On generalized parton distributions of spin-3/2 particles

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PRD 107, 116021
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Outline

1, Introduction: Form factors (FFs) and GPDs

2, ★Spin-3/2 particle (selected) and the basic properties

3, ●Numerical calculation: (Framework: Covariant quark-diquark model)
●Results (EMFFs and GFFs, and some others)

4, Summary and Discussions

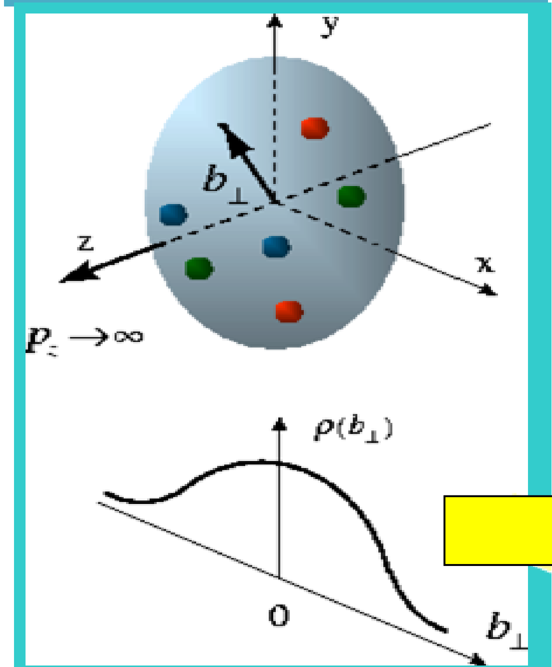
1, Introduction: Form factors and GPDs

● Electromagnetic probe is useful tool

- Electric and magnetic proton form factors$_{2D}$
- Proton and Neutron charge distributions
- Nucleon spin structure$_{2D}$
- Nucleon-Delta transition (other resonances)
- Parton distributions$_{2D}$
- Pion and deuteron form factors ……(others)
- ★ Generalized parton distributions (GPDs$_{3D}$)

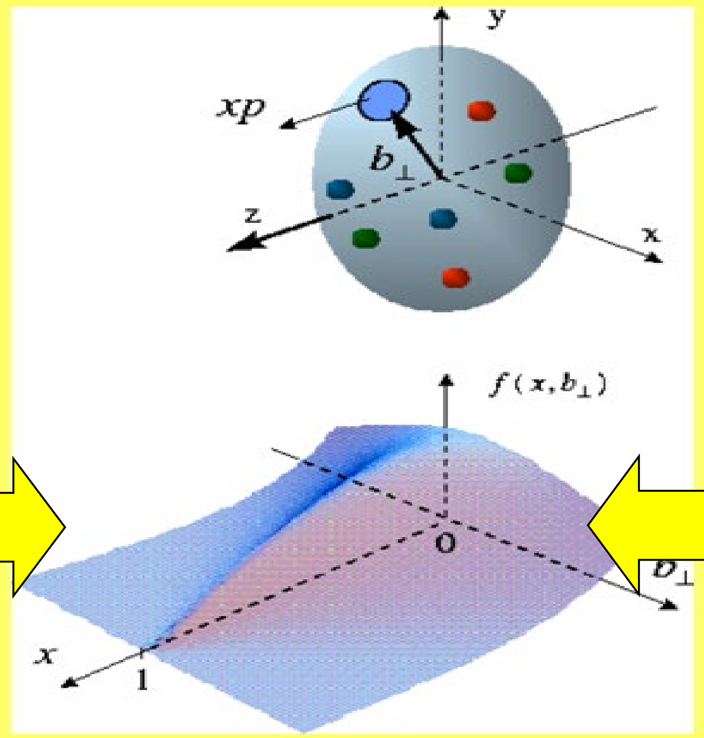
Experiments and theoretical studies

♦Last 50 years



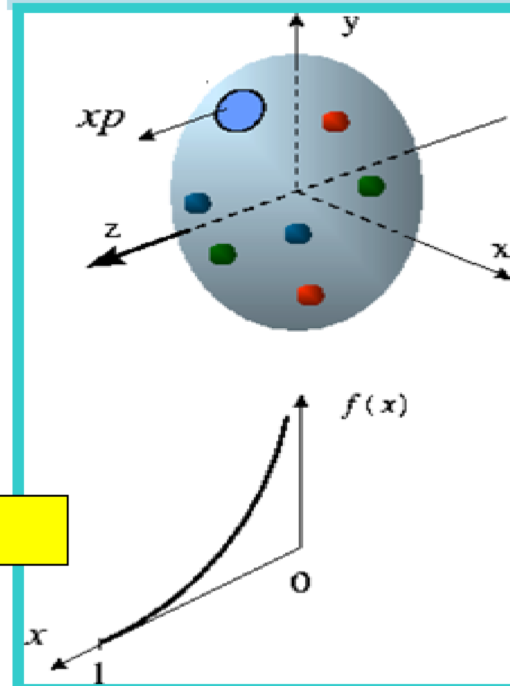
form factors, transverse charge & current densities

♠Last 10 years



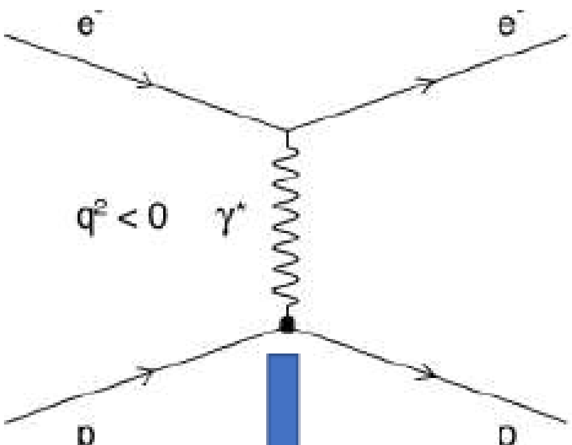
correlated quark momentum and helicity distributions in transverse space --- GPDs

♣Last 40 years

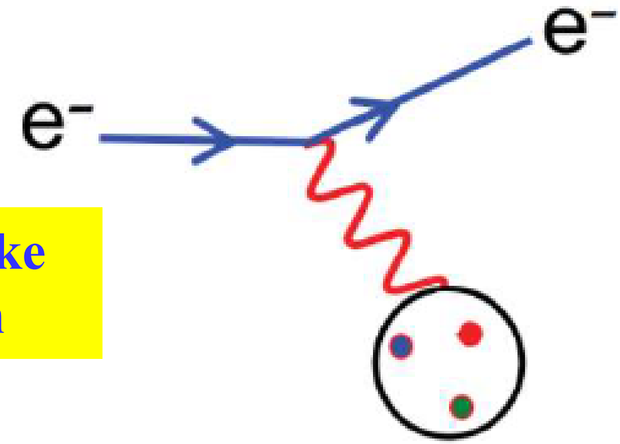


structure functions, quark longitudinal momentum & spin distributions

★ Electromagnetic form factors (space-like) nucleon(1/2)



space-like region



Point? Structure?

★:Local

$$\langle p_f | \hat{J}^{\mu}(0) | p_i \rangle = \overline{u}(p_f) \left[F_1(q^2)\gamma^{\mu} - F_2(q^2)\frac{\sigma^{\mu\nu}q_{\nu}}{2M} \right] u(p_i)$$

$$\Gamma^{\mu}(q^2) = \gamma^{\mu} F_1^{p}(q^2) + i\frac{F_2^{p}(q^2)}{2M_p}\sigma^{\mu\nu}q_{\nu}$$

F_1^N : Dirac form factor

F_2^N: Pauli form factor

$$G_E^N(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2)\ ,\ \ G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)\ ,\ \ \tau = \frac{Q^2}{4M_N^2}$$

$$F_1^p(0) = 1\ ,\ F_1^n(0) = 0\ ,\ \ F_2^p(0) = \kappa_p\ ,\ F_2^n(0) = \kappa_n$$

S. Pacetti, R. Baldini Ferroli and E. Tomasi-Gustafsson, ``Proton electromagnetic form factors: Basic notions, present achievements and future perspectives," **Phys. Rept. 550-551, 1-103 (2015).**

Other observables (Like Gravitational FFs) and a Global Description of nucleon (s=1/2):

last global unknown: How do we learn about hadrons?

$|N\rangle$ = strong interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\|J^\mu_{\text{em}}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\|J^\mu_{\text{weak}}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\text{grav}}\|N\rangle$	$\longrightarrow$	$M,\ J,\ D,\ \ldots$

global properties:

$$Q_{\text{prot}} = 1.602176487(40)\times 10^{-19}\text{C}$$
$$\mu_{\text{prot}} = 2.792847356(23)\mu_N$$
$$g_A = 1.2694(28)$$
$$g_p = 8.06(0.55)$$
$$M = 938.272013(23)\,\text{MeV}$$
$$J = \tfrac{1}{2}$$
$$D = \;??$$

and more:

[Maxim Polyakov, proposed, 1998]

$\hookrightarrow$ $D =$ *"last" global unknown*

which value does it have?

Gravitational and a Global Description of nucleon mehanical properties :

GPDs → Gravitational FFs,

<|Energy-momentum tensor|>

Mechanics Observables:

(Energy density, mass radius, Spin) Distributions

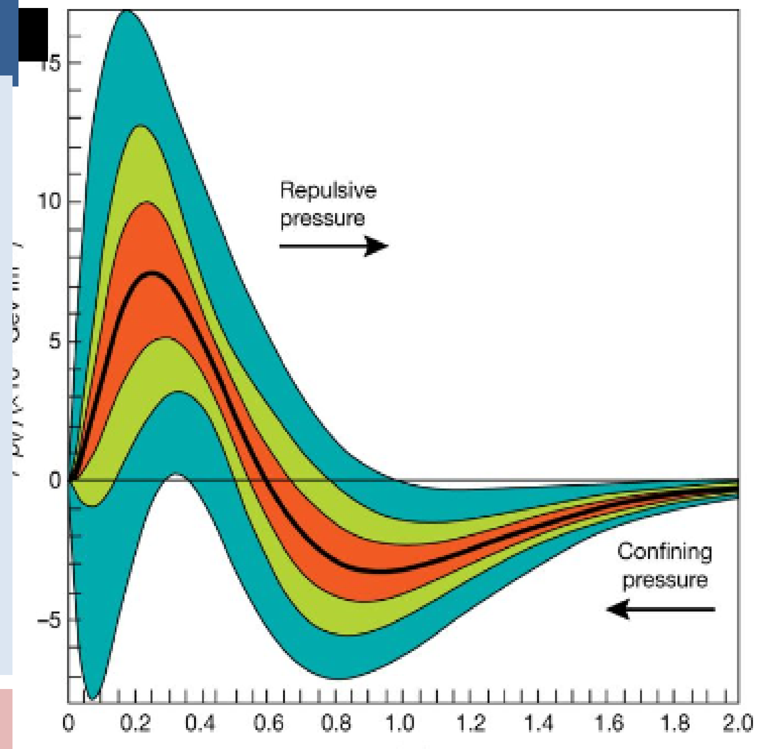
- ***"pressure",***
- ***"shear force",***

★"D-term"★

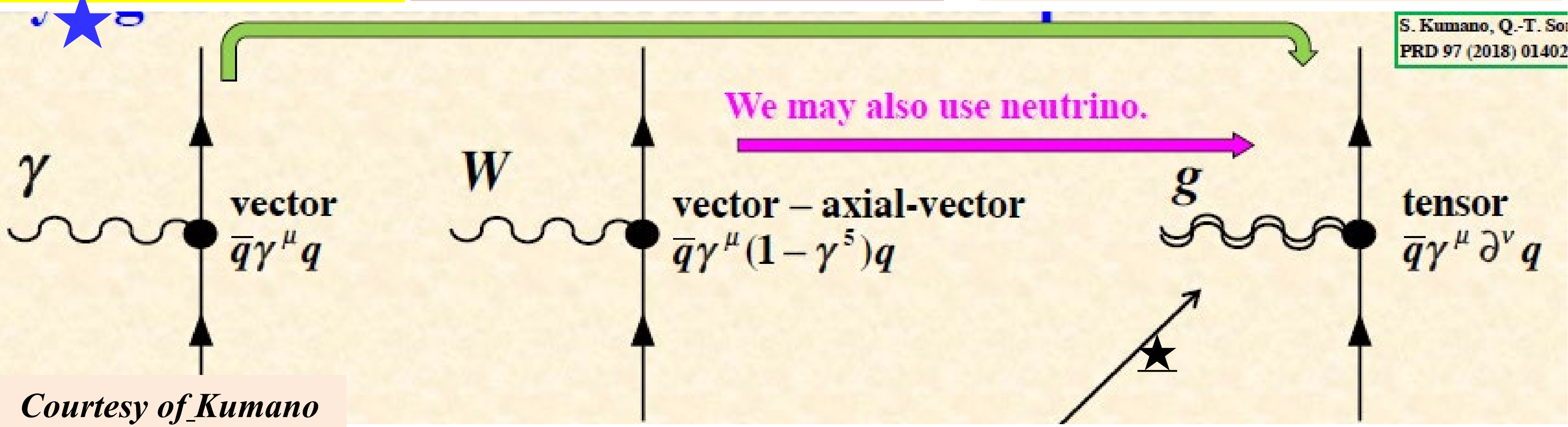
Gravitation form factors (GFFs)

[Polyakov, proposed, 1998]

ig. 1: Radial pressure distribution in the proton.



