QCD sum rules for hadron spin decomposition

ref) PLB 843 (2023) 137986

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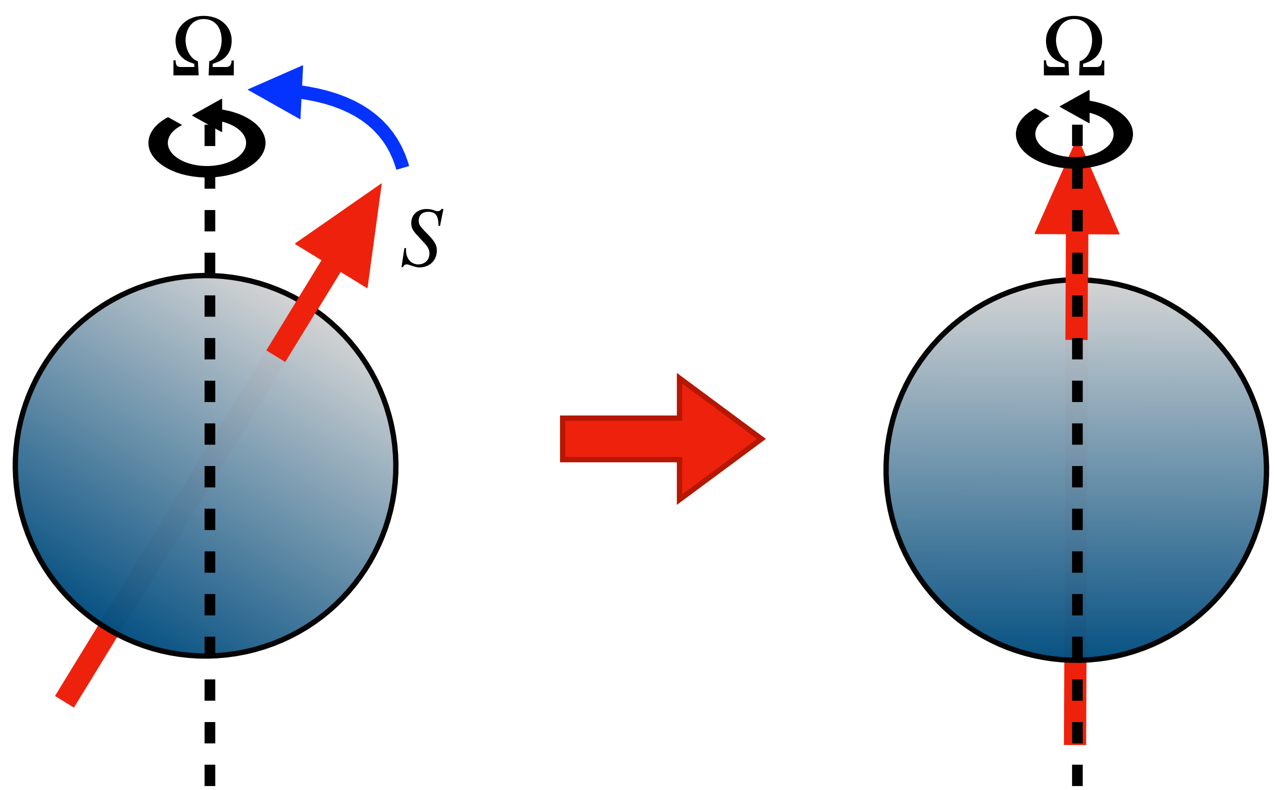
Spin-Rotation Coupling(SRC)



$$H_{SRC} = -S \cdot \Omega$$

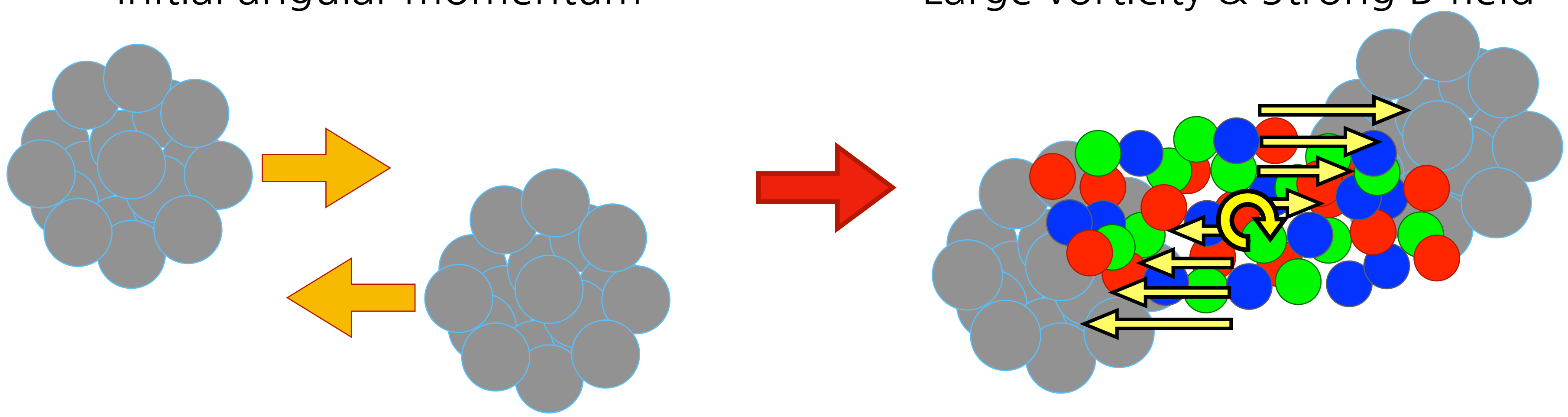
Q. *SRC is valid for any spin? for any composite particle?*

=> Spin decomposition of

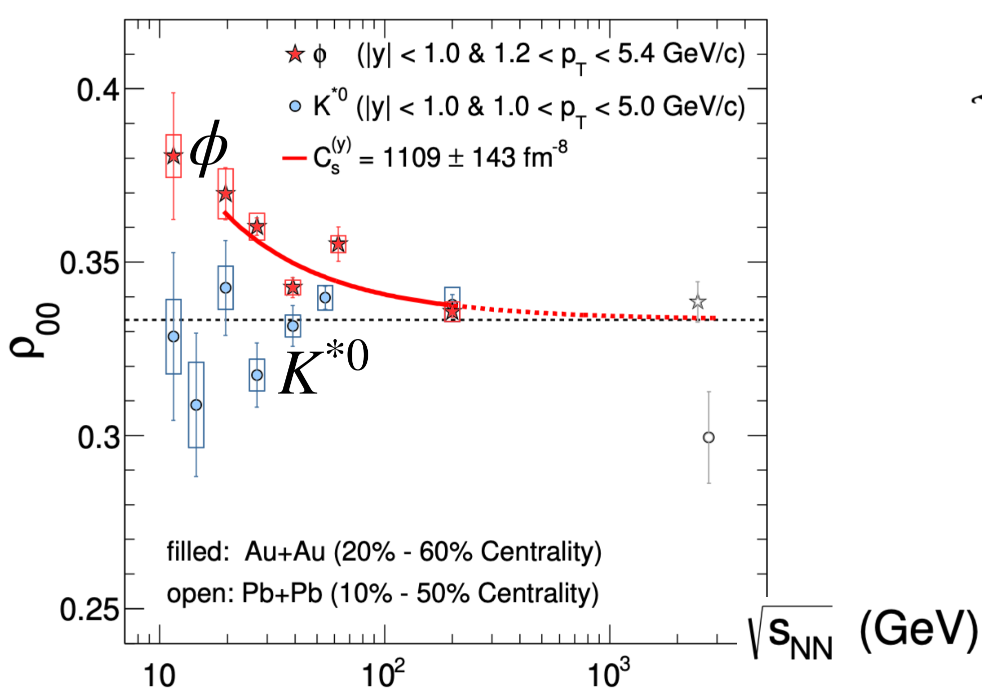
- Spin-1 quarkonium
- Spin-1/2 proton (on-going)

