Parton distribution functions in tensor-polarized spin-1 hadrons

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PacificSpin2024, November 12, 2024

Outline

- Tensor-polarized disitributions
- Gluon transversity: one leading-twist PDF
- Twist-3 correction in proton-deuteron Drell-Yan process
- Wandzura-Wilczek (WW) and Lorentz Invariant Relation (LIR) relations for PDFs and FFs

Spin parameters for spin-1 hadrons

See Professor Bressan's talk for vector-polarized spin-1 deuteron.

Spin parameters for spin-1 hadrons

- Vector polarization *S* (3 parameters): same as the case of proton
- Tensor polarization *T* (5 Parameters): unique in spin-1 hadrons

Covariant form of the matrix *T*:

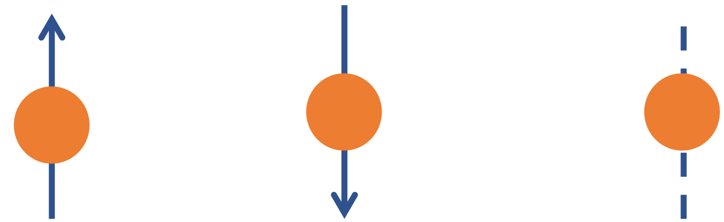
$$T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3}S_{LL}\frac{(P^+)^2}{M^2}\bar{n}^\mu\bar{n}^\nu - \frac{2}{3}S_{LL}(\bar{n}^{\{\mu}n^{\nu\}} - g_T^{\mu\nu}) + \frac{1}{3}S_{LL}\frac{M^2}{(P^+)^2}n^\mu n^\nu \right.$$
$$\left. + \frac{P^+}{M}\bar{n}^{\{\mu}S_{LT}^{\nu\}} - \frac{M}{2P^+}n^{\{\mu}S_{LT}^{\nu\}} + S_{TT}^{\mu\nu}\right],$$

$$n^\mu = \frac{1}{\sqrt{2}}(\,1,\,0,\,0,\,-1\,),\quad \bar{n}^\mu = \frac{1}{\sqrt{2}}(\,1,\,0,\,0,\,1\,)$$

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000)

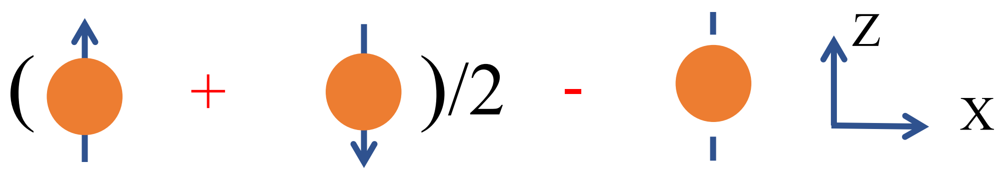
Interpretation for parameters of tensor polarization

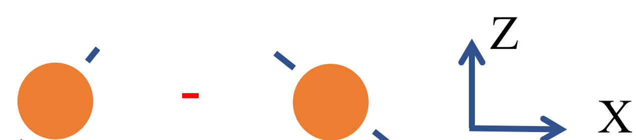
Three spin projections along z axis
for spin-1 hadrons:

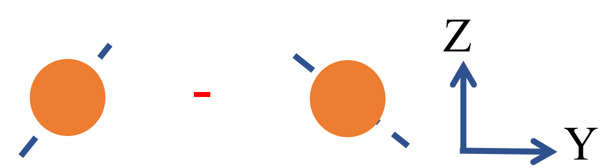


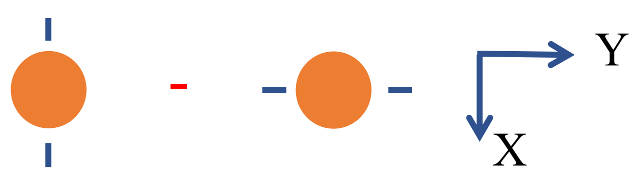
spin up spin down spin projection of 0

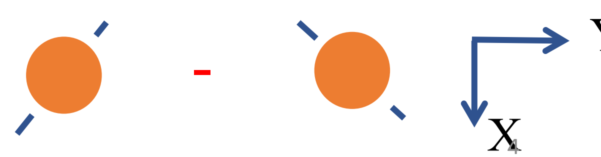
In comparison with a spin-1 hadron, the spin projection of a spin-1/2 hadron can not be 0, thus, one can infer that the tensor polarization must be related to the state with a spin projection of 0.

$S_{LL} =$ 

$S_{LT}^{x} =$ 

$S_{LT}^{y} =$ 

$S_{TT}^{xx} =$ 

$S_{TT}^{xy} =$ 

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000)

Quark correlation function of spin-1 hadrons

$$\Phi_{ij}^{[c]}(k,P,S,T) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi} \times \langle P,S,T \,|\, \bar{\psi}_j(0)\, W^{[c]}(0,\xi)\psi_i(\xi) \,|\, P,\, S,T \rangle$$

A. Bacchetta and P. Mulders, PRD 62, 114004 (2000).
S. Kumano and Qin-Tao Song, PRD 103 (2021) 014025.

Terms of A_{13} –A_{20} are given by Bacchetta and Mulders. The new terms of B_{21} –B_{52} terms are dependent on the lightcone vector *n* due to the gauge link.