Burkert V D, Nature, 2018



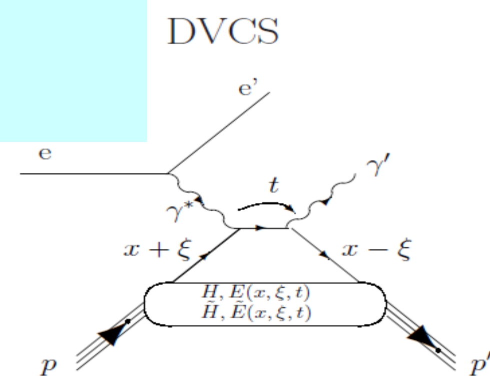
Courtesy of Kumano

★ GPDs (generalized parton distributions)$_{for\ nucleon}$

GPDs $H_{(q,g)}(x,\xi,Q^2)$ naturally embody the information of both PDFs and FFs, and therefore, display the unique properties to present a "3-D" description for a system.

$$V_{\lambda'\lambda}^{S=1/2} = \frac{1}{2}\int \frac{dz^- e^{ix(P\cdot z)}}{(2\pi)} \langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not{n} q \left(+\frac{z}{2}\right) | p,\lambda \rangle \Big|_{z^+=0,\vec{z}=0} \quad \text{for nucleon } (S=1/2)$$

$$= H^q(x,\xi,t)\bar{u}(p')\not{n}\, u(p) + \frac{i}{2M_N} E^q(x,\xi,t)\bar{u}(p')\sigma^{\alpha\beta} n_\alpha q_\beta u(p)$$



example:

(1) In the forward limit they reduce to conventional PDFs

$$H_q(x,0,0) = q(x)\,,$$

$$\tilde{H}_q(x,0,0) = \Delta q(x)\,.$$

Parton distributions (PDFs)
Hadronic level

★GPDs

(2) When one integrates GPDs over x they reduce to the usual form factors, e.g. the Dirac form factors[a]

$$\sum_q e_q \int dx\, H_q(x,\xi,t) = F_1(t)\,,$$

$$\sum_q e_q \int dx\, E_q(x,\xi,t) = F_2(t)\,.$$

Form Factors(FFs)
Hadronic level

★ GPDs (generalized parton distributions) $_{(S<3/2)}$

① ***for pion*** **(*S*=0)**

Broniowski, PLB 574, PRD78; Choi et al., PRD64; Fanelli, EPJC76; ……

② ***for nucleon*** **(*proton and neutron, S*=1/2)**

Diehl et al., EPJC 73; Kroll, EPJA53; Pire et al., PRD79; Selyugin, PRD91;……

③ ***Light Nuclei: He-3*** **(*S*=1/2)**

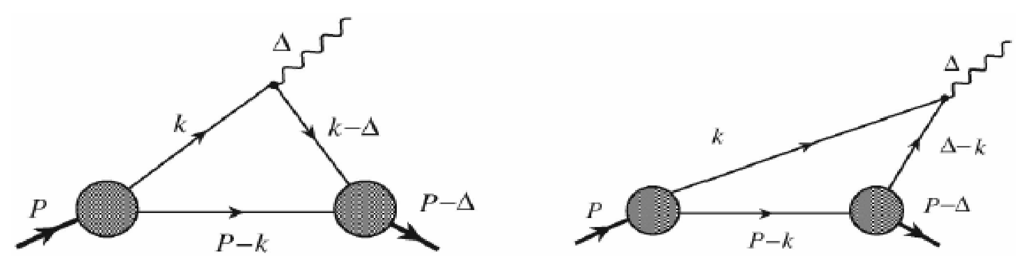
Rinaldi et al., PRC87……

$$V^{S=1/2}_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^- e^{ix(P\cdot z)}}{(2\pi)} \langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not{n} q\left(+\frac{z}{2}\right) | p,\lambda\rangle \Big|_{z^+=0,\vec{z}=0}$$

④ ***Deuteron, ρ meson*** **(*S*=1)**

Cano et al., PRL87, YBD et al., JPG19,……

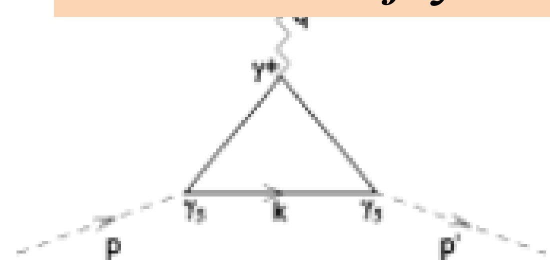
Generalized Parton distributions for pion, e. g.



Covariant amplitude with a reduced photon vertex for pion GPD (left diagram) and its nonvalence $x<\zeta$ part (right diagram).

PRD73, 114013

Broniowski, PLB574, In the limit of $\xi = 0$



Fi… he diagram for the evaluation of the generalized parton distribution of the pion in chiral quark models.

★ Spin-1 particle and basic properties *example*

♠ Form factor: decomposition of Local current ⇨ EMFFS

$$I^{\mu}_{\lambda'\lambda} = \langle p',\lambda' | \bar{q}(0)\gamma^{\mu} q(0) | p,\lambda \rangle$$
$$= \varepsilon'^{\beta *}\left[-\left(G^q_1 g_{\beta\alpha} + G^q_3 \frac{P_\beta P_\alpha}{2M^2}\right)P^\mu + G^q_2\left(g^\mu_\alpha P_\beta + g^\mu_\beta P_\alpha\right)\right]\varepsilon^\alpha$$

$$\begin{cases} G_C(t) = G_1(t) + 2\eta/3 \cdot G_Q(t) \\ G_M(t) = G_2(t) \\ G_Q(t) = G_1(t) - G_2(t) + (1+\eta)G_3(t) \end{cases}$$

[Hoodbhoy '89, Frederico '97, Berger '01, Broniowski '08 Cosyn'17]

$$P = \frac{p'+p}{2}, \qquad t = \Delta^2 = (p'-p)^2$$
$$\varepsilon = \varepsilon(p,\lambda'), \qquad \varepsilon' = \varepsilon(p',\lambda')$$

Definitions of GPDs (spin −1)

- *Unpolarized*

$$V_{\mu\nu} : \{g_{\mu\nu}, P_\mu n_\nu, P_\nu n_\mu, P_\mu P_\nu, n_\mu n_\nu\}$$

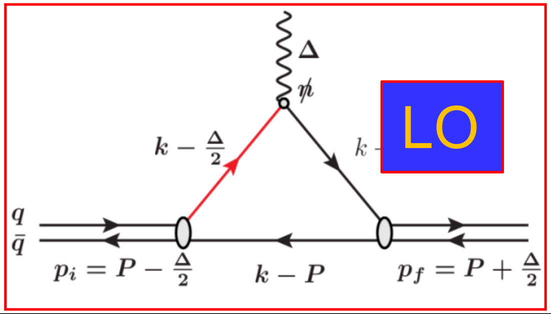
$$V^{S=1}_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^- e^{ix(P\cdot z)}}{(2\pi)} \langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not{n} q\left(+\frac{z}{2}\right) | p,\lambda \rangle \Big|_{z^+=0,\vec{z}=0}$$
$$= \sum_{i=1}^{5} (\varepsilon'^{\beta})^* V^i_{\beta\alpha}\varepsilon^\alpha H^q_i(x,\xi,t) \qquad \text{for } S=1$$

★ Spin-1 particle and basic properties *example*

$$V^{S=1}_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^- e^{ix(P\cdot z)}}{(2\pi)} \langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not{n} q\left(+\frac{z}{2}\right) | p,\lambda \rangle \Big|_{z^+=0,\vec{z}=0}$$

for $S=1$

$$= \sum_{i=1}^{5} (\varepsilon'^{\beta})^* V^i_{\beta\alpha} \varepsilon^{\alpha} H^q_i(x,\xi,t) = \sum_{i=1}^{5} (\varepsilon'^{\beta})^* \tilde{V}^i_{\beta\alpha} \varepsilon^{\alpha}$$



$$\tilde{V}_{\lambda'\lambda} = -(\epsilon'^* \cdot \epsilon) H_1 + \frac{(\epsilon\cdot n)(\epsilon'^*\cdot P) + (\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n} H_2 - \frac{(\epsilon\cdot P)(\epsilon'^*\cdot P)}{2M^2} H_3$$

$$+ \frac{(\epsilon\cdot n)(\epsilon'^*\cdot P) - (\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n} H_4 + \left\{4M^2 \frac{(\epsilon\cdot n)(\epsilon'^*\cdot n)}{(P\cdot n)^2} + \frac{1}{3}(\epsilon'^*\cdot\epsilon)\right\} H_5 ,$$

$$P = \frac{p'+p}{2}, \quad t = \Delta^2 = (p'-p)^2 ,$$

$$n^2 = 0 , \text{ (lightlike four-vector)}$$

$$\xi = (n\cdot\Delta)/(n\cdot P) , \quad \text{skewness parameter} ,$$

$$\epsilon = \epsilon(p,\lambda) , \ \epsilon' = \epsilon'(p',\lambda') , \text{ polarizations} ,$$

Asymmetry in Longitudinal direction

● **EMT (Energy Momentum tensor), Gravitational Form Factors (GFFs)**

Mellin Moment: $\begin{cases} \alpha = 0, & EMFFs \\ \alpha = 1, & GFFs \end{cases}$

$$(P\cdot n)^{\alpha+1} \int dx\, x^{\alpha} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{\psi}\left(-\frac{z}{2}\right) \not{n} \psi\left(+\frac{z}{2}\right)\right]\Big|^{z^+=0}_{\vec{z}=0}$$

$$= \left(i\frac{d}{dz^-}\right)^{\alpha} \left[\bar{\psi}\left(-\frac{z}{2}\right) \not{n} \psi\left(+\frac{z}{2}\right)\right]\Big|_{z=0} = \bar{\psi}(0) \not{n} \left(i\overleftrightarrow{\partial}^+\right)^{\alpha} \psi(0)$$

●2, Spin (1or3/2)$_{\text{high-spin}}$ particles (selected) and the basic properties

★ Spin-1: deuteron target is accessible in some facilities

★ Spin-3/2: particles, theoretically necessary

★ Spin-3/2 target may be accessible in future EIC (EicC), and some other Facilities

The comparison between the parameters of the electron-ion colliders proposed in China and in the US [redacted].

Facility	CoM energy	Lum./10^{33} ($\text{cm}^{-2}\cdot\text{s}^{-1}$)	Ions	Polarization
EicC	15–20	2–3	$p \to$U	e^-, p, and light nuclei
EIC-US	30–140	2–15	$p \to$U	e^-, p, ^{3}He

[◆[Electron-ion collider in China Fronties of Physics, 16, 64701]]

Li-7 (3/2) stable

★ Spin-3$^-$/2 (Ω hyperon) target might be possible in future

$$\begin{cases} e^+e^- \to (\Omega\bar{\Omega})\ pair \ \Big|_{c\tau=1.261cm} \\ p+A \\ Heavy\ ion\ collisions\ (RHIC) \end{cases}$$

★ 2.1, Spin-3/2 particle and basic properties

Spin-3/2 ---- Rarita–Schwinger spinor: $u^\alpha(p,\lambda)$

(DYF, BDS, YBD), PRD105, 096002, PRD106,116012, 2305.02680

- $u^\alpha(p,\lambda) = \sum_{\rho,\sigma} C^{\frac{3}{2}\lambda}_{1\rho,\frac{1}{2}\sigma}\epsilon^\alpha(p,\rho)u(p,\sigma) \quad u(p,\sigma) = \frac{(\not p + M)}{\sqrt{2p\cdot n}}\not n\chi_\sigma;$

$$\epsilon^\alpha(p,0) = \frac{1}{M}\left(p^+, p^- - \frac{2M^2}{p^+}, \boldsymbol{\epsilon}_\perp(p,0)\right)^{\mathrm{T}}, \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,0) = (p_1, p_2),$$

$$\epsilon^\alpha(p,+1) = -\left(0, \frac{\sqrt{2}(p_1+ip_2)}{p^+}, \boldsymbol{\epsilon}_\perp(p,+1)\right)^{\mathrm{T}}, \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,+1) = (\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}),$$

$$\epsilon^\alpha(p,-1) = \left(0, \frac{\sqrt{2}(p_1-ip_2)}{p^+}, \boldsymbol{\epsilon}_\perp(p,-1)\right)^{\mathrm{T}}, \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,-1) = (\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}).$$

Light-Cone $v = (v^+, v^-, \mathbf{v}), \quad \text{with} \quad v^\pm = v^0 \pm v^3 \text{ and } \mathbf{v} = (v^1, v^2)$

