New Way of Generating Odderon Contribution to Single Spin Asymmetry

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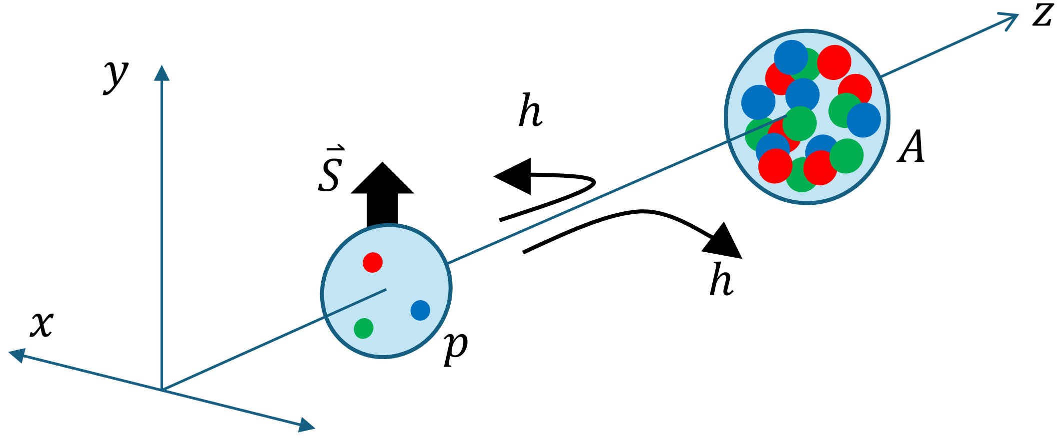
Hefei, November, 11th 2024.

S. Benić and EAV, in preparation

Transverse Single Spin Asymmetries (TSSA)

- Left-right asymmetry of produced particles in collisions involving polarized hadrons



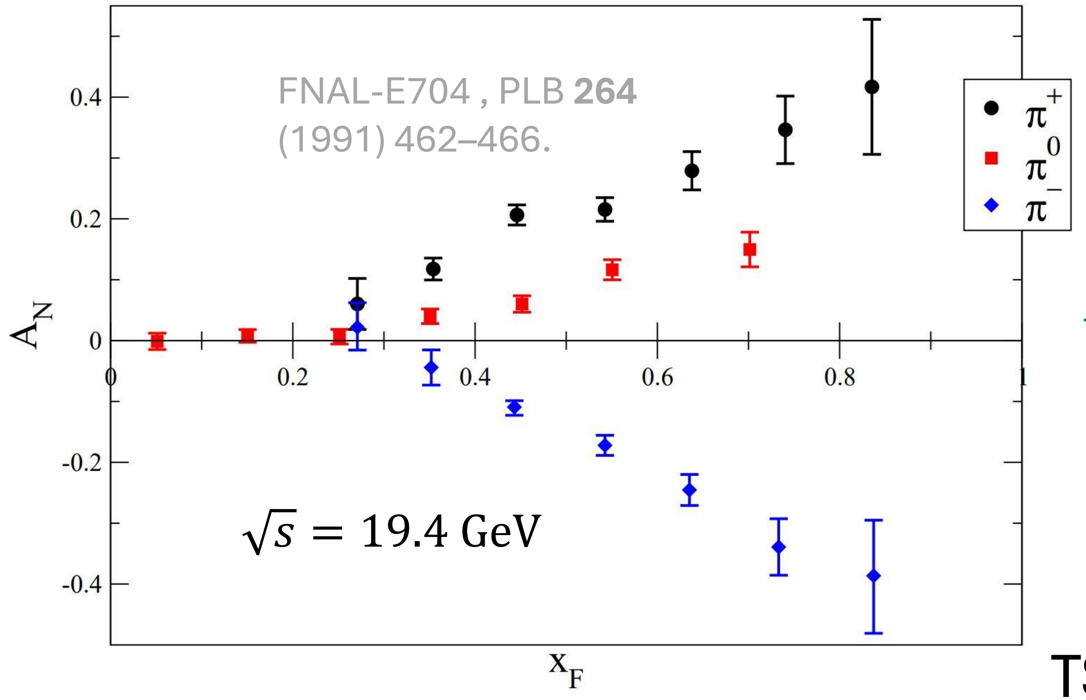
$$A_N \equiv \frac{N_L - N_R}{N_L + N_R} = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} = \frac{\mathrm{d}\Delta\sigma}{2\mathrm{d}\sigma_{unp}}$$

$$A_N \sim (\vec{S} \times \vec{P}_h) \cdot \vec{P} \sim \sin(\varphi_h - \varphi_S)$$

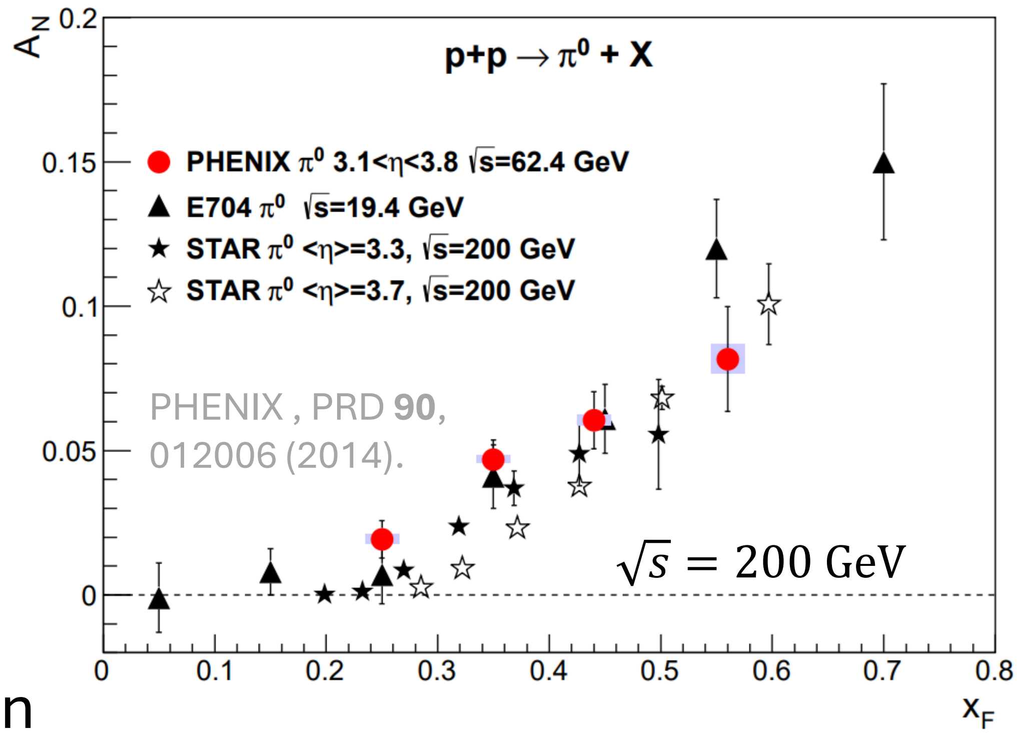
Naively T-odd quantity!

- Requires helicity flip → $A_N \sim \frac{m_q}{P_{h\perp}}$ → in pQCD there is no TSSA?