Vector meson spin alignment

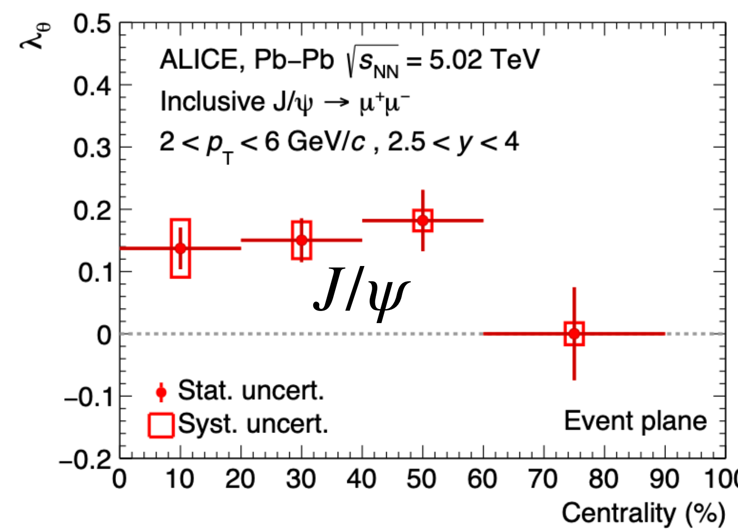
In non-central HICs,



STAR, arXiv:2204.02302 (2022)



ALICE, arXiv:2204.10171 (2022)



=> Detailed mechanism is complex and still not clearly understood.

G.R. based derivations for SRC

- Spin-1/2: Dirac eq. in a rotating frame using G.R.

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma}\cdot\mathbf{\Omega} - m\right]\Psi(x) = 0 \text{ where } \mathbf{\Sigma} = \gamma^0(\mathbf{S}+\mathbf{L})$$

=> $\mathrm{H_{SRC}} = -\,\mathrm{S}\cdot\Omega$ for spin-1/2

- Spin-1: No strict derivation based on G.R. until recently

- ***PRD102(2020)12,125028*** - J.Kapusta, E.Rrapaj, S.Rudaz
  - Proca eq. for massive spin-1 particle using G.R.
  - $\mathrm{H_{SRC}} = -\,\mathbf{\frac{1}{2}}\,\mathrm{S}\cdot\Omega$ for spin-1!
  - contradictory to naive expectation and quark model

- **Motivation**

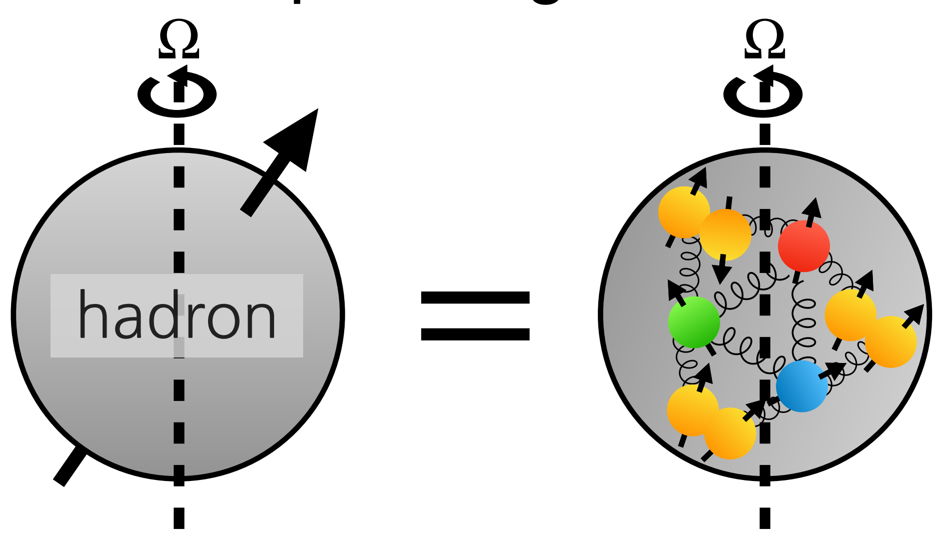
=> Clarify the strength of SRC for spin-1 particle in a different way!

Outline

- We study SRC of (the simplest) spin-1 heavy $\mathrm{Q\bar{Q}}$ system
- Introduce a free parameter "g_Ω" which indicates the strength of SRC,

$$\mathrm{H_{SRC}} = -\, \mathrm{g}_\Omega \, \mathrm{S} \cdot \Omega$$

- "Total SRC = All reaction of quark + gluon in a rot frame"



- We prove that $\mathrm{g}_\Omega = \mathrm{g}_\Omega^{\mathrm{quark}}(\mathrm{Q}^2) + \mathrm{g}_\Omega^{\mathrm{gluon}}(\mathrm{Q}^2) = 1$ for spin-1 $\mathrm{Q\bar{Q}}$ system
- Each component of g_Ω carried by quarks and gluons = Spin content
- We study spin contents of J/ψ, $\Upsilon(1\mathrm{S})$ for V and $\chi_{\mathrm{c1}}, \chi_{\mathrm{b1}}$ for AV

How to extract g_Ω?

1. Describe the correlation function in a rotating frame

$$\Pi^{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|\,\mathrm{T}\{\mathrm{j}^{\mu}(\mathrm{x})\mathrm{j}^{\nu}(0)\}\,|0\rangle$$

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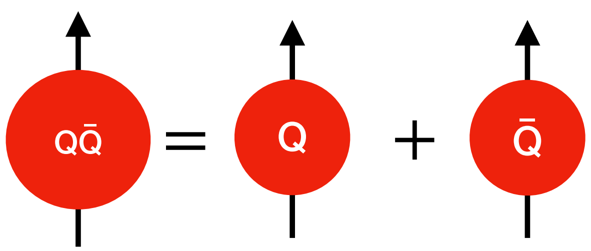
$$\epsilon_\mu^+ = (0,1,i,0)/\sqrt{2}$$

2. Pick out a right circularly polarized state => $\Pi^+(\omega) = \epsilon_\mu^+\epsilon_\nu^{+*}\Pi^{\mu\nu}(\omega,0)$



$$|11\rangle = |\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2}\rangle$$
$$|10\rangle = |\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}-\frac{1}{2}\rangle$$
$$|1-1\rangle = |\frac{1}{2}-\frac{1}{2}\rangle + |\frac{1}{2}-\frac{1}{2}\rangle$$

Energy of $|11\rangle$ will be shifted by '$-\Omega$' i.e. $\omega \to \omega - \Omega$

