

# Fragmentation Functions at Belle and Belle II

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Simon Schneider

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**Duke**  
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Science

# Fragmentation Functions at Belle and Belle II

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- How the Belle and Belle II experiments are designed
- Fragmentation function results from Belle data
- Currently ongoing studies
- Near-future opportunities

# Long history of studying QCD in $e^+e^-$ experiments

E.g. PETRA at DESY: Discovery of the gluon (1979)

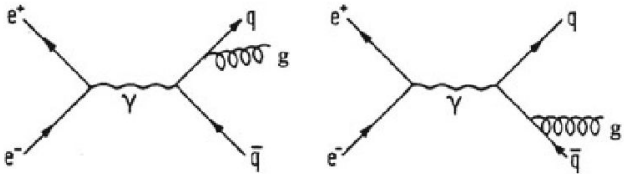
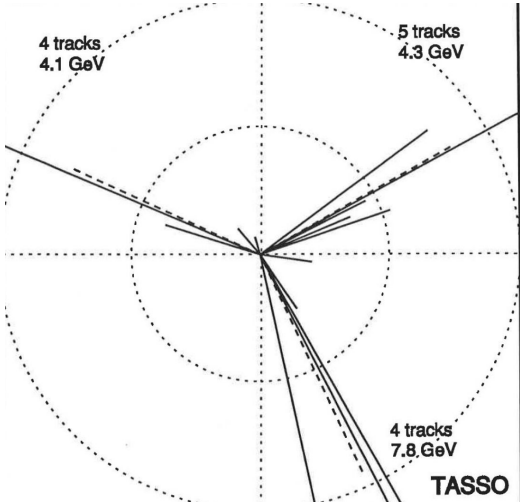
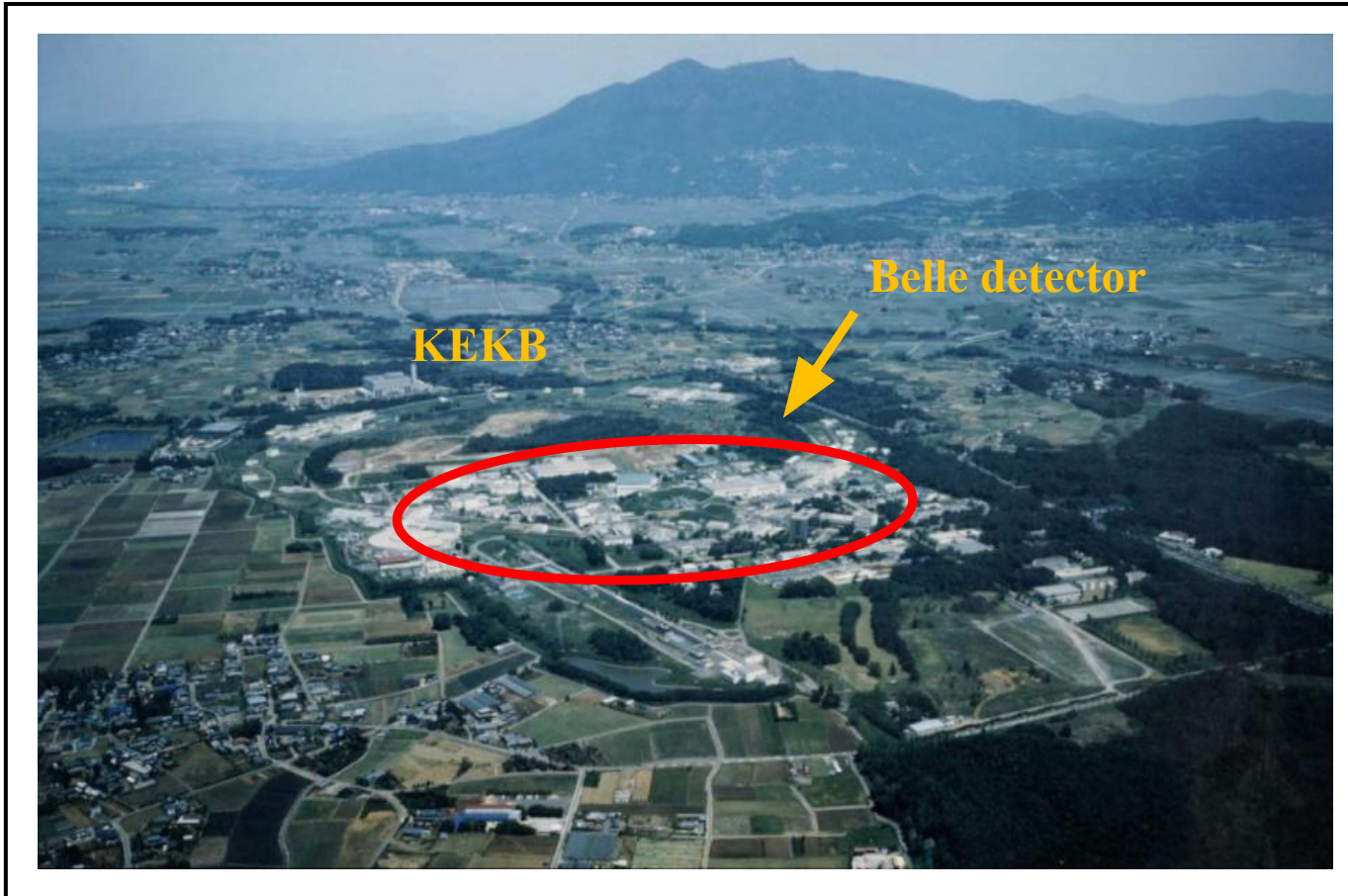


Image source: Söding, P. (2010). On the discovery of the gluon. *The European Physical Journal H*, 35(1), 3-28.

Fig. 1. Feynman diagrams for hard gluon bremsstrahlung  $e^+e^- \rightarrow q\bar{q}g$ .

