Spin physics at small x

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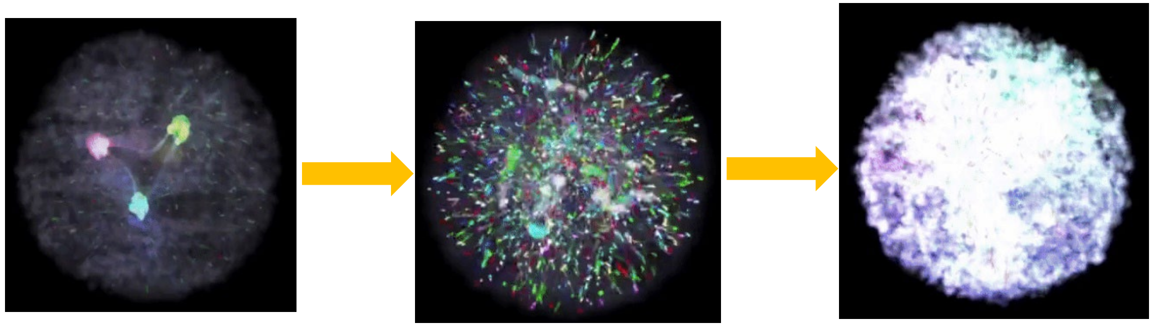
Outline:

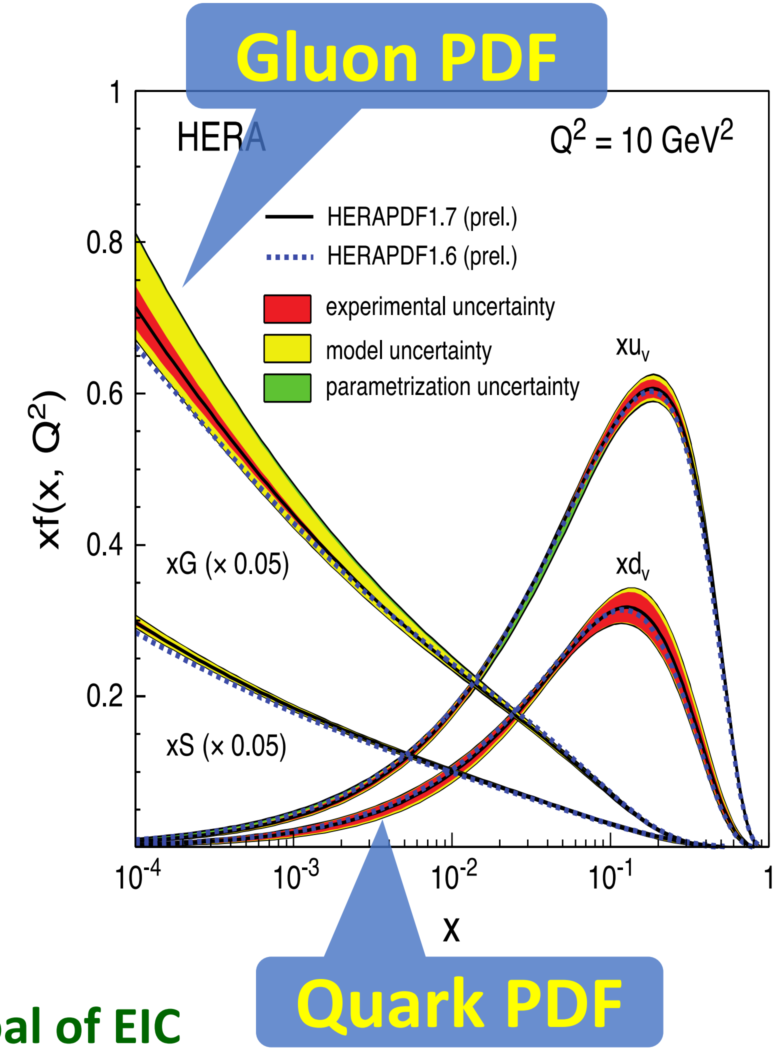
- ➢ **Linearly polarized gluons at small x**
- ➢ **Spin dependent odderon**
- ➢ **Gluon OAM at small x**
- ➢ **QED analogy in UPCs**
- ➢ **Summary**

Background I

X: Longitudinal momentum fraction carried by gluon



x=0.3 x=0.01 x=0.0001



Nucleon structure dominated by gluons at small x:

- ➢ **Extreme dense gluonic matter： CGC**
- ➢ **Stimulate the development of pQCD theoretical tools**

◆ **Important physics program at LHC and RHIC， the core scientific goal of EIC**

Background II

MV model: Classical gluon fields:

- For $Q_s >> \Lambda_{QCD}$, $\alpha_s(Q_s^2) \ll 1$

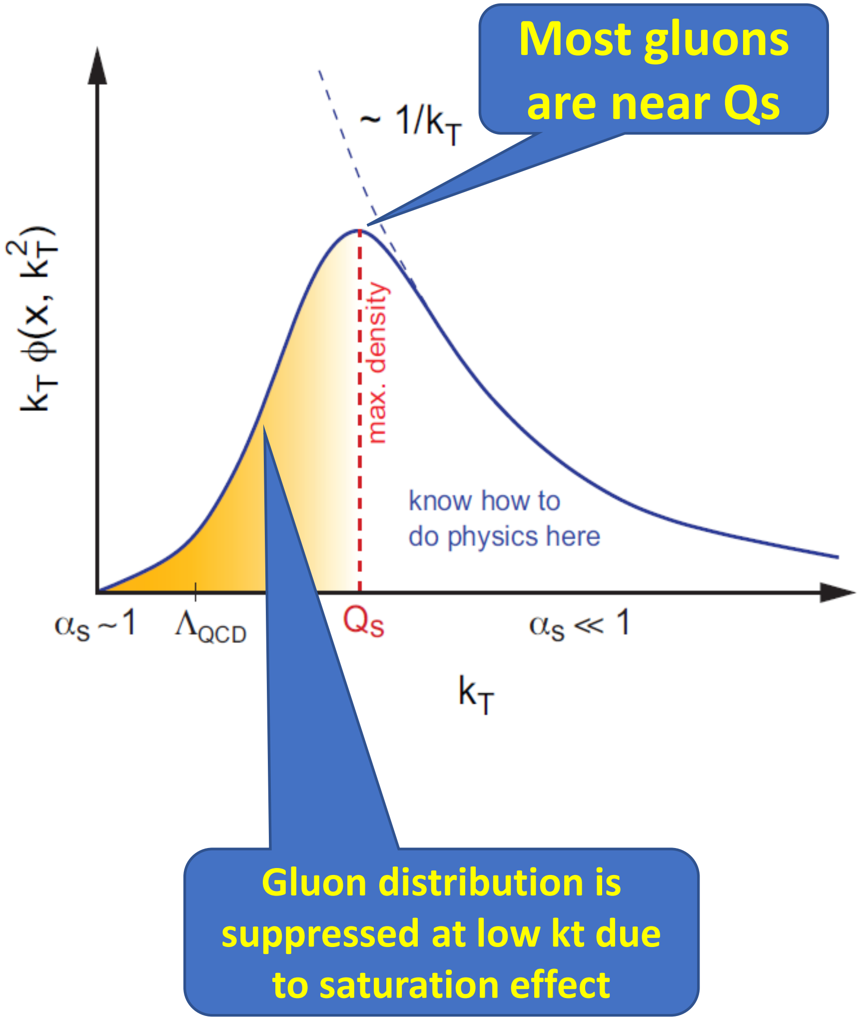
Perturbative treatment is justified!

- In the small x limit, high occupation number

A semi-classical treatment is justified

MV model & Glauble-Mueller model applied at x=0.0001

Dipole distribution amplitude



Linearly polarized gluons at small x

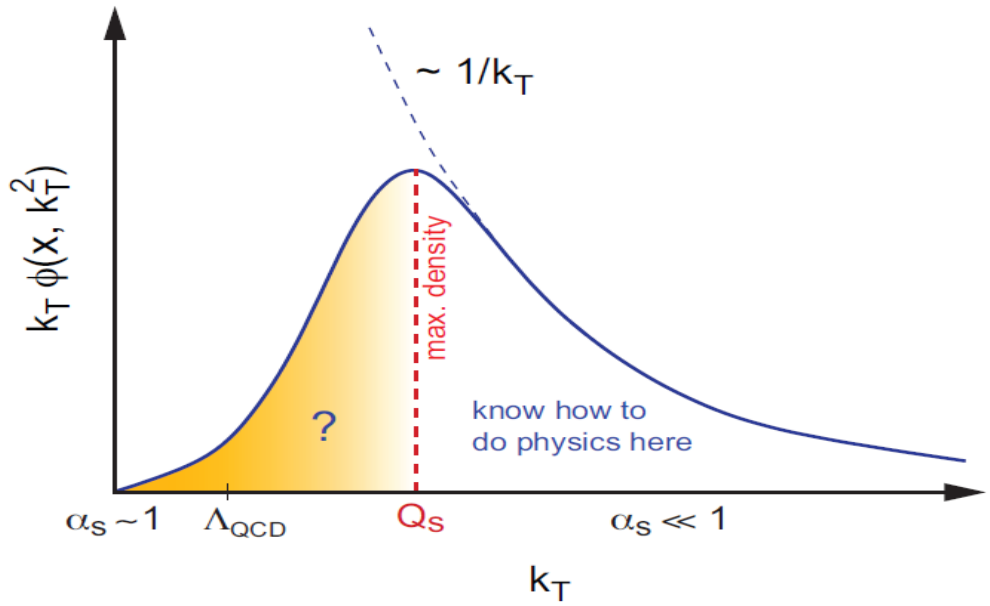
Linearly polarized gluon TMD

$$\int \frac{dr^- d^2 r_\perp}{(2\pi)^3 P^+} e^{-ix_1 P^+ r^- + i\vec{k}_{1\perp}\cdot\vec{r}_\perp} \langle A | F^{+i}(r^- + y^-, r_\perp + y_\perp)\, L^\dagger\, L\, F^{+j}(y^-, y_\perp) | A \rangle$$