- $(\not p - M)\,u^\alpha(p,\lambda) = 0, \quad \gamma_\alpha u^\alpha(p,\lambda) = 0, \quad \partial_\alpha u^\alpha(p,\lambda) = 0.$
- $\bar{u}_\alpha(p,\lambda')u^\alpha(p,\lambda) = -2M\delta_{\lambda'\lambda}$ $\quad n^2 = 0$, *lightlike four vector*

● EM Form factors of a Spin−3/2 particle : _conventional (2S+1)

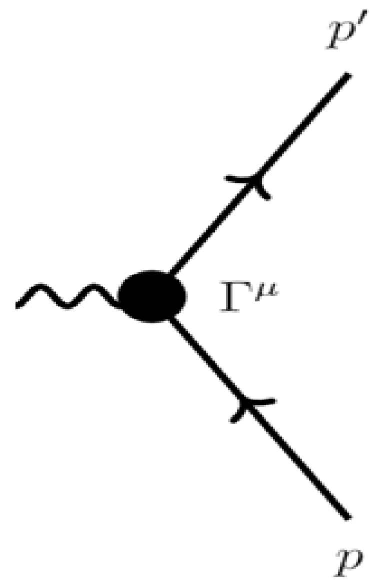
$$\langle p',\lambda'|\bar{\psi}(0)\gamma^{\mu}\psi(0)|p,\lambda\rangle = -\vec{u}_{\alpha'}(p',\lambda')\left[\frac{P^{\mu}}{M}\left(g^{\alpha'\alpha}F^{V,a}_{1,0}(t)-\frac{q^{\alpha'}q^{\alpha}}{2M^{2}}F^{V,a}_{1,1}(t)\right)\right.$$

★

$$\left.+\frac{i\sigma^{\mu\nu}q_{\nu}}{2M}\left(g^{\alpha'\alpha}F^{V,a}_{2,0}(t)-\frac{q^{\alpha'}q^{\alpha}}{2M^{2}}F^{V,a}_{2,1}(t)\right)\right]u_{\alpha}(p,\lambda),$$

$$\langle p',\lambda'|\bar{\psi}(0)\gamma^{\mu}\gamma^{5}\psi(0)|p,\lambda\rangle = -\vec{u}_{\alpha'}(p',\lambda')\left[\gamma^{\mu}\left(g^{\alpha'\alpha}\tilde{F}^{V,a}_{1,0}(t)+\frac{P^{\alpha'}P^{\alpha}}{M^{2}}F^{V,a}_{1,0}(t)\right)\right.$$

$$\left.-\frac{q^{\mu}}{2M}\left(-g^{\alpha'\alpha}\tilde{F}^{V,a}_{2,0}(t)+\frac{P^{\alpha'}P^{\alpha}}{M^{2}}\tilde{F}^{V,a}_{2,1}(t)\right)\right]\gamma^{5}u_{\alpha}(p,\lambda)$$



$$\left(G_{E0}(t),G_{M1}(t),G_{E2}(t),G_{M3}(t)\right)\Leftarrow\left(F_{10}(t),F_{11}(t),F_{20}(t),F_{21}(t)\right)$$

$$G_{E0}(t)=\left(1+\frac{2}{3}\tau\right)[F^{V}_{2,0}(t)+(1+\tau)(F^{V}_{1,0}(t)-F^{V}_{2,0}(t))]$$
$$+\frac{2}{3}\tau(1+\tau)[F^{V}_{2,1}(t)+(1+\tau)(F^{V}_{1,1}(t)-F^{V}_{2,1}(t))],$$

$$\tau=\frac{Q^{2}}{4M^{2}}$$

$$G_{E2}(t)=[F^{V}_{2,0}(t)+(1+\tau)(F^{V}_{1,0}(t)-F^{V}_{2,0}(t))]+(1+\tau)[F^{V}_{2,1}(t)+(1+\tau)(F^{V}_{1,1}(t)-F^{V}_{2,1}(t))],$$

$$G_{M1}(t)=\left(1+\frac{4}{5}\tau\right)F^{V}_{2,0}(t)+\frac{4}{5}\tau(\tau+1)F^{V}_{2,1}(t),$$

$$G_{M3}(t)=F^{V}_{2,0}(t)+(\tau+1)F^{V}_{2,1}(t),$$

One can select a reference frame, (say Breit frame), to proceed a calculation for EM-multipole form factors ★

GPDs for a spin-3/2 particle

$$V_{\lambda'\lambda}^{S=3/2} = \frac{1}{2}\int \frac{dz^- e^{ix(P\cdot z)}}{(2\pi)} \langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not{n} q\left(+\frac{z}{2}\right) | p,\lambda \rangle \Big|_{z^+=0,\vec{z}=0}^{S=3/2}$$

Rarita-Schwinger spinor

$$u^\alpha(p,\lambda) = \sum_{\rho,\sigma} C^{\frac{3}{2}\lambda}_{1\rho,\frac{1}{2}\sigma} \epsilon^\alpha(p,\rho) u(p,\sigma)$$

$$\begin{cases} (1, \not{n}) & S = 1/2 \\ (g^{\alpha\alpha'}, P^\alpha P^{\alpha'}, n^{[\alpha'} P^{\alpha]}, n^{[\alpha} P^{\alpha']}, n^{\alpha'} n^\alpha) & S = 1 \end{cases}$$

Direct product

$$(g^{\alpha\alpha'}, P^\alpha P^{\alpha'}, n^{[\alpha'} P^{\alpha]}, n^{[\alpha} P^{\alpha']}, n^{\alpha'} n^\alpha, g^{\alpha\alpha'} \not{n}, P^\alpha P^{\alpha'} \not{n}, n^{[\alpha'} P^{\alpha]} \not{n}, n^{[\alpha} P^{\alpha']} \not{n}, n^{\alpha'} n^\alpha \not{n},)$$

$$a^{[\mu} b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$$
$$a^{\{\mu} b^{\nu\}} = a^\mu b^\nu - a^\nu b^\mu$$

Conservations: (time-reversal, parity……)
+some other on-shell relations

Independent

$$\bar{u}_{\alpha'}(p',\lambda')\gamma^\mu \vec{u}_{\alpha'}(p,\lambda) = \bar{u}_{\alpha'}(p',\lambda')\left[\frac{P^\mu}{M} + \frac{i\sigma^{\mu\nu} q_\nu}{2M}\right]\gamma^\mu \vec{u}_{\alpha'}(p,\lambda) \Rightarrow \text{Identities}$$

$$i\varepsilon^{\mu\nu\rho\sigma} g^{\tau\lambda} + i\varepsilon^{\nu\rho\sigma\tau} g^{\mu\lambda} + i\varepsilon^{\rho\sigma\tau\mu} g^{\nu\lambda} + i\varepsilon^{\sigma\tau\mu\nu} g^{\rho\lambda} + i\varepsilon^{\tau\mu\nu\rho} g^{\sigma\lambda} = 0 \Rightarrow \text{Schouten Identity}$$

● GPDs for spin-3/2 particles

Some other on-shell relations +conservations

$$1 \doteq \frac{\not{P}}{M}, \qquad 0 \doteq \not{q}, \qquad \gamma^\mu\gamma_5 \doteq \frac{q^\mu\gamma_5}{2M} + \frac{i\sigma^{\mu P}}{M}, \qquad 0 \doteq P^\mu\gamma_5 + \frac{i\sigma^{\mu q}\gamma_5}{2},$$

$$\gamma_5 \doteq \frac{\not{q}\gamma_5}{2M}, \qquad 0 \doteq \not{P}\gamma_5, \qquad i\sigma^{\mu\nu} \doteq -\frac{q^{[\mu}\gamma^{\nu]}}{2M} + \frac{i\epsilon^{\mu\nu P\lambda}\gamma_\lambda\gamma_5}{M}, \qquad 0 \doteq -P^{[\mu}\gamma^{\nu]} + \frac{i\epsilon^{\mu\nu q\lambda}\gamma_\lambda\gamma_5}{2},$$

$$\gamma^\mu \doteq \frac{P^\mu}{M} + \frac{i\sigma^{\mu q}}{2M}, \qquad 0 \doteq \frac{q^\mu}{2} + i\sigma^{\mu P}$$

$$i\sigma^{\mu\nu}\gamma_5 \doteq -\frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{M} + \frac{i\epsilon^{\mu\nu q\lambda}\gamma_\lambda}{2M}, \qquad 0 \doteq -\frac{q^{[\mu}\gamma^{\nu]}\gamma_5}{2} + i\epsilon^{\mu\nu P\lambda}\gamma_\lambda,$$

$$\overline{u}^\alpha \frac{\not{P}}{M} u^\beta \doteq \overline{u}^\alpha I u^\beta$$

where $\doteq$ represents the on shell equality

like the well-known identities for S=1/2 fermion

● GPDs of a Spin-3/2 particle: Definitions of GPDs (spin −3/2)

★ Unpolarized

$$V_{\lambda'\lambda}^{S=3/2} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \left\langle p',\lambda' \middle| \bar{\psi}\left(-\frac{1}{2}z\right) \not{n}\, \psi\left(\frac{1}{2}z\right) \middle| p,\lambda \right\rangle \Bigg|_{z^+=0,\vec{z}=0}$$

$$= -\bar{u}_{\alpha'}(p',\lambda')\mathcal{H}^{\alpha'\alpha}(x,\xi,t)u_{\alpha}(p,\lambda),$$

$$\mathcal{H}^{\alpha'\alpha} = H_1 g^{\alpha'\alpha} + H_2 \frac{P^{\alpha'}P^{\alpha}}{M^2} + H_3 \frac{n^{[\alpha'}P^{\alpha]}}{(P\cdot n)} + H_4 \frac{M^2 n^{\alpha'}n^{\alpha}}{(P\cdot n)^2} + H_5 \frac{M g^{\alpha'\alpha}\not{n}}{(P\cdot n)}$$

$$+ H_6 \frac{P^{\alpha'}P^{\alpha}\not{n}}{M(P\cdot n)} + H_7 \frac{M n^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2} + H_8 \frac{M^3 n^{\alpha'}n^{\alpha}\not{n}}{(P\cdot n)^3}$$

$$a^{[\mu}b^{\nu]} = a^{\mu}b^{\nu} - a^{\nu}b^{\mu}$$

$$a^{\{\mu}b^{\nu\}} = a^{\mu}b^{\nu} - a^{\nu}b^{\mu}$$

★ Polarized

$$\tilde{V}_{\lambda'\lambda}^{S=3/2} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \left\langle p',\lambda' \middle| \bar{\psi}\left(-\frac{1}{2}z\right) \not{n}\,\gamma_5 \psi\left(\frac{1}{2}z\right) \middle| p,\lambda \right\rangle \Bigg|_{z^+=0,\vec{z}=0}$$

$$= -\bar{u}_{\alpha'}(p',\lambda')\tilde{\mathcal{H}}^{\alpha'\alpha}(x,\xi,t)u_{\alpha}(p,\lambda),$$

$$\tilde{\mathcal{H}}^{\alpha'\alpha} = \tilde{H}_1 g^{\alpha'\alpha}\gamma_5 + \tilde{H}_2 \frac{P^{\alpha'}P^{\alpha}}{M^2}\gamma_5 + \tilde{H}_3 \frac{n^{[\alpha'}P^{\alpha]}}{(P\cdot n)}\gamma_5 + \tilde{H}_4 \frac{M^2 n^{\alpha'}n^{\alpha}}{(P\cdot n)^2}\gamma_5 + \tilde{H}_5 \frac{M g^{\alpha'\alpha}\not{n}}{(P\cdot n)}\gamma_5$$

$$+ \tilde{H}_6 \frac{P^{\alpha'}P^{\alpha}\not{n}}{M(P\cdot n)}\gamma_5 + \tilde{H}_7 \frac{M n^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2}\gamma_5 + \tilde{H}_8 \frac{M^3 n^{\alpha'}n^{\alpha}\not{n}}{(P\cdot n)^3}\gamma_5$$

GPDs and EMFFs

Sum rules

$$M\int_{-1}^{1} \mathrm{d}x\, H_i(x,\xi,t) = G_i(t) \quad \text{with} \quad i=1,2,5,6,$$

$$M\int_{-1}^{1} \mathrm{d}x\, \tilde{H}_i(x,\xi,t) = \xi\tilde{G}_i(t) \quad \text{with} \quad i=1,2,$$

$$M\int_{-1}^{1} \mathrm{d}x\, \tilde{H}_i(x,\xi,t) = \tilde{G}_i(t) \quad \text{with} \quad i=5,6,$$

$$M\int_{-1}^{1} \mathrm{d}x\, H_j(x,\xi,t) = M\int_{-1}^{1} \mathrm{d}x\, \tilde{H}_j(x,\xi,t) = 0 \quad \text{with} \quad j=3,4,7,8,$$

Properties:

$$H_i(x,\xi,t) = H_i(x,-\xi,t) \quad \text{with} \quad i=1,2,4,5,6,8,$$

$$H_i(x,\xi,t) = -H_i(x,-\xi,t) \quad \text{with} \quad i=3,7,$$

$$\tilde{H}_i(x,\xi,t) = -\tilde{H}_i(x,-\xi,t) \quad \text{with} \quad i=1,2,3,4,$$

$$\tilde{H}_i(x,\xi,t) = \tilde{H}_i(x,-\xi,t) \quad \text{with} \quad i=5,6,7,8.$$