# The less distant past: Belle at the KEKB $e^+e^-$ collider (1999-2010)

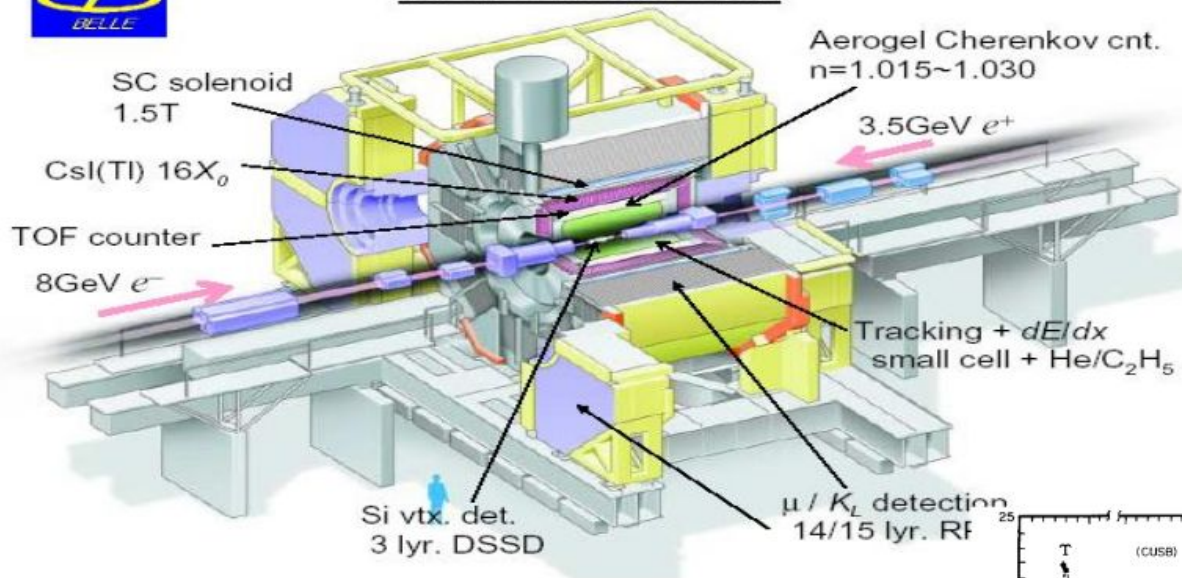
2



AERIAL VIEW OF EXPERIMENT

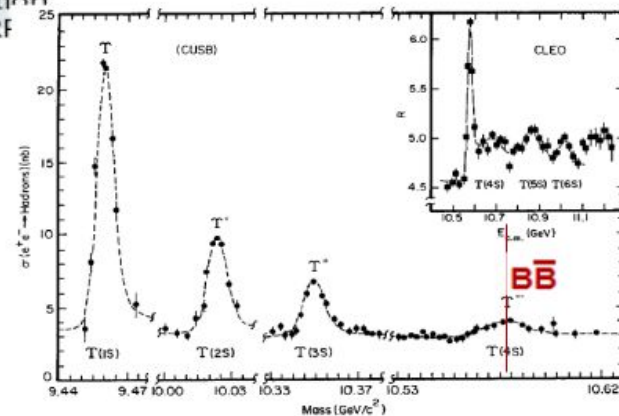


# Belle Detector

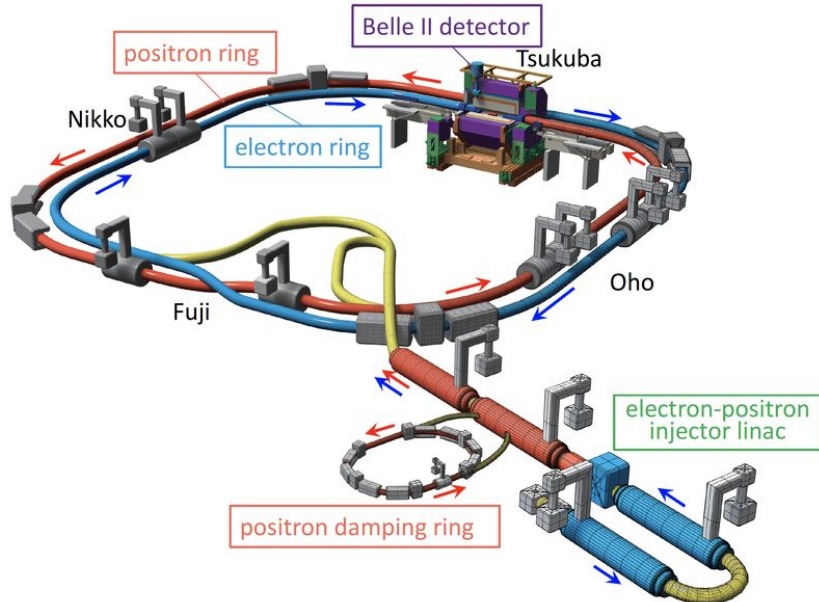


3

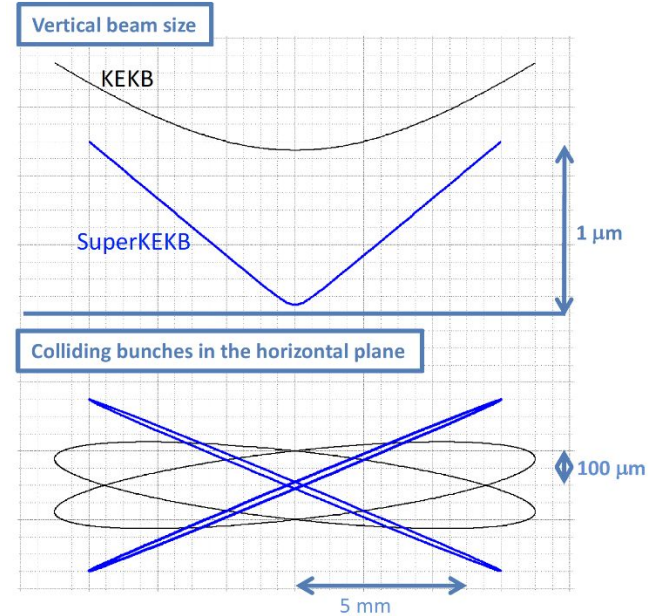
Exp.	Scans / Off-res. $fb^{-1}$	$\Upsilon(5S)$		$\Upsilon(4S)$		$\Upsilon(3S)$		$\Upsilon(2S)$		$\Upsilon(1S)$	
		10876 MeV $fb^{-1}$	10 <sup>6</sup>	10580 MeV $fb^{-1}$	10 <sup>6</sup>	10355 MeV $fb^{-1}$	10 <sup>6</sup>	10023 MeV $fb^{-1}$	10 <sup>6</sup>	9460 MeV $fb^{-1}$	10 <sup>6</sup>
CLEO	17.1	0.4	0.1	16	17.1	1.2	5	1.2	10	1.2	21
BaBar	54	$R_s$ scan		433	471	30	122	14	99		
Belle	100	121	36	711	772	3	12	25	158	6	102



# SuperKEKB



## Nanobeam collision scheme



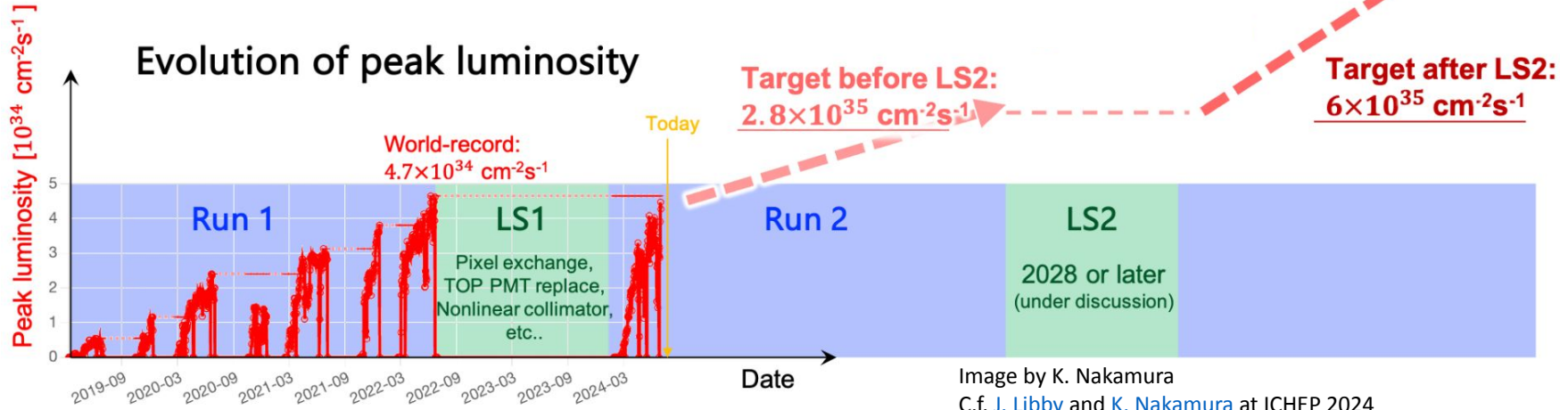
Upgrade from KEKB to increase luminosity  
Design beam currents increased  
Main increase from much smaller beam sizes

image source: Akai, K., Furukawa, K., & Koiso, H. (2018). SuperKEKB collider. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 907, 188-199.

# Next Generation B-factory

BELLE II at SuperKEKB

5



SuperKEKB delivered  $4.7 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$   
world record instantaneous luminosity

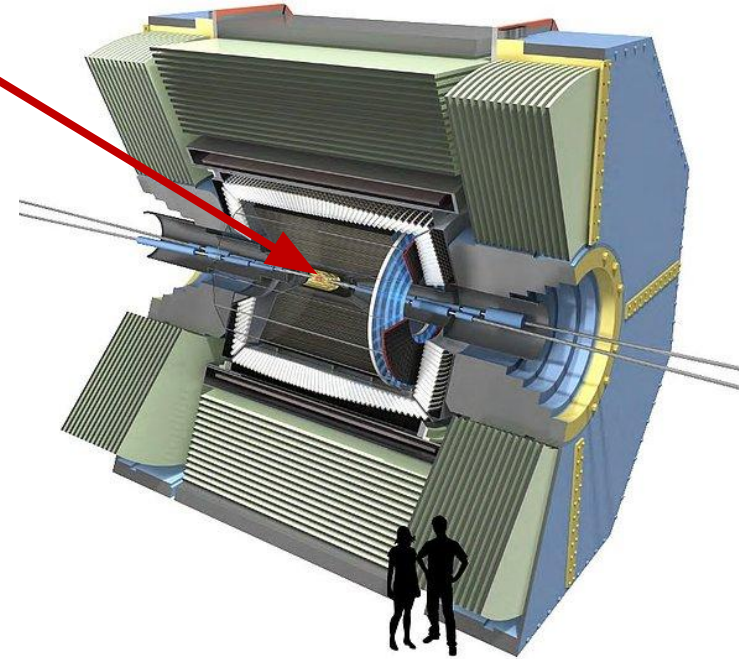
Belle II scheduled to drastically increase  
available data over the next few years

# BELLE II EXPERIMENT

## THE DETECTOR

6

- The upgraded Belle II began taking data in 2019:
  - **Vertex Detector (VXD)**: two innermost silicon strips replaced with pixel detectors
  - **Central Drift Chamber (CDC)**: smaller drift cells, reaches smaller radii
  - **Time Of Propagation (TOP)**: replaced TOF
  - **Aerogel Ring Imaging Cherenkov Detector (ARICH)**: replaced endcap ACC
  - **Electromagnetic Calorimeter (ECL)**: electronics upgrade
  - **$K_L$  Muon Detector (KLM)**: two innermost barrel RPCs and endcaps replaced with scintillators



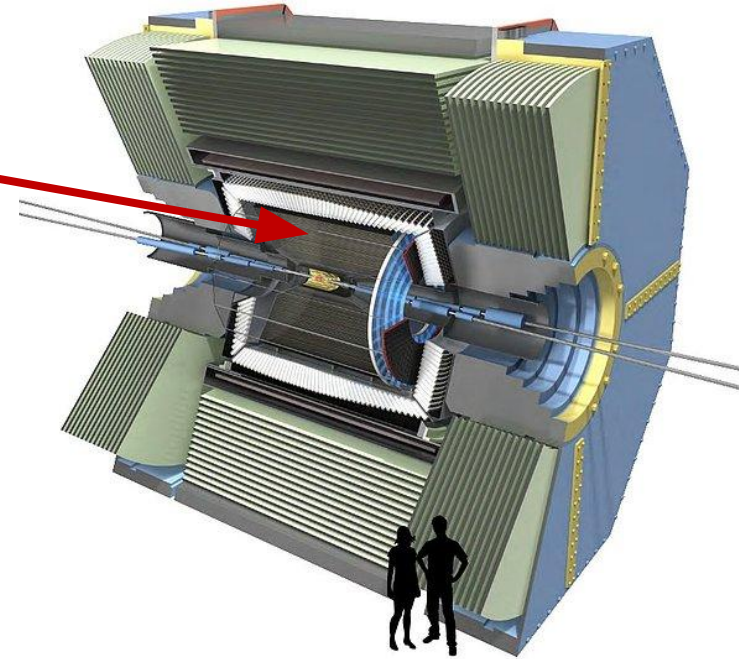


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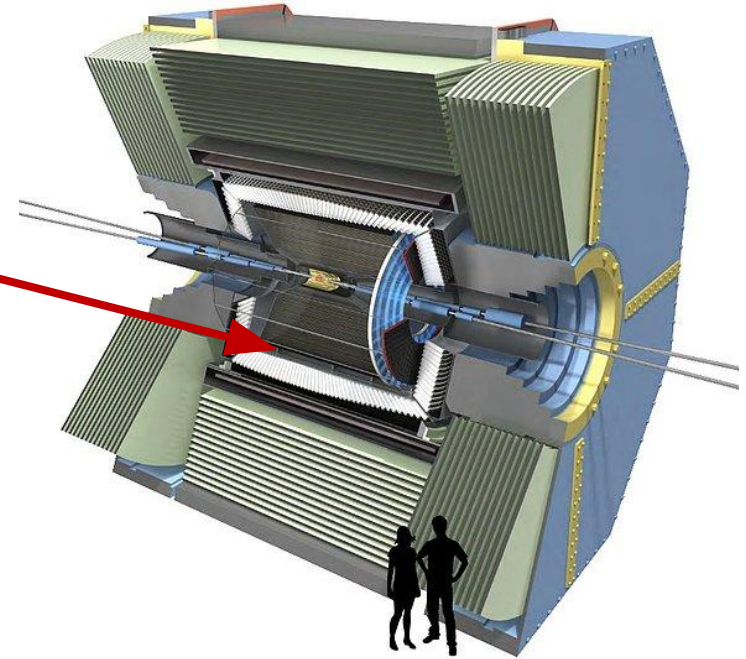