$$\begin{aligned}
\Phi(k,P,T\,|n) =\; & \frac{A_{13}}{M} T_{kk} + \frac{A_{14}}{M^2} T_{kk} \not{P} + \frac{A_{15}}{M^2} T_{kk} \not{k} + \frac{A_{16}}{M^3} \sigma_{Pk} T_{kk} + A_{17} T^{k\nu}\gamma_\nu + \frac{A_{18}}{M}\sigma_{\nu P} T^{k\nu} + \frac{A_{19}}{M}\sigma_{\nu k} T^{k\nu} \\
& + \frac{A_{20}}{M^2}\varepsilon^{\mu\nu Pk}\gamma_\mu\gamma_5 T_{\nu k} + \frac{B_{21}M}{P\cdot n} T_{kn} + \frac{B_{22}M^3}{(P\cdot n)^2} T_{nn} + \frac{B_{23}}{P\cdot nM}\varepsilon^{\mu kPn} T_{\mu k}(i\gamma_5) + \frac{B_{24}M}{(P\cdot n)^2}\varepsilon^{\mu kPn} T_{\mu n}(i\gamma_5) + \frac{B_{25}}{P\cdot n}\not{n} T_{kk} \\
& + \frac{B_{26}M^2}{(P\cdot n)^2}\not{n} T_{kn} + \frac{B_{27}M^4}{(P\cdot n)^3}\not{n} T_{nn} + \frac{B_{28}}{P\cdot n}\not{P} T_{kn} + \frac{B_{29}M^2}{(P\cdot n)^2}\not{P} T_{nn} + \frac{B_{30}}{P\cdot n}\not{k} T_{kn} + \frac{B_{31}M^2}{(P\cdot n)^2}\not{k} T_{nn} + \frac{B_{32}M^2}{P\cdot n}\gamma_\mu T^{\mu n} \\
& + \frac{B_{33}}{P\cdot n}\varepsilon^{\mu\nu Pk}\gamma_\mu\gamma_5 T_{\nu n} + \frac{B_{34}}{P\cdot n}\varepsilon^{\mu\nu Pn}\gamma_\mu\gamma_5 T_{\nu k} + \frac{B_{35}M^2}{(P\cdot n)^2}\varepsilon^{\mu\nu Pn}\gamma_\mu\gamma_5 T_{\nu n} + \frac{B_{36}}{P\cdot nM^2}\varepsilon^{\mu kPn}\gamma_\mu\gamma_5 T_{kk} \\
& + \frac{B_{37}}{(P\cdot n)^2}\varepsilon^{\mu kPn}\gamma_\mu\gamma_5 T_{kn} + \frac{B_{38}M^2}{(P\cdot n)^3}\varepsilon^{\mu kPn}\gamma_\mu\gamma_5 T_{nn} + \frac{B_{39}}{(P\cdot n)^2}\not{n}\gamma_5 T_{\mu k}\varepsilon^{\mu kPn} + \frac{B_{40}M^2}{(P\cdot n)^3}\not{n}\gamma_5 T_{\mu n}\varepsilon^{\mu kPn} \\
& + \frac{B_{41}}{P\cdot nM}\sigma_{Pk} T_{kn} + \frac{B_{42}M}{(P\cdot n)^2}\sigma_{Pk} T_{nn} + \frac{B_{43}}{P\cdot nM}\sigma_{Pn} T_{kk} + \frac{B_{44}M}{(P\cdot n)^2}\sigma_{Pn} T_{kn} + \frac{B_{45}M^3}{(P\cdot n)^3}\sigma_{Pn} T_{nn} + \frac{B_{46}}{P\cdot nM}\sigma_{kn} T_{kk} \\
& + \frac{B_{47}M}{(P\cdot n)^2}\sigma_{kn} T_{kn} + \frac{B_{48}M^3}{(P\cdot n)^3}\sigma_{kn} T_{nn} + \frac{B_{49}M}{P\cdot n}\sigma_{\mu n} T^{\mu k} + \frac{B_{50}M^3}{(P\cdot n)^2}\sigma_{\mu n} T^{\mu n} + \frac{B_{51}M}{P\cdot n}\sigma_{\mu P} T^{\mu n} + \frac{B_{52}M}{P\cdot n}\sigma_{\mu k} T^{\mu n}, \quad (20)
\end{aligned}$$

Tensor-polarized PDFs and FFs

TMDs:

There are 40 Tensor-polarized TMD PDFs (FFs) in spin-1 hadrons.
10 twist-2 TMDs; 20 twist-3 TMDs; 10 twist-4 TMDs.

A. Bacchetta and P. J. Mulders, PRD 62 (2000), 114004
K. B. Chen, W. H. Yang, S. Y. Wei and Z. T. Liang, PRD 94(2016), 034003
S. Kumano and Qin-Tao Song, PRD 103 (2021) 014025.

Collinear PDFs:

$$\Phi(x,P,T)=\frac{1}{2}\left[S_{LL}\bar{\not{n}}\,f_{1LL}(x)+\frac{M}{P^+}S_{LL}\,e_{LL}(x)+\frac{M}{P^+}\not{S}_{LT}\,f_{LT}(x)+\frac{M^2}{(P^+)^2}S_{LL}\not{n}\,f_{3LL}(x)\right]$$

Twist 2: f_{1LL}; Twist 3: e_{LL}, f_{LT}; Twist 4: f_{3LL}.

Collinear FFs:

$$\Delta(z,P_h,T)=\frac{1}{z}\Big\{S_{LL}\not{n}F_{1LL}(z)+\frac{M}{P_h^-}\left[\not{S}_{LT}F_{LT}(z)+S_{LL}E_{LL}(z)\right]+\frac{M^2}{(P_h^-)^2}S_{LL}\bar{\not{n}}F_{3LL}(z)$$

$$\boxed{+\sigma^{i+}S_{LT,i}H_{1LT}+\frac{M}{P_h^-}\left[S_{LL}\sigma^{-+}H_{LL}(z)+\gamma_5\gamma_i\epsilon_T^{ij}S_{LT,j}G_{LT}\right]+\frac{M^2}{(P_h^-)^2}\sigma^{i-}S_{LT,i}H_{3LT}\Big\}}$$

T-odd FFs

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571
L. L. Frankfurt and M. I. Strikman, NP A405 (1983) 557.
J. P. Ma, C. Wang and G. P. Zhang, arXiv:1306.6693 [hep-ph].
X. D. Ji, PRD 49 (1994), 114-124

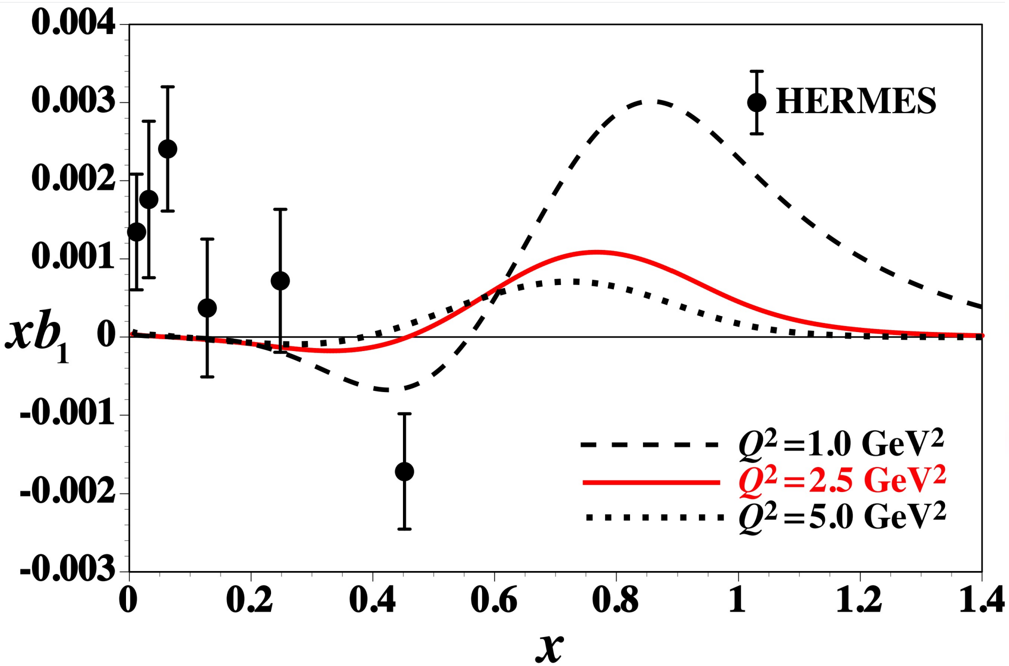
Twist 2: F_{1LL}, H_{1LT};
Twist 3: E_{LL}, F_{LT}, H_{LL}, G_{LT};
Twist 4: f_{3LL}, H_{3LT}

