-leading order

We are working on to push the global analysis with full next-to-leading order setup.

Table 3: *Values of χ^2/n_{pts} for each LO or NLO model (columns) for the total DIS + DVCS + DVMP dataset and for subsets corresponding to different processes (rows). (The values denoted by $\gg 1$ are greater than 10.)*

Dataset	Refs.	n_{pts}	LO-DVCS	LO-DVMP	LO-DVCS-DVMP	NLO-DVCS	NLO-DVMP	NLO-DVCS-DVMP
DIS	[90]	85	0.6	0.6	0.6	0.8	0.8	0.8
DVCS	[92, 93, 94, 95]	27	0.4	$\gg 1$	0.6	0.6	$\gg 1$	0.8
DVMP	[88, 89]	45	$\gg 1$	3.1	3.3	$\gg 1$	1.5	1.8
Total		157	$\gg 1$	$\gg 1$	1.4	3.7	$\gg 1$	1.1

Marija Cuic et.al. JHEP 12 192 (2023)

Recent work in the Kumericki-Muller (KM) model also show the importance of NLO corrections.

Summary

Summary

- The application of moment-space parameterization to the global analysis of GPDs
- Report some efforts in constraining/extracting GPDs from global analysis at LO/NLO
  - DVCS at LO for the quark GPD
  - DVJ/psi P at NLO for the gluon GPD at small-x

Outlook

- Actively working on a complete NLO global analysis including DVCS, DVMP and different kinds of other input, including lattice simulation results of course.
- More theoretical efforts in understanding the perturbative/mass/relativistic corrections for exclusive J/psi or light-meson photo- and lepto- productions and DVCS.

Thank you!

Energy-momentum tensor form factors

The energy-momentum tensor (EMT) is the tool to study the mechanical properties of the nucleon. Its nucleon matrix element can be written as:

$$\langle P'|\, T^{\mu\nu}_{q,g}\,|P\rangle = \bar{u}(P')\left[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M_N} + C_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M_N} + \bar{C}_{q,g}(t)M_N g^{\mu\nu}\right]u(P)$$

X. Ji Phys. Rev. Lett. 78, 610 (1997)

Momentum form factors: $A_{q,g}(t)$

Angular momentum form factors: $J_{q,g}(t) = \frac{1}{2}\left(A_{q,g}(t) + B_{q,g}(t)\right)$

Stress tensor form factors: $C_{q,g}(t)$