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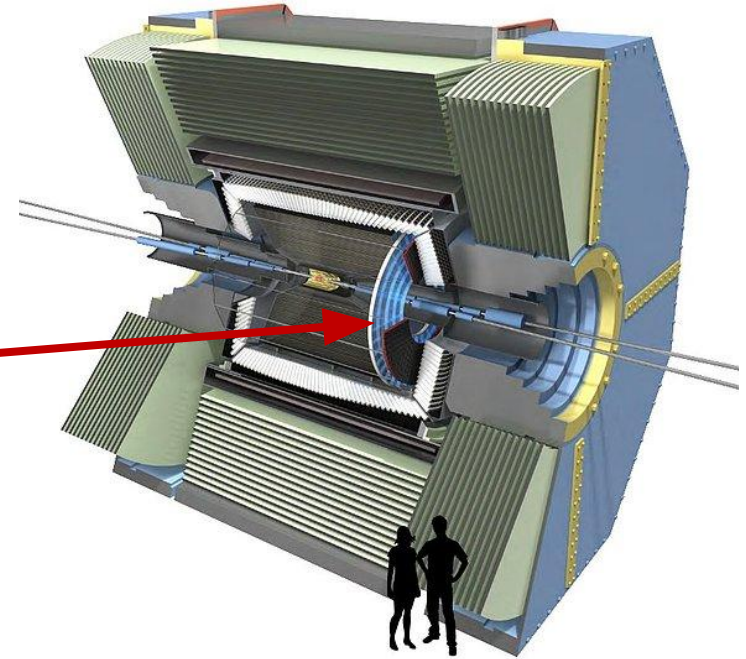


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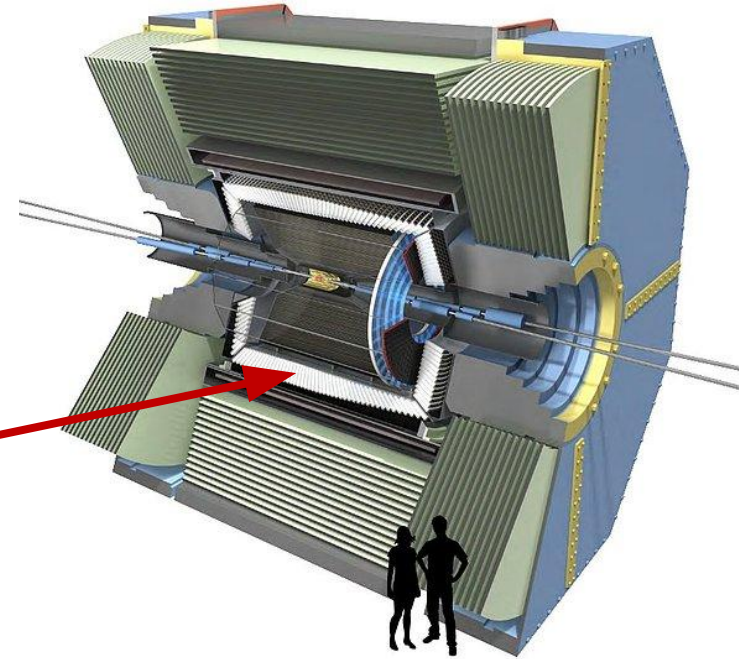


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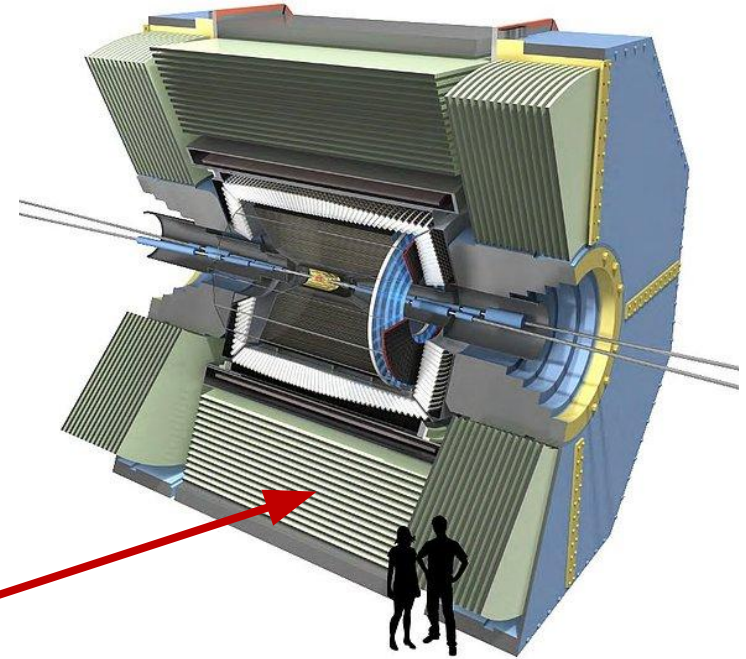


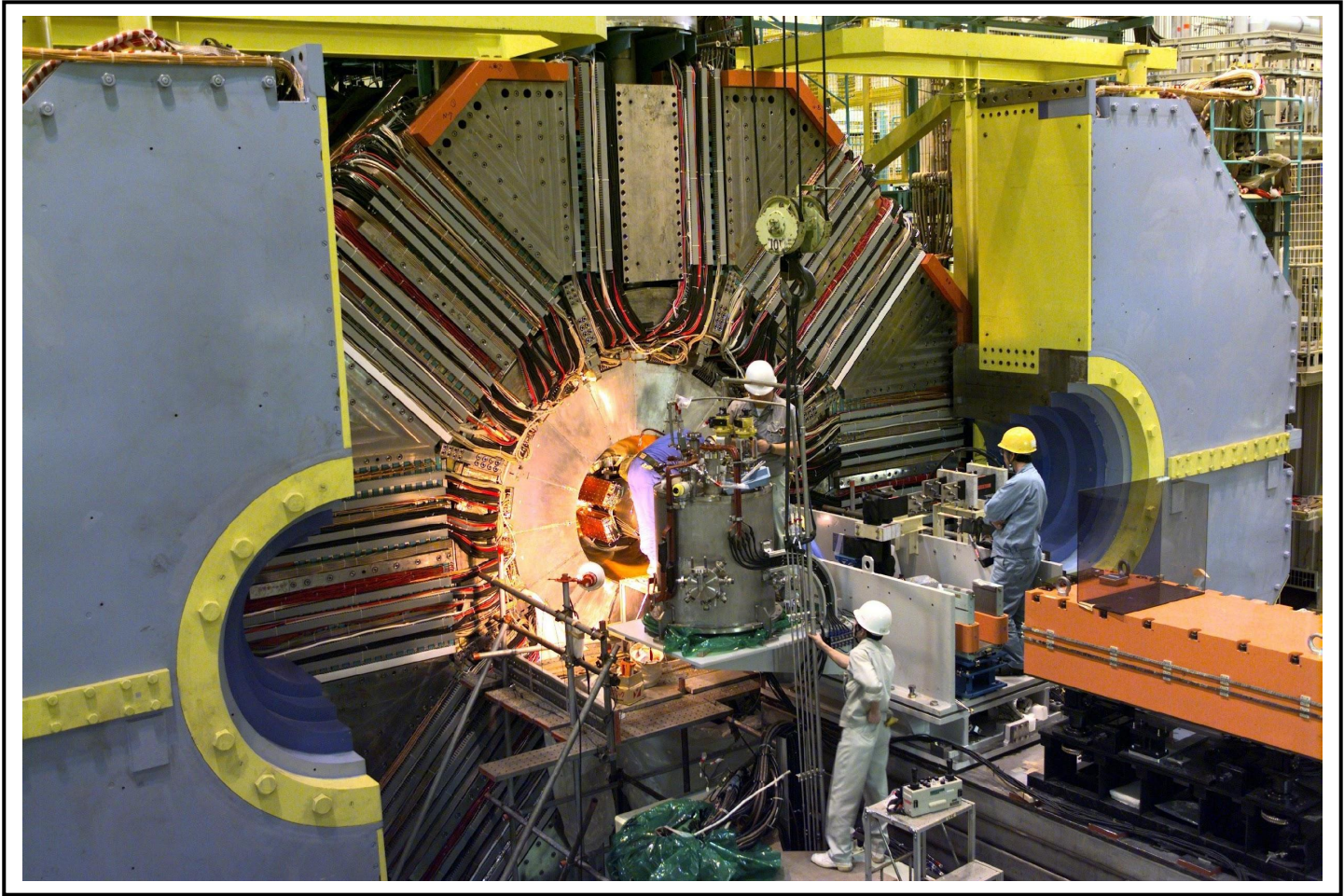
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BELLE

# FRAGMENTATION FUNCTIONS

## INTRODUCTION

Encode the non-perturbative link between perturbative QCD processes and the observed final state particles

(light hadron) FFs can't be calculated from pQCD

→ Need to observe in experiment

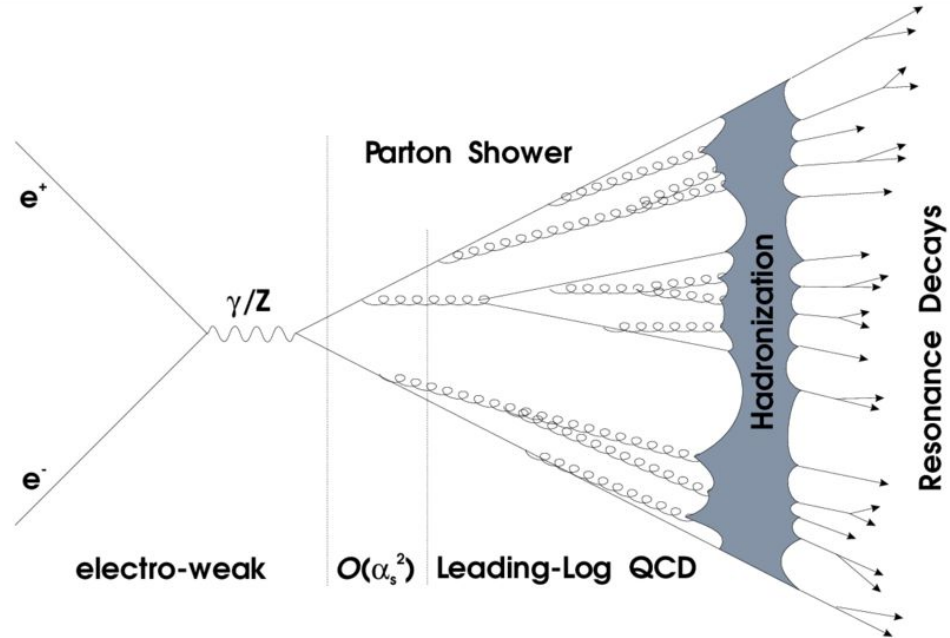


image source: Buskulic et al. (1995)