● GPDs and Structure Functions (S=3/2)

In Forward limit

$$F_1^q(x) = H_1(x,0,0) = \frac{q_\uparrow^{\frac{3}{2}}(x) + q_\uparrow^{-\frac{3}{2}}(x) + q_\uparrow^{\frac{1}{2}}(x) + q_\uparrow^{-\frac{1}{2}}(x)}{2},$$

$$b_1^q(x) = H_4(x,0,0) = \frac{(q_\uparrow^{\frac{3}{2}}(x) + q_\uparrow^{-\frac{3}{2}}(x)) - (q_\uparrow^{\frac{1}{2}}(x) + q_\uparrow^{-\frac{1}{2}}(x))}{2},$$

$$g_1^q(x) = \tilde{H}_5(x,0,0) = \frac{3(q_\uparrow^{\frac{3}{2}}(x) - q_\uparrow^{-\frac{3}{2}}(x)) + (q_\uparrow^{\frac{1}{2}}(x) - q_\uparrow^{-\frac{1}{2}}(x))}{\sqrt{20}},$$

$$g_2^q(x) = \tilde{H}_8(x,0,0) = \frac{(q_\uparrow^{\frac{3}{2}}(x) - q_\uparrow^{-\frac{3}{2}}(x)) - 3(q_\uparrow^{\frac{1}{2}}(x) - q_\uparrow^{-\frac{1}{2}}(x))}{\sqrt{20}}.$$

EMFFs, Structure functions, and PDFs

⟺ GPDs (S=3/2)

GPDs and EMT Energy-Momentum Tensor

Decomposition of EMT

$$\left\langle p',\lambda'\left|\hat{T}^{\mu\nu}(0)\right|p,\lambda\right\rangle$$

$$= -\,\overline{u}_{\alpha'}(p',\lambda')\left[\frac{P^\mu P^\nu}{M}\left(g^{\alpha'\alpha}F^T_{1,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{1,1}(t)\right)\right.$$

$$+\frac{(q^\mu q^\nu-g^{\mu\nu}q^2)}{4M}\left(g^{\alpha'\alpha}F^T_{2,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{2,1}(t)\right)$$

$$+Mg^{\mu\nu}\left(g^{\alpha'\alpha}F^T_{3,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{3,1}(t)\right)+\frac{P^{\{\mu}i\sigma^{\nu\}q}}{2M}\left(g^{\alpha'\alpha}F^T_{4,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{4,1}(t)\right)$$

$$\left.-\frac{1}{M}\left(2q^{\{\mu}g^{\nu\}[\alpha'}P^{\alpha]}+8g^{\mu\nu}P^{\alpha'}P^\alpha-g^{\alpha'\{\mu}g^{\nu\}\alpha}q^2\right)F^T_{5,0}(t)+Mg^{\alpha'\{\mu}g^{\nu\}\alpha}F^T_{6,0}(t)\right]u_\alpha(p,\lambda)$$

$$u^\alpha(p,\lambda)=\sum_{\rho,\sigma}C^{\frac{3}{2}\lambda}_{1\rho,\frac{1}{2}\sigma}\epsilon^\alpha(p,\rho)u(p,\sigma)$$

- **EMT (Energy Momentum tensor), Gravitational Form Factors (GFFs)**

Mellin Moment: $\begin{cases}\alpha=0, & \text{EMFFs}\\ \alpha=1, & \text{GFFs}\end{cases}$

GPDs and (EMT, GFFs, GMFFs)

Mellin Moment: $(P\cdot n)^{\alpha+1}\int dx x^{\alpha}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\left[\bar{\psi}\left(-\frac{z}{2}\right)\not{n}\psi\left(+\frac{z}{2}\right)\right]\Big|_{\vec{z}=0}^{z^+=0}$

$$=\left(i\frac{d}{dz^-}\right)^{\alpha}\left[\bar{\psi}\left(-\frac{z}{2}\right)\not{n}\psi\left(+\frac{z}{2}\right)\right]\Big|_{z=0}=\bar{\psi}(0)\not{n}\left(i\overleftrightarrow{\partial}^+\right)^{\alpha}\psi(0)$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_1(x,\xi,t)=F^T_{1,0}(t)+\xi^2F^T_{2,0}(t)-2F^T_{4,0}(t),$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_2(x,\xi,t)=2F^T_{1,1}(t)+2\xi^2F^T_{2,1}(t)-4F^T_{4,1}(t),$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_3(x,\xi,t)=8\xi F^T_{5,0}(t),$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_4(x,\xi,t)=\frac{2t}{M^2}F^T_{5,0}(t)+2F^T_{6,0}(t),$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_5(x,\xi,t)=2F^T_{4,0}(t),$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_6(x,\xi,t)=4F^T_{4,1}(t),$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_i(x,\xi,t)=0,\quad\text{with}\quad i=7,8.$$

One can select a reference frame to proceed a calculation (say Breit frame) for gravitational multi-pole form factors

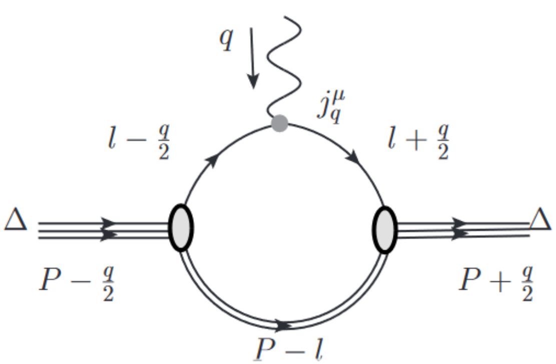
3. Numerical calculation and Results

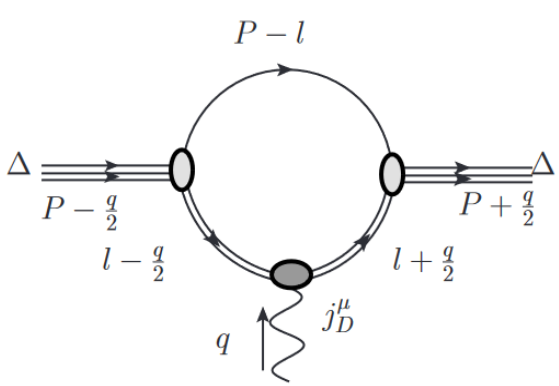
3.1), Framework: Covariant quark-diquark model (S=3/2)

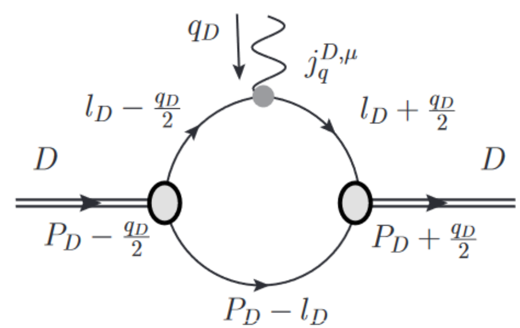
$$\begin{cases}\Delta^+ \\ \Omega^-\end{cases} \Rightarrow \begin{pmatrix} q\left(1^+/2\right) \\ D_{qq}\left(1^+\right)\end{pmatrix}$$

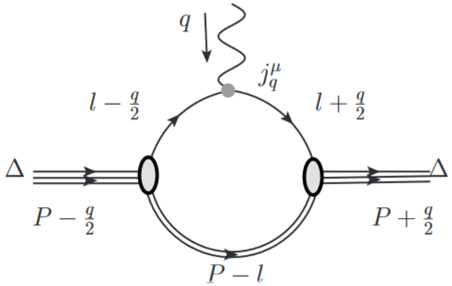
$$\begin{cases} j_q^\mu = -iQe\gamma^\mu \\ T_q^{\mu\nu} = \dfrac{i}{4}\left(\bar{\psi}_q\gamma^\mu \overleftrightarrow{\partial}^\nu \psi_q + \bar{\psi}_q\gamma^\nu \overleftrightarrow{\partial}^\mu \psi_q\right)\end{cases}$$

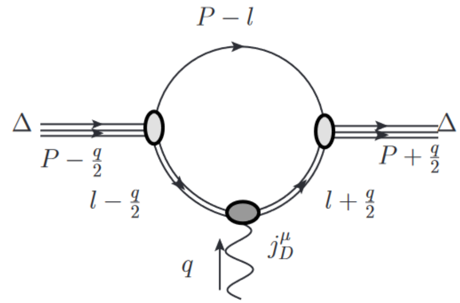
$$\mathcal{H}^{\alpha'\alpha} = H_1 g^{\alpha'\alpha} + H_2\frac{P^{\alpha'}P^{\alpha}}{M^2} + H_3\frac{n^{[\alpha'}P^{\alpha]}}{(P\cdot n)} + H_4\frac{M^2 n^{\alpha'}n^{\alpha}}{(P\cdot n)^2}$$
$$+ H_5\frac{Mg^{\alpha'\alpha}\not{n}}{(P\cdot n)} + H_6\frac{P^{\alpha'}P^{\alpha}\not{n}}{M(P\cdot n)} + H_7\frac{Mn^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2} + H_8\frac{M^3 n^{\alpha'}n^{\alpha}\not{n}}{(P\cdot n)^3}$$

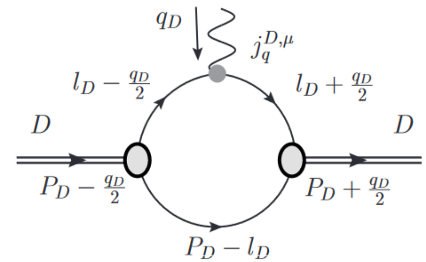












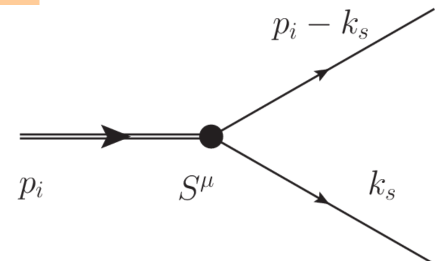
Phenomenological vertex For ⬭ :

[Choi '04, Frederico '09] [Scadon, PR165,1640]

$$\Gamma^{\alpha\beta} = c_1\left[g^{\alpha\beta} + g_2\gamma^\beta \Lambda^\alpha + g_3\Lambda^\alpha\Lambda^\beta\right]$$

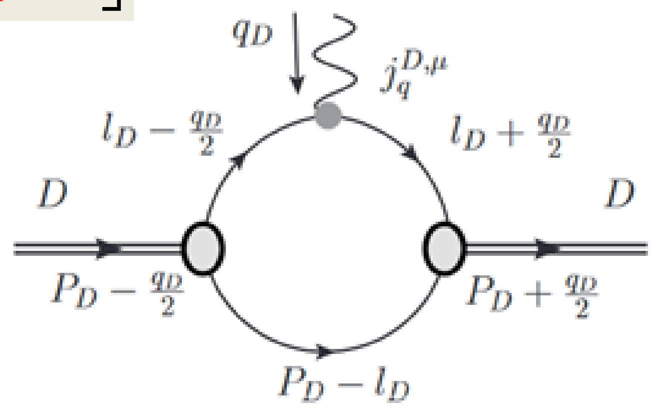
Phenomenal vertex:

$$\tilde{\Gamma}^{\alpha\beta} = \Gamma^{\alpha\beta} \cdot \Xi(p_1, p_2; m_R)$$



correlation amplitude:

$$\Xi(p_1, p_2; m_R) = \frac{c}{\left[p_1^2 - m_R^2 + i\varepsilon\right]\left[p_2^2 - m_R^2 + i\varepsilon\right]}$$



Explicit Diquark and its FFs

$$\mathcal{L}_{D\to qq} = c_D \Psi_q^T C^{-1} \gamma^\mu \Psi_q \epsilon_{\mu,D}(p_D, \lambda) \Xi_D + \text{H.c.},$$

$$j_D^{\mu,\beta'\beta} = \left[g^{\beta'\beta} F_{D;1}^V(t) - \frac{q^{\beta'} q^\beta}{2m_D^2} F_{D;2}^V(t)\right](p'_D + p_D)^\mu - (q^{\beta'} g^{\mu\beta} - q^\beta g^{\mu\beta'}) F_{D;3}^V(t)$$

3.2), Results: a), EMFFs of Δ

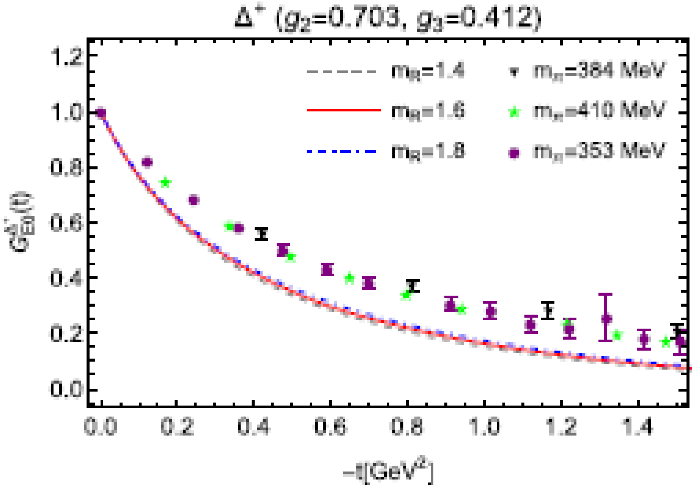
$$\left\{\begin{matrix}\Delta \\ \Omega^-\end{matrix}\right. \Rightarrow \begin{pmatrix} q\left(\frac{1^+}{2}\right) \\ D_{qq}\left(1^+\right)\end{pmatrix}$$

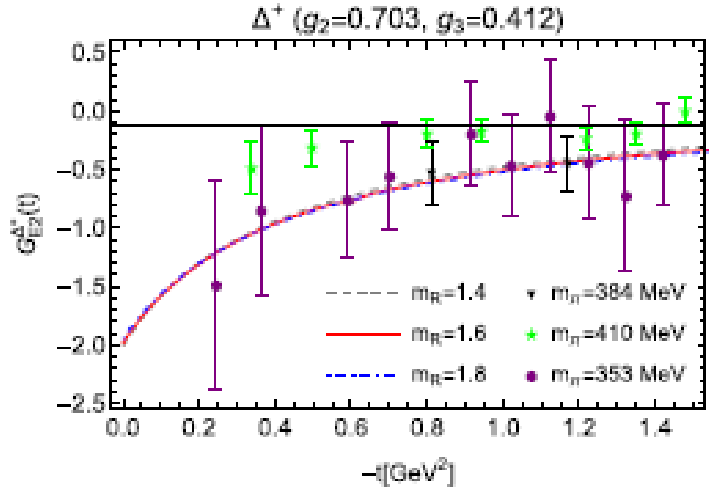
No Direct Measurement (So far)

- ****LQCDs***
- ***Models***

TABLE I. The parameters used in our approach.