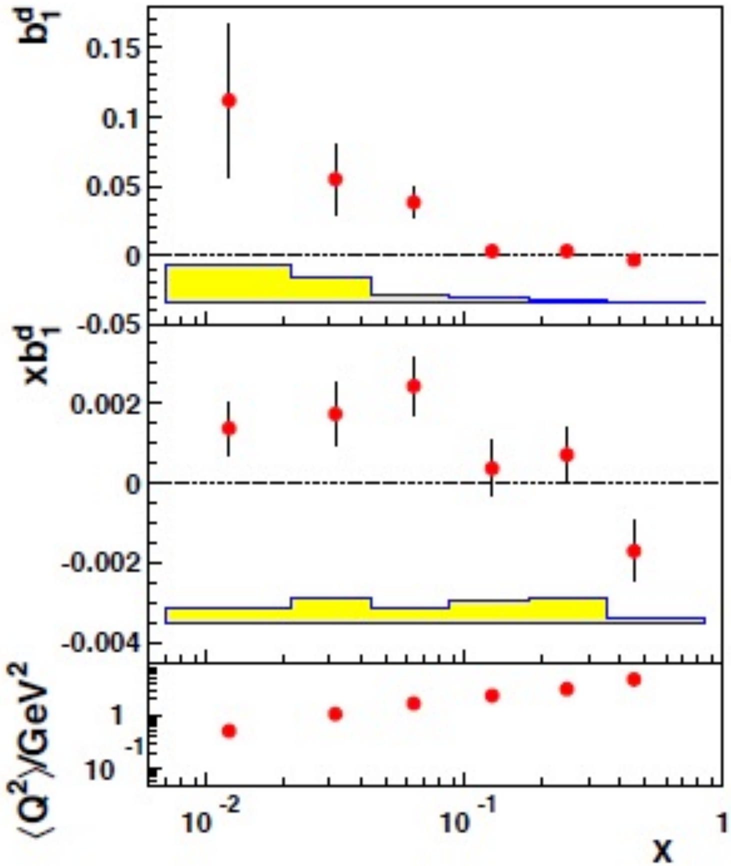
See recent review on the tensor-polarized distributions: S. Kumano, EPJA 60 (2024), 205.

First experimental measurement of b_1

Hermes data show that b_1 (f_{1LL}) is not as small as the theoretical predictions based on that deuteron is a S-D mixture.

H.Khan and P. Hoodhoy, PRC 44 (1991) 1219.
A. Y. Umnikov, PLB 391(1997), 177-184.
A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.
W. Cosyn, *et al* PRD 95 (2017), 074036.





Hermes measurements of b_1

Hermes measurements are much larger than theoretical predictions.

The large b_1 could indicate the existence of exotic structures such as six-quark and hidden-color components in deuteron.

G. A. Miller, PRC 89(2014), 045203.

Puzzle in tensor-polarized structure of deuteron

To solve this puzzle, b_1 (f_{1LL}) will be measured at JLab(DIS), Fermilab (Drell-Yan process), and NICA(See Professor Guskov's talk) in the near future!

PR12-13-011

Full approval in 2023

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

E12-13-011: The b_1 experiment, there is also a tensor meeting everyweek at JLab.

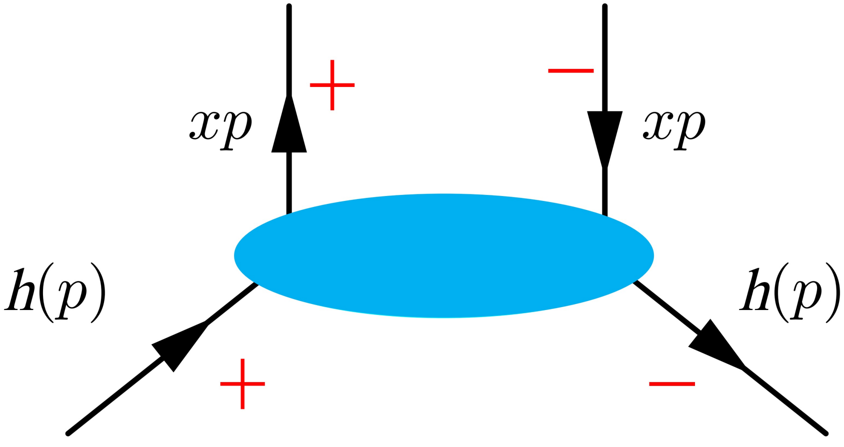
In 2023, the first conference of "Tensor spin observables" was held at ECT*, Italy

E1039 experiment at Fermilab: Drell-Yan process with unpolarizd 120 GeV proton beam and tensor-polarized deuteron target.

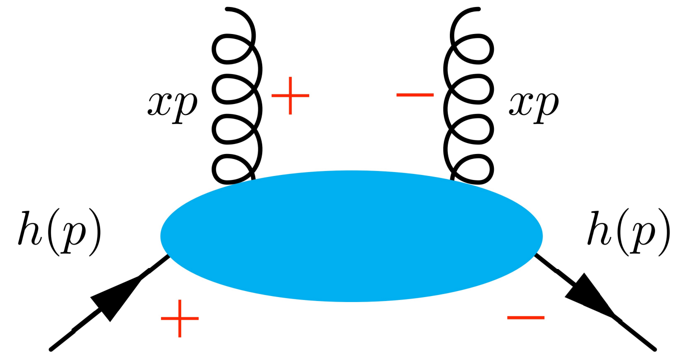
The SpinQuest Collaboration, arXiv:2205.01249v1.
D. Keller, D. Crabb, and D. Day, Nucl. Instrum. Meth. A 981(2020), 164504.
J. Clement and D. Keller, Nucl. Instrum. Meth. A 1050 (2023), 168177

Gluon transversity of deuteron

Gluon transversity distribution



Quark transversity in the proton



Gluon transversity in the proton

Gluon transversity can not exist in the proton due to the angular momentum conservation.

$$\lambda_1+\Lambda_2/2=\lambda_2+\Lambda_1/2$$

λ_1 and λ_2: gluon helicities, Λ_1 and Λ_2: proton helicities

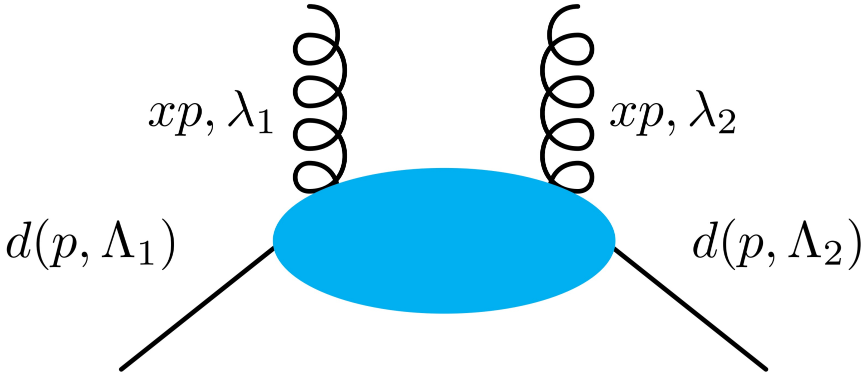
Gluon transversity exists in the hadrons with spin ≥ 1 such as deuteron.

Why gluon transversity is interesting?