TSSA in $p^{\uparrow}p \to hX$ - experiments



Larger energy



TSSA is largest in forward region!

$$x_F = \frac{P_h^z}{\sqrt{s}}$$

- Measured at different $\sqrt{s}$

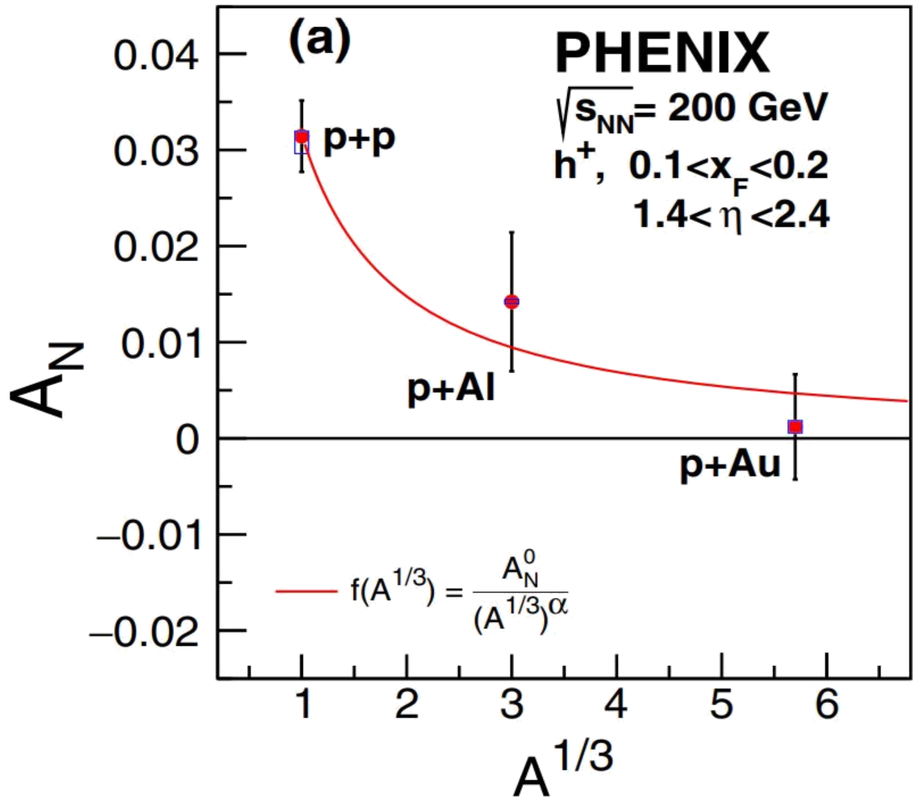
Fixed target!

- PRL, **36**, 929 (1976). $\sqrt{s} = 4.9$ GeV
- PRD, **65**, 092008 (2002). $\sqrt{s} = 6.6$ GeV
- PLB, **264**, 462 (1991). $\sqrt{s} = 19.4$ GeV

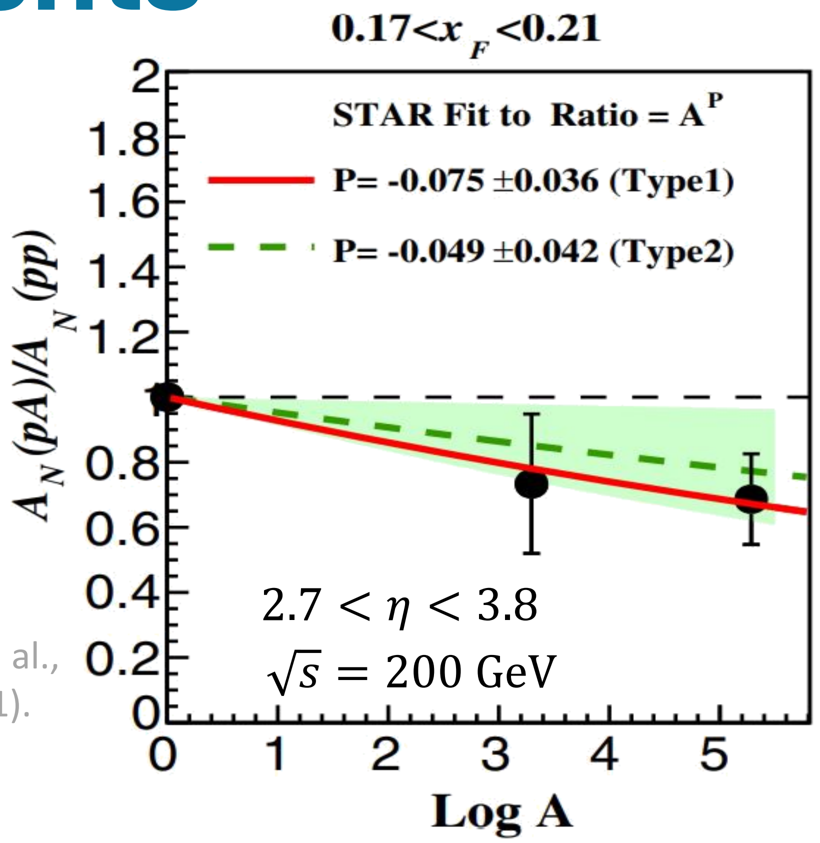
Collider!

- PRL, **101**, 042001 (2008). $\sqrt{s} = 62.4$ GeV
- PRD, **90**, 012006 (2014). $\sqrt{s} = 200$ GeV

TSSA in $p^{\uparrow}A \to hX$ - experiments



PHENIX Collaboration, C. Aidala et. al., Phys.Rev.Lett. **123**, 122001 (2019).



$2.7 < \eta < 3.8$

$\sqrt{s} = 200$ GeV

Star Collaboration, J. Adam et. al., Phys.Rev.D. **103**, 072005 (2021).

$1.8 < P_{hT} < 7.0$ GeV (integrated)

$0.004 \leq x \leq 0.1$

$A_N \sim A^{(-1/3)\alpha}$

$\alpha = 1.10^{+0.75}_{-0.41}$

Significant A dependence!

$1.5 < P_{hT} < 2.0$ GeV

$2.0 < P_{hT} < 3.0$ GeV…

$P_{hT} < 6.0$ GeV

$x < 0.005$

$A_N \sim A^{-0.027 \pm 0.005}$

TSSA as a quest for an i

M.D.Sievert, arxiv:1407.4047

- By using CPT symmetry, the Dirac structure of the cross section has the form:

$$\sigma_n \sim a + \chi b$$

a, spin independent part, is real!

b, spin dependent part, is completely imaginary!

Spin always comes with γ_5

- Cross section is real → we need another i
- Twist-3 contribution to polarized cross section:

Loop corections are higher order in α_S

$$\Delta\sigma \sim D_2 \otimes G_{F3} \otimes G_2 \otimes H_{pole} + iD_3 \otimes h_2 \otimes G_2 \otimes H + D_2 \otimes h_2 \otimes G_3 \otimes H_{pole}$$

- Real twist-3 ETQS functions for polarized projectile
- Phase from propagator cut

C. Kouvaris et. all, PRD **74**, 114013 (2006).

- Transversity PDF for polarized projectile
- Phase from imaginary part of twist-3 fragmentation function

A. Metz and D. Pitonyak, PLB **723**, 365 (2013).

- Transversity PDF for polarized projectile and twist-3 in target
- Phase from propagator cut

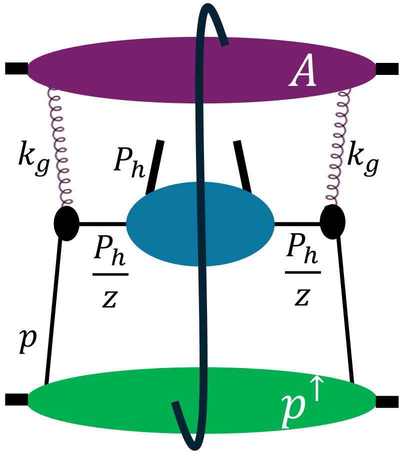
Y. Kanazawa and Y. Koike, PLB **478**, 121-126 (2000).