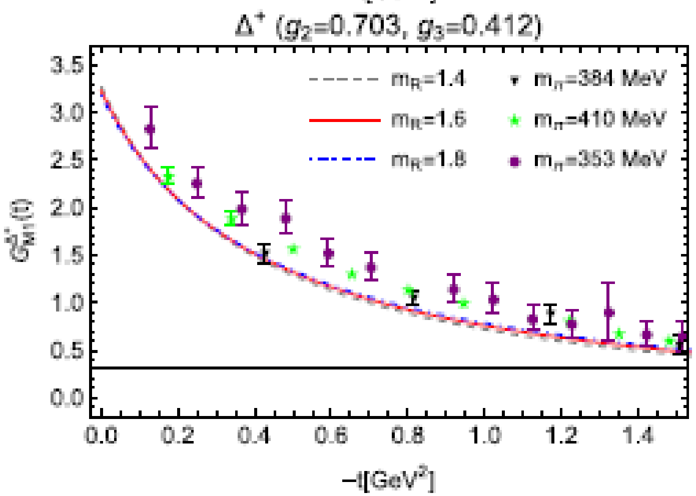
M/GeV	m_q/GeV	m_D/GeV	m_R/GeV	g_2/GeV^{-1}	g_3/GeV^{-2}
1.085	0.4	0.76	1.6	0.703	0.412

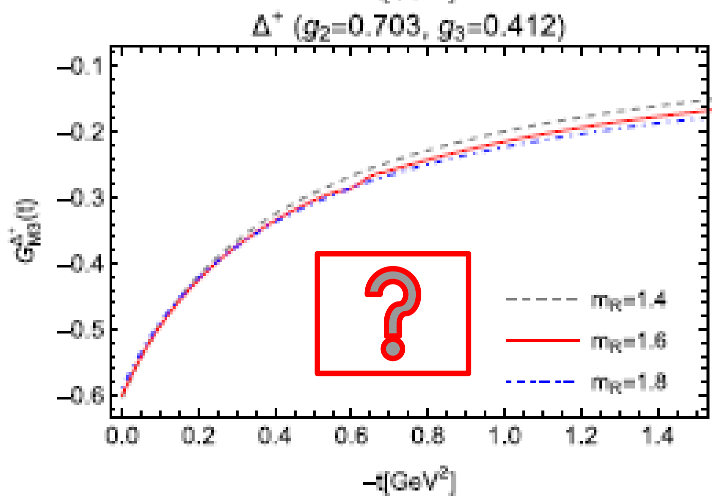




Our EMFFs v.s. LQCD Consistent

$$\Gamma^{\alpha\beta} = c_1\left[g^{\alpha\beta} + g_2\gamma^\beta\Lambda^\alpha + g_3\Lambda^\alpha\Lambda^\beta\right]$$





Δ

TABLE II. A comparison of our magnetic-dipole moment with other models.

$G_{M1}(t=0)$

$G_{M1}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
This work	6.04	3.02	0.00	−3.02
NQM [68]	5.56	2.73	−0.09	−2.92
RQM [71]	4.76	2.38	0.00	−2.38
QCDSR [72–74]	4.39 ± 1.00	2.19 ± 0.50	0.00	-2.19 ± 0.50
LCQSR [76]	4.4 ± 0.8	2.2 ± 0.4	0.0	-2.2 ± 0.4
Large N_c [77–79]	5.9(4)	2.9(2)	...	−2.9(2)
χQMEC[80,81]	6.93	3.47	0.00	−3.47
QCDQM [82,83]	5.689	2.778	−0.134	−3.045
CBM [84]	4.52	2.12	−0.29	−2.69
EMS [87,88]	4.56	2.28	0	−2.28
χPT [89,90]	5.390	2.383	−0.625	−3.632
LQCD [92–94]	4.91 ± 0.61	2.46 ± 0.31	0.00	-2.46 ± 0.31
χCQM[95]	5.82 ± 0.08	2.63 ± 0.06	-0.56 ± 0.09	-3.75 ± 0.08

Consistent

TABLE III. A comparison of our electric-quadrupole moment with other models.

$G_{E2}(t=0)$

$G_{E2}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
This work	−3.86	−1.93	0.00	1.93
NQM [69]	−3.82	−1.91	0	1.91
NQM [70]	−3.63	−1.79	0	1.79
χPT [91]	-3.12 ± 1.95	-1.17 ± 0.78	0.47 ± 0.20	2.34 ± 1.17
χQSM [86]	...	−2.15	...	...
QCDSR [75]	-0.0452 ± 0.0113	-0.0226 ± 0.0057	0	0.0226 ± 0.0057

$r_E^2(\Delta^+) = 0.665\ fm^2$

$G_{E2}^{\Delta^+} = -1.93$

Oblate deformed

$G_A^{\Delta^+} = 0.727$

Oblate

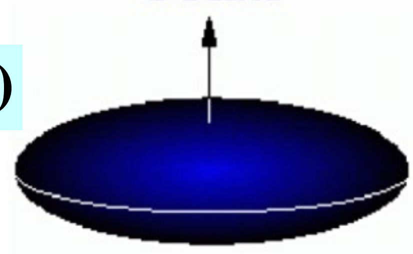


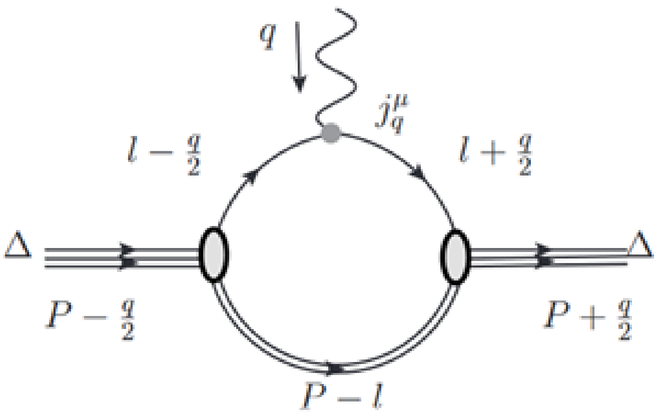
TABLE IV. A comparison of our magnetic-octupole moment with other model calculations.

$G_{M3}(t=0)$

$G_{M3}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
This work	−1.12	−0.56	0.00	0.56
GPQCD [85]	−11.68	−5.84	0	5.84
QCDSR [75]	-0.0925 ± 0.0234	-0.0462 ± 0.0117	0	0.0462 ± 0.0117

PRD105, 096002

3.2), Results: b), GPDs of Δ :



$$P = \frac{p' + p}{2}\ , \quad t = \Delta^2 = (p' - p)^2\ ,$$
$$n^2 = 0\ ,\ \text{(lightlike four-vector)}$$
$$\xi = (n\cdot\Delta)/(n\cdot P)\ ,\ \text{skewness parameter}\ ,$$
$$\epsilon = \epsilon(p,\lambda)\ ,\ \epsilon' = \epsilon'(p',\lambda')\ ,\ \text{polarizations}\ ,$$

Asymmetry in Longitudinal direction

• Example : PRC80, 045210

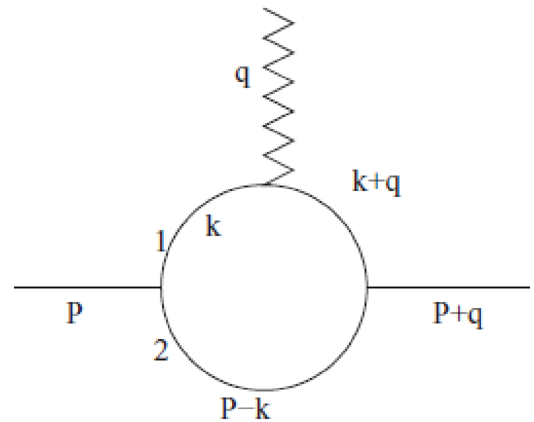


FIG. 1. Feynman diagram for the form factor with the photon coupling to the ϕ particle of mass m_1. The initial and final hadrons Ψ carry momentum P and $P+q$. The ξ is a spectator.

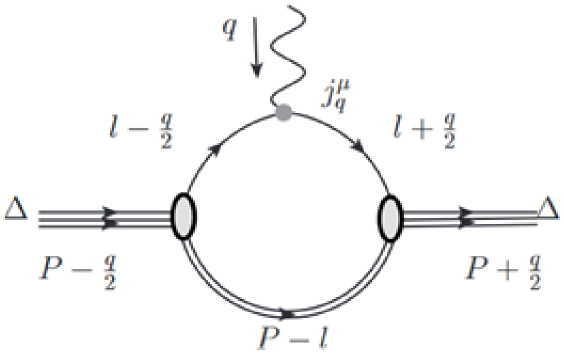
given by

$$J^\mu = \phi\ \overleftrightarrow{\partial^\mu}\ \phi$$

and find

$$\langle P+q|J^\mu(0)|P\rangle \equiv\ F(Q^2)(2P^\mu + q^\mu)$$

3.2), Results: b), GPDs of Δ :



$$V^{S=3/2}_{\lambda'\lambda} = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ix(P\cdot z)}\left\langle p',\lambda'\left|\bar{\psi}\left(-\frac{1}{2}z\right)\not{n}\,\psi\left(\frac{1}{2}z\right)\right|p,\lambda\right\rangle\Big|_{z^+=0,\vec{z}=0}$$

$$= -\bar{u}_{\alpha'}(p',\lambda')\mathcal{H}^{\alpha'\alpha}(x,\xi,t)u_\alpha(p,\lambda),$$

$$\mathcal{H}^{\alpha'\alpha} = H_1 g^{\alpha'\alpha} + H_2\frac{P^{\alpha'}P^{\alpha}}{M^2} + H_3\frac{n^{[\alpha'}P^{\alpha]}}{(P\cdot n)} + H_4\frac{M^2 n^{\alpha'}n^{\alpha}}{(P\cdot n)^2} + H_5\frac{Mg^{\alpha'\alpha}\not{n}}{(P\cdot n)}$$

$$+ H_6\frac{P^{\alpha'}P^{\alpha}\not{n}}{M(P\cdot n)} + H_7\frac{Mn^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2} + H_8\frac{M^3 n^{\alpha'}n^{\alpha}\not{n}}{(P\cdot n)^3}$$