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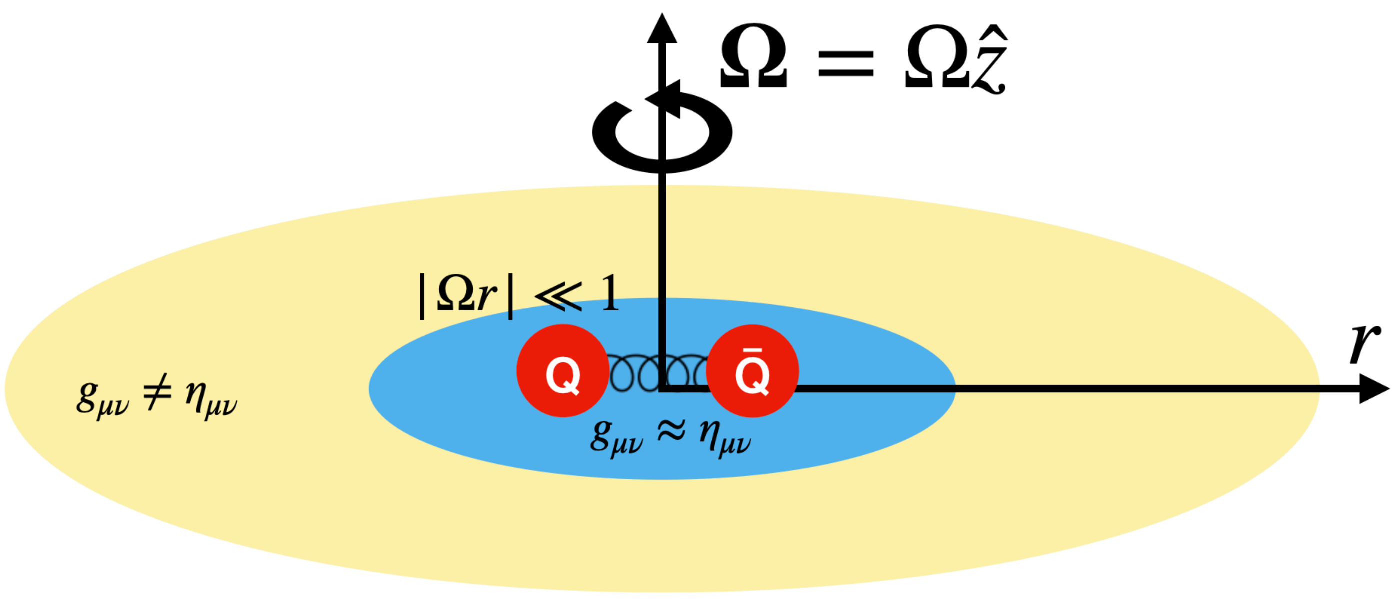

- No external OAM
- Expand $\mathbf{\Omega}$ linear term

$$ds^2 = g_{\mu\nu}x^\mu x^\nu = -dt^2 + d\mathbf{r}'^2 = (-1 + (\mathbf{\Omega}\times\mathbf{r})^2)dt^2 + 2(\mathbf{\Omega}\times\mathbf{r})d\mathbf{r}dt + d\mathbf{r}^2$$

How to extract g_Ω?

1. Describe the correlation function in a rotating frame

$$\Pi^{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0\,|\,\mathrm{T}\{\mathrm{j}^\mu(\mathrm{x})\mathrm{j}^\nu(0)\}\,|0\rangle$$

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3. Put the system at the center of the rotation => $q_\mu = (\omega, \vec{0})$

4. Up to linear terms in Ω

$$\Pi^+(\omega) = \omega^2\,\Pi^{vac}(\omega^2) + \omega\,\Pi^{rot}(\omega^2)\,\Omega + \mathcal{O}(\Omega^2)$$

Π^{vac} : ordinary vacuum invariant ftn. vacuum properties ex) mass

Π^{rot} : new function appearing in a rotating frame. spin information

How to extract g_Ω?

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Π^{vac} : ordinary vacuum invariant ftn. vacuum properties ex) mass

Π^{rot} : new function appearing in a rotating frame. spin information

5. Extract g_Ω by comparing two different descriptions of Π^{rot}

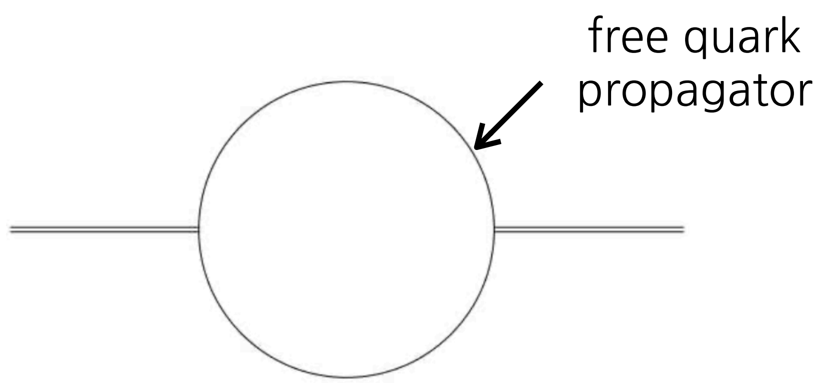
(a) Directly compute Feynman diagrams in a rotating frame

(b) Phenomenological derivation from Π^{vac}

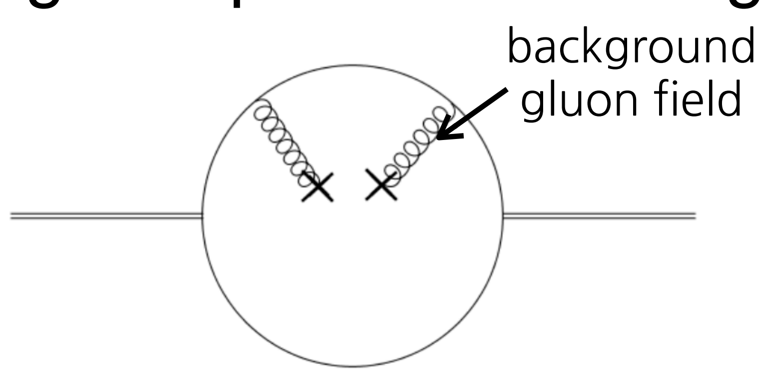
Method (a) - direct computation in a rotating frame

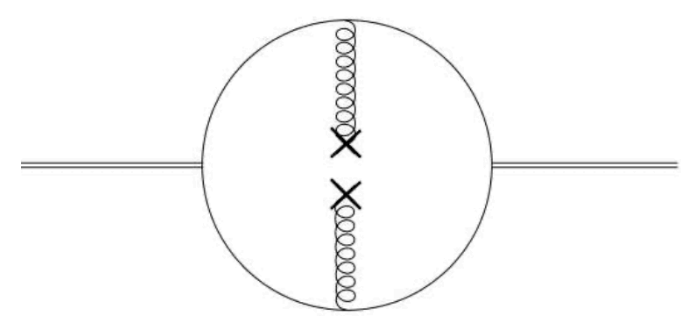
Feynman diagrams in Operator Product Expansion(OPE)

- Leading perturbative diagram



- Leading non-perturbative diagrams : Gluon condensates $\langle(\alpha_s/\pi)G^2\rangle$





- Compute in an inertial frame → Ω independent terms → Π^{vac}
- Compute in a rotating frame → collect Ω linear terms → Π^{rot}