Angular momentum conservation

$$\lambda_1+\Lambda_2=\lambda_2+\Lambda_1$$

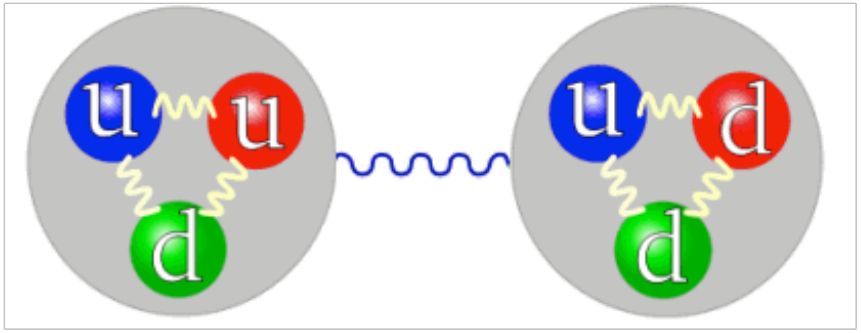
R. L. Jaffe and A. Manohar, PLB 223 (1989), 218-224.
M. Nzar and P. Hoodbhoy, PRD 45 (1992), 2264-2268.



Gluon transversity in the deuteron

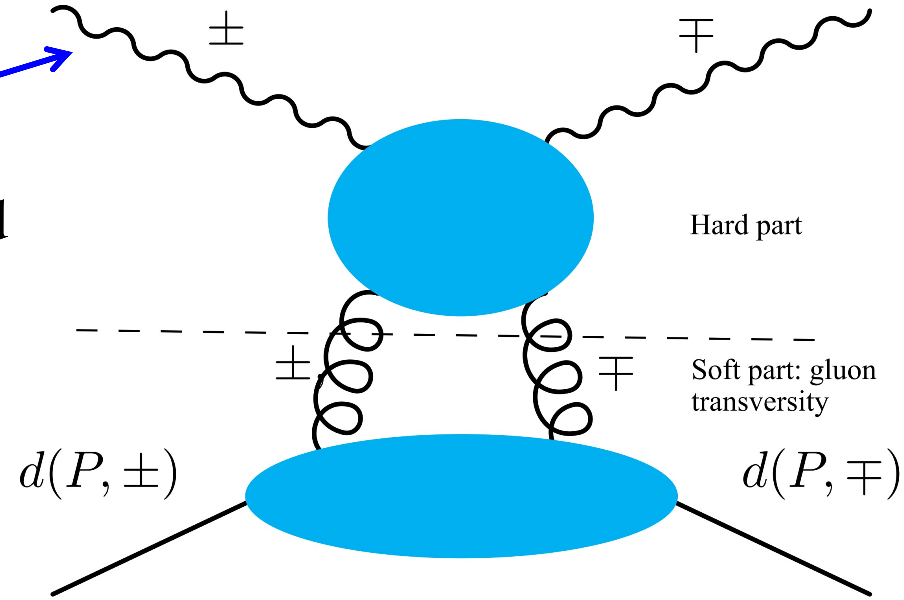
① New distribution which does not exist in spin ½ hadrons.
② Twist-2, important physical quantity, dominant in the cross section.
③ It could be related to non-nucleonic components such as ΔΔ in the deuteron.

The deuteron is often considered as a loosely bounded system of proton and neutron, and gluon transversity does not exist in nucleons.



Experimental measurement by DIS

Off-shell photon



Gluon transversity can be measured by DIS at JLab.

R. L. Jaffe and A. Manohar, PLB 223 (1989) 218.
J. P. Ma, C. Wang and G. P. Zhang, arXiv:1306.6693.

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins

Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan

Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

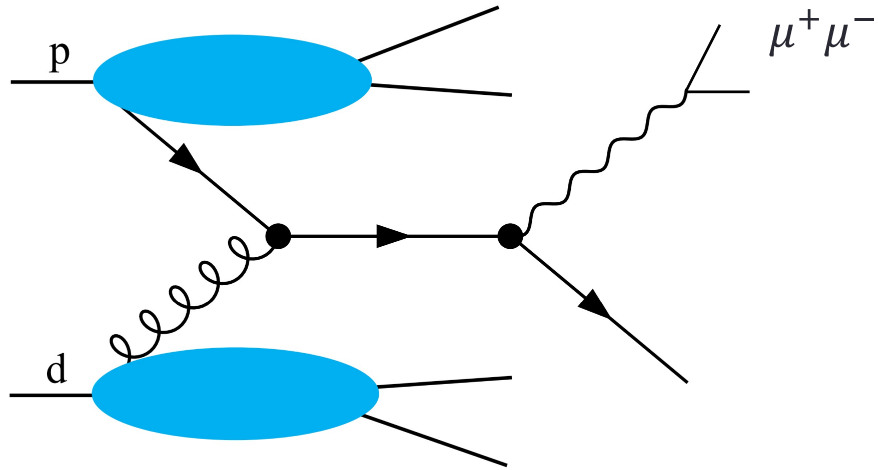
Oak Ridge National Laboratory, Oak Ridge, TN 37831

A Letter of Intent to JLab

arXiv:1803.11206v1

Gluon transversity in proton-deuteron collisions?

Proton-deuteron
Drell-Yan process



p+d$\rightarrow\mu^+\mu^-$+X

Proton is
unpolarized

Lineally polarized
deuteron

$$d\sigma_\tau=[d\sigma(\cdot\,\varepsilon_x)-d\sigma(\cdot\,\varepsilon_y)]$$

Unpolarized proton PDF will contribute.

➔ Deuteron gluon transversity also appears.

The cross section for Drell-Yan process

Amplitude squared of the subprocess.

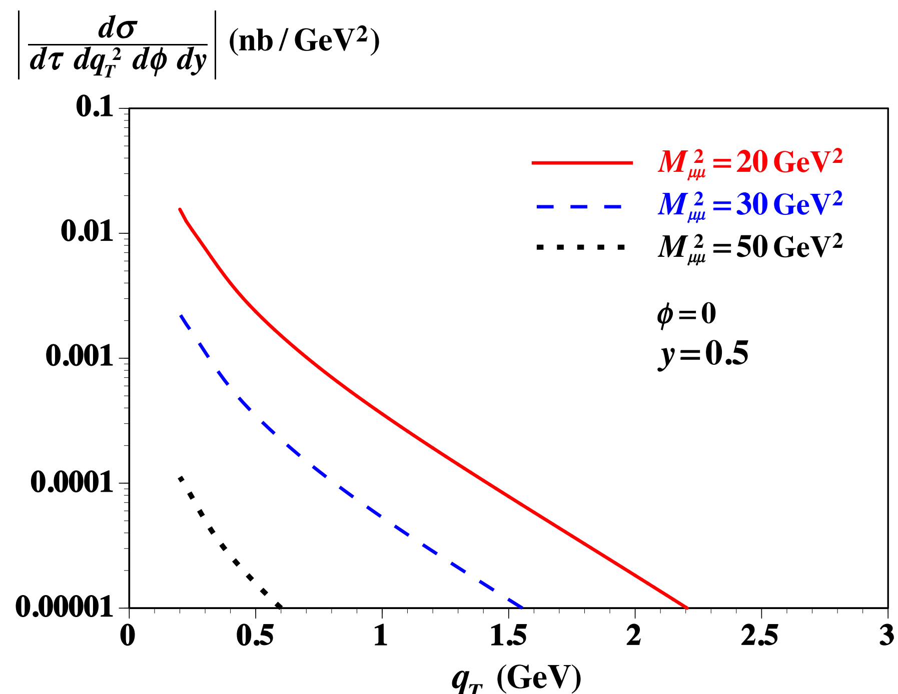
$$\frac{d\sigma_\tau}{dQ^2 d\phi dy dp_T^2} = \int dx_a q(x_a) \Delta_T g(x_b) \frac{\alpha}{96\pi^3 Q^2 \hat{s}} \frac{1}{x_a s - (Q^2 - u)} |\bar{M}|^2$$

Unpolarized PDF in proton, well determined by other experiments.