# FRAGMENTATION FUNCTIONS

## FACTORIZATION

9

Scattering amplitudes can be factorized into a perturbatively calculable part,  $\hat{\sigma}$ , and a non-perturbatively calculable part:

Single-inclusive annihilation:

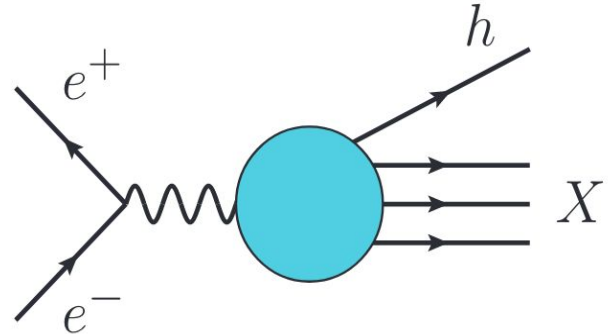
$$\sigma^{e^+e^- \rightarrow hX} = \hat{\sigma} \otimes FF$$

Semi-inclusive deep inelastic scattering (SIDIS):

$$\sigma^{\ell N \rightarrow \ell hX} = \hat{\sigma} \otimes PDF \otimes FF$$

Single-inclusive proton-proton scattering:

$$\sigma^{pp \rightarrow hX} = \hat{\sigma} \otimes PDF \otimes PDF \otimes FF$$





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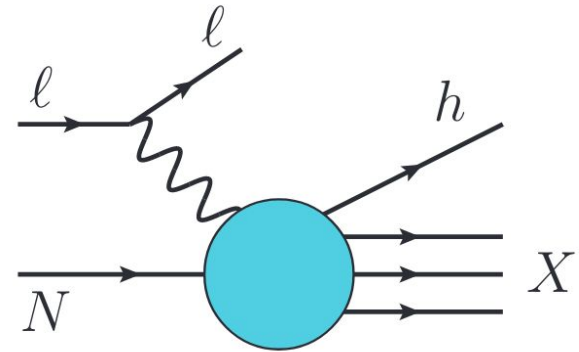
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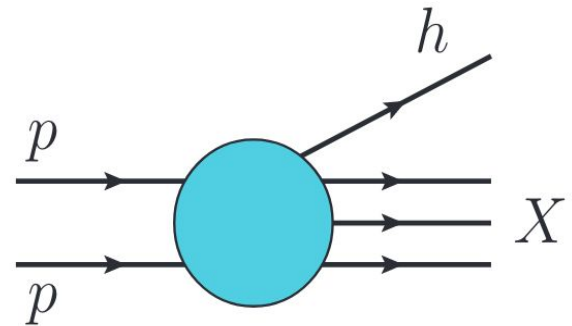
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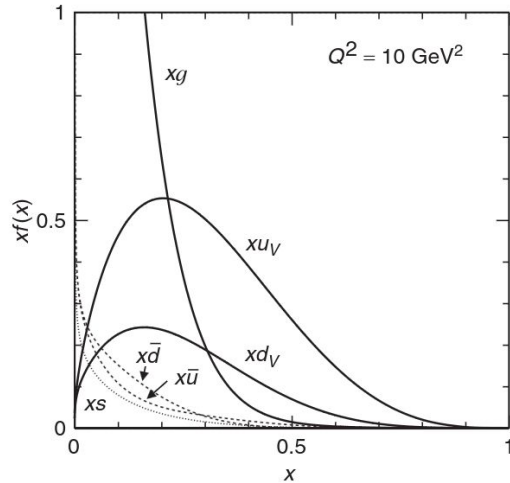
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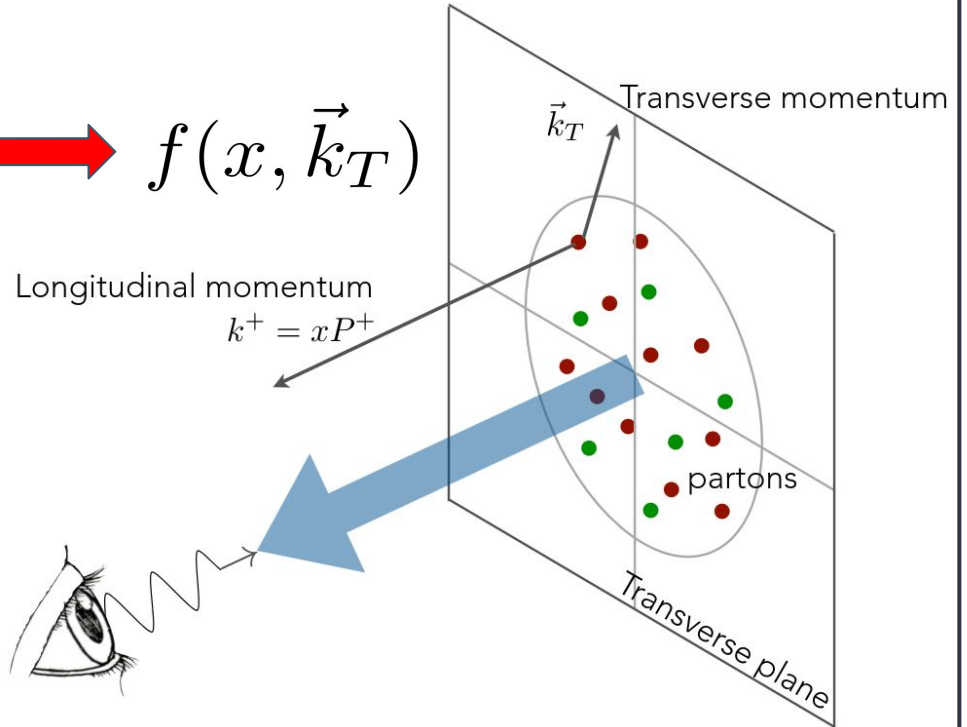
# FRAGMENTATION FUNCTIONS

## TRANSVERSE MOMENTUM DEPENDENCE

More than one degree of freedom



$$f(x) \longrightarrow f(x, \vec{k}_T)$$



source: Mark Thomson "Modern Particle Physics"

source: <https://indico.cern.ch/event/797767/contributions/3682621/>

# Leading Quark TMDFFs



		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarized Hadrons	L		$G_1 = \text{circle with red arrow right} - \text{circle with red arrow right}$ Helicity	$H_{1L}^\perp = \text{circle with red arrow up-right} - \text{circle with red arrow up-right}$
	T	$D_{1T}^\perp = \text{circle with red dot and arrow up} - \text{circle with red dot and arrow down}$ Polarizing FF	$G_{1T}^\perp = \text{circle with red dot and arrow up} - \text{circle with red dot and arrow up}$	$H_1 = \text{circle with red dot and arrow up} - \text{circle with red dot and arrow up}$ Transversity $H_{1T}^\perp = \text{circle with red dot and arrow up-right} - \text{circle with red dot and arrow up-right}$
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{circle with red dot}$ Unpolarized		$H_1^\perp = \text{circle with red dot and arrow up} - \text{circle with red dot and arrow up}$ Collins

Image source: TMD handbook

Figure 2.6: Leading power quark TMD fragmentation functions for a spin-1/2 (or for an unpolarized or spin 0) hadron.

Determining final state polarization needs self-analyzing decays ( $\Lambda$ ) -> Belle II Strength!

# BELLE RESULTS

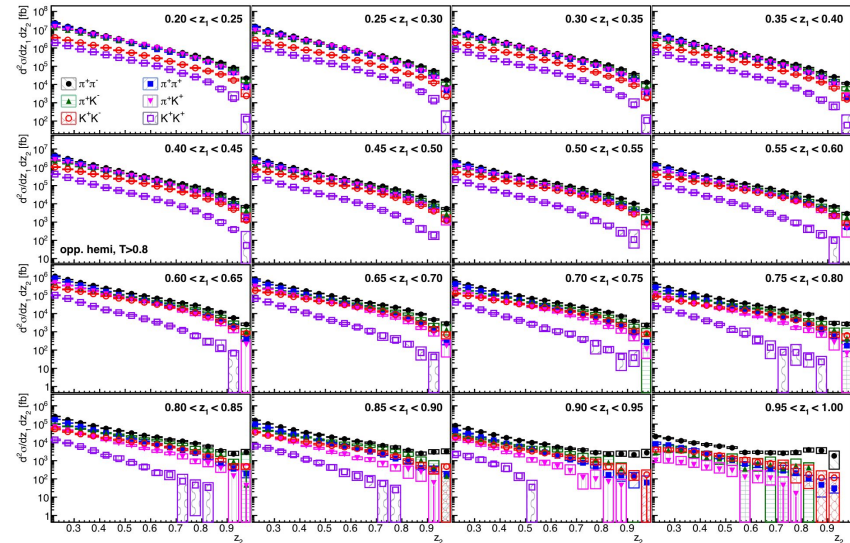
## RECENT PUBLICATIONS

12

R. Seidl et al., “Update of inclusive cross sections of single and pairs of identified light charged hadrons”, *Phys.Rev.D* 101 (2020) 9, 092004

- Used an updated ISR correction procedure
- Alternative fractional energy definitions were considered

- Conventional:  $z_i = 2E_{h,i} / \sqrt{S}$
- AEMP:  $z_1 = \frac{2P_1 \cdot q}{q^2}$ ,  $z_2 = \frac{P_1 \cdot P_2}{P_1 \cdot q}$
- MVH:  $z_1 = \left( P_1 \cdot P_2 - \frac{M_{h1}^2 M_{h2}^2}{P_1 \cdot P_2} \right) \frac{1}{P_2 \cdot q - M_{h2}^2 \frac{P_1 \cdot q}{P_1 \cdot P_2}}$ ,  
 $z_2 = \left( P_2 \cdot P_1 - \frac{M_{h2}^2 M_{h1}^2}{P_2 \cdot P_1} \right) \frac{1}{P_1 \cdot q - M_{h1}^2 \frac{P_2 \cdot q}{P_2 \cdot P_1}}$



Differential cross sections of charged hadron pairs in opposite hemispheres, binned by conventional fractional energy