Quarks in a rotating frame

- Recall Dirac eq. in a rotating frame

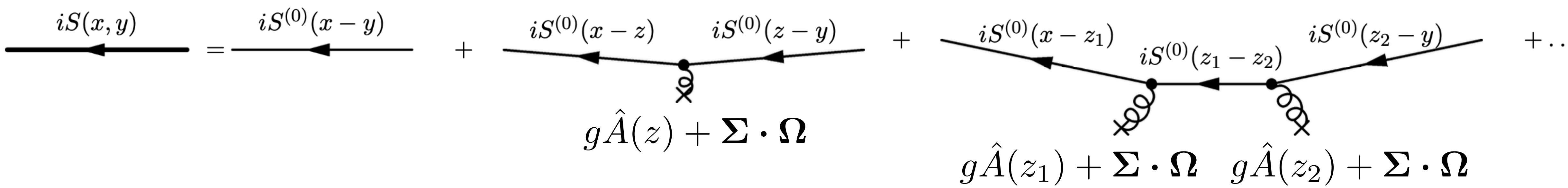
$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma}\cdot\mathbf{\Omega} - m\right]\Psi(x) = 0 \quad \text{where} \quad \mathbf{\Sigma} = \gamma^0(\mathbf{S}+\mathbf{L})$$

- Quark propagator in a rotating frame

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma}\cdot\mathbf{\Omega} - m\right]S(x) = \delta(x)$$

- It is difficult to find full propagator
- We can expand in terms of 'g' and 'Ω'

$$S^{\text{full}} \approx S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + \cdots$$



Gluons in a rotating frame

- Covariant derivatives in curved space-time(Γ^a_{bc} : Christoffel symbols)

$$D_c G_{ab} = \partial_c G_{ab} - \Gamma^d_{ca} G_{db} - \Gamma^d_{cb} G_{ad}$$

- Fock-Schwinger gauge($x^\mu A_\mu = 0$) in curved space-time

$$A_\mu(x) = -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha \underline{\partial_\alpha} G_{\mu\nu} + \cdots$$

$$= -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha \underline{D_\alpha} G_{\mu\nu} \underline{- \frac{1}{3} x^\nu x^\alpha (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d})} + \cdots$$

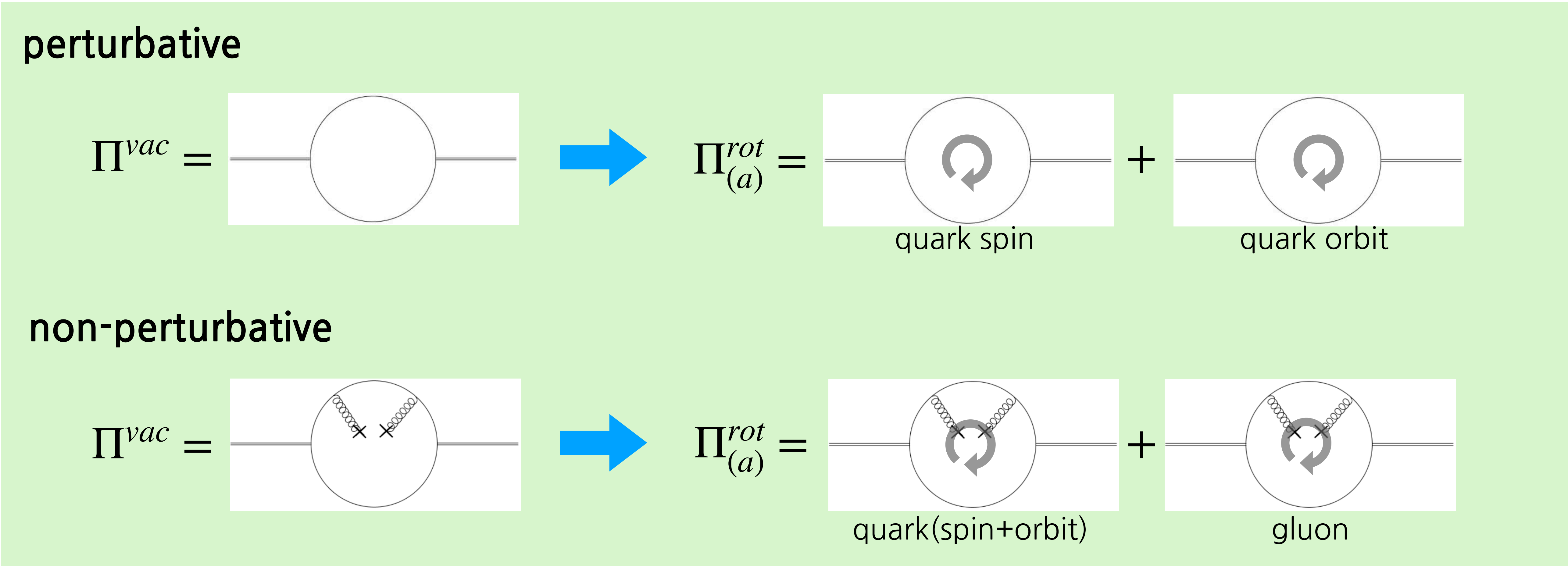
additional contribution in curved space-time

- $\Gamma^2_{01} = \Omega, \Gamma^1_{02} = -\Omega$ in a rotating frame.

$$\not{A}_\Omega(x) = -\frac{1}{3} x^\nu x^\alpha \gamma^\mu (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d}) \propto \vec{x} \times (\vec{E} \times \vec{B}) \cdot \Omega = J_g \cdot \Omega$$

- Kapusta et al. thought that $D_c G_{ab} = \partial_c G_{ab}$ in a rotating frame.
(Their result might be wrong)

Spin decomposition of method (a)



In a rot frame, we can compute $\Pi^{rot}_{(a)}$ using

$$S_{\text{quark}} \approx S^{(0)} + S^{(0)}\left[g\not{A}_{\Omega} + \Sigma \cdot \Omega\right]S^{(0)} + \cdots$$

$$\uparrow$$

$$(\mathrm{S_q} + \mathrm{L_q} + \mathrm{J_g})\,\Omega$$

We can decompose the given diagrams into quark and gluon AM contributions depending on their origin => advantage of method(a)

Method (b) - Phenomenological derivation

- In an inertial frame

$$\Pi^{+}(\omega) = \epsilon_{+}^{\mu *}\epsilon_{+}^{\nu}\Pi_{\mu\nu}(\omega,0) = \omega^2\,\Pi^{vac}(\omega^2)$$

- Energy shift of all right circularly polarized state in a rotating frame

$$\Rightarrow \omega \to \omega - g_{\Omega}\,\Omega \qquad (\because H_{\mathrm{SRC}} = -\,g_{\Omega}\,S\cdot\Omega)$$

$$\begin{aligned}\Pi^{+}(\omega + g_{\Omega}\Omega) &= (\omega + g_{\Omega}\Omega)^2\,\Pi^{vac}((\omega + g_{\Omega}\Omega)^2) \\ &= \omega^2\,\Pi^{vac}(\omega^2) + \omega\,\Pi^{rot}(\omega^2)\,\Omega + \mathcal{O}(\Omega^2)\end{aligned}$$

- Simple expression of rotating part in terms of vacuum invariant ftn.

$$\Pi^{rot}_{(b)}(\omega^2) = 2\underline{g_{\Omega}}\left\{\Pi^{vac}(\omega^2) + \omega^2\frac{\partial\Pi^{vac}(\omega^2)}{\partial\omega^2}\right\}$$