$$= \frac{\delta_\perp^{ij}}{2} x_1 G(x_1, k_{1\perp}) + \left(\hat{k}_{1\perp}^i \hat{k}_{1\perp}^j - \frac{1}{2}\delta_\perp^{ij} \right) x_1 h_1^{\perp g}(x_1, k_{1\perp}) \,,$$

Mulders, Rodrigues, 2001

Unpolarized gluon TMD
computed in the MV model

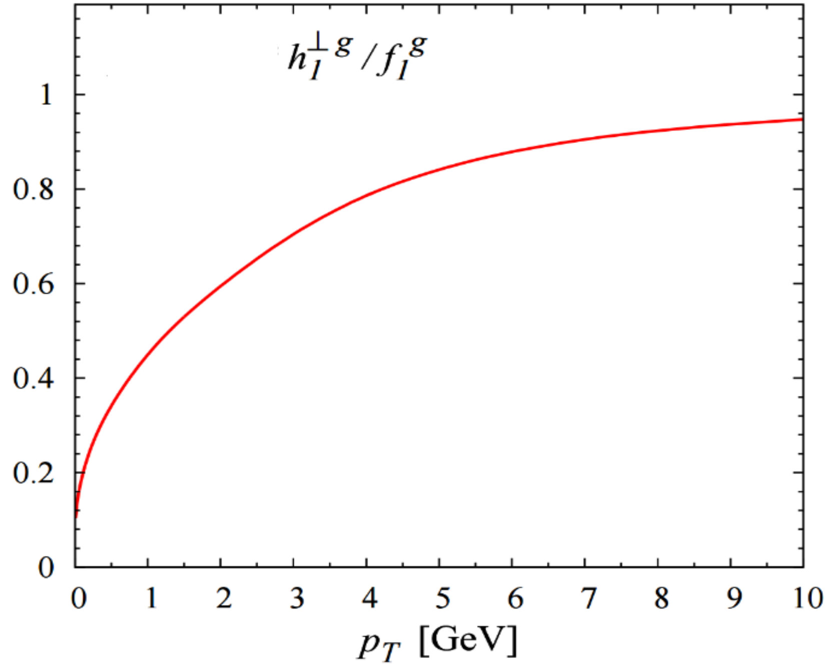


Kovchegov, 96
J. Marian, Kovner, Mclerran & Weigert, 97

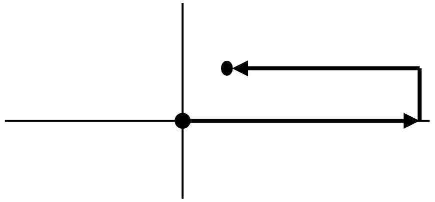
Gluon TMDs in the MV model

➢ The linearly polarized gluon TMDs in the MV model,

Metz & ZJ, 2011

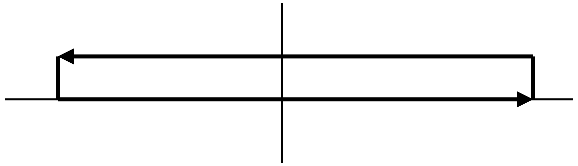


Weizsäcker-Williams(WW) distribution:



$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3} S_\perp \int d\xi_\perp \frac{K_2(k_\perp \xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

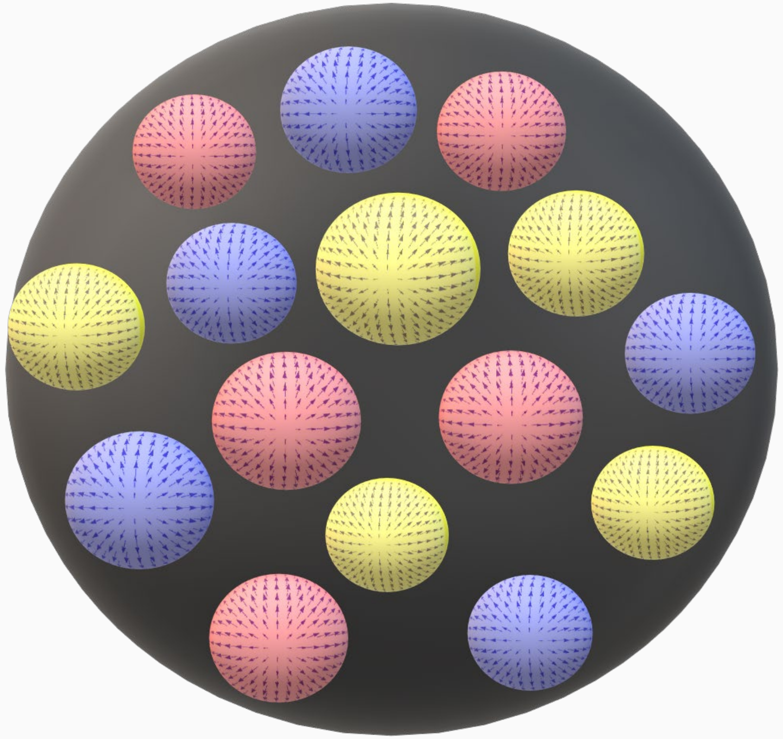
Dipole distribution:



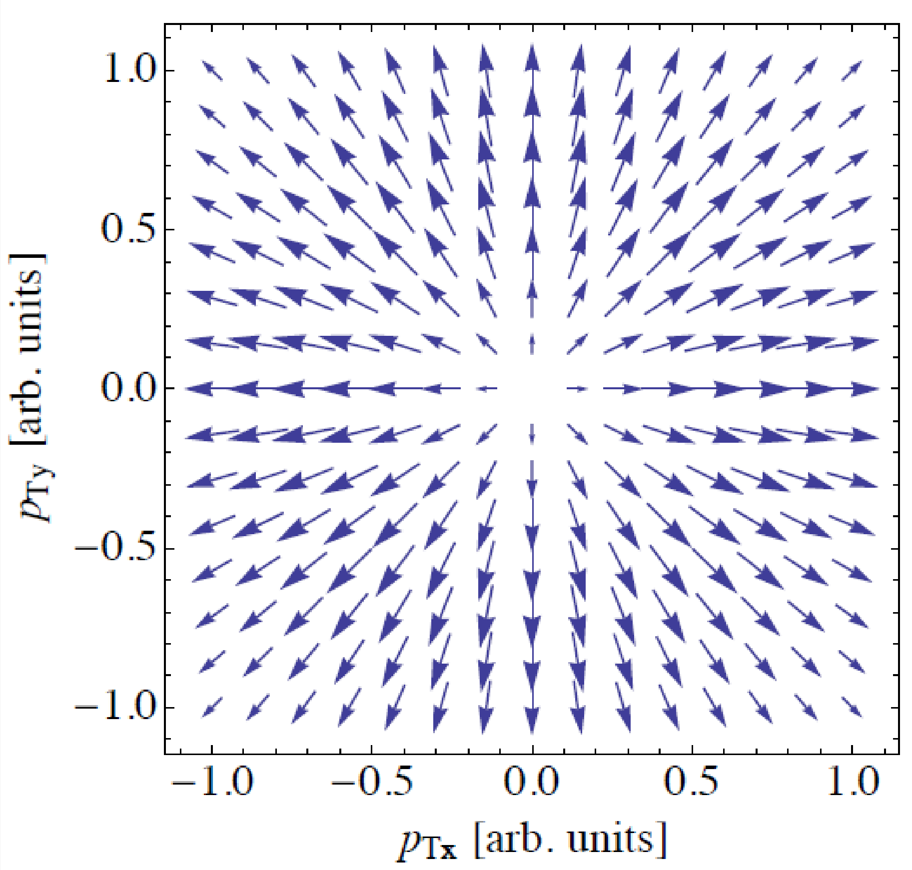
$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}^g(x,k_\perp) = \frac{k_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2\xi_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp \cdot \vec{\xi}_\perp} e^{-\frac{Q_{sq}^2 \xi_\perp^2}{4}}$$

positivity bound saturated for any value of k_t

Transverse position space



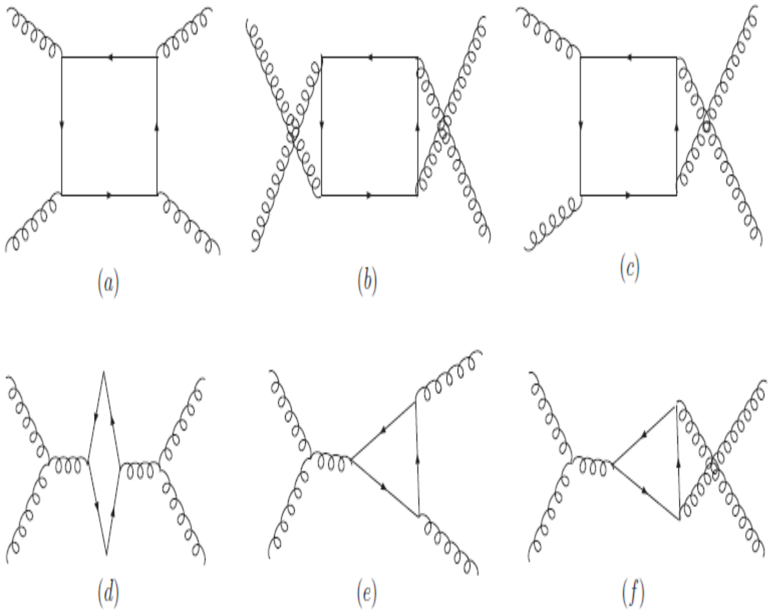
Transverse momentum space



CGC is a highly linearly polarized matter state.