- ● **Advantage**
- ◆ **Difference**

b.1), *3-D plots for d-quark unpolarized GPDs of Δ^+ with ($\xi = 0$, or -0.4)*

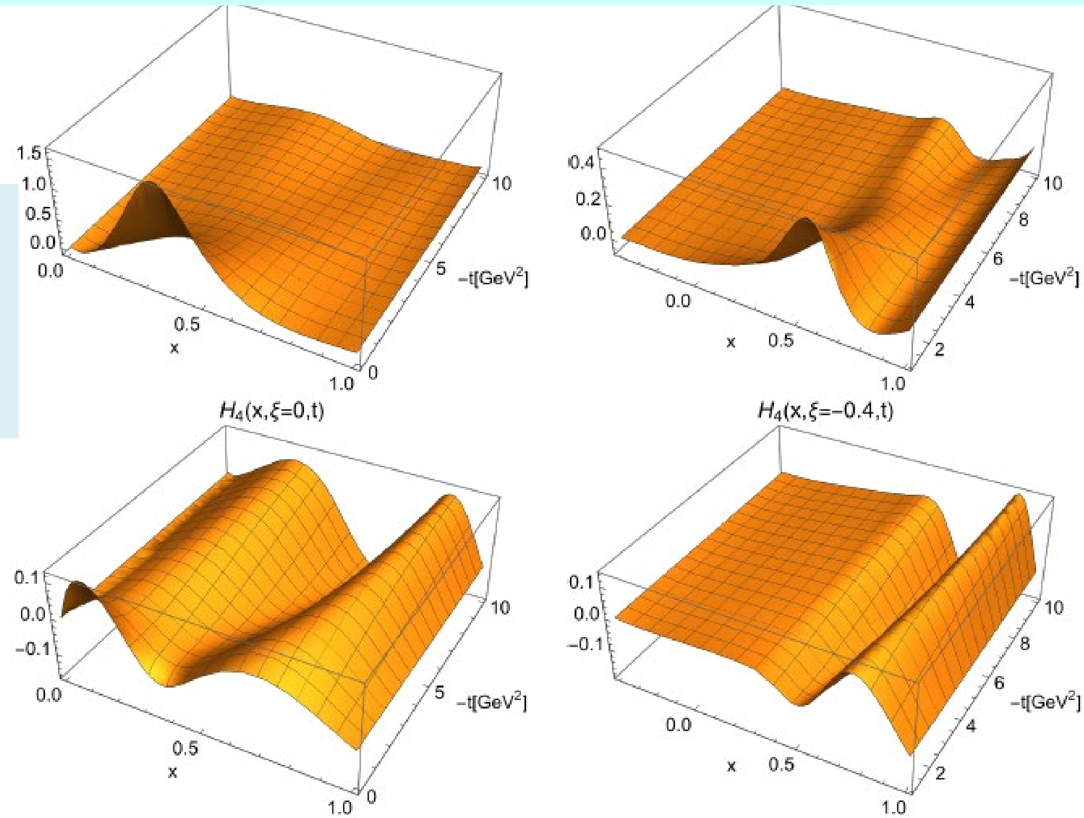
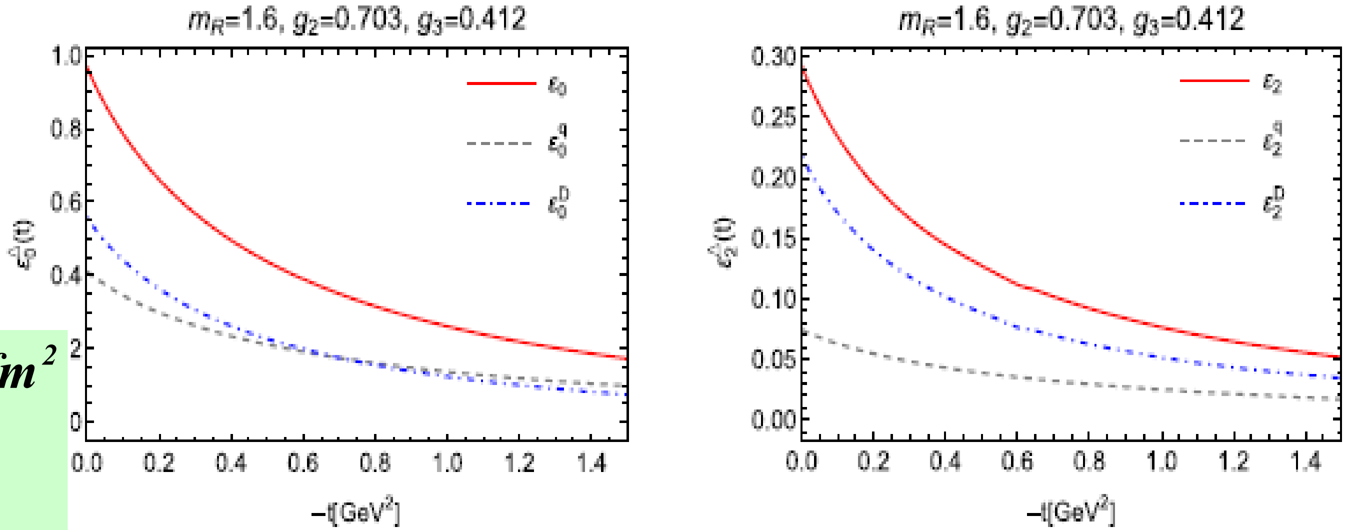


Figure 3: The 3D d quark unpolarized GPDs of Δ^+ H_1 and H_4 as functions of x and $-t$ at $\xi = 0$ and $\xi = -0.4$.

c), GFFs of Δ



$r_M^2(\Delta) = 0.529\ fm^2$

$\varepsilon_0(t=0) \sim 1$

$S \sim 3/2$

FIG. 7. The calculated energy-monopole form factor of the Δ as a function of $-t$ (left panel) and the energy quadrupole (right panel).

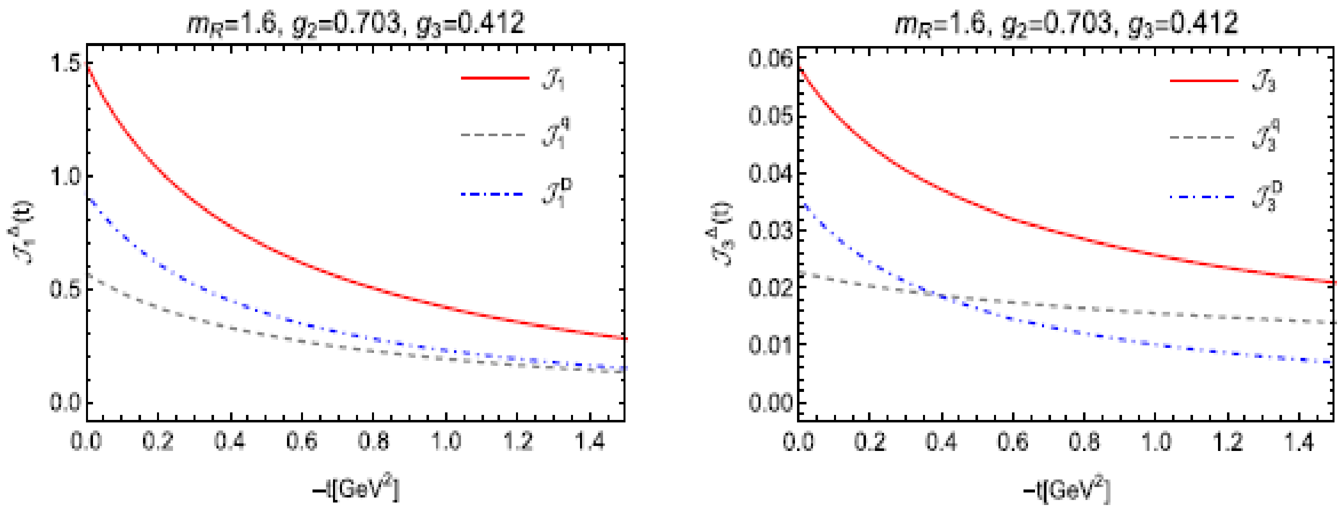


FIG. 8. The angular-momentum form factor of the Δ as a function of $-t$ (left panel), and the octupole-angular momentum form factor

All the numerical results!!

4, Summary

① Focus on spin-3/2 particles (Δ & Ω) and their GPDs, EMFFs, GFFs, and some other properties;

★ GPDs of the systems with spin-3/2 are defined and given.

② Calculation: framework (Quark-diquark (1^+) approach or quark diquark spectator approach)

③ Results: electromagnetic form factors of the example look okay (at least qualitatively) √

④ Some properties (static) of the systems are obtained (√)

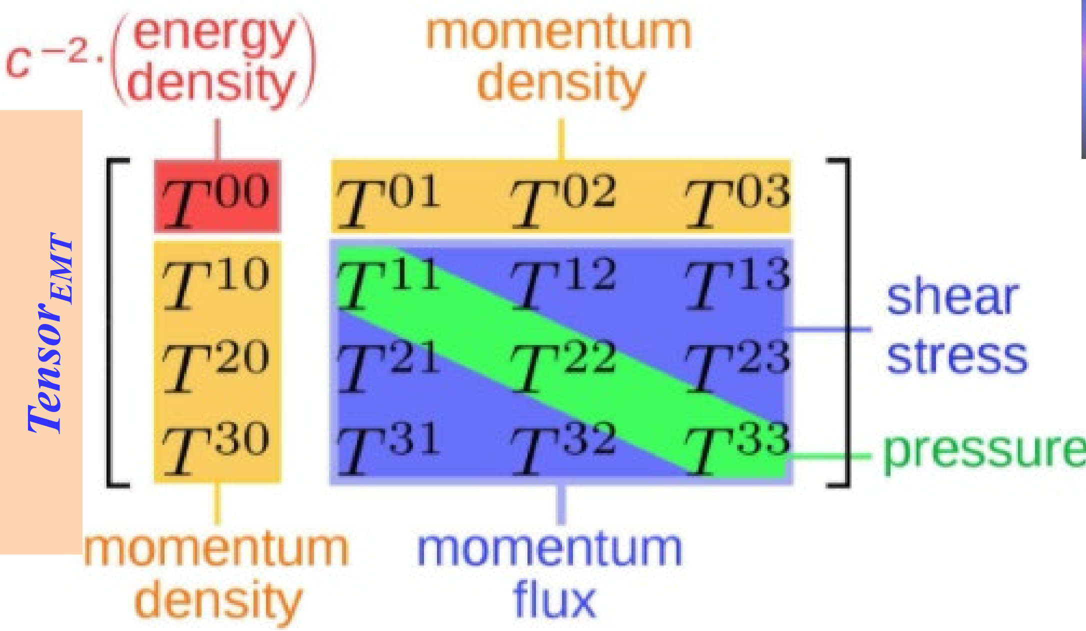
⑤ The calculations and analyses maybe useful for EicC (EIC)…

4.2, Discussions (GFFs)

I. Gravitational form factors of the systems (a system governed by the strong interaction) are also discussed through the GPDs and their moments.

II. Understanding the mechanical properties of the systems is necessary.

Energy-Momentum Tensor$_{EMT}$



Gravitational and a Global Description of nucleon mehanical properties :

GPDs → Gravitational FFs,

<|Energy-momentum tensor|>

(Energy density, mass radius, Spin)

Distributions

● *"pressure",*

● *"shear force",*

★*"D-term"*★

Classical Systems

⇕

Quantum Systems

♣ **In continuum media theory**

4.2, Discussions and questions (GFFs)

$$D(t)\left(\tilde{D}_n(r)\right) - term$$

$$\begin{cases} \text{"Shear Force"}: s_n(r) = -\dfrac{1}{4M} r \dfrac{d}{dr}\dfrac{1}{r}\dfrac{d}{dr}\tilde{D}_n(r) \\ \text{"Pressure"}: p_n(r) = \dfrac{1}{6M}\dfrac{1}{r^2}\dfrac{d}{dr} r^2 \dfrac{d}{dr}\tilde{D}_n(r) \end{cases}$$

Moreover, there is an equilibrium relation between the pressure and shear force densities

$$\frac{2}{3}\frac{ds_n(r)}{dr} + 2\frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0$$

von Laue condition is indeed satisfied

$$\int_0^\infty r^2 p_0(r)\, dr = 0$$

- × **But not inequality**

$$p_0(r) + \frac{2}{3}s_0(r) > 0$$

[Polyakov Proposed, 1998]

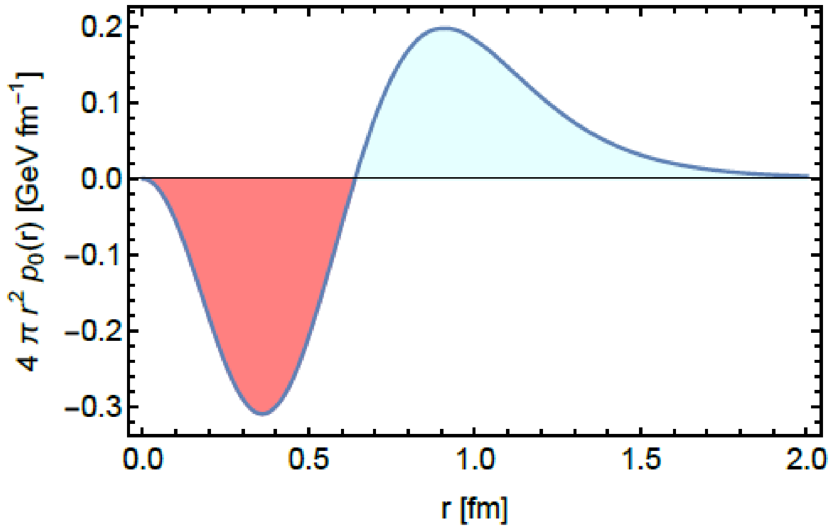


Figure 8: The physical quantity $4\pi r^2 p_0(r)$ as a function of r.