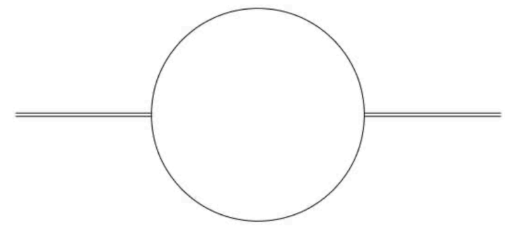
unknown

=> We can directly derive Π^{rot} from Π^{vac} but it includes unknown g_{Ω}

g_Ω in perturbative region

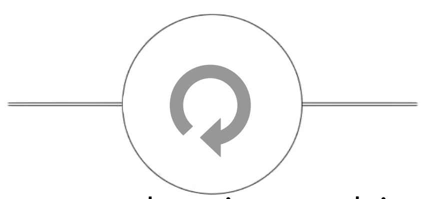
Inertial frame

$$\Pi^{vac}_{pert} =$$



description (a)

$$\Pi^{rot}_{(a)} =$$



quark spin + orbit

all responses of quarks in a rot frame
= **Sq + Lq**

=

description (b)

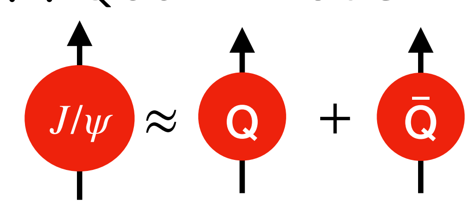
$$\Pi^{rot}_{(b)} = 2g_\Omega \left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$$

phen. derivation based on the energy shift of total system by "$-g_\Omega \Omega$".

$g_\Omega = 1$ in the perturbative region

When two free quarks form a spin-1 state in a rel. way, they follow $\mathrm{H_{SRC}} = -\mathrm{S} \cdot \Omega$

c.f. Quark Model

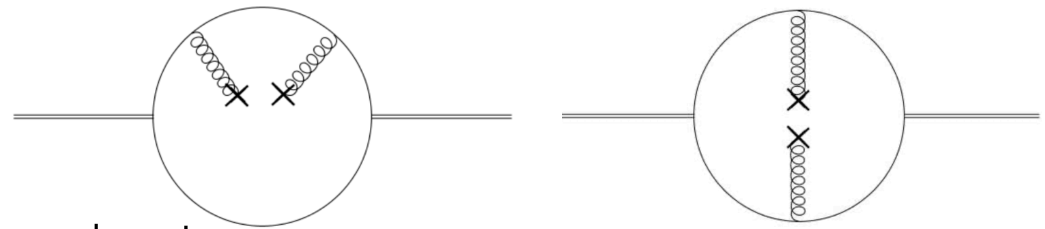


$$|11\rangle \sim |\frac{1}{2}\frac{1}{2}\rangle + |\frac{1}{2}\frac{1}{2}\rangle$$

$$H_r = m_{J/\psi} - \Omega$$

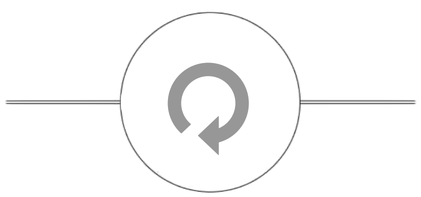
g_Ω in non-perturbative region

Inertial frame

$\Pi^{vac}_{G_0} =$ 

G_0 : gluon condensate

description (a)

$\Pi^{rot}_{(a)} =$ 

quark + gluon

all responses of quarks and gluons

= Sq + Lq + Jg

=

description (b)

$$\Pi^{rot}_{(b)} = 2g_\Omega \left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$$

phen. derivation based on the energy shift of total system by “-$g_\Omega \Omega$”.

$g_\Omega = 1$ in the non-perturbative region

Even in non-pert region, spin-1 system follows $\mathrm{H_{SRC}} = -\mathrm{S} \cdot \Omega$

Physical meaning of $g_\Omega = 1$?

Method (b)

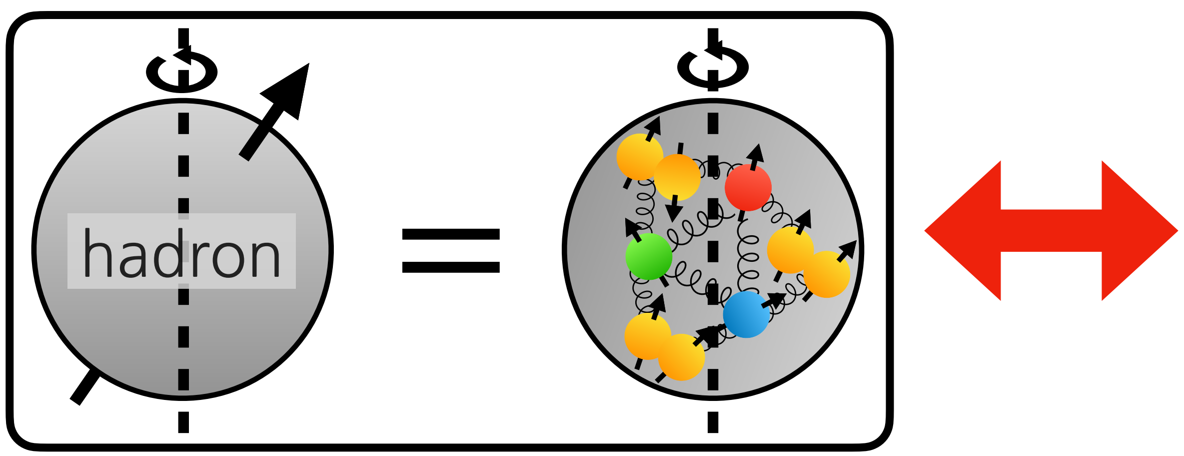
= SRC of the total system

= $g_\Omega \langle \vec{S} \rangle$ where $\vec{S}$ is spin-1 operator where $\langle \cdots \rangle = \int d^4x e^{iq\cdot x} \langle 0 | T[j(x) \cdots j(0)] | 0 \rangle$

Method (a)

= Ω linear terms in all responses of quarks and gluon in a rotating frame

= $\langle \vec{J}_{\rm QCD} \rangle$ where $\vec{J}_{\rm QCD} = \int d^3x (\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \psi^\dagger(\vec{x}\times(-i\vec{D}))\psi + \vec{x}\times(\vec{E}\times\vec{B}))$



$g_\Omega \langle \vec{S} \rangle = \langle \vec{J}_{\rm QCD} \rangle$

Therefore, we can conclude that $g_\Omega = \langle \vec{J}_{\rm QCD} \rangle / \langle \vec{S} \rangle$

- $g_\Omega = 1$ means $\langle \vec{S} \rangle = \langle \vec{J}_{\rm QCD} \rangle$
- This should be valid for any Feynman diagram ($\because$ AM conservation)

Application - g_Ω of ground states

From Kallen-Lehmann(or spectral) rep,

"$g_\Omega = 1$" is universal for all physical states that can couple to $j^\mu(x)$.