Metz & Zhou, 2011

Azimuthal asymmetries in heavy quark pair production



$$
\begin{aligned}
\frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} \approx \; & \frac{\alpha_s}{(N_c^2-1)} \int \frac{d^2k_{1\perp}d^2k_{2\perp}}{(2\pi)^4}\delta^2(k_{1\perp}+k_{2\perp}-q_\perp)x_2 g(x_2,k_{2\perp}) \int d^2x_\perp d^2x'_\perp e^{-ik_{1\perp}\cdot(x_\perp-x'_\perp)} \\
& \times \Bigg\{ \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^A_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{A\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp \partial x'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^A_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{B\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp \partial y'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^B_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{A\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial y^i_\perp \partial x'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^B_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{B\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,y_\perp,y'_\perp,x'_\perp)}{\partial y^i_\perp \partial y'^j_\perp}\right]_{x_\perp=y_\perp,x'_\perp=y'_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^A_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{\dagger'}_{g,j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,y_\perp,x'_\perp,x'_\perp)}{\partial x^i_\perp \partial x'^j_\perp}\right]_{x_\perp=y_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}^B_{q\bar{q},i}(\not{l}_2-m)\gamma^0\tilde{T}^{\dagger'}_{g,j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,y_\perp,x'_\perp,x'_\perp)}{\partial y^i_\perp \partial x'^j_\perp}\right]_{x_\perp=y_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}_{g,i}(\not{l}_2-m)\gamma^0\tilde{T}^{A\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,x_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp \partial x'^j_\perp}\right]_{x'_\perp=y'_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}_{g,i}(\not{l}_2-m)\gamma^0\tilde{T}^{B\dagger'}_{q\bar{q},j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,x_\perp,y'_\perp,x'_\perp)}{\partial x^i_\perp \partial y'^j_\perp}\right]_{x'_\perp=y'_\perp} \\
& + \mathrm{Tr}\left[(\not{l}_1+m)\tilde{T}_{g,i}(\not{l}_2-m)\gamma^0\tilde{T}^{\dagger'}_{g,j}\gamma^0\right]_{k_{2\perp},k_{1\perp}=0}\left[\frac{\partial^2 C(x_\perp,x_\perp,x'_\perp,x'_\perp)}{\partial x^i_\perp \partial x'^j_\perp}\right]\Bigg\}\;, \qquad (27)
\end{aligned}
$$

TMD factorization at small x

- Employing power expansion $k_\perp << P_\perp$

$$\frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} = \frac{\alpha_s^2 N_c}{\hat{s}^2(N_c^2-1)}\left[\mathcal{A}(q_\perp^2) + \frac{m^2}{P_\perp^2}\mathcal{B}(q_\perp^2)\cos 2\phi + \mathcal{C}(q_\perp^2)\cos 4\phi\right]$$

Akcakaya, Schafer, ZJ 2012

$$\mathcal{A}(q_\perp^2) = x_2 g(x_2)\frac{(\hat{u}^2+\hat{t}^2)}{4\hat{u}\hat{t}}\left\{\frac{(\hat{t}-\hat{u})^2}{\hat{s}^2}x_1 G_{DP}(x_1,q_\perp) + x_1 G_{q\bar{q}}(x_1,q_\perp)\right\}$$

$$\mathcal{B}(q_\perp^2) = x_2 g(x_2)\left\{\frac{(\hat{t}-\hat{u})^2}{\hat{s}^2}x_1 h_{1,DP}^{\perp g}(x_1,q_\perp) + x_1 h_{1,q\bar{q}}^{\perp g}(x_1,q_\perp)\right\}$$