Twist-3 FF in forward pA

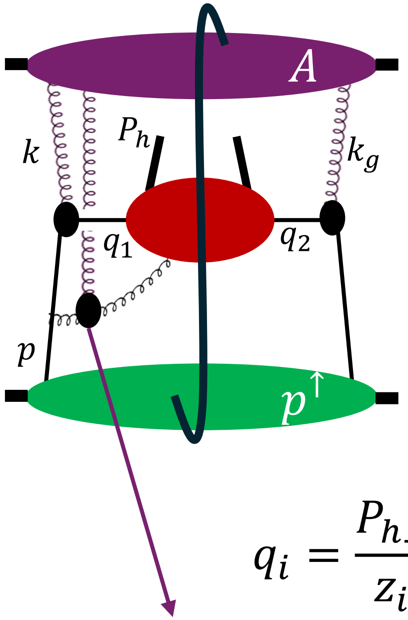
K. Kanazawa and Y. Koike, Phys. Rev. D 88, 074022 (2013).

Y. Hatta, B.-W. Xiao, S. Yoshida, and F. Yuan, Phys. Rev. D 95, 014008 (2017).

- Hybrid approach: polarized proton described by transversity distribution, and target described by Color Glass Condensate (CGC) ($Q_S^2 \propto A^{1/3}$)



$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{1}{2(2\pi)^3}\left(\int \frac{\mathrm{d}z}{z^2}\mathrm{Tr}[\Delta(z)S^{(0)}(z)] + \int \frac{\mathrm{d}z}{z^2}\mathrm{Im\ Tr}\left[\Delta_\partial^\alpha(z)\frac{\partial S^{(0)}(K)}{\partial K^\alpha}\right]_{K=\frac{P_h}{z}} - \int \frac{\mathrm{d}z_1\mathrm{d}z_2}{z_1^2 z_2^2}\mathrm{P}\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1}\mathrm{Im\ Tr}[\Delta_F^\alpha(z_1,z_2)S_\alpha^{1L}(z_1,z_2) + \overline{\Delta}_F^\alpha(z_2,z_1)S_\alpha^{1R}(z_1,z_2)]\right)$$



$$\langle P|\bar{\psi}\psi|P\rangle = \Phi(x_q) = -\frac{P^+ S_{\perp i}}{2} h_1(x_q) i\gamma_5\sigma^{-i} + \cdots$$

$$q_i = \frac{P_{h\perp}}{z_i}$$

- $S^{(0)}$ and $S_\alpha^{(1)}$ are scattering kernels containing projectile and target distributions; the twist-3 fragmentation functions are contained in Δ correlators:

Eikonalized interaction!

INTRINSIC $\langle 0|\psi|\rangle\langle|\bar{\psi}|0\rangle \propto \Delta(z) = \frac{M_N}{z}\hat{e}_1(\mathrm{z}) + \frac{M_N}{2z}\sigma_{\lambda\alpha} i\gamma_5\epsilon^{\lambda\alpha w P_h}\hat{e}_{\bar{1}}(\mathrm{z}) + \cdots$

A. Metz and D. Pitonyak, PLB 723, 365 (2013).

KINEMATICAL $\langle 0|\partial_\perp\psi|\rangle\langle|\bar{\psi}|0\rangle \propto \Delta_\partial^\alpha = \frac{M_N}{2}\gamma_5\frac{\not{P}_h}{z}\gamma_\lambda\epsilon^{\lambda\alpha w P_h}\tilde{e}(z) + \cdots$

$$w^2 = 0$$
$$P_h \cdot w = 1$$

DYNAMICAL $\langle 0|F_\perp\bar{\psi}|\rangle\langle|\bar{\psi}|0\rangle \propto \Delta_F^\alpha(z_1,z_2) = \frac{M_N}{2}\gamma_5\frac{\not{P}_h}{z_2}\gamma_\lambda\epsilon^{\lambda\alpha w P_h}\hat{E}_F(z_1,z_2) + \cdots$

X.-D. Ji, Phys. Rev. D 49, 114 (1994).

Phase obtained by taking imaginary part of FFs!

- In forward region the polarized cross section is:

$$\frac{d\Delta\sigma}{dy_h d^2\boldsymbol{P}_{h\perp}} = \frac{M}{2} S_{\perp i}\epsilon^{ij} \int \frac{dz_2}{z_2^2} x_q h_1(x_q) \Bigg\{ -\mathrm{Im}\, \tilde{e}(z_2) \frac{d}{dP_h^j/z_2} F\left(x_g, \frac{P_{h\perp}}{z_2}\right)$$

Y. Hatta, B.-W. Xiao, S. Yoshida, and F. Yuan, Phys. Rev. D 95, 014008 (2017).

$$+4\frac{P_{hj}}{P_{h\perp}^2}\int_z^\infty \frac{dz_1}{z_1^2} P\left(\frac{z_2}{1/z_2 - 1/z_1}\right) \frac{\mathrm{Im}\, \hat{E}_F(z_1, z_2)}{N_C^2 - 1} \left(\frac{2\pi N_C^2}{\pi R_A^2} \int_0^{P_{h\perp}/z_1} l_\perp dl_\perp F(x_g, l_\perp) + \frac{1}{z_1(1/z_2 - 1/z_1)}\right) F\left(x_g, \frac{P_{h\perp}}{z_2}\right) \Bigg\}$$

- Distribution $F\left(x_g, \frac{P_{h\perp}}{z}\right)$ is Fourier transform of the CGC Dipole distribution

$$F(x_g, \kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2} \int_{r_\perp} e^{i\boldsymbol{\kappa}_\perp \cdot \boldsymbol{x}_\perp} \frac{1}{N_C} \mathrm{tr}\langle V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{0}_\perp)\rangle_{x_g}$$

$$V(\boldsymbol{x}_\perp) = \mathcal{P}\exp\left[ig\int_{-\infty}^{\infty} dx^+ A^-(x^-, \boldsymbol{x}_\perp)\right]$$