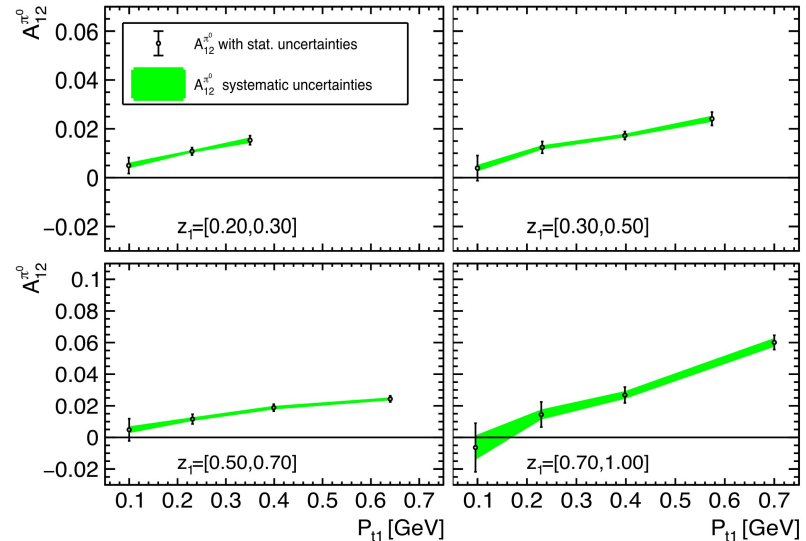
# BELLE RESULTS

## RECENT PUBLICATIONS

13

H. Li, A. Vossen, et al., “Azimuthal asymmetries of back-to-back  $\pi^\pm - (\pi^0, \eta, \pi^\pm)$  pairs in  $e^+e^-$  annihilation”, *Phys.Rev.D* 100 (2019) 9, 092008

- The first observation of azimuthal asymmetries of pairs of back-to-back hadrons, one a charged pion and the other a  $\pi^0$ ,  $\pi^\pm$ , or  $\eta$  meson
- Measurements are sensitive to the Collins fragmentation function,  $H_1^\perp$



Azimuthal asymmetry of  $\pi^0$  events binned by fractional energy,  $z$

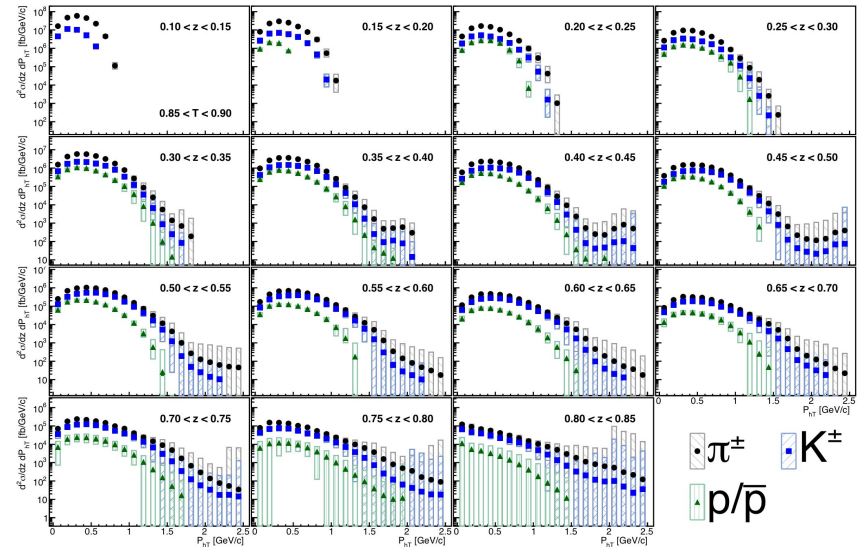
# BELLE RESULTS

## RECENT PUBLICATIONS

14

R. Seidl et al., “Transverse momentum dependent production cross sections of charged pions, kaons and protons produced in inclusive  $e^+e^-$  annihilation at  $\sqrt{s} = 10.58$  GeV”, *Phys.Rev.D* 99 (2019) 11, 112006

- Production cross sections of charged pions, kaons, and protons as a function of fractional energy, thrust, and transverse momentum
- Measurements access the transverse momenta created during fragmentation



Differential cross sections as a function of transverse momentum

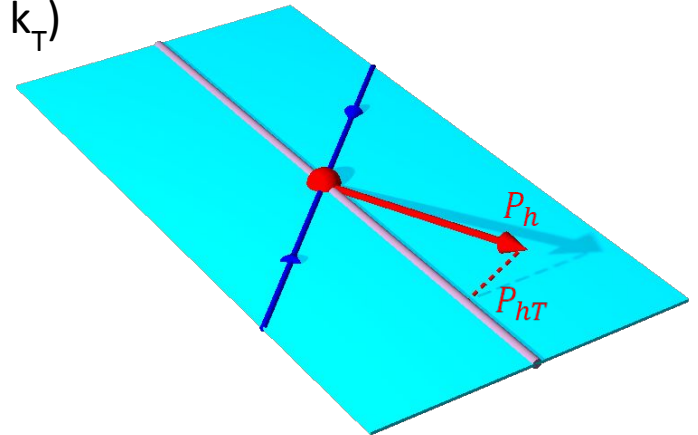
BELLE

$H_1^\times$  DiFF

JET FUNCTIONS

## Back-to-back di-hadron TMD measurements

- A. Vossen, C. van Hulse
- Second hadron provides axis to measure  $P_{hT}$
- Sensitive to the single hadron TMD FF  $D_1(z, k_T)$





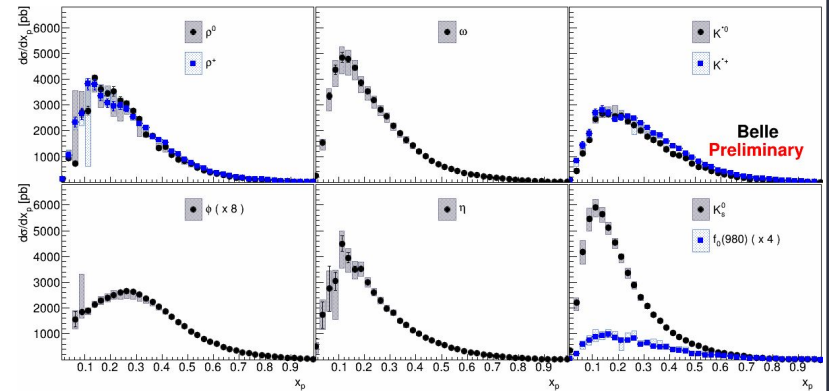
BELLE

$H_1^{\chi}$  DIFF

JET FUNCTIONS

Production cross-sections of light and charmed mesons (R. Seidl)

- Comprehensive cross section measurements, differential in final hadron momentum
- First measurement of  $\eta$ ,  $K_s^0$  at any B-factory, first  $D_s^{+*}$  cross-sections at Belle
- Detailed comparison with various MC tunes
- Improved ISR corrections for D-mesons
- Important for SIDIS studies