polarization piece

$$\mathcal{C}(q_\perp^2) = 0\ ,$$

TMD evolution effect

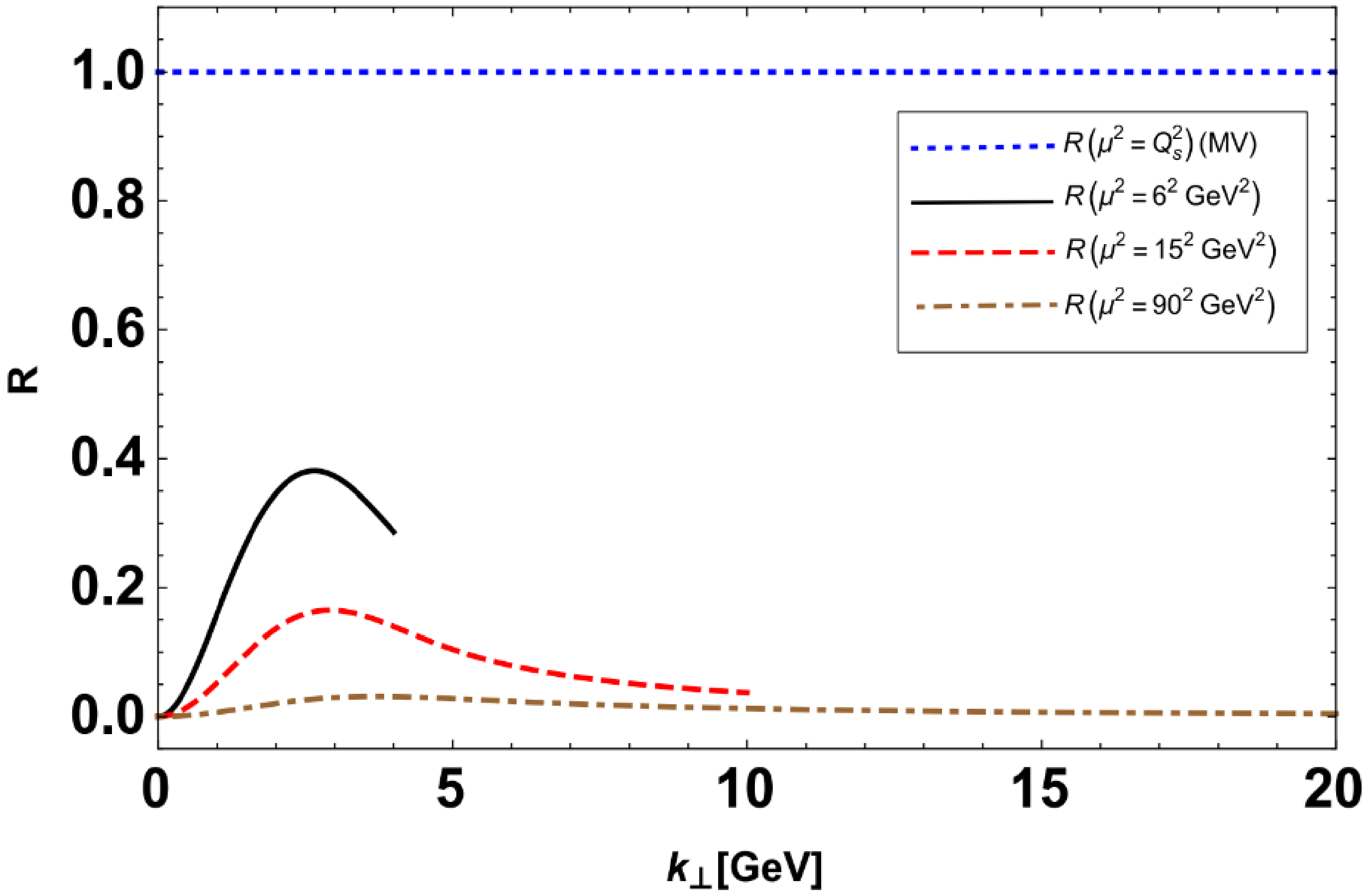


FIG. 2: The ratio $R = h_1^{\perp g}/f_1^g$ as function of $k_\perp$, at $x = 0.01$ for $\mu = 6$, 15 and 90 GeV.

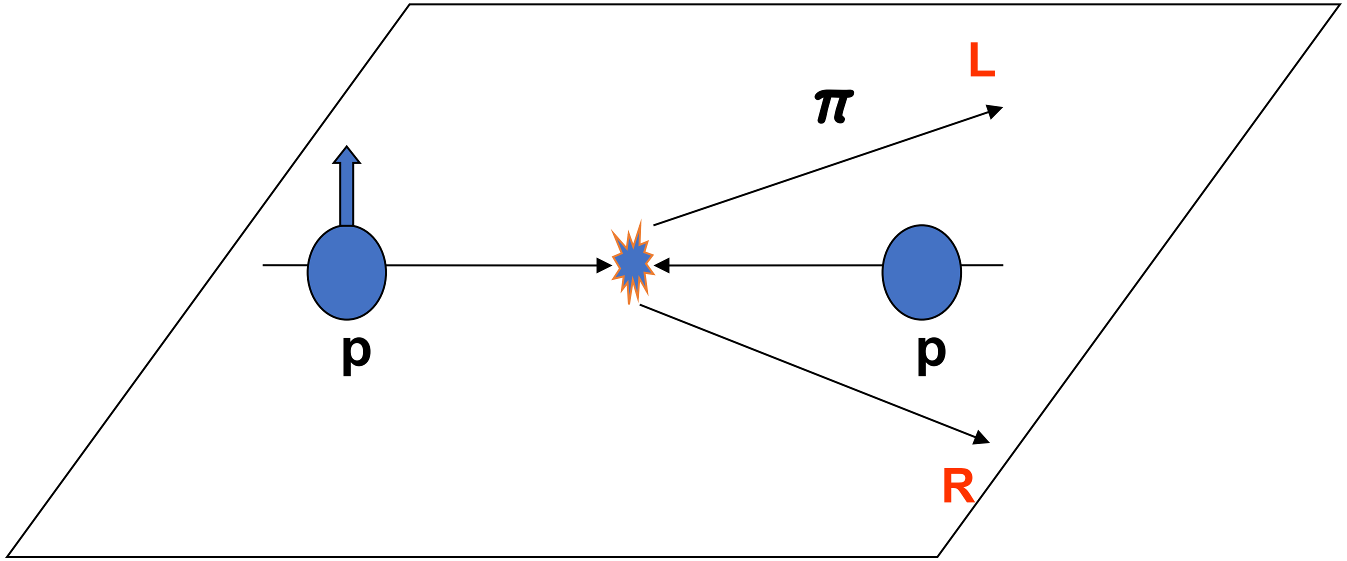
D. Boer, P. Mulders, JZ, Y. J. Zhou, 2017