If we can extract the ground state,

=> Fraction of g_Ω carried by each a.m. inside the ground state

$$g_\Omega^{\text{ground}} = \frac{\langle J_{\text{QCD}} \rangle}{\langle S_{\text{tot}} \rangle} = \frac{\langle S_q \rangle + \langle L_k \rangle + \langle L_p \rangle + \langle J_g \rangle}{\langle S_{\text{tot}} \rangle} = 1$$

$S_q = \frac{1}{2}\gamma^1\gamma^2$: quark spin,

$L_k = r \times p$: kinetic part of quark orbital a.m,

$L_p = r \times gA$: potential part of quark orbital a.m,

$J_g = r \times (E \times B)$: gluon total a.m.

=> Spin content of the ground state

How to extract the ground state?

⟨QCD sum rules⟩

$$\Pi(q^2) = i\int d^4x e^{iqx}\langle \mathrm{T}\{\phi(x)\phi(0)\}\rangle$$

$$Q^2 = -q^2 \gg 0$$

$$\Pi^{\mathrm{OPE}}(Q^2) = \sum_n C_n \langle \mathcal{O}_n \rangle$$

$$\Pi^{\mathrm{phen}}(q^2) = \frac{|\langle 0|\phi|n_0\rangle|^2}{q^2 - m_0^2} + \cdots$$

$$\Pi^{\mathrm{OPE}}(Q^2) = \int_0^\infty ds \frac{\mathrm{Im}\Pi^{\mathrm{phen}}(s)}{s+Q^2}$$

$$\hat{f}(M^2) \equiv \lim_{\substack{Q^2, n\to\infty \\ Q^2/n = M^2}} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n f(Q^2)$$

$$\mathrm{Im}\Pi^{\mathrm{phen}}(s) \approx \text{ground state pole} + \text{continuum}$$

$$\hat{\Pi}^{\mathrm{OPE}}(M^2) = \int_0^\infty ds e^{-s/M^2}\,\mathrm{Im}\Pi^{\mathrm{phen}}(s)$$

=> spectral parameters are expressed as a ftn of Borel mass 'M' with QCD condensates.
But, approximate relation. Reliable only inside a limited range of M

QCDSR analysis: vector channel

$$g_{\Omega}^{\text{ground}} = \frac{\langle J_{\text{QCD}} \rangle}{\langle S_{\text{tot}} \rangle} = \frac{\langle S_q \rangle + \langle L_k \rangle + \langle L_p \rangle + \langle J_g \rangle}{\langle S_{\text{tot}} \rangle} = 1$$ as a ftn of M

Take average over a reliable range of M

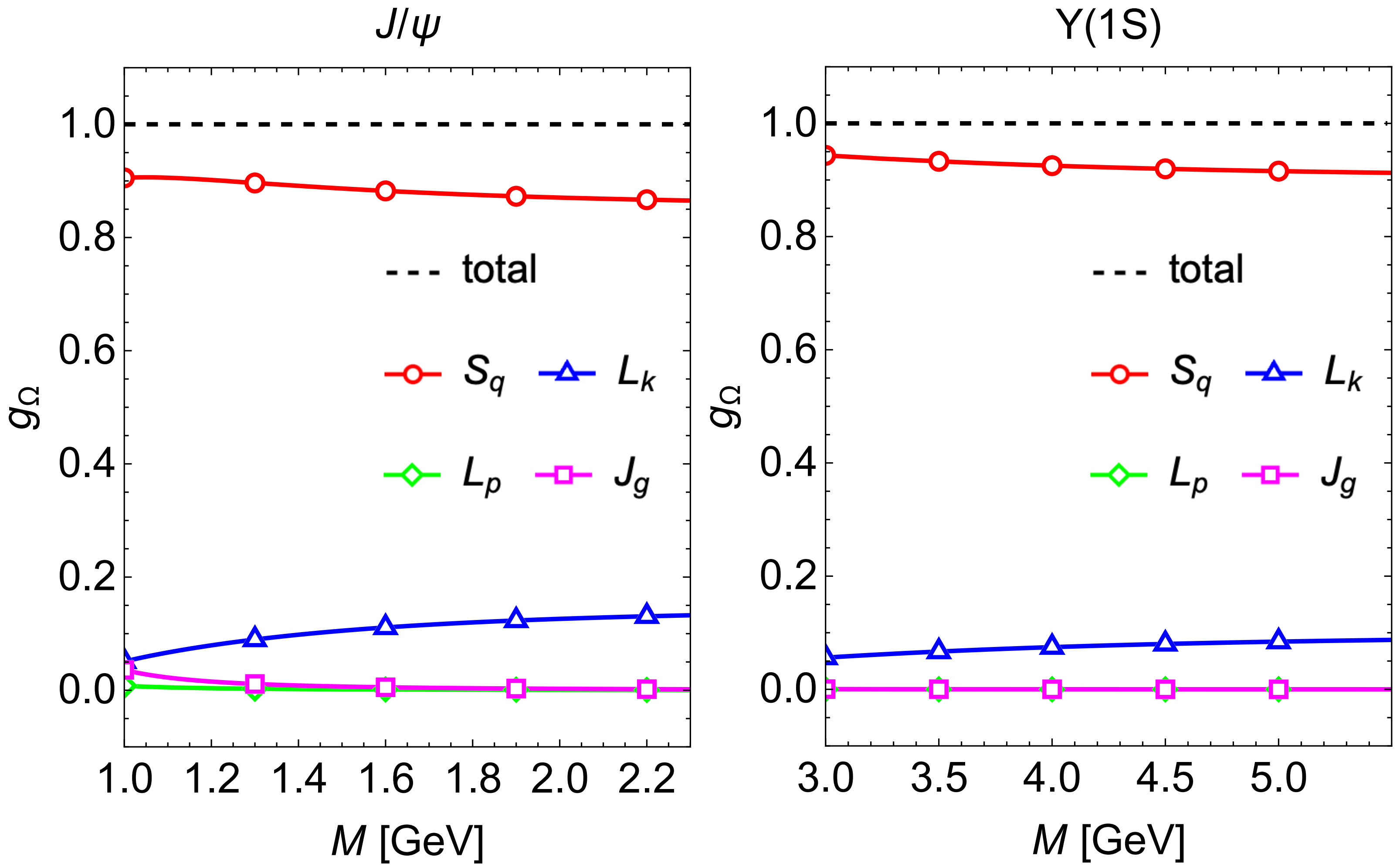


TABLE I: $\sqrt{\bar{s}_0}$ and Borel window for spin-1 quarkonia

	J/ψ	χ_{c1}	$\Upsilon(1S)$	χ_{b1}
$\sqrt{\bar{s}_0}$ [GeV]	3.5	4.0	10.3	11
$(M_{\min}, M_{\max})$ [GeV]	(1,2.3)	(1.4,2.3)	(3,5.5)	(3.6,4.9)

spin contents of spin-1 quarkonia

With the help of ‘QCD sum rule’ + simple ‘pole+continuum’ ansatz.

		Vector (%)			Axial (%)		
		S-wave	$\Upsilon(1S)$	J/ψ	P-wave	χ_{b1}	χ_{c1}
Quark: spin	S_q	**100**	92	88	**50**	43	40
Quark: $r \times p$	L_k	0	7.6	11	**50**	57	61
Quark: $r \times gA$	L_p	0	0.003	0.2	0	-0.001	0.08
Gluon $r \times (E \times B)$	J_g	0	0.015	0.8	0	-0.005	-1.5

- Total sum of 4 pieces = 100 %
- Classical picture from the naive Q.M.
  S-wave: quark spin(100%) , P-wave: quark spin(50%) quark oam(50%)
- Spin contents are slightly different from the classical picture.
  As the quark mass becomes lighter, spin contents deviate more from the classical picture
  ex) J/ψ is considered as S-wave but quarks do not carry all of the total spin
  $\Upsilon(1S)$ is still comparable with the classical picture

Light quark system?