# ONGOING RESEARCH

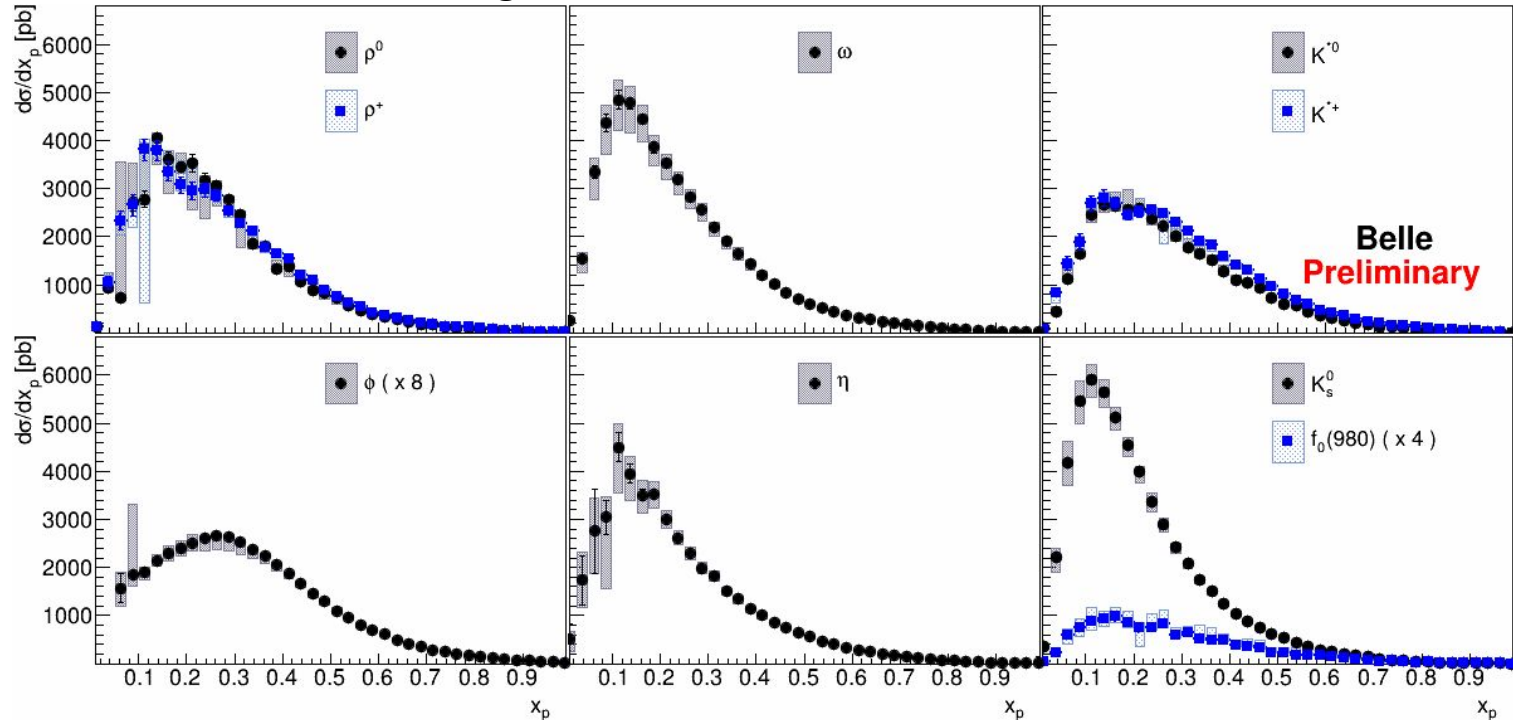
17

BELLE

$H_1^x$  DiFF

JET FUNCTIONS

## Production cross-sections of light mesons

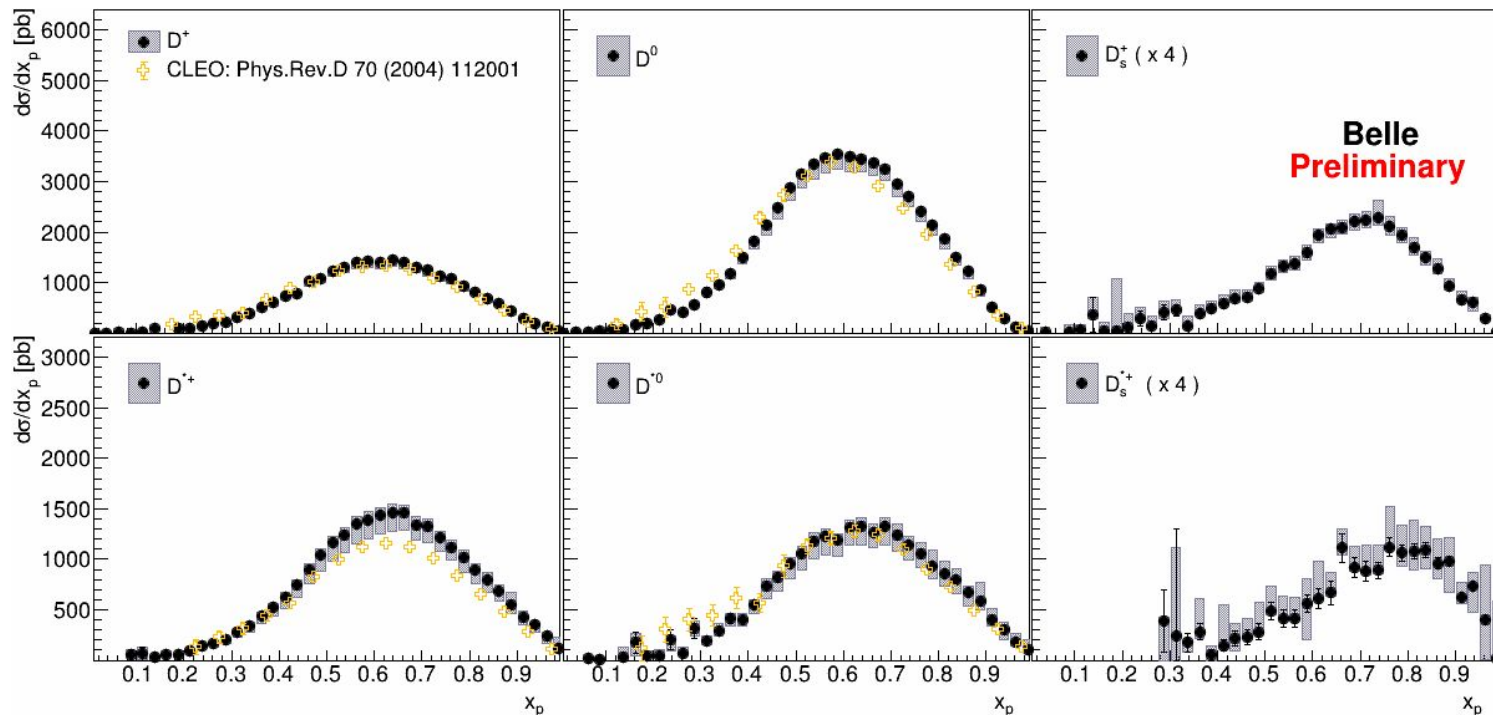


BELLE

$H_1^{\chi}$  DiFF

JET FUNCTIONS

## Production cross-sections of charmed mesons



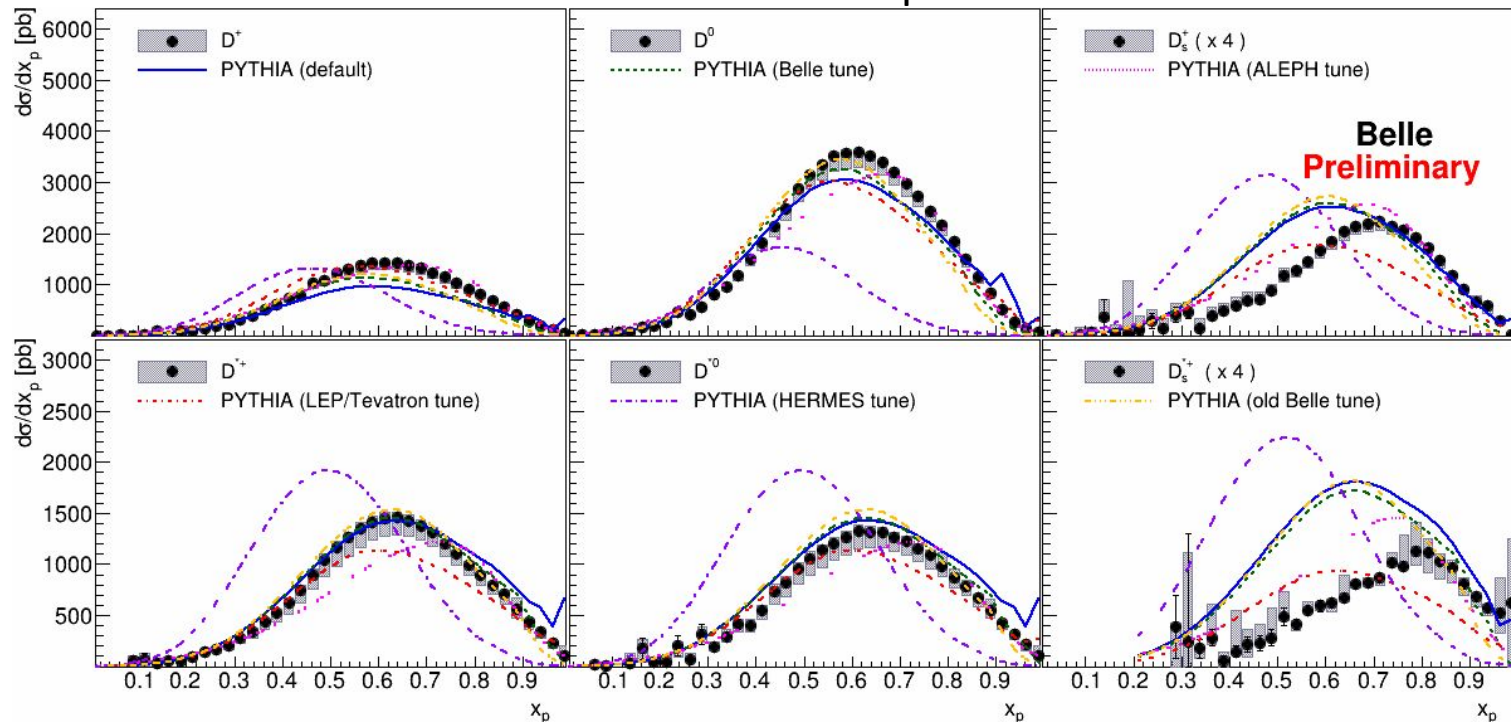
# ONGOING RESEARCH

BELLE

 $H_1^x$  DiFF

JET FUNCTIONS

Production cross-sections of charmed mesons compared to various PYTHIA tunes

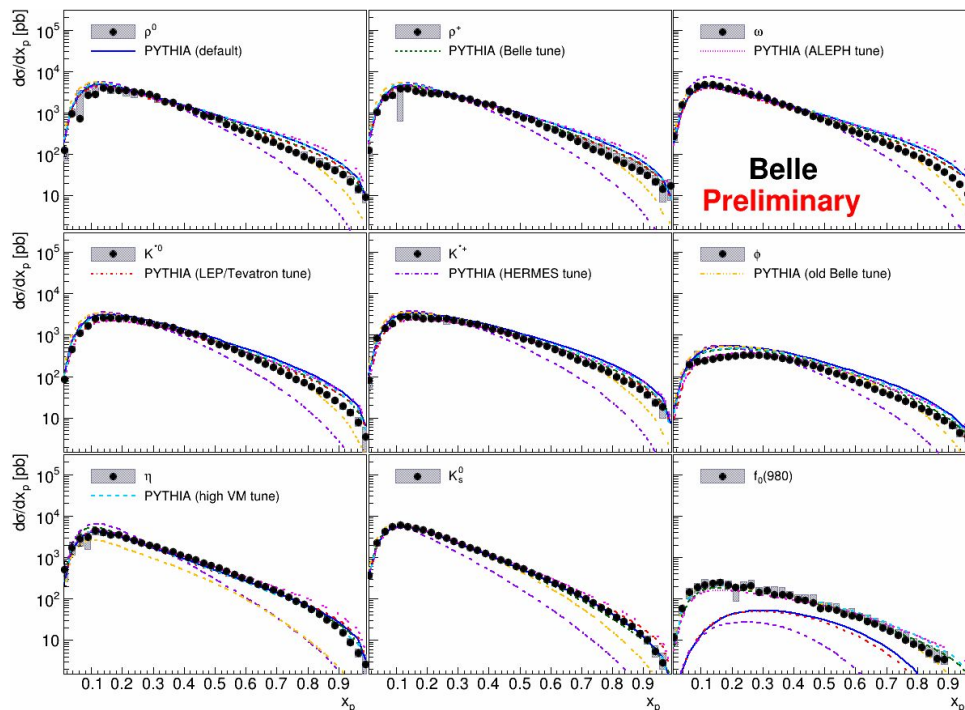


BELLE

$H_1^{\chi}$  DIFF

JET FUNCTIONS

Production cross-sections of light mesons compared to various PYTHIA tunes



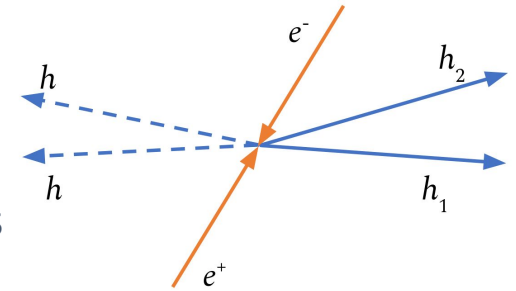
# BELLE II STRENGTH: COMPLEX FINAL STATES




## DIHADRON FRAGMENTATION FUNCTIONS

21

Relative momentum between hadrons as additional d.o.f.  
 More d.o.f.'s  $\rightarrow$  more information about final state correlations  
 $\rightarrow$  See e.g. recent extraction of twist-3  $e(x)$  (e-Print: 2203.14975 [hep-ph])

Orientation of two hadrons w.r.t. each other and jet direction as indicator of quark transverse spin