Gluon transversity in deuteron can be obtained by this Drell-Yan process.

Numerical analysis of Drell-Yan process at Fermilab

p_T dependence of the cross section with fixed $y, \tau = \frac{M_{\mu\mu}^2}{s}$ and ϕ



S. Kumano and Qin-Tao Song, PRD 101 (2020), 054011; PRD 101 (2020), 094013.

Experimental measurement of gluon transversity

The measurement of gluon transversity by Drell-Yan process is discussed in the proposal of Fermilab E1039 experiment:

The Transverse Structure of the Deuteron with Drell-Yan

The SpinQuest Collaboration[a]

We propose to measure neutron and deuteron transversity TMDs. The quark transversity distributions of the nucleon are decoupled from the deuteron gluon transversity in the Q^2 evolution due to the chiral-odd property in the transversely-polarized target. The gluon transversity TMD only exists for targets of spin greater or equal to 1 and does not mix with quark distributions at leading twist, thereby providing a particularly clean probe of gluonic degrees of freedom. This experiment would be the first of its kind and would probe the gluonic structure of the deuteron, investigating exotic glue contributions in the nucleus not associated with individual nucleons. This experiment can be performed with the SpinQuest polarized target recently assembled for experiment E1039 and the spectrometer already in place in NM4. This new experimental setup would require very minimal modification to the target system and no modification to the detector package. An additional

The Transverse Structure of the Deuteron with Drell-Yan, arXiv:2205.01249v1

Tensor-polarized deuteron target is prepared at Fermilab.

D. Keller, D. Crabb, and D. Day, Nucl. Instrum. Meth. A 981, 164504 (2020).
J. Clement and D. Keller, Nucl. Instrum. Meth. A 1050 (2023), 168177

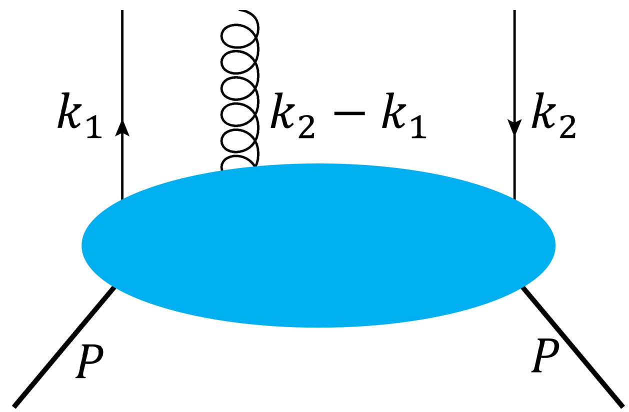
Twist-3 correction in proton-deuteron Drell-Yan process

Proton-deuteron Drell-Yan process

Cross section of proton-deuteron Drell-Yan process with polarized deuteron target, and its measurement is possible at Fermilab.

Cross section
- unpolarized part
- vector-polarized part
- tensor-polarized part

$\}$ Same as proton-proton Drell-Yan process



Quark-gluon-quark distributions:

$$(\Phi_G^\alpha)_{ij}(x_1, x_2) = \int \frac{d\xi_1^-}{2\pi} \frac{d\xi_2^-}{2\pi} e^{ix_1 P^+ \xi_1^-} e^{i(x_2 - x_1)P^+ \xi_2^-} \langle P, T \,|\, \bar{\psi}_j(0)\, g\, G^{+\alpha}(\xi_2^-)\, \psi_i(\xi_1^-) \,|\, P, T \rangle,$$

$$\Phi_G^\alpha(x_1, x_2) = \frac{M}{2} \Big\{ i\gamma^\alpha \not{n} E(x_1, x_2) + \Big[iS_{LT}^\alpha \not{n} F_{LT}(x_1, x_2) - \tilde{S}_{LT}^\alpha \gamma_5 \not{n} G_{LT}(x_1, x_2)$$
$$+ iS_{LL}\gamma^\alpha \not{n} H_{LL}^\perp(x_1, x_2) + iS_{TT}^{\alpha\mu}\gamma_\mu \not{n} H_{TT}(x_1, x_2) \Big] \Big\}.$$

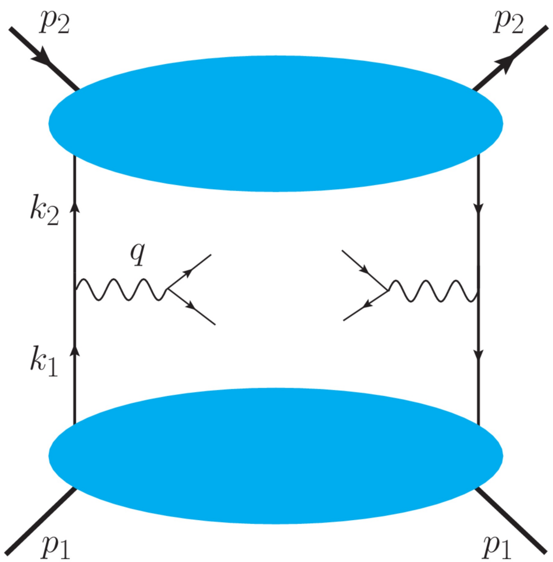
Kinematical distributions:

$$\int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, T \,|\, \bar{\psi}_j(0)\, i\partial^\alpha\, \psi_i(\xi^-) \,|\, P, T \rangle$$
$$= \frac{M}{2} \Big\{ ih_1^{\perp(1)}(x)\gamma_T^\alpha \not{n} + \Big[f_{1LT}^{(1)}(x)\, S_{LT}^\alpha \not{n} + g_{1LT}^{(1)}(x)\, \tilde{S}_{LT}^\alpha \gamma_5 \not{n} - h_{1LL}^{\perp(1)}(x) S_{LL} \sigma^{\alpha\mu} \bar{n}_\mu + h_{1TT}^{\prime(1)}(x)\, S_{TT}^{\alpha\beta} \sigma_{\beta\mu} \bar{n}^\mu \Big] \Big\},$$

Hadronic tensor contributed by quark-quark distributions

The higher-twist correction of proton-proton Drell-Yan process has been well investigated. Here, we use the moehtod that was developed by J. P. Ma *et al.* to study the proton-deuteron Drell-Yan process.