Questions:

0. Numerical results: model-dependent ($D > 0$ or $D < 0$)

1. The interpretation of "pressure" and "shear force" in this quantum few-body system?

2. EMT and the $\vec{\nabla}_i \langle T^{ij} \rangle = 0$ is sufficient? momentum current density?

Classical Systems
⇳
Quantum Systems

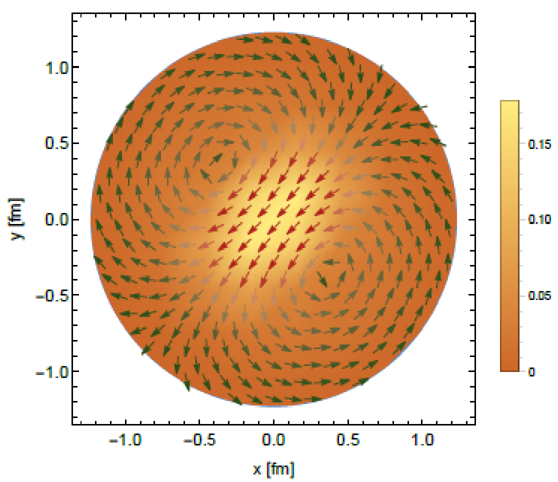


Figure 9: The momentum current with the unit GeV fm^{-3} on the $x - y$ plane with $z = 0$.

END－－Thanks for your attention!

Talk

“On electromagnetic and gravitational form factors

and generalized parton distributions (GPDs)

of spin-3/2 particles”

$$2P^+F(Q^2) = -ig^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{2k^+}{(k^2 - m_1^2 + i\epsilon)} \frac{1}{[(k+q)^2 - m_1^2 + i\epsilon]} \frac{1}{[(P-k)^2 - m_2^2 + i\epsilon]} \right]$$

$$= -ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{2k^+}{k^{+2}(P^+ - k^+)} \frac{1}{\left[k^- - \frac{(\mathbf{k}^2+m_1^2)}{k^+} + \frac{i\epsilon}{k^+}\right]} \frac{1}{\left[k^- - \frac{(\mathbf{k}+\mathbf{q})^2+m_1^2}{k^+} + \frac{i\epsilon}{k^+}\right]} \frac{1}{\left[P^- - k^- - \frac{(\mathbf{P}-\mathbf{k})^2+m_2^2}{P^+-k^+} + \frac{i\epsilon}{P^+-k^+}\right]}.$$

If we integrate over the upper half of the complex k^- plane we find a nonzero contribution only for the case $0 < k^+ < P^+$. Carrying out the integral leads to

$$2P^+F(Q^2) = \frac{g^2}{(2\pi)^3} \int d^2\mathbf{k} \int \frac{dk^+}{k^+(P^+ - k^+)} \frac{1}{P^- - \frac{\mathbf{k}^2+m_1^2}{k^+} - \frac{(\mathbf{P}-\mathbf{k})^2+m_2^2}{P^+-k^+}} \frac{1}{P^- - \frac{(\mathbf{k}+\mathbf{q})^2+m_1^2}{k^+} - \frac{(\mathbf{P}-\mathbf{k})^2+m_2^2}{P^+-k^+}}.$$

Next we change variables by defining

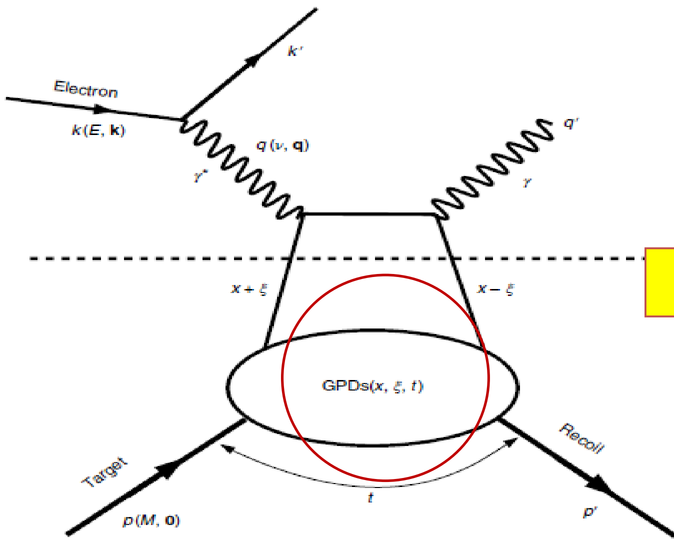
$$x \equiv \frac{k^+}{P^+}, \tag{12}$$

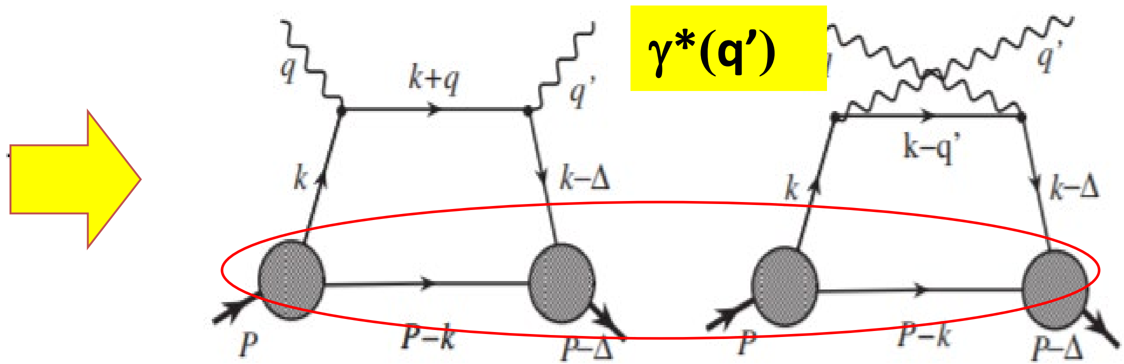
so that

$$F(Q^2) = \frac{g^2}{2(2\pi)^3} \int d^2\mathbf{k} \int_0^1 \frac{dx}{x(1-x)} \frac{1}{P^+P^- - \frac{\mathbf{k}^2+m_1^2}{x} - \frac{(\mathbf{P}-\mathbf{k})^2+m_2^2}{1-x}} \frac{1}{P^+P^- - \frac{(\mathbf{k}+\mathbf{q})^2+m_1^2}{x} - \frac{(\mathbf{P}-\mathbf{k})^2+m_2^2}{1-x}}.$$

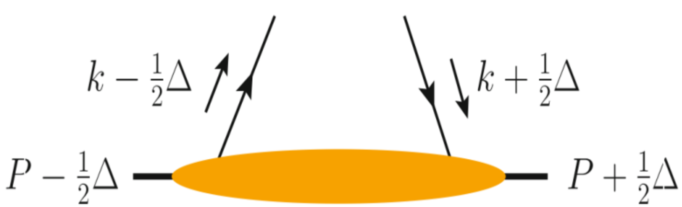
GPDs (generalized parton distributions)

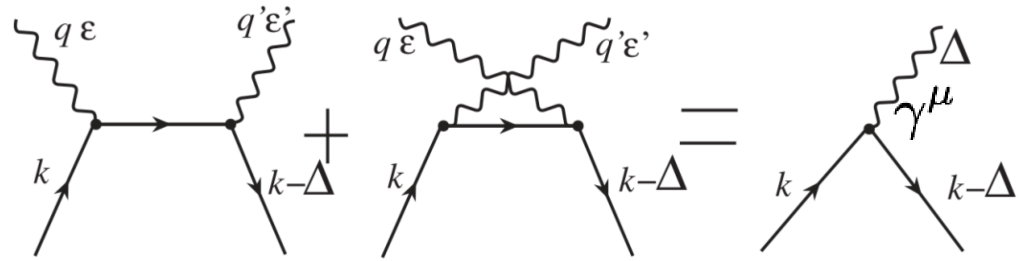
Deep virtual Compton Scattering (DVCS) PRD73, 114013





[Chueng-Ryong Ji '06, Diehl '16]





Parton correlation function:

GPDs (generalized parton distributions)

Deep virtual Compton Scattering(DVCS)
PRD73, 114013

[Chueng-Ryong Ji '06, Diehl '16]

A GPD factorization formula:

$$\mathcal{A}(\xi, \Delta^2, Q^2) = \sum_i \int_{-1}^{1} \mathrm{d}x\, C_i\big(x, \xi; \log(Q/\mu)\big) H_i(x, \xi, \Delta^2; \mu)$$

DVCS, TCS, meson productions

$$H(k, P, \Delta) = (2\pi)^{-4} \int d^4z\, e^{izk} \times \langle p(P + \tfrac{1}{2}\Delta)|\bar{q}(-\tfrac{1}{2}z)\Gamma q(\tfrac{1}{2}z)|p(P - \tfrac{1}{2}\Delta)\rangle$$

flavor by flavor

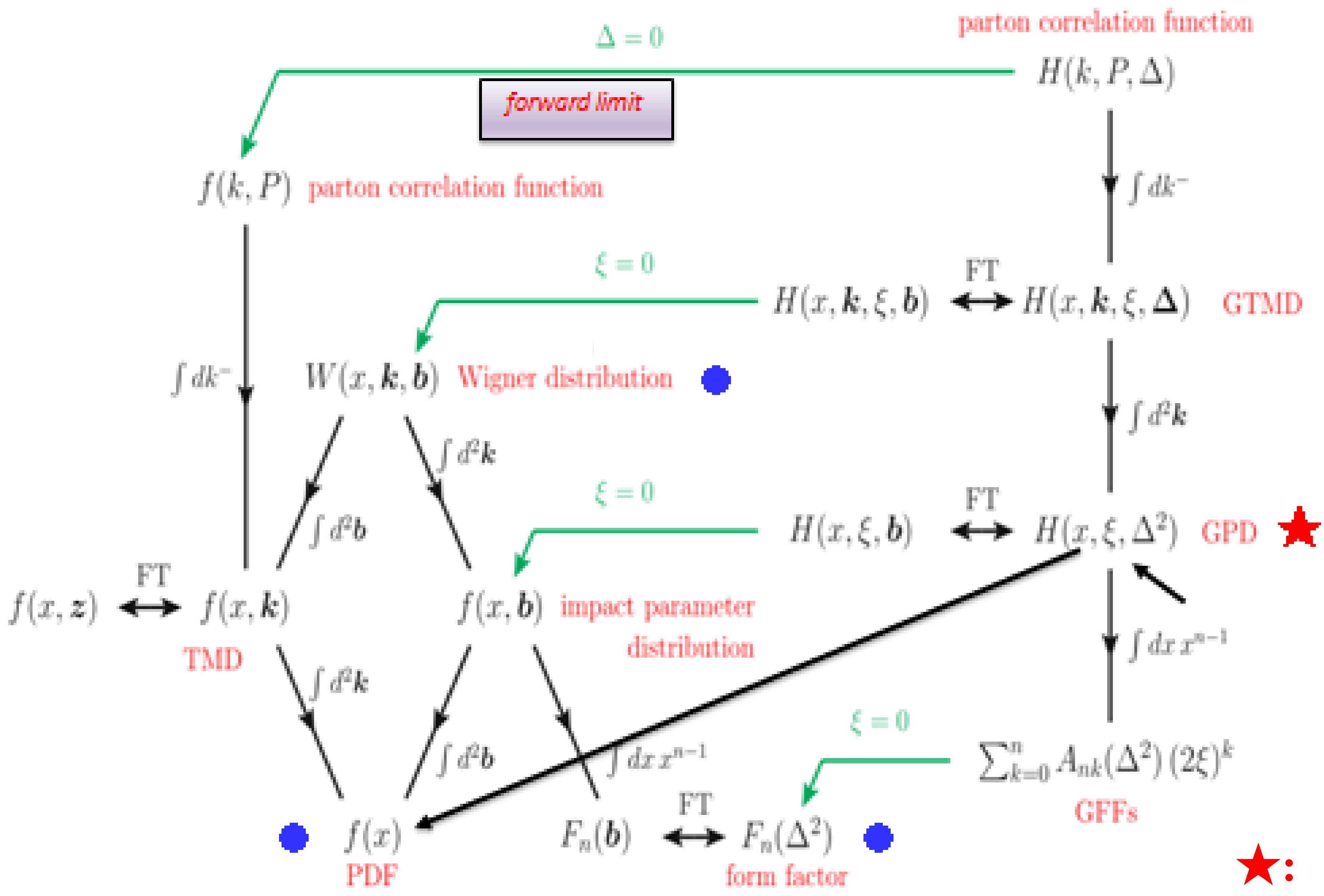
GPDs may be measured by
Deeply virtual Compton scattering
OR
Deeply virtual meson electro-productions