- In forward region the TSSA scales as $A^{-1/3}$ for $P_{h\perp} < Q_S$ (saturation scale)

$$Q_S^2 = Q_{S0}^2 A^{1/3}$$

- Nuclear suppression gets washed away by high energy evolution

S. Benić and Y. Hatta, Phys. Rev. D 99, 094012 (2019).

CGC-odderon mechanism for TSSA

- Odderon in CGC = imaginary part of dipole distribution: TALK BY JIAN ZHOU

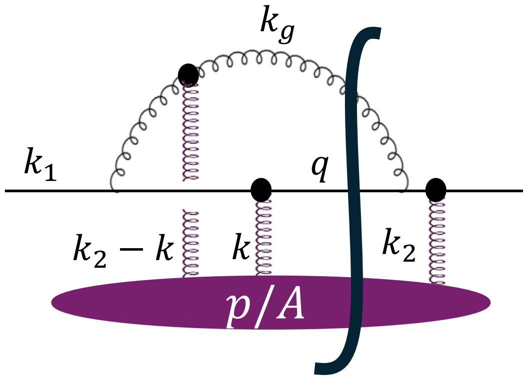
$$\mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp) \equiv \frac{1}{N_C}\mathrm{tr}\langle V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{x'}_\perp)\rangle \qquad \mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp) \equiv \mathcal{P}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp) + i\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp)$$

- Phase from odderon? Y. V. Kovchegov and M. D. Sievert, PRD **86**, 034028 (2012).
- Asymmetry calculated at parton level (up to NLO), i.e. $q^\uparrow A$ collisions
- Polarized cross section:

$$E_q \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 q} \propto i\alpha_S \int_{\boldsymbol{k}_\perp \boldsymbol{k}_{2\perp}} \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp \boldsymbol{r'}_\perp} \mathcal{H}(\boldsymbol{r}_\perp, \boldsymbol{r'}_\perp, \boldsymbol{S}_\perp)$$
$$\times [\mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r'}_\perp, \boldsymbol{b'}_\perp) - \mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{P}(\boldsymbol{r'}_\perp, \boldsymbol{b'}_\perp)]$$

- For $P_{h\perp} \approx Q_S$ the TSSA has significant nuclear suppression:

$$A_N \propto A^{-7/6}$$



$$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$$
$$\boldsymbol{b}_\perp = \frac{1}{2}(\boldsymbol{x}_\perp + \boldsymbol{y}_\perp)$$
$$\boldsymbol{r}'_\perp = \boldsymbol{y}_\perp - \boldsymbol{x}'_\perp$$
$$\boldsymbol{b}'_\perp = \frac{1}{2}(\boldsymbol{x}'_\perp + \boldsymbol{y}_\perp)$$

➢ In Wandzura-Wilczek approximation, odderon does not contribute to TSSA in pA collisions!

S. Benić, D. Horvatić, A. Kaushik and EAV, PRD, **106**, 114025 (2022).

This work: Combining FFs with the Odderon

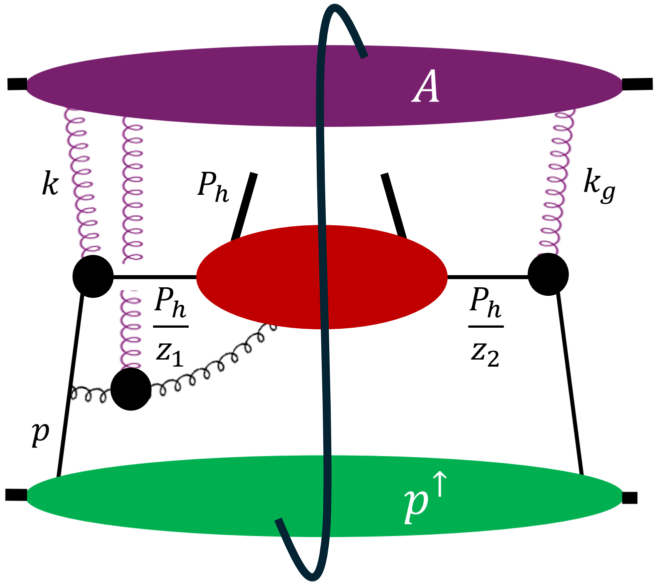
S. Benić and EAV, in preparation

- Combining the **real part** of twist-3 FF with the **Odderon**!
- Only dynamical twist-3 FF may contribute (they have real and imaginary part):

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = -\frac{1}{2(2\pi)^3}\frac{M_N}{2}\epsilon^{\lambda\alpha w P_h}\int \frac{\mathrm{d}z_1\mathrm{d}z_2}{z_1^2 z_2^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} \times \left(\mathrm{Re}\, \hat{E}_F(z_1,z_2) \times \mathrm{Im}\,\mathrm{Tr}\left(\gamma_5 \frac{\not{P}_h}{z_2}\gamma_\lambda S_\alpha^{(1)L}(z_1,z_2) + \gamma_\lambda \frac{\not{P}_h}{z_2}\gamma_5 \bar{S}_\alpha^{(1)L}(z_1,z_2)\right)\right)$$

- Transversity and target distributions are contained in $S_\alpha^{(1)L}$:

$$S_\alpha^{(1)L}(z_1,z_2) = \frac{1}{2P^+}\frac{2}{(N_C^2-1)}\int \mathrm{d}x_q (2\pi)\delta\left(\frac{P_h^+}{z_2} - x_q P^+\right)\langle \mathrm{tr}_{\mathrm{C}}(\mathcal{M}_\alpha^a \Phi(x_q)\bar{\mathcal{M}} t^a)\rangle$$



- Amplitudes:

$$\mathcal{M}_\alpha^a(z_1,z_2) = i\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} \mathrm{e}^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\mathrm{e}^{i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}_\perp\right)\cdot\boldsymbol{y}_\perp} T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2) V(\boldsymbol{x}_\perp) t^b U^{ab}(\boldsymbol{y}_\perp)$$

$$T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2) = i\int_{-\infty}^{+\infty}\frac{\mathrm{d}k^-}{(2\pi)}\gamma^+ \frac{\frac{\not{P}_h}{z_1}-\not{k}}{\left(\frac{P_h}{z_1}-k\right)^2 + i\epsilon}\gamma^\beta \frac{V_{\alpha\beta}}{\left(x_q P + k - \frac{P_h}{z_1}\right)^2 + i\epsilon}$$

$$\mathcal{M} = -i\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} \mathrm{e}^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\mathrm{e}^{i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}_\perp\right)\cdot\boldsymbol{y}_\perp}\gamma^+ V(\boldsymbol{x}_\perp)$$

- Object to calculate: color and Dirac traces

$$\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}\left(\gamma_5\frac{\not P_h}{z_2}\gamma_\lambda S_\alpha^{(1)L}(z_1,z_2)\right)=\frac{S_{\perp i}}{2(N_C^2-1)}\int \mathrm{d}x_q h_1(x_q)(2\pi)\delta\left(\frac{P_h^+}{z_2}-x_qP^+\right)\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{k}'_\perp,\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}e^{i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}_\perp\right)\cdot\boldsymbol{y}_\perp}e^{-i\boldsymbol{k}'_\perp\cdot\boldsymbol{x}'_\perp}e^{-i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}'_\perp\right)\cdot\boldsymbol{y}'_\perp}$$

$$\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}\left((i\gamma_5\sigma^{-i})\gamma^+\gamma_5\frac{\not P_h}{z_2}\gamma_\lambda T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2)\right)\left\langle \mathrm{tr_C}\left(V^\dagger(\boldsymbol{x}'_\perp)t^bV(\boldsymbol{x}_\perp)t^a\right)U^{ba}(\boldsymbol{y}_\perp)\right\rangle$$