J. P. Ma G. P. Zhang, JHEP 02 (2015) 163
A. P. Chen, J. P. Ma, G. P. Zhang, PRD 95 (2017), 074005
M. C. Hu, J. P. Ma, Z. Y. Pang, G. P. Zhang, PRD 105 (2022), 014009



The contribution of quark-quark distributions

$$d(p_1) + p(p_2) \rightarrow \mu^-(l_1)\mu^+(l_2) + X,$$

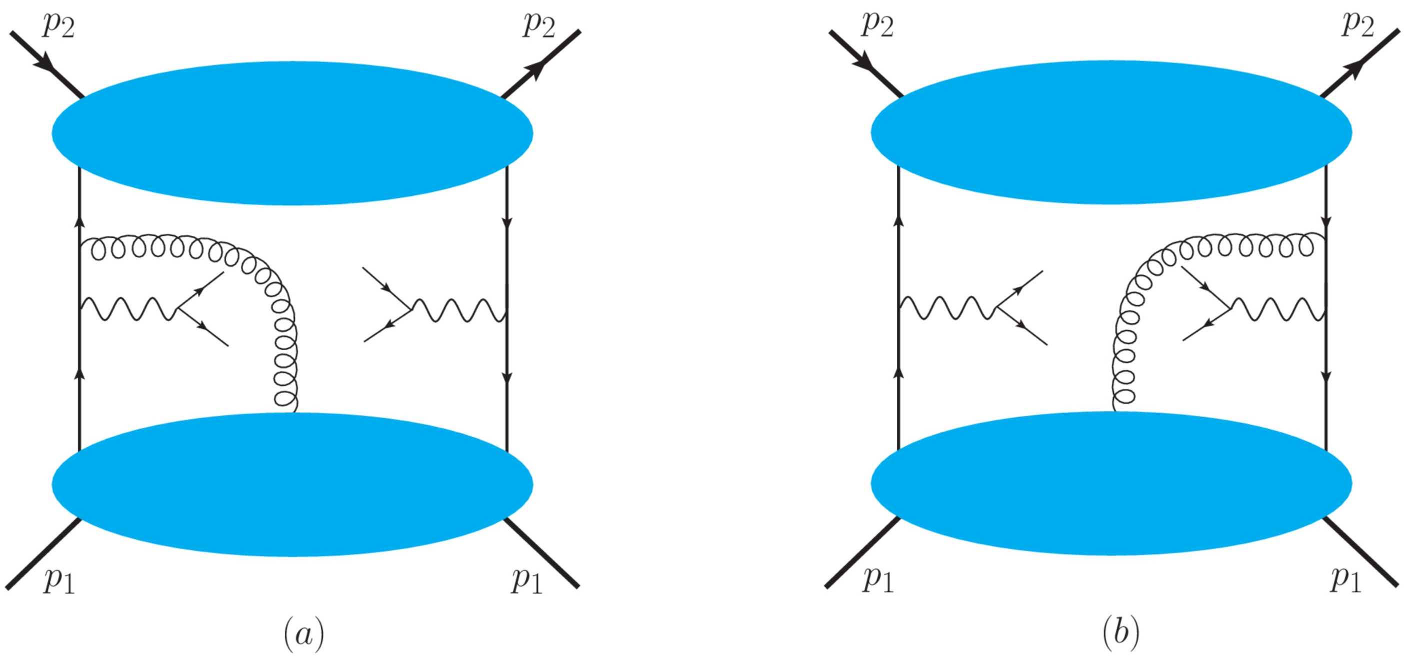
Twist-2 contribution:

$$W_1^{\mu\nu}\Big|_{\text{twist 2}} = -\frac{1}{N_c}\delta^2(q_T) f_1^{\bar{q}}(y) f_{1LL}^{q}(x) S_{LL} g_T^{\mu\nu},$$

Twist-3 contribution:

$$W_1^{\mu\nu}\Big|_{\text{twist 3}} = \frac{M}{N_c} f_1^{\bar{q}}(y)\left[\delta^2(q_T)\frac{p_2^{\{\mu}S_{LT}^{\nu\}}}{p_1\cdot p_2} f_{LT}^{q}(x) + g_T^{\mu\nu}\frac{\partial\delta^2(q_T)}{\partial q_T^{\rho}} S_{LT}^{\rho} f_{1LT}^{(1)q}(x)\right],$$

The contribution of quark-gluon-quark distributions



The contribution of quark-gluon-quark distributions

$$W_2^{\mu\nu} = -\frac{M}{N_c} f_1^{\bar{q}}(y)\delta^2(q_T)\frac{p_1^{\{\mu} S_{LT}^{\nu\}}}{y p_1 \cdot p_2}\mathcal{P}\int dy_1 \frac{F_{LT}^q(x,y_1)+G_{LT}^q(x,y_1)}{x-y_1},$$

The quark-gluon-quark distributions can be replaced with the twist-3 quark-quark distributions by the QCD e.o.m relation.

Hadronic tensor and gauge invarance

The complete expression of hadronic tensor(up to twist 3):

Unpolarized PDF in proton

$$W^{\mu\nu} = \frac{1}{N_c} f_1^{\bar{q}}(y) \Big\{ -\delta^2(q_T) S_{LL} g_T^{\mu\nu} f_{1LL}^q(x) + \boxed{\frac{M}{p_1\cdot p_2}\delta^2(q_T)\left[p_2^{\{\mu} S_{LT}^{\nu\}} f_{LT}^q(x) - p_1^{\{\mu} S_{LT}^{\nu\}} \frac{1}{y}\left(x f_{LT}^q(x) - f_{1LT}^{(1)q}(x)\right)\right] + M g_T^{\mu\nu} \frac{\partial \delta^2(q_T)}{\partial q_T^\rho} S_{LT}^\rho f_{1LT}^{(1)q}(x)} \Big\},$$

Twist-2 contribution

Twist-3 correction

The $U(1)$-gauge invariance is satisfied for both the twist-2 and twist-3 contributions in the hadronic tensor.

$$\int d^2 q_T W^{\mu\nu} q_\mu \mathcal{F}(q_T) = 0,$$

where $\mathcal{F}(q_T)$ is a test function of q_T.

S. Y. Qiao and Q. T. Song, arXiv:2410.13225 [hep-ph].

Cross section of proton-deuteron Drell-Yan process

$$\frac{d\hat{\sigma}}{dQ^2 d\Omega} = \sum_q \frac{\alpha^2 e_q^2}{4N_c Q^2} \int dx dy \delta(xyS - Q^2) \Big\{ S_{LL} \left[f_{1LL}^q(x) f_1^{\bar{q}}(y) + (q \leftrightarrow \bar{q}) \right] (1 + \cos^2\theta)$$

$$+ |S_{LT}| \frac{M}{Q} \left[\boxed{\left(2x f_{LT}^q(x) - f_{1LT}^{(1)q}(x)\right)} f_1^{\bar{q}}(y) + (q \leftrightarrow \bar{q}) \right] \sin(2\theta) \cos\hat{\phi} \Big\},$$

Combination of two twist-3 PDFs of deuteron

After the integration over the solid angle, there is only twist-2 contribution.

$$\frac{d\hat{\sigma}}{dQ^2} = S_{LL} \sum_q \frac{4\pi\alpha^2 e_q^2}{3N_c Q^2} \int dx dy \delta(xyS - Q^2) \left[f_{1LL}^q(x) f_1^{\bar{q}}(y) + (q \leftrightarrow \bar{q}) \right],$$

S. Hino and S. Kumano, PRD 59 (1999), 094026.
S. Kumano and Q. T. Song, PRD 94 (2016), 054022.

In order to separate f_{LT} from $f_{1LT}^{(1)}$ in the cross section, we define the weighted differential cross section,

$$\frac{d\hat{\sigma}\langle\mathcal{F}_1\rangle}{dQ^2 d\Omega} = \frac{\alpha^2 e_q^2}{4SQ^4} \int d^4q \, \delta(q^2 - Q^2) W^{\mu\nu} L_{\mu\nu} \mathcal{F}_1(q_T),$$

$$\mathcal{F}_1(q_T) = q_T \cdot S_{LT}/Q$$

Cross section of proton-deuteron Drell-Yan process

We find that only $f_{1LT}^{(1)}$ surivives in the weighted differential cross section.

$$\frac{d\hat{\sigma}\langle\mathcal{F}_1\rangle}{dQ^2} = -|S_{LT}|^2 \frac{4\pi\alpha^2 e_q^2 M}{3N_c Q^3} \int dx dy \delta(xyS - Q^2) \left[f_{1LT}^{(1)q}(x) f_1^{\bar{q}}(y) + (q \leftrightarrow \bar{q}) \right].$$

Thus, the tensor-polarized PDFs f_{1LL}, f_{LT} and $f_{1LT}^{(1)}$ can be extracted from the cross sections of the Drell-Yan process. Moreover, one can test the Wandzura-Wilczek (WW) type relation:

$$f_{LT}(x) = \frac{3}{2} \int_x^1 \frac{dy}{y} f_{1LL}(y).$$

This WW type relation will be discussed in the following part.