Parton polarization $\rightarrow$ Hadron Polarization $\downarrow$	Spin averaged	longitudinal	transverse
spin averaged	$D_1^{h/q}(z, M)$ 		$H_1^{\perp h/q}(z, p_T M, (\mathbf{P}_h), \theta)$ 'Di-hadron Collins'
longitudinal			
Transverse		$G_1^{\perp}(z, M, \mathbf{P}_h, \theta) =$ T-odd, chiral-even $\rightarrow$ jet handedness QCD vacuum structure 	$H_1^{\ast}(z, M, (\mathbf{P}_h), \theta) =$ T-odd, chiral-odd Collinear 

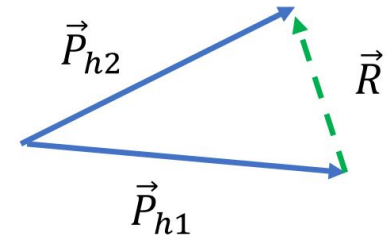
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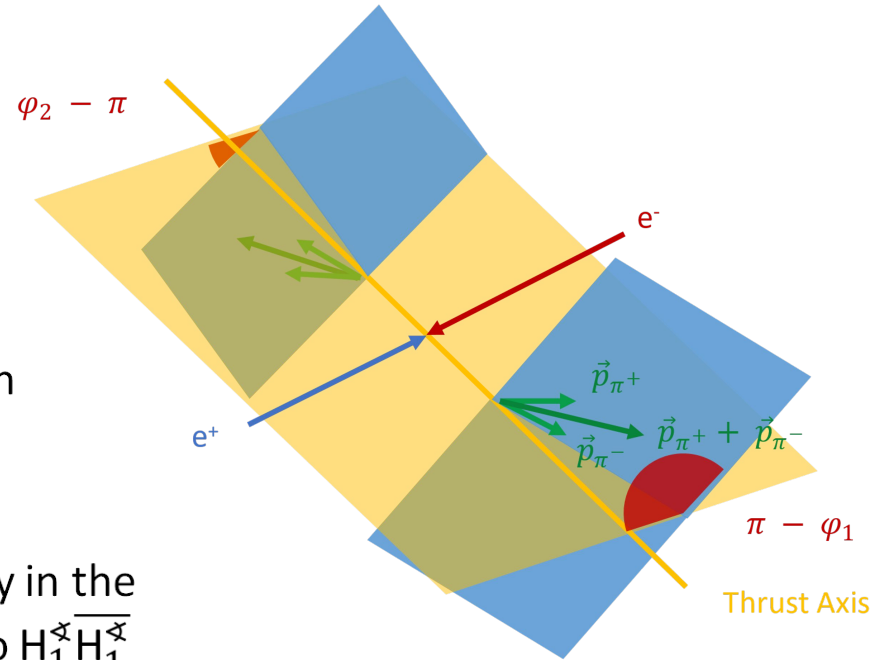
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Work by K. Parham

- The  $H_1^{\chi}$  FF describes the production of a pair of spin-0 hadrons
- Conservation of angular momentum leads to left and right asymmetry
- The spin correlation between a quark-antiquark pair leads to a correlation between the azimuthal angle of the di-hadron pairs it produces
- The amplitude of the resulting asymmetry in the azimuthal angle is directly proportional to  $H_1^{\chi} \overline{H_1^{\chi}}$





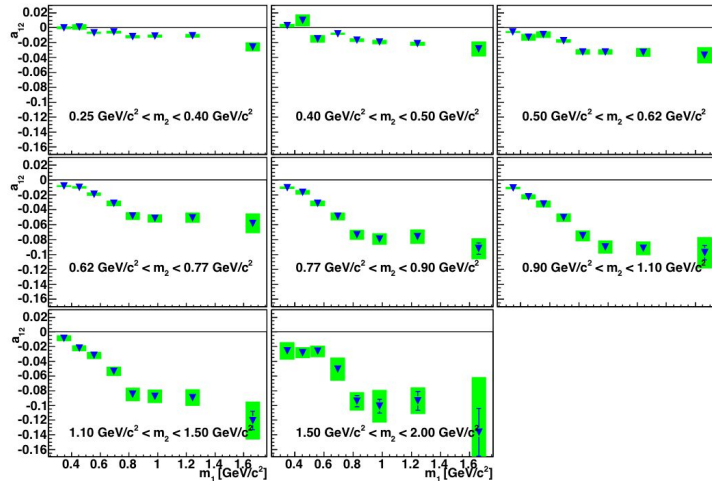
BELLE

$H_1^{\chi}$  DiFF

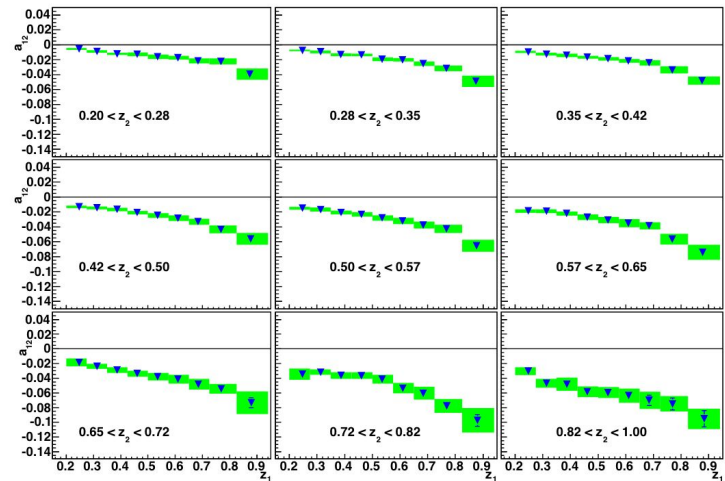
JET FUNCTIONS

Azimuthal Asymmetries in  $e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)$  Belle Results ( $670 fb^{-1}$ ):

Asymmetry by Invariant Mass Bins:



Asymmetry by Pion Fractional Energy Bins:



Phys. Rev. Lett. 107, 072004 (2011).

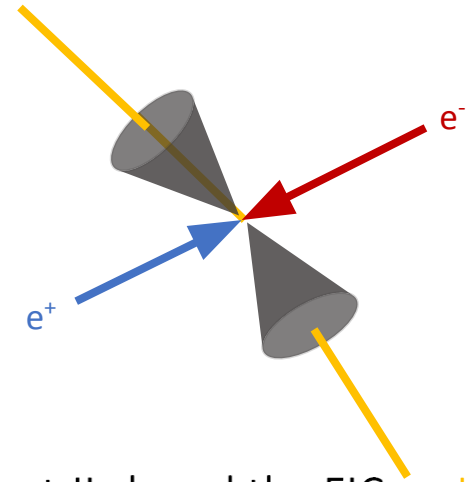
BELLE

$H_1^{\chi}$  DiFF

JET FUNCTIONS

Work by K. Parham

- Kaon Inclusive
  - The same measurement of  $H_1^{\chi}$  can be made with  $K^+ K^-$ ,  $K^+ \pi^-$ , or  $\pi^+ K^-$  pairs
  - Resulting  $H_1^{\chi}$  measurements could be used to describe the distribution of strange quarks within the nucleon
  
- Jet Axis
  - Jets are well defined observables and produce better results than the naïve  $q\bar{q}$  axis
  - Resulting FFs in  $e^+e^-$  are directly connected to FFs in SIDIS
  
- New measurements are critical to upcoming experiments at JLab and the EIC

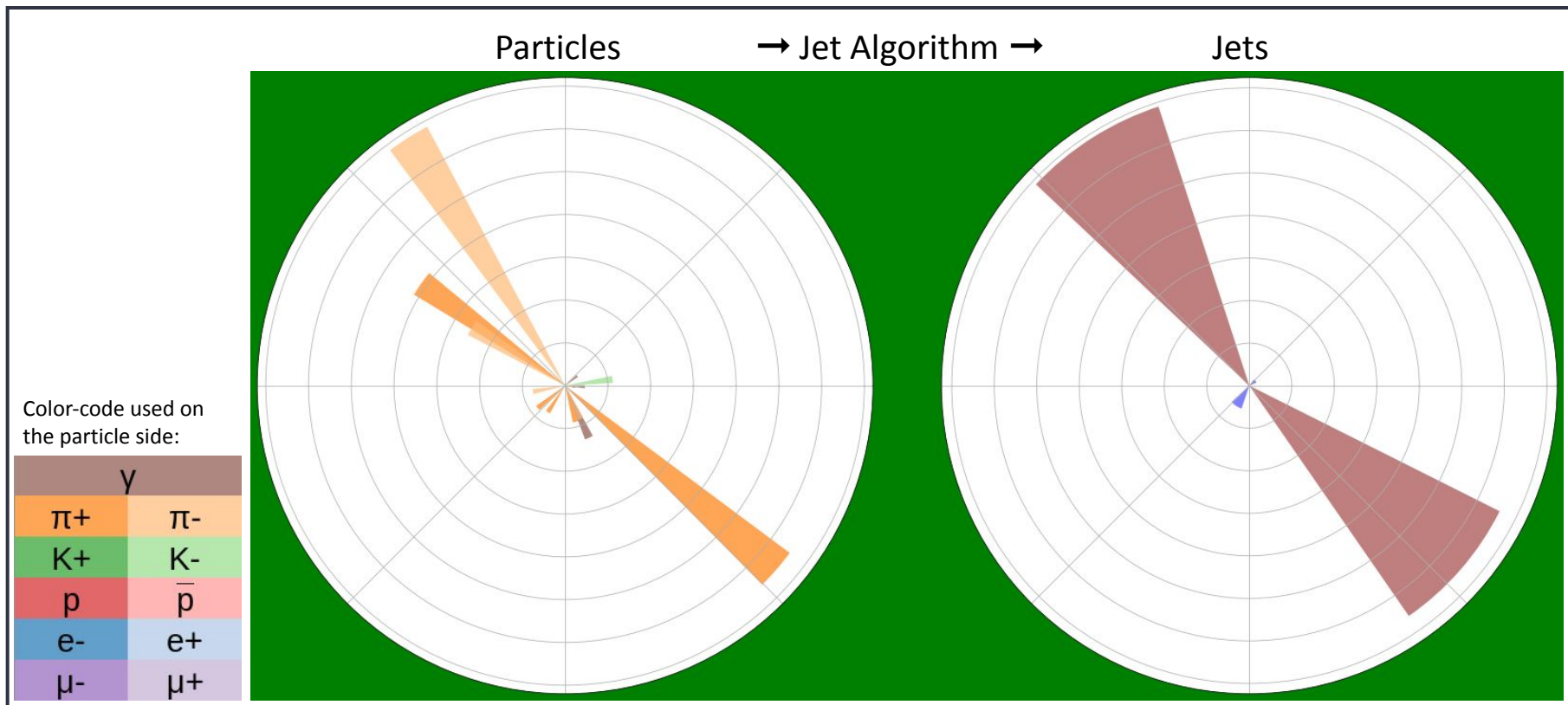


Jet  
Axes

# JETS AT B-FACTORIES

DIFFERENCES TO HIGH-ENERGY HADRON COLLISIONS

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The illustration shows a simulated event projected transverse to the beam direction. The length of the wedges corresponds to the particle's/jet's momentum.