- Color structure calculated with SU(N) Fierz identitiy; Large N_C:

$$\left\langle \mathrm{tr_C}\left(V^\dagger(\boldsymbol{x}'_\perp)t^bV(\boldsymbol{x}_\perp)t^a\right)U^{ba}(\boldsymbol{y}_\perp)\right\rangle=\frac{1}{2}\left(N_C^2\mathcal{D}(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp)\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)-\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp)\right)$$

Imaginary part vanishes under transverse integration: NO ODDERON

- Dirac trace is calculated in forward limit; keeping leading power of P_h^+:

$$\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}\left((i\gamma_5\sigma^{-i})\gamma^+\gamma_5\frac{\not P_h}{z_2}\gamma_\lambda T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2)\right)=-16\frac{P_h^+}{z_1}\epsilon^{ij}\frac{\left(\frac{P_h}{z_1}-k\right)_j}{\left(\frac{P_{h\perp}}{z_1}-k_\perp\right)^2}$$

The support properties of $\hat{E}_F$ ($0<z_2<1, z_2<z_1<\infty$) enabled straightforward k^- integration

- After calculation of the mirror diagram (gluon on right side of the cut):

$$\frac{d\Delta\sigma}{dy_h d^2 \boldsymbol{P}_{h\perp}} = \frac{M}{2\pi^2}\frac{N_C^2}{N_C^2-1}\int \frac{dz_1 dz_2}{z_1^2 z_2^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} x_q h_1(x_q) \mathrm{Re}\, \hat{E}_F(z_1,z_2)$$

$$\mathcal{H}(z_1,z_2,\boldsymbol{k}_\perp) = \frac{z_2}{z_1}\frac{\epsilon^{ij} S_{\perp i}\left(\frac{P_h}{z_1}-k\right)_j}{\left(\frac{\boldsymbol{P}_{h\perp}}{z_1}-\boldsymbol{k}_\perp\right)^2}$$

$$\int_{\boldsymbol{k}_\perp,\boldsymbol{r}_\perp,\boldsymbol{b}_\perp,\boldsymbol{r}'_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\frac{\boldsymbol{P}_{h\perp}}{z_2}\cdot\boldsymbol{r}'_\perp} \mathcal{H}(z_1,z_2,\boldsymbol{k}_\perp)[\mathcal{P}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}'_\perp,\boldsymbol{b}'_\perp)+\mathcal{P}(\boldsymbol{r}'_\perp,\boldsymbol{b}'_\perp)\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)]$$

- Fourier transforms of Pomeron and Odderon:

$$\mathcal{P}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) = \int_{\boldsymbol{r}_\perp,\boldsymbol{b}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\mathcal{P}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp) \qquad \mathcal{O}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) = \int_{\boldsymbol{r}_\perp,\boldsymbol{b}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)$$

- As a function of $b_\perp$, Odderon peaks around R_A (small $\Delta_\perp$ approximation)

S. Benić, D. Horvatić, A. Kaushik, and E. A. Vivoda, Phys. Rev. D 108, 074005 (2023).

TALK BY JIAN ZHOU

$$\frac{d\Delta\sigma}{dy_h d^2 \boldsymbol{P}_{h\perp}} = \frac{M}{2\pi^2}\frac{N_C^2}{N_C^2-1}\int \frac{dz_1 dz_2}{z_1^2 z_2^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} x_q h_1(x_q) \mathrm{Re}\, \hat{E}_F(z_1,z_2)$$

$$\int_{\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp} \mathcal{H}^{(1)}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)\left[\mathcal{P}\left(\frac{\boldsymbol{P}_{h\perp}}{z_2},\boldsymbol{\Delta}_\perp\right)\mathcal{O}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) - \mathcal{P}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)\mathcal{O}\left(\frac{\boldsymbol{P}_{h\perp}}{z_2},\boldsymbol{\Delta}_\perp\right)\right]$$

$$\mathcal{H} = \mathcal{H}(\Delta_\perp = 0) + \Delta_\perp \frac{\partial\mathcal{H}}{\partial\Delta_\perp}(\Delta_\perp = 0)$$

Vanishes under angular integral due to Odderon cosine modulation

$\mathcal{H}^{(1)}$

Separable Pomeron and Odderon

S. Benić and EAV, in preparation

- We assume separable form of Pomeron and Odderon:

$$\mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = \mathcal{P}(r_\perp)T(b_\perp)$$

T. Lappi and H. Mäntysaari, PRD **88**, 114020 (2013).

$T(b_\perp)$ is a profile function normalized as: $\int_{\boldsymbol{b}_\perp} T(b_\perp) = \pi R_A^2$

$$\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = R_A \mathcal{O}(r_\perp)\frac{dT(b_\perp)}{db_\perp}\cos(\phi_{rb})$$

S. Jeon and R. Venugopalan, Phys. Rev. D 71, 125003 (2005).

- After performing angular integrations ϕ_κ and ϕ_Δ cross section becomes:

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = -\frac{M_N}{2}\frac{N_C^2}{N_C^2-1}\frac{\epsilon^{ij}S_{\perp i}P_{hj}}{P_{h\perp}^3}\frac{1}{\pi^2 R_A^3}\int_0^\infty \Delta_\perp^3 T^2(\Delta_\perp)\mathrm{d}\Delta_\perp$$
$$\int_{z_{min}}^1 \frac{\mathrm{d}z_2}{z_2^2}\int_{z_2}^\infty \frac{\mathrm{d}z_1}{z_1^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} \mathrm{Re}\,\hat{E}_F(z_1,z_2)x_q h_1(x_q) z_2 G\left(\frac{P_{h\perp}}{z_2}\right) z_1 \int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp)\mathrm{d}\kappa_\perp$$

$$F(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2}\int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp}\mathcal{P}(r_\perp)$$

$$i\cos(\phi_{\kappa\Delta})G(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2}\int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp}\mathcal{O}(r_\perp)\cos(\phi_{r\Delta})$$

Separable Pomeron and Odderon

S. Benić and EAV, in preparation

Dipole-square piece from: Y. Hatta, B.-W. Xiao, S. Yoshida, and F. Yuan, Phys. Rev. D 95, 014008 (2017).

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2 \boldsymbol{P}_{h\perp}} = 4\pi M \frac{N_C^2}{N_C^2-1} \frac{\epsilon^{ij} S_{\perp i} P_{hj}}{P_{h\perp}^2} \frac{1}{\pi R_A^2} \int_{z_{min}}^{1} \frac{\mathrm{d}z_2}{z_2^2} \int_{z_2}^{\infty} \frac{dz_1}{z_1^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} \mathrm{Im}\, \hat{E}_F(z_1,z_2) x_q h_1(x_q)\, z_2 F\left(\frac{P_{h\perp}}{z_2}\right) \int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp$$

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2 \boldsymbol{P}_{h\perp}} = -\frac{M_N}{2} \frac{N_C^2}{N_C^2-1} \frac{\epsilon^{ij} S_{\perp i} P_{hj}}{P_{h\perp}^3} \frac{1}{\pi^2 R_A^3} \int_0^{\infty} \Delta_\perp^3 T^2(\Delta_\perp) \mathrm{d}\Delta_\perp$$
$$\int_{z_{min}}^{1} \frac{\mathrm{d}z_2}{z_2^2} \int_{z_2}^{\infty} \frac{\mathrm{d}z_1}{z_1^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} \mathrm{Re}\, \hat{E}_F(z_1,z_2) x_q h_1(x_q) z_2 G\left(\frac{P_{h\perp}}{z_2}\right) z_1 \int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp$$

$$F(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2} \int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp \cdot \boldsymbol{r}_\perp} \mathcal{P}(r_\perp)$$

$$i\cos(\phi_{\kappa\Delta}) G(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2} \int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp \cdot \boldsymbol{r}_\perp} \mathcal{O}(r_\perp) \cos(\phi_{r\Delta})$$