Spin dependent odderon

Transverse single spin asymmetries

$$p(\uparrow) + p \rightarrow \pi + X$$

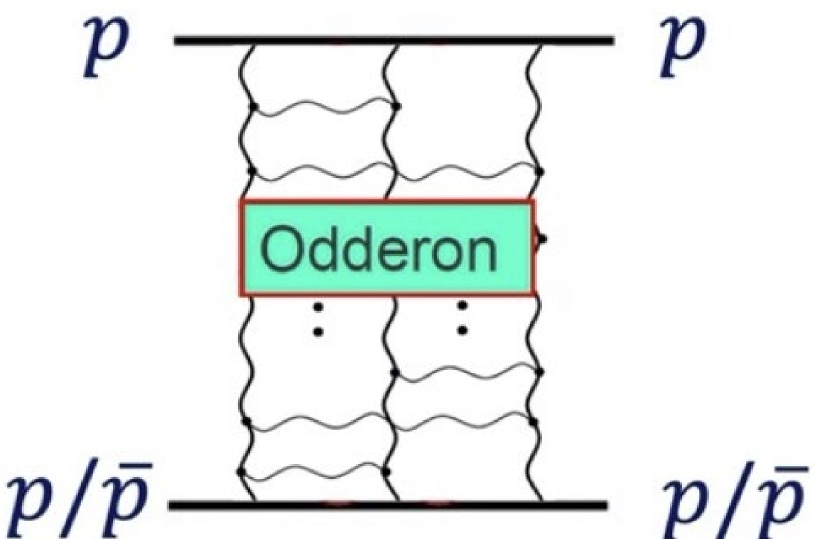
$$A_N \equiv (\sigma(S_\perp) - \sigma(-S_\perp))/(\sigma(S_\perp) + \sigma(-S_\perp))$$

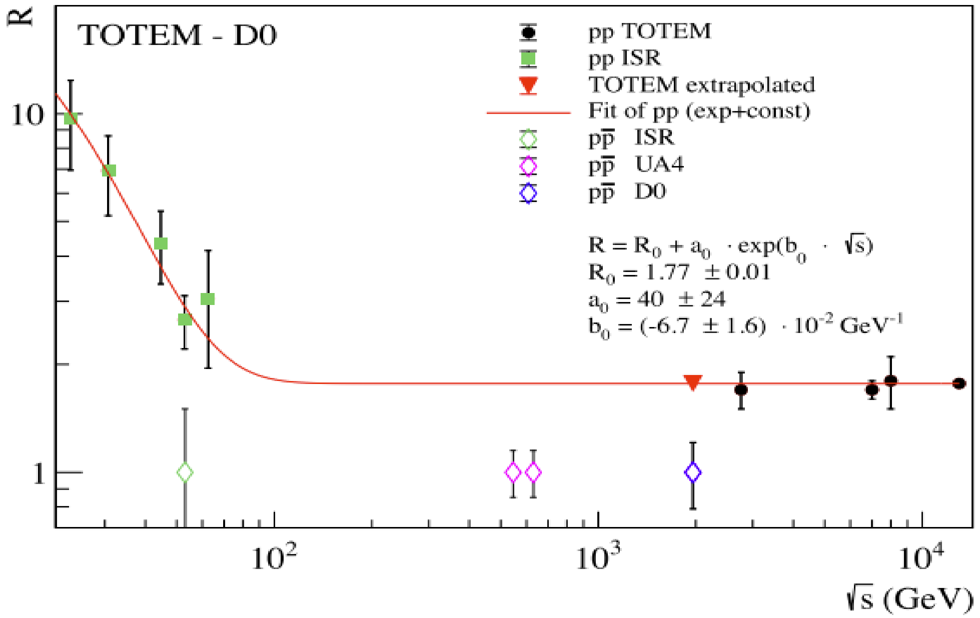


Odderon and proton-anti-proton elastic scattering

- Odderon, a colorless exchange with C=-1 Łukaszuk, L.; Nicolescu, 1973
  accounts for the difference between the cross section pp scattering and p-anitp scatterings.