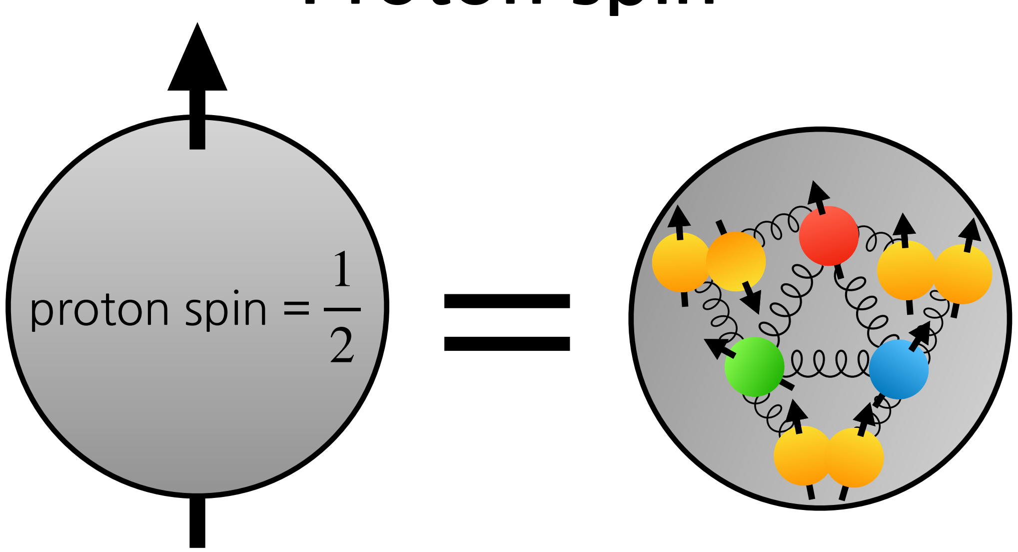
vector mesons

	Q.M.	$\Upsilon(1S)$	J/ψ	ρ, ω, ϕ
S_q	100	92	88	?
L_k	0	7.6	11	?
L_p	0	0.003	0.2	?
J_g	0	0.015	0.8	?

nucleons

	Q.M.	p, n
S_q	100	?
L_k	0	?
L_p	0	?
J_g	0	?

Proton spin



$$\frac{1}{2} = \langle S_q \rangle + \langle L_q \rangle + \langle J_g \rangle$$

quark spin $\sim 30\,\%$ | quark orbit $(\,?\,\%\,)$ | gluon $(\,?\,\%\,)$

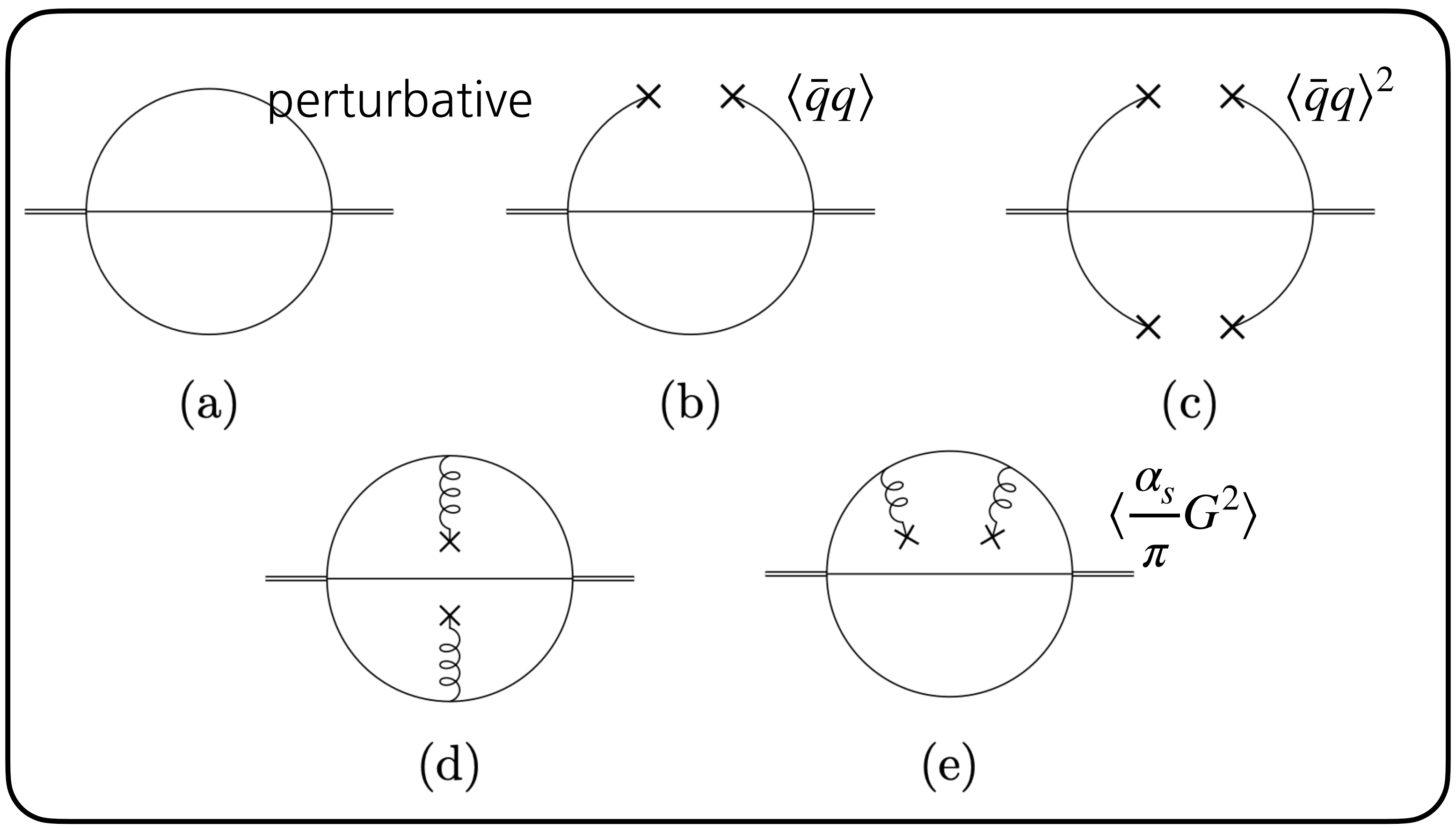
Proton in an inertial frame

$$\Pi(q) = i\int d^4x e^{iqx}\langle \mathrm{T}\{\eta(\mathrm{x})\bar{\eta}(0)\}\rangle$$