S. Y. Qiao and Q. T. Song, arXiv:2410.13225 [hep-ph].

Wandzura-Wilczek (WW) and Lorentz Invariant Relation (LIR) relations for tensor-polarized PDFs and FFs

Tensor-polarized PDFs of spin-1 hadrons

Twist-3

$$\Phi(x,P,T) = \frac{1}{2}\Big[S_{LL}\, \bar{\not{n}}\, f_{1LL}(x) + \frac{M}{P^+} S_{LL}\, e_{LL}(x) + \frac{M}{P^+} \not{S}_{LT}\, f_{LT}(x) + \frac{M^2}{(P^+)^2} S_{LL}\, \not{n}\, f_{3LL}(x) \Big]$$

Twist-2

Twist-4

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571
L. L. Frankfurt and M. I. Strikman, NP A405 (1983) 557.
S. Kumano and Qin-Tao Song, PRD 103 (2021) 014025.
J. P. Ma, C. Wang and G. P. Zhang, arXiv:1306.6693 [hep-ph].

simliar as g_1 in proton

simliar as g_T in proton

Can we derive similar relations as the proton case?

1) Decomposition of g_T

2) Wandzura-Wilczek (WW) relation

Twist-3 level operator relation

The identity between the quark-quark and quark-gluon-quark operators

$$\xi_\mu \bar{\psi}(0)(\partial^\mu \gamma^\alpha - \partial^\alpha \gamma^\mu)\psi(\xi) = g\int_0^1 dt\, \bar{\psi}(0)\left[i\left(t-\frac{1}{2}\right) G^{\alpha\mu}(t\xi) - \frac{1}{2}\gamma_5 \widetilde{G}^{\alpha\mu}(t\xi)\right]\xi_\mu \not{\xi}\, \psi(\xi)$$

The matrix elements of this identity is expressed in terms of PDFs.

$$x\,\frac{df_{LT}(x)}{dx} = -\frac{3}{2} f_{1LL}(x) - f_{LT}^{(HT)}(x)$$

$$f_{LT}^{(HT)}(x) = -\mathscr{P}\int_{-1}^{1} dy \frac{1}{x-y}\left[\frac{\partial}{\partial x}\{F_{LT}(x,y)+G_{LT}(x,y)\} + \frac{\partial}{\partial y}\{F_{LT}(y,x)+G_{LT}(y,x)\}\right].$$

Quark mass term does not exist.

Decomposition of f_{LT} at twist-3 level

$$f_{LT}(x) = \frac{3}{2}\int_x^{\epsilon(x)} \frac{dy}{y} f_{1LL}(y) + \int_x^{\epsilon(x)} \frac{dy}{y} f_{LT}^{(HT)}(y)$$

Twist-3 PDF

Twist-2 PDF

Twist-3 quark-gluon-quark distributions

Wandzura-Wilczek (WW) relation: if we neglect the quark-gluon-quark distributions

S. Kumano and Q. T. Song, JHEP 09 (2021), 141

In case of g_1 and g_2 in proton, quark-gluon-quark distributions account for ~20% of the twist-3 PDF g_2.

A. Accardi, A. Bacchetta, W. Melnitchouk and M. Schlegel, JHEP 11 (2009), 093.

Lorentz Invariant Relation (LIR) relation

Decomposition of f_{LT} at twist-3 level

$$f_{LT}(x) = \frac{3}{2}\int_x^{\epsilon(x)} \frac{dy}{y} f_{1LL}(y) + \int_x^{\epsilon(x)} \frac{dy}{y} f_{LT}^{(HT)}(y)$$

Combine with the QCD e.o.m relation:

$$x f_{LT}(x) - f_{1LT}^{(1)}(x) - \mathscr{P}\int_{-1}^{1} dy \, \frac{F_{G,LT}(x,y) + G_{G,LT}(x,y)}{x-y} = 0.$$

One can obtain the LIR realtion:

$$\frac{d f_{1LT}^{(1)}(x)}{dx} - f_{LT}(x) + \frac{3}{2} f_{1LL}(x) - 2\mathscr{P}\int_{-1}^{1} dy \, \frac{F_{G,LT}(x,y)}{(x-y)^2} = 0.$$

S. Kumano and Qin-Tao Song, PLB 826(2022), 136908.

The Lorentz-invariance relations (LIRs) can guarantee the frame independence of the twist-3 spin observables.

K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak and M.Schlegel, PRD 93(2016), 054024

LIRs for tensor-polarized FFs

Three identities between the nonlocal quark-quark and quark-gluon-quark operators are used, one of them is

$$\epsilon^{\alpha\mu\rho S_{LT}}\xi_\rho \frac{\partial}{\partial\xi^\alpha}\langle 0|q(-\xi)|P_h,T;X\rangle\langle P_h,T;X|\overline{q}(\xi)\gamma_\mu\gamma_5|0\rangle$$

$$=\int_{-1}^{\infty} dt\langle 0\,|gF_{\xi S_{LT}}(t\xi)q(-\xi)|\,P_h,T;X\rangle\langle P_h,T;X|\,\overline{q}(\xi)\not{\xi}|0\rangle + \int_{\infty}^{1} dt\langle 0|q(-\xi)|P_h,T;X\rangle\langle P_h,T;X\,|\,\overline{q}(\xi)\not{\xi}gF_{\xi S_{LT}}(t\xi)|\,0\rangle$$

$$+\,i\epsilon^{\alpha\mu\xi S_{LT}}\Big[\int_{-1}^{\infty} dtt\langle 0\,|gF_{\alpha\xi}(t\xi)q(-\xi)|\,P_h,T;X\rangle\langle P_h,T;X|\,\overline{q}(\xi)\gamma_\mu\gamma_5|0\rangle + \int_{\infty}^{1} dtt\langle 0|q(-\xi)|P_h,T;X\rangle$$