Model for $\mathcal{P}(r_\perp)$ and $\mathcal{O}(r_\perp)$

F. Salazar, B. Schenke, and A. Soto-Ontoso, Phys. Lett. B 827, 136952 (2022).

- For $r_\perp$ dependent part of the Pomeron and the Odderon we use a fit to our solution of Balitsky-Kovchegov equation from:

S. Benić, D. Horvatić, A. Kaushik, and E. A. Vivoda, Phys. Rev. D 108, 074005 (2023).

I. Balitsky, Nucl. Phys. B **463**,99 (1996). Y. V. Kovchegov, Phys. Rev. D **60**,034008 (1999).

$$\mathcal{P}_Y(r_\perp) = \mathrm{e}^{-\frac{r^2}{4}C_{0P}A^{1/3}\ln\left(\frac{1}{r_\perp\Lambda_{QCD}}+e\right)\mathrm{e}^{0.3Y\frac{\exp(-C_1 r_\perp)}{(C_2 r_\perp)^{C_3}}}}$$

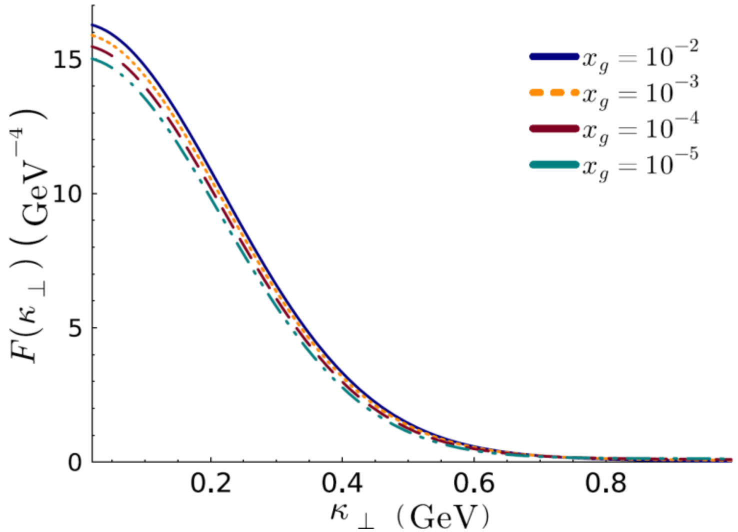
$$\mathcal{O}_Y(r_\perp) = \Lambda e^{-BY}(Cr)^{3(1+\gamma Y)}\ln\left(\frac{1}{r_\perp\Lambda_{QCD}}+e\right)\mathrm{e}^{-\frac{r^2}{4}C_{0O}A^{1/3}\ln\left(\frac{1}{r_\perp\Lambda_{QCD}}+e\right)\mathrm{e}^{0.15Y\frac{\exp(-C_1 r_\perp)}{(C_2 r_\perp)^{C_3}}}}$$

- Maximal Λ (SU(3) constraint) $(1+3\mathcal{P})(1-\mathcal{P})^3 - 6(6\mathcal{P}+(1-\mathcal{P})^2)\mathcal{O}^2 - 3\mathcal{O}^4 \geq 0$

(valid for all $r_\perp$ and $b_\perp$)

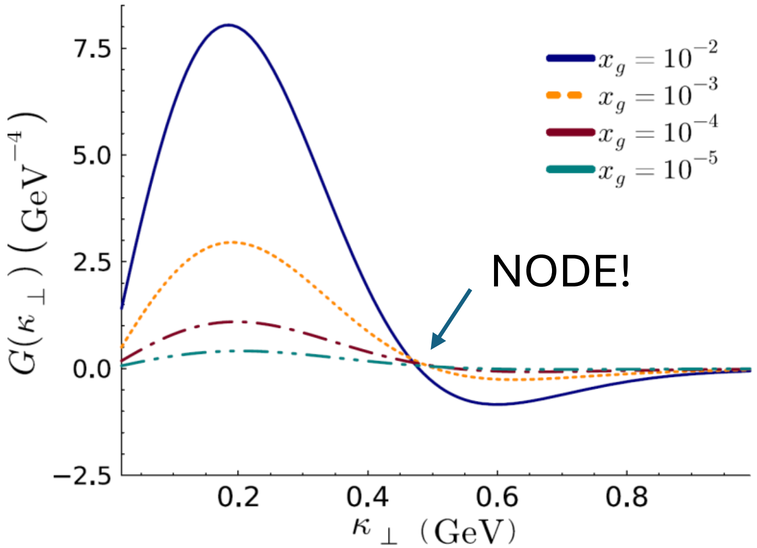
N. Kaiser, Journal of Physics A: Mathematical and General 39, 15287 (2006).

T. Lappi, A. Ramnath, K. Rummukainen, and H. Weigert, Phys. Rev. D 94, 054014 (2016).



$A = 1$

$$Y = \ln\frac{0.01}{x_g}$$



Fitting parameters:

- $C_{0P} = 0.087$ GeV^2
- $C_1 = 0.80$ GeV
- $C_2 = 1$ GeV
- $C_3 = -0.77$
- $B = 0.13$
- $C = 1.21$ GeV
- $\gamma = -0.04$
- $C_{0O} = 0.1$ GeV^2

$\Lambda_{QCD} = 0.2$ GeV

Estimation of nuclear dependence of TSSA

- At initial condition and for outgoing hadron transvesre momenta around saturation scale: $P_{h\perp} \propto \sqrt{(C_{0P} A^{1/3}}$, polarized cross section scales as:

$$\mathrm{d}\Delta\sigma \propto \frac{1}{P_{h\perp}^2 R_A^3} G\left(\frac{P_{h\perp}}{z_2}\right) \int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp \propto A^{-5/6}$$

$$R_A \propto A^{1/3}$$

$$G\left(\frac{P_{h\perp}}{z_2}\right) \propto A^{-1/6}$$

$$\int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp \propto A^{2/3}$$

A. Dumitru, A. Hayashigaki, and J. Jalilian-Marian, Nucl. Phys. A 765, 464 (2006)

- Unpolarized cross section is:

$$\frac{\mathrm{d}\sigma^{unpol}}{\mathrm{d}y_h \mathrm{d}^2 \boldsymbol{P}_{h\perp}} = \int \frac{\mathrm{d}z_2}{z_2^2} D(z_2) x_q f(x_q) F\left(\frac{P_{h\perp}}{z_2}\right) \propto F\left(\frac{P_{h\perp}}{z_2}\right) \propto A^{1/3}$$

$D(z_2)$ is a twist-2 FF, and $f(x_q)$ is a collinear twist-2 PDF

- TSSA scales as:

$$A_N \propto \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}\sigma^{unpol}} \propto A^{-7/6}$$

Same nuclear scaling as in:

Y. V. Kovchegov and M. D. Sievert, PRD **86**, 034028 (2012).

Setup for numerical calculations

- Due to support properties of twist-3 FFs we approximate:

S. Benić and Y. Hatta, Phys. Rev. D 99, 094012 (2019).

$$z_1 \int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp = z_2 \int_0^{\frac{P_{h\perp}}{z_2}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp$$

S. Benić and EAV, in preparation

- Remaining z_1 dependence is of the form to use QCD EOM relation:

$$\frac{\hat{e}_1(z_2)}{z_2} = -\int_{z_2}^{\infty} \frac{\mathrm{d}z_1}{z_1^2} P\left(\frac{1}{1/z_2 - 1/z_1}\right) \mathrm{Re}\hat{E}_F(z_1, z_2)$$

Y. Koike, D. Pitonyak, Y. Takagi, and S. Yoshida, Phys. Lett. B 752, 95 (2016).

K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, and M. Schlegel, Phys. Rev. D 93, 054024 (2016).