- Theoretically, TMD FFs can be replaced by TMD Jet Functions in factorization equations.  
(Gutierrez-Reyes, D., Scimemi, I., Waalewijn, W.J. *et al.*)
- TMD Jet Functions are perturbatively calculable, thus removing a source of uncertainty and allowing greater sensitivity when used to extract PDFs in SIDIS.
- At low momenta, non-perturbative corrections are larger.

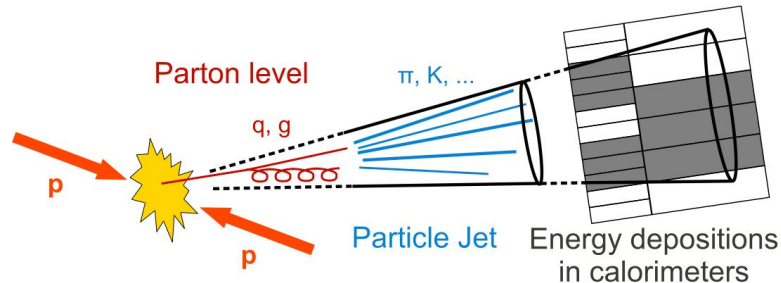


image source: <https://cms.cern/news/jets-cms-and-determination-their-energy-scale>

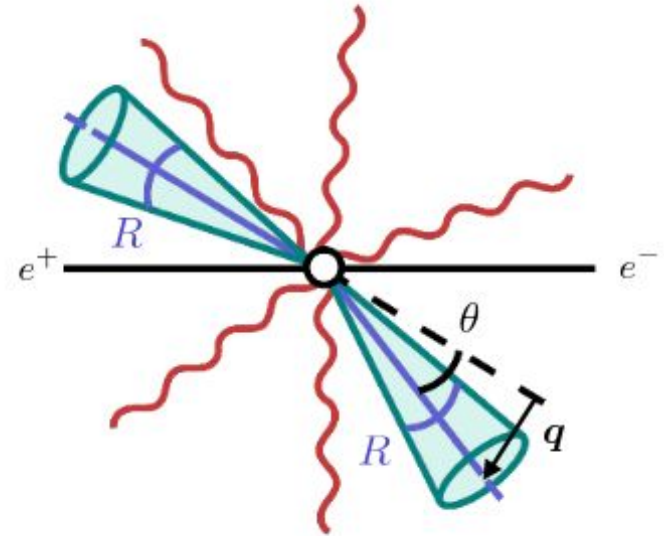
Measuring the jet  $q_T$  spectrum:

- Measure the transverse momentum decorrelation,  $\mathbf{q}$ , where  $\mathbf{p}_i$  are the jet transverse momenta and  $z_i$  are their fractional energy:

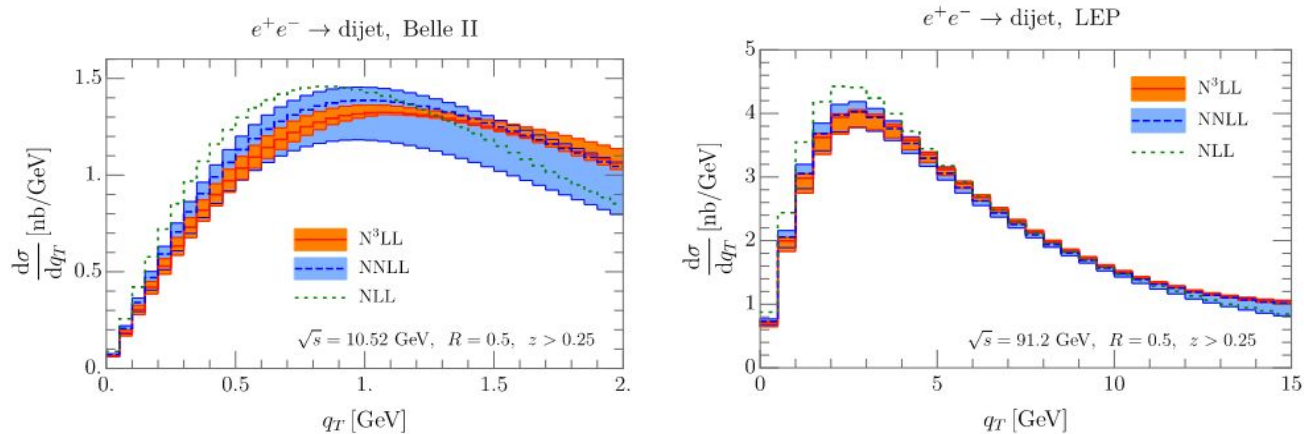
$$\mathbf{q} = \frac{\mathbf{p}_1}{z_1} + \frac{\mathbf{p}_2}{z_2}$$

- Factorization requires the momentum decorrelation to be small:

$$q_T \equiv |\mathbf{q}| \ll \frac{\sqrt{s}}{2}$$

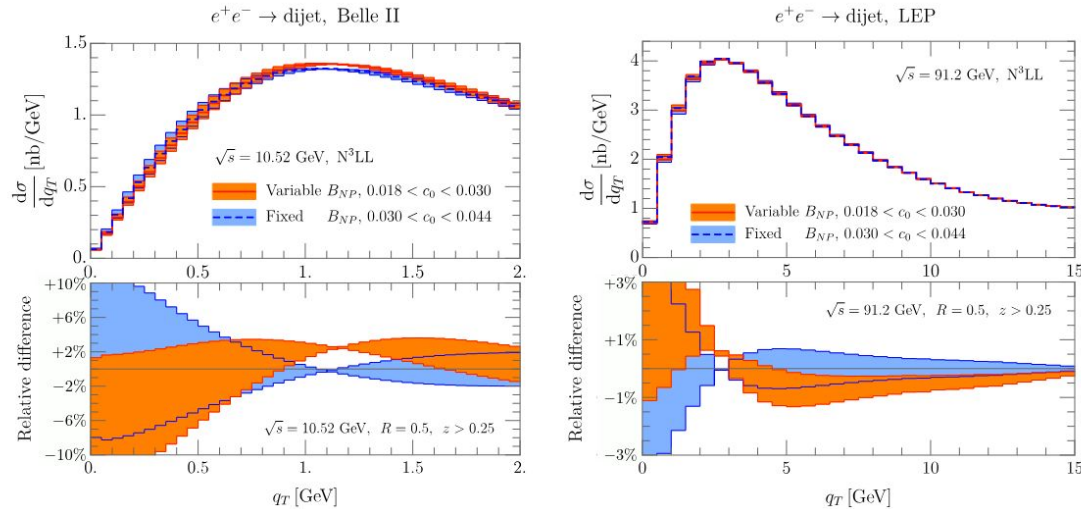


## Theoretical $q_T$ Spectrum at Belle II and LEP



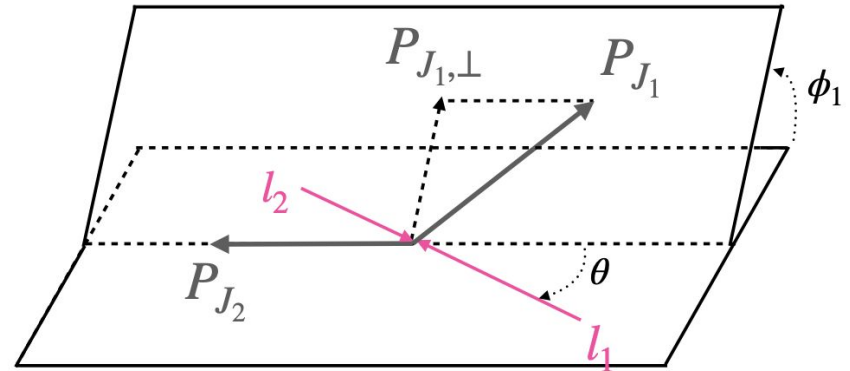
**Figure 5.** Perturbative convergence of the cross section differential in transverse momentum decorrelation, for Belle II (left) and LEP (right), for jet radius  $R = 0.5$  and jet energy fraction  $z > 0.25$ . The  $N^3LL$  result is obtained with the prescription in eq. (6.1). The bands encode the perturbative uncertainty, as described in the text.

## Theoretical $q_T$ Spectrum at Belle II and LEP



**Figure 7.** Estimate of the sensitivity of the TMD to nonperturbative effects in the rapidity resummation at Belle II (left) and LEP (right). We vary the parameter  $c_0$  in the range of its statistical uncertainty, testing both the fixed and variable  $B_{NP}$  schemes of ref. [56]. Results are obtained with the prescription in eq. (6.1).

- The time reversal-odd (T-odd) components of a jet were thought to vanish, but were recently found to survive due to non-perturbative effects.
- T-odd jet structure plays an important role in accessing nucleon spin structures.
- In particular, the T-odd constituent can couple to the proton transversity at the EIC.



$$R^{J_1 J_2} = 1 + \cos(2\phi_1) \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{F_T(q_T)}{F_U(q_T)}$$



# Near-future opportunities with Belle II data

Some highlights

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- 1<sup>st</sup> generation B-factory data insufficient for EIC program
  - e.g. higher dim. binnings
- Improved vertexing to allow separation of charm contributions
- Heavy flavor fragmentation studies
- Di-hadron production
  - → Including partial wave analysis
- Interrelations with MC modelling
- For more information, see recent [Belle II QCD whitepaper](#):
  - (arXiv:2204.02280 [hep-ex])

- Belle and Belle II data plays important role in the direct study of QCD
  - Demonstrated e.g. by recent Belle publications
  
- Key ingredient to extract nucleon structure in SIDIS (e.g. at the EIC)
  - Planned datasets to improve on 1<sup>st</sup> gen by order of magnitude
  
- Advances in theory and phenomenology motivate analyses with new tools
  
- A rich QCD program has been formulated for Belle II, with several analyses under way

Thanks for help in preparing this presentation to K. Parham, A. Vossen, R. Seidl,  
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