Three LIR relations for FFs:

$$\frac{3}{2}\tilde{F}_{1LL}(z) - \tilde{F}_{LT}(z) - (1-z\frac{d}{dz})F_{1LT}^{(1)}(z) = -2\int_z^\infty \frac{dz_1}{(z_1)^2}\frac{\mathrm{Re}\left[\tilde{F}_{LT}(z,z_1)\right]}{(\frac{1}{z}-\frac{1}{z_1})^2}$$

$$\tilde{H}_{LL}(z) + 2\tilde{H}_{1LT}(z) + (1-z\frac{d}{dz})H_{1LL}^{(1)}(z) = -2\int_z^\infty \frac{dz_1}{(z_1)^2}\frac{\mathrm{Im}\left[\tilde{H}_{LL}^{\perp}(z,z_1)\right]}{(\frac{1}{z}-\frac{1}{z_1})^2},$$

$$\tilde{G}_{LT}(z) + (1-z\frac{d}{dz})G_{1LT}^{(1)}(z) = -2\int_z^\infty \frac{dz_1}{(z_1)^2}\frac{\mathrm{Im}\left[\tilde{G}_{LT}(z,z_1)\right]}{(\frac{1}{z}-\frac{1}{z_1})^2}.$$

Qin-Tao Song, PRD 108 (2023), 094041

Decomposition of 6 twist-3 FFs:

Twist 3 Twist 2

$$F_{LT}(z) = -\frac{3z}{2}\int_z^1 dz_1 \frac{F_{1LL}(z_1)}{(z_1)^2} + z\int_z^1 \frac{dz_1}{z_1}\int_{z_1}^{\infty}\frac{dz_2}{(z_2)^2}\left\{\frac{\left[1+\frac{1}{z_1}\delta(\frac{1}{z_1}-\frac{1}{z})\right]\mathrm{Re}\left[\hat{G}_{LT}(z_1,z_2)\right]}{\frac{1}{z_1}-\frac{1}{z_2}} - \frac{\left[\frac{3}{z_1}-\frac{1}{z_2}+\frac{1}{z_1}(\frac{1}{z_1}-\frac{1}{z_2})\delta(\frac{1}{z_1}-\frac{1}{z})\right]\mathrm{Re}\left[\hat{F}_{LT}(z_1,z_2)\right]}{}\right\}$$

$$F^{(1)}_{1LT}(z) = \frac{3}{2}\int_z^1 dz_1 \frac{F_{1LL}(z_1)}{(z_1)^2} + \int_z^1 \frac{dz_1}{z_1}\int_{z_1}^{\infty}\frac{dz_2}{(z_2)^2}\left\{\frac{\left(\frac{3}{z_1}-\frac{1}{z_2}\right)\mathrm{Re}\left[\hat{F}_{LT}(z_1,z_2)\right]}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2} - \frac{\mathrm{Re}\left[\hat{G}_{LT}(z_1,z_2)\right]}{\frac{1}{z_1}-\frac{1}{z_2}}\right\}.$$

$$G_{LT}(z) = -\frac{m_q}{M}\left[zH_{1LT}(z) + z\int_z^1 dz_1 \frac{H_{1LT}(z_1)}{z_1}\right] - z\int_z^1 \frac{dz_1}{z_1}\int_{z_1}^{\infty}\frac{dz_2}{(z_2)^2}\left\{\frac{\left[1+\frac{1}{z_1}\delta(\frac{1}{z_1}-\frac{1}{z})\right]\mathrm{Im}\left[\hat{F}_{LT}(z_1,z_2)\right]}{\frac{1}{z_1}-\frac{1}{z_2}} - \frac{\left[\frac{3}{z_1}-\frac{1}{z_2}+\frac{1}{z_1}(\frac{1}{z_1}-\frac{1}{z_2})\delta(\frac{1}{z_1}-\frac{1}{z})\right]\mathrm{Im}\left[\hat{G}_{LT}(z_1,z_2)\right]}{(\frac{1}{z_1}-\frac{1}{z_2})^2}\right\},$$

$$G^{(1)}_{1LT}(z) = \frac{m_q}{M}\int_z^1 dz_1 \frac{H_{1LT}(z_1)}{z_1} + \int_z^1 \frac{dz_1}{z_1}\int_{z_1}^{\infty}\frac{dz_2}{(z_2)^2}\left\{\frac{\mathrm{Im}\left[\hat{F}_{LT}(z_1,z_2)\right]}{\frac{1}{z_1}-\frac{1}{z_2}} - \frac{(\frac{3}{z_1}-\frac{1}{z_2})\mathrm{Im}\left[\hat{G}_{LT}(z_1,z_2)\right]}{(\frac{1}{z_1}-\frac{1}{z_2})^2}\right\}.$$

Decomposition of 6 twist-3 FFs and WW relations:

Twist 3 Twist 2

$$\boxed{H_{LL}(z) = 4\int_z^1 dz_1 \frac{H_{1LT}(z_1)}{z_1}} + 4\int_z^1 dz_1 \int_{z_1}^{\infty} \frac{dz_2}{(z_2)^2} \frac{\frac{2}{z_1} - \frac{1}{z_2} + \frac{1}{2z_1}(\frac{1}{z_1} - \frac{1}{z_2})\delta(\frac{1}{z_1} - \frac{1}{z})}{(\frac{1}{z_1} - \frac{1}{z_2})^2} \mathrm{Im}\left[\hat{H}_{LL}^{\perp}(z_1, z_2)\right],$$

WW relations for FFs

Simon Schneider

$$\boxed{H_{1LL}^{(1)}(z) = -\frac{2}{z}\int_z^1 dz_1 \frac{H_{1LT}(z_1)}{z_1}} - \frac{2}{z}\int_z^1 dz_1 \int_{z_1}^{\infty} \frac{dz_2}{(z_2)^2} \frac{\frac{2}{z_1} - \frac{1}{z_2}}{(\frac{1}{z_1} - \frac{1}{z_2})^2} \mathrm{Im}\left[\hat{H}_{LL}^{\perp}(z_1, z_2)\right],$$

See Simon's talk.

Qin-Tao Song, PRD 108 (2023), 094041

Measurements of tensor-polarized FFs at BESIII and Belle:

e^+e^- →phi+X, preliminary results have been obtained!

These theoretical relations can be useful for the future analysis of the tensor-polarized distributions.

Summary

- Tensor-polarized PDFs, FFs, and TMDs are discussed.

- We propose that gluon transversity can be measured in proton-deuteon Drell-Yan process, and this measurement will be possible at Fermilab.

- We present twist-3 correction for the proton-deuteon Drell-Yan process.

- Wandzura-Wilczek (WW) and Lorentz Invariant Relation (LIR) relations are derived for tensor-polarized PDFs and FFs.

Thank you very much

Backup

Polarizations	$\vec{E}$	S_T^x	S_T^y	S_L	S_{LL}	S_{TT}^{xx}
Longitudinal $+z$	$\frac{1}{\sqrt{2}}(-1, -i, 0)$	0	0	+1	$+\frac{1}{2}$	0
Longitudinal $-z$	$\frac{1}{\sqrt{2}}(+1, -i, 0)$	0	0	−1	$+\frac{1}{2}$	0
Transverse $+x$	$\frac{1}{\sqrt{2}}(0, -1, -i)$	+1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $-x$	$\frac{1}{\sqrt{2}}(0, +1, -i)$	−1	0	0	$-\frac{1}{4}$	$+\frac{1}{2}$
Transverse $+y$	$\frac{1}{\sqrt{2}}(-i, 0, -1)$	0	+1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Transverse $-y$	$\frac{1}{\sqrt{2}}(-i, 0, +1)$	0	−1	0	$-\frac{1}{4}$	$-\frac{1}{2}$
Linear x	$(1, 0, 0)$	0	0	0	$+\frac{1}{2}$	−1
Linear y	$(0, 1, 0)$	0	0	0	$+\frac{1}{2}$	+1