- For the profile function we take the Gaussian form: $T(b_\perp) = e^{-b^2/R_A^2}$

$$\int_0^\infty \Delta_\perp^3 T^2(\Delta_\perp) \mathrm{d}\Delta_\perp = 2\pi^2$$

- Polarized cross section used for numerics is thus:

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = M_N \frac{N_C^2}{N_C^2 - 1} \frac{\epsilon^{ij} S_{\perp i} P_{hj}}{P_{h\perp}^3} \frac{1}{R_A^3} \int_{z_{min}}^1 \frac{\mathrm{d}z}{z} \hat{e}_1(z, P_{h\perp}^2) x_q h_1(x_q, P_{h\perp}^2)\, G\left(\frac{P_{h\perp}}{z}\right) \int_0^{\frac{P_{h\perp}}{z}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp$$

➢ All FFs and PDFs evaluated at the transverse momenta of produced hadron! Same in unpolarized case!

Used PDFs and FFs:

- Collinear twist-2 PDF: CJ12 dataset

J. J. Ethier et. All,
Phys. Rev. Lett. 119, 132001 (2017).

- Twist-2 fragmentation function: JAM17 global analysis result

J. F. Owens et All. (JAM),
Phys. Rev. D 87, 094012 (2013).

- Transversity PDF: JAM22 global analyis result

L. Gamberg et. All, (JAM),
Phys. Rev. D 106, 034014 (2022).

Fragmentation function $\hat{e}_1(z)$

- Chiral odd FF usually studied in the context of double spin asymmetries

Y. Koike, D. Pitonyak, Y. Takagi,and S. Yoshida, Phys. Lett. B 752, 95 (2016).
R. L. Jaffe and X.-D. Ji, Phys. Rev. Lett. 71, 2547 (1993).
F. Yuan, Phys. Lett. B 589, 28 (2004).
K. Kanazawa, A. Metz, D. Pitonyak, and M. Schlegel, Phys. Lett. B 742, 340 (2015).

- Can be used to measure transversity distribution in processes like: $\vec{l}p^{\uparrow} \to hX$

R. L. Jaffe and X.-D. Ji,
Phys. Rev. Lett. 71, 2547 (1993).

Mass of constituent quark

- Chiral quark model: $\hat{e}_1(z) = \frac{z}{1-z}\frac{m_q}{M_N}D(z)$

X. Ji and Z. K. Zhu,
arXiv:hep-ph/9402303.

- Recent input by B. Bauer, D. Pitonyak, and C. Shay, Phys. Rev. D 107, 014013 (2023).

$$\hat{e}_1(z) = \begin{cases} \tilde{H}(z) \\ 0 \\ -\tilde{H}(z) \end{cases}$$

$$\tilde{H}(z) = \frac{M_N}{M_h} z \int_z^{\infty} \frac{\mathrm{d}z_1}{z_1^2} \frac{\mathrm{Im}\,\hat{E}_F(z, z_1)}{1/z - 1/z_1}$$

JAM22!

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  K. Kanazawa, A. Metz, D. Pitonyak, and M. Schlegel, Phys. Lett. B 742, 340 (2015).

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- Chiral quark model: $\hat{e}_1(z) = \frac{z}{1-z}\frac{m_q}{M_N}D(z)$ (Mass of constituent quark: m_q)

  X. Ji and Z. K. Zhu, arXiv:hep-ph/9402303.

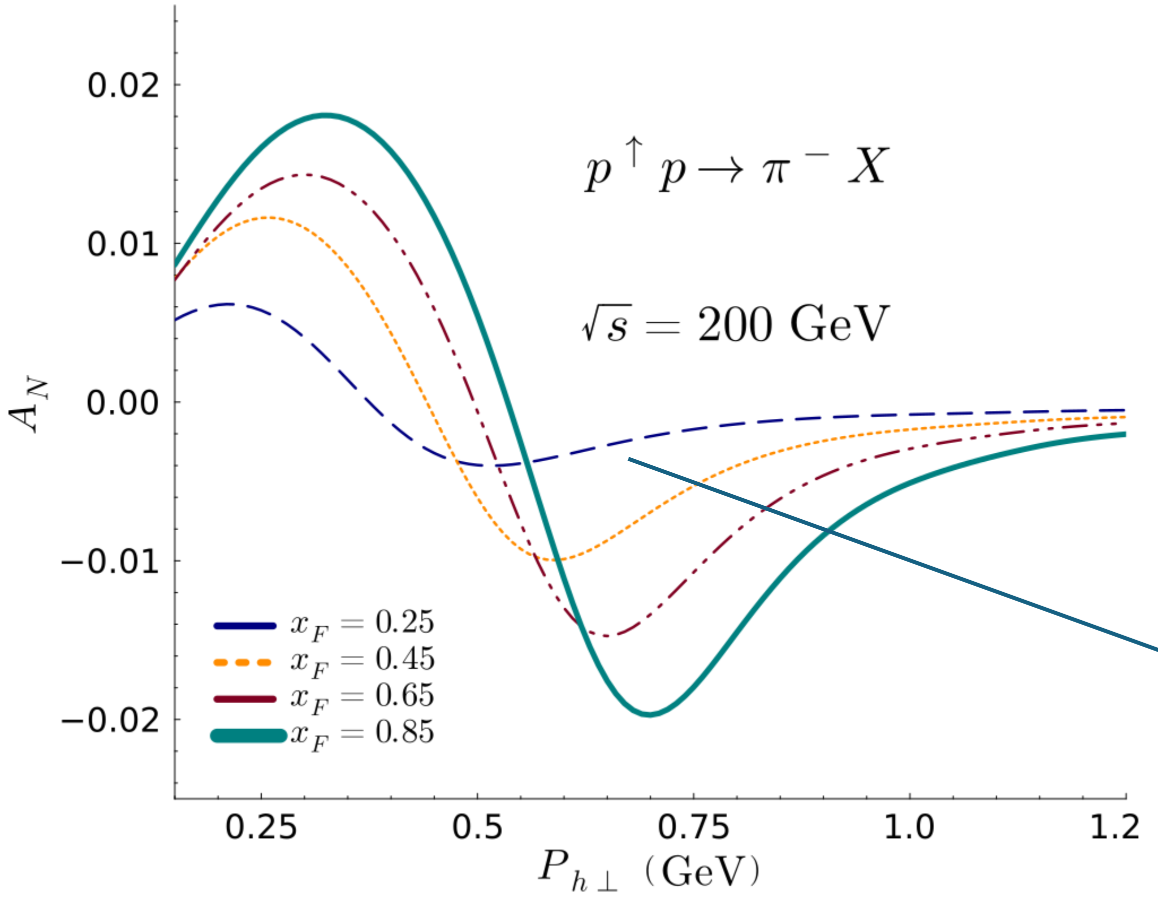
- Recent input by B. Bauer, D. Pitonyak, and C. Shay, Phys. Rev. D 107, 014013 (2023).