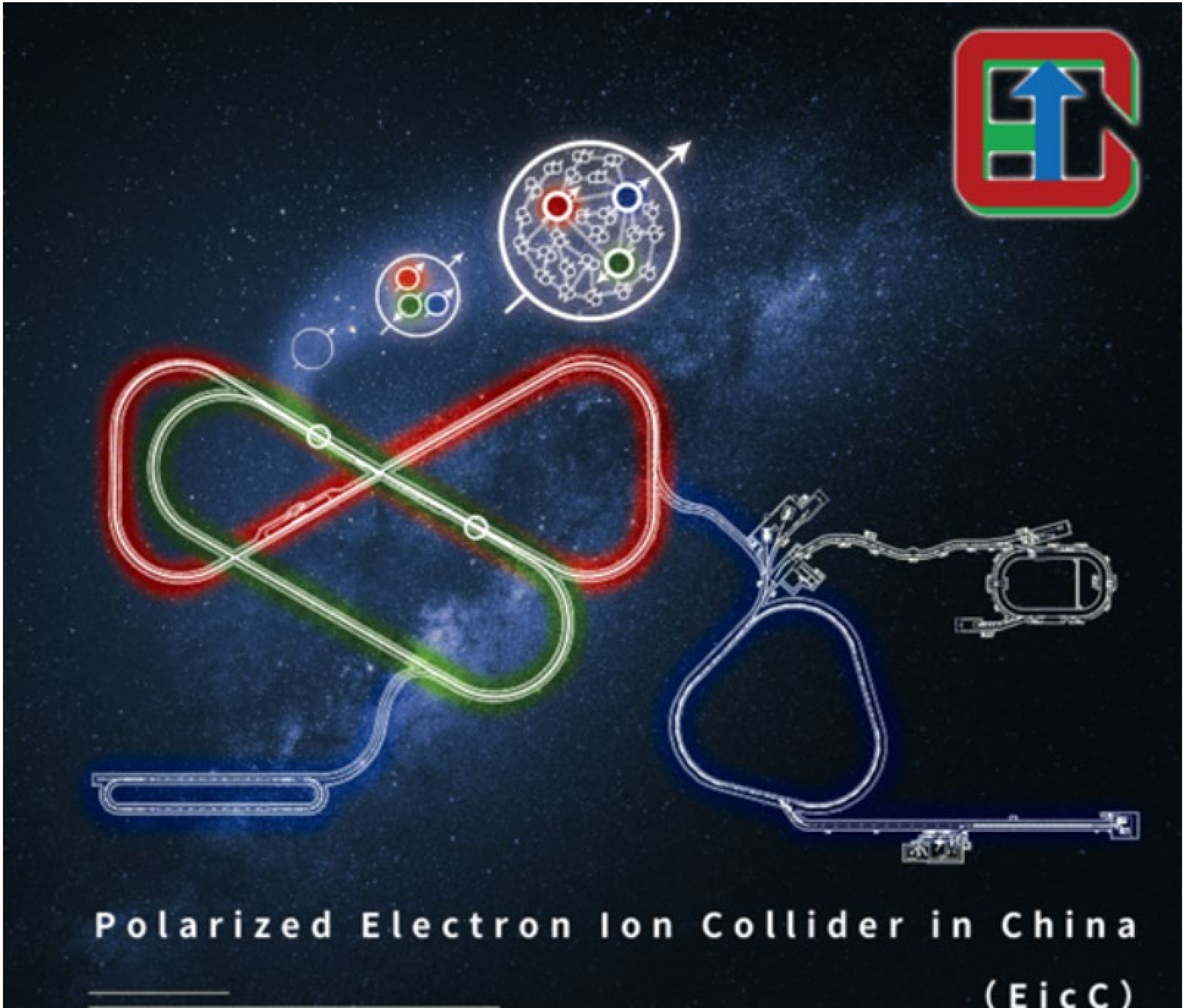
The Dirac matrix Γ selects the twist and the parton spin degrees of freedom.

$$\Gamma^\mu \to \gamma^\mu$$

3-D GPDs Schemes★: give rich information

[Diehl '16]




Polarized Electron Ion Collider in China
(EicC)

Axial vector form factors of Δ

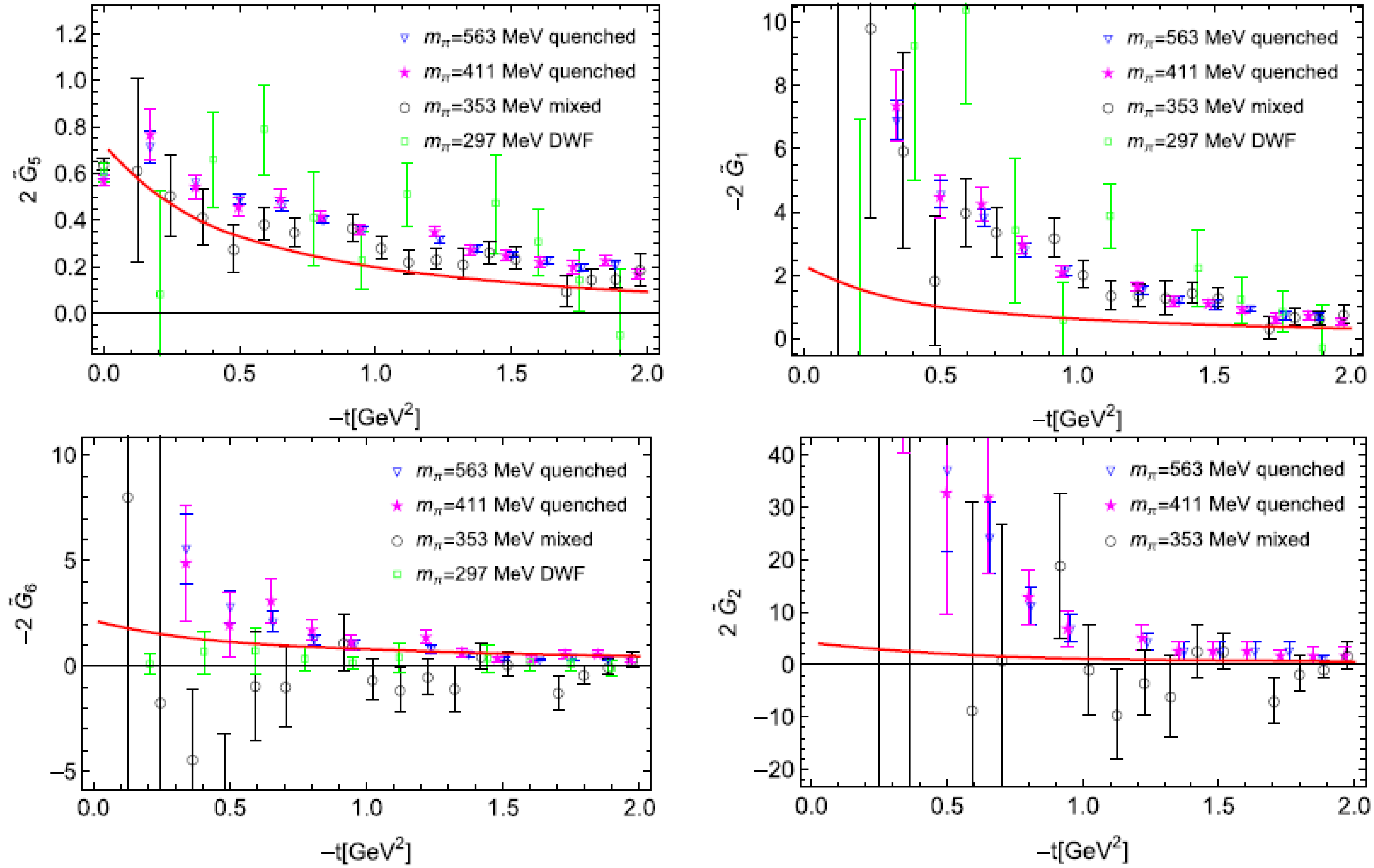
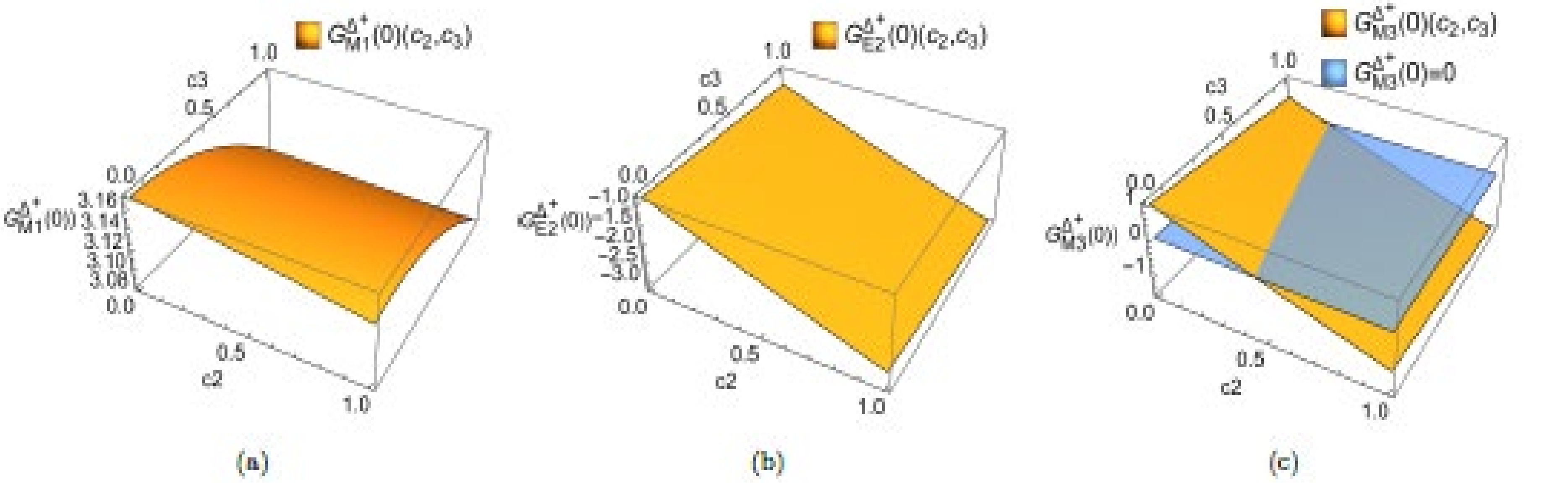


FIG. 6. The axial vector form factors of Δ^+ as functions of $-t$ in comparison with lattice QCD results [21].

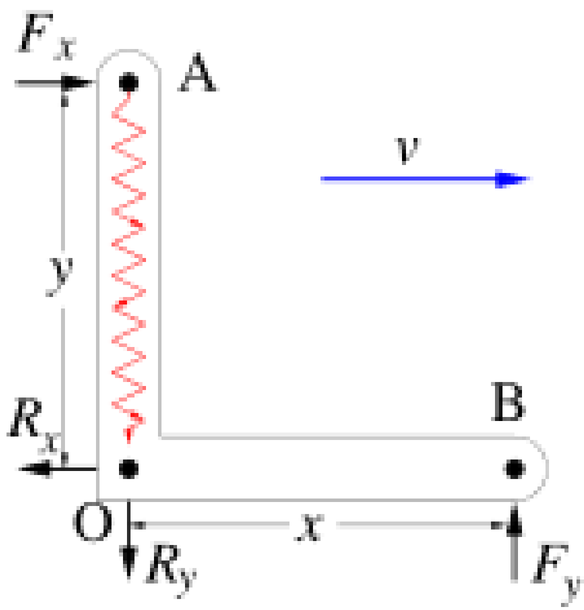
Parameters: c_2 and c_3



The parameters c_2 and c_3 dependence : $G_{M1,E2,M3}$

$$\Gamma^{\alpha\beta} = \mathbf{c}_1\left[g^{\alpha\beta} + \mathbf{c}_2 \gamma^{\beta}\Lambda^{\alpha} + \mathbf{c}_3 \Lambda^{\alpha}\Lambda^{\beta} \right]$$

“A Lewis-Tolman-like Paradox”



Moving Reference'

$$f'^{0} = \beta\gamma f^{(1)}, \qquad f'^{1} = \gamma f^{(1)}, \qquad f'^{2} = f^{(2)}, \qquad f'^{3} = f^{3} = 0$$

$$f'_A = (\beta\gamma F, \gamma F, 0, 0), \qquad f'_B = (0, 0, F, 0)$$

Total clockwise torque is

$$y'F'_{Ax} - x'F'_{By} = LF(1-\gamma^{-2}) = \beta^2 LF \neq 0$$

Energy – Momentum – Tensor

$$\begin{cases} T^{00} = \text{energy} - \text{density} \\ T^{j0} = c \times \text{density of } (\vec{P})_j, j = 1,2,3 \\ T^{0j} = \dfrac{1}{c} \times \text{ flux of energy in the } j \text{ direction} \\ T^{ij} = \text{flux of } (\vec{P})_i, \text{ in the } j \text{ direction} \end{cases}$$

The angular momentum is given by the integrating the mements of the momentum density

$$\begin{cases} L^{\mu\nu}(t) = \dfrac{1}{c}\int d^3x \mathcal{M}^{\mu\nu 0}(t,\vec{x}) \\ \mathcal{M}^{\mu\nu\rho}(t,\vec{x}) = x^{\nu}T^{\mu\rho} - x^{\mu}T^{\nu\rho} \\ \partial_{\rho}\mathcal{M}^{\mu\nu\rho}(t,\vec{x}) = 0 \end{cases}$$

The angular momentum conservation requires that the stress-energy tensor is symmetric.