spin 1/2 nucleon current

massless limit : $m_{u,d} \to 0$

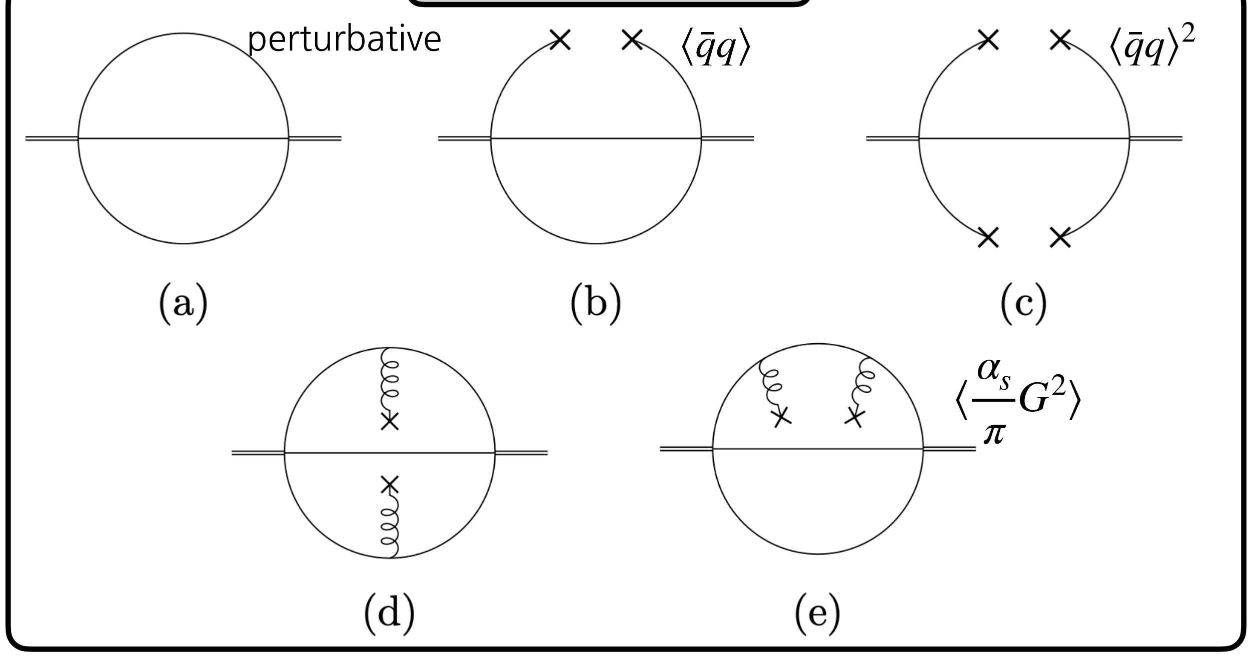
Feynman diagrams (the most essential)



$$\langle\bar{q}q\rangle = -(240\,\mathrm{MeV})^3, \langle\frac{\alpha_s}{\pi}G^2\rangle = (330\,\mathrm{MeV})^4$$

Repeat!

Inertial frame



description (a)

all responses of quarks and gluons

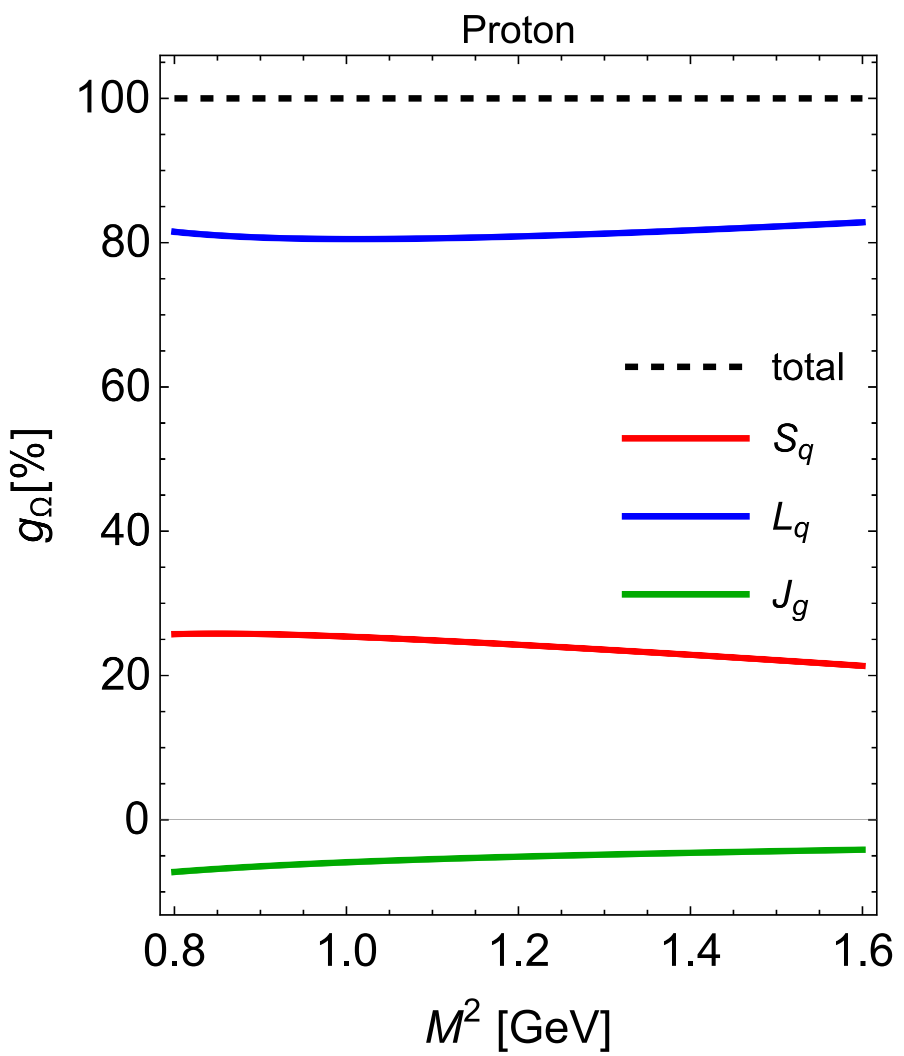
=

description (b)

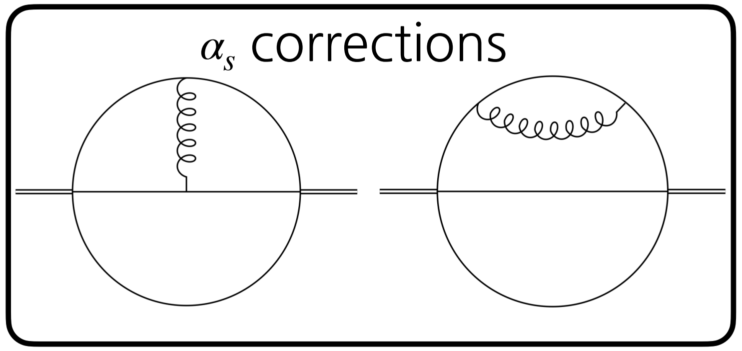
phen. derivation based on the energy shift of total system by "$-g_\Omega \Omega$".

$g_\Omega = 1$ for spin1/2 composite system

Intermediate result



- Roughly, at M~1 GeV, $\langle S_q \rangle : \langle L_q \rangle \approx 1:4$
- This naive analysis captures the important feature that Sq is small
- More accurate analysis requires more diagrams with finite quark masses.
- ⟨Jg⟩ is small and negative => α_s-corrections



Summary

- We proved that spin-1 composite systems follow $\mathrm{H_{SRC}} = -\,\mathrm{S} \cdot \Omega$
- Inspired by SRC, we proposed a way to study hadron spin decomposition.
- Using QCD Sum Rules, we examined spin contents of spin-1 quarkonia and are currently working on the proton.