$$\hat{e}_1(z) = \begin{cases} \widetilde{H}(z) \\ 0 \\ -\widetilde{H}(z) \end{cases}$$

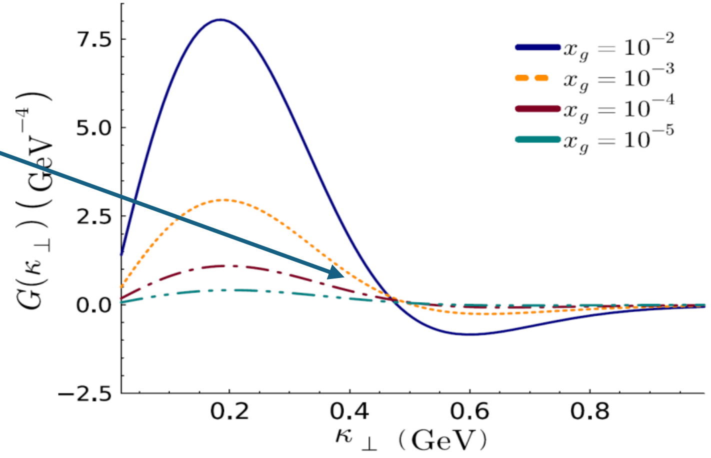
$$\widetilde{H}(z) = \frac{M_N}{M_h} z \int_z^{\infty} \frac{\mathrm{d}z_1}{z_1^2} \frac{\mathrm{Im}\,\hat{E}_F(z,z_1)}{1/z - 1/z_1}$$

JAM22!

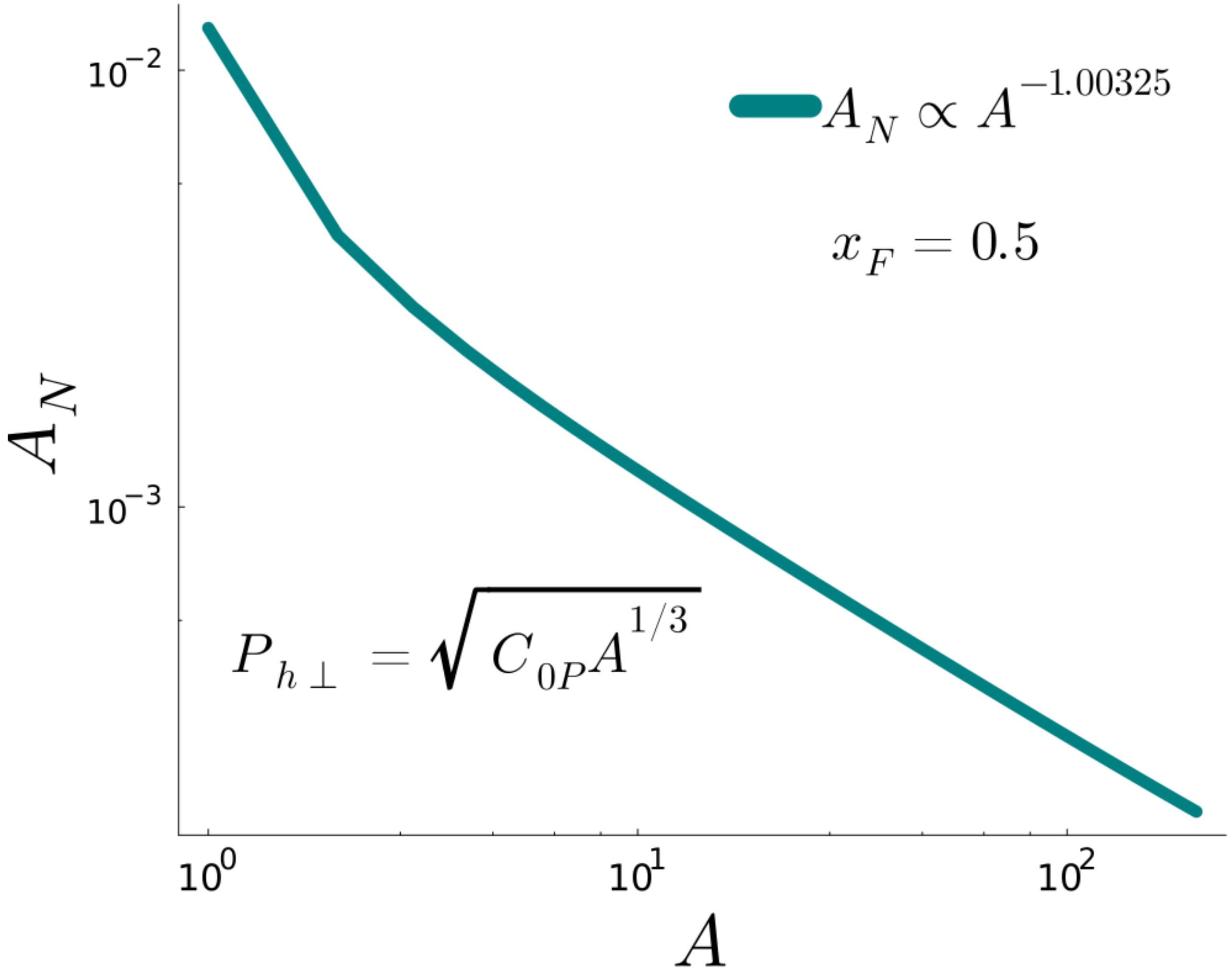
Preliminarly results for $p^{\uparrow}p \to \pi^{-}X$



- TSSA grows as a function of Feynamn-x
- This mechanism is significant for low transverse momenta of produced hadron
  - Smaller $P_{h\perp}$ than STAR and PHENIX
- TSSA obeys a sign change! Consequence of the node in the Odderon (small change of node position is consequence of $\kappa_{\perp} = P_{h\perp}/z$)



Preliminarly results for nuclear dependence



- Obtained nuclear dependence scales as:

$$A_N \propto A^{-1.00325}$$

- Since fitting parameter C has dimension of GeV, at higher rapidities the Odderon scales as:

$$G(\kappa_\perp) \propto \frac{R_A^2}{P_{h\perp}^{5+3\gamma Y}} \propto A^{-1/6-(1/2)\gamma Y}$$

- For RHIC energies ($x_g \approx 10^{-4}$) TSSA scales as:

$$A_N \propto A^{-1.07}$$

On RHIC energies the suppression is not as strong as $A^{-7/6}$

Summary:

1. Completely new contribution to TSSA coming from the combination of the real part of genuine twist-3 FF and the Odderon

2. Cross section calculated in small-Δ approximation

3. TSSA changes sign as a function of $P_{h\perp}$

4. TSSA is largest for $P_{h\perp} < Q_S$ (saturation scale)

5. Numerically confirmed nuclear scaling