  - In QCD, it is dominated by a three gluon exchange in the color symmetric state.





D0 & TOTEM collaborations, first evidence of odderon exchange, 2020

Formulations in pQCD inspired models

- Formulation in Mueller's dipole model **Kovchegov, Szymanowski & Wallon 2004**

- Formulation in the CGC **Hatta, Iancu, Itakura & McLerran 2005**

$$\hat{D}(R_\perp, r_\perp) = \frac{1}{N_c}\mathrm{Tr}\left[U(R_\perp + \frac{r_\perp}{2})U^\dagger(R_\perp - \frac{r_\perp}{2})\right]$$

$$= \hat{S}(R_\perp, r_\perp) + i\hat{O}(R_\perp, r_\perp)$$

Wilson line:

$$U(x_\perp) = \mathrm{P}\exp\left(ig\int dz^- A^+(z^-, x_\perp)\right)$$

Antisymmetric part $$\hat{O}(R_\perp, r_\perp) = \frac{1}{2i}\left[\hat{D}(R_\perp, r_\perp) - \hat{D}(R_\perp, -r_\perp)\right]$$

Symmetric part $$\hat{S}(R_\perp, r_\perp) = \frac{1}{2}\left[\hat{D}(R_\perp, r_\perp) + \hat{D}(R_\perp, -r_\perp)\right]$$

Odderon in the MV model

- The expectation value of the odderon operator

Valence quark distribution

$$\int d^2R_\perp \theta(R_0 - |R_\perp|) < \hat{O}(R_\perp, r_\perp) >$$

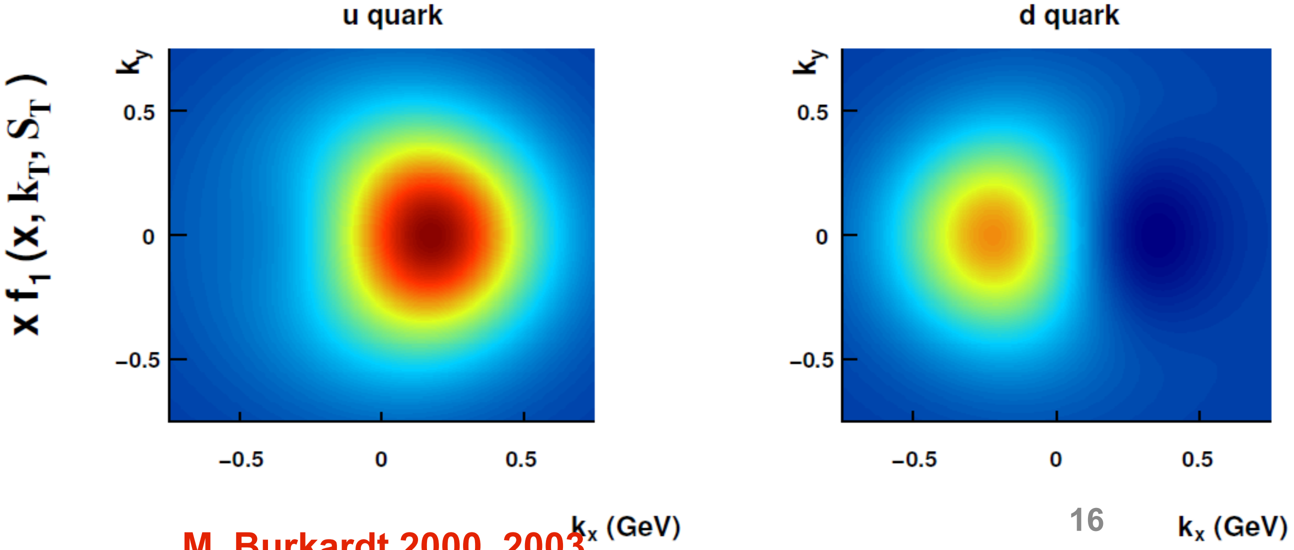
$$= c_0 \alpha_s^3 \int d^2R_\perp \theta(R_0 - |R_\perp|) \int d^2 z_\perp \ln^3 \frac{|R_\perp + r_\perp/2 - z_\perp|}{|R_\perp - r_\perp/2 - z_\perp|} e^{-\frac{1}{4} r_\perp^2 Q_s^2} \frac{1}{3} \int dx_q f_q(x_q, z_\perp)$$

JZ 2013

$$\approx \frac{c_0 \alpha_s^3 \pi}{4R_0^2} r_\perp^2 e^{-\frac{1}{4} r_\perp^2 Q_s^2} \int dx_q d^2 z_\perp (r_\perp \cdot z_\perp) f_q(x_q, z_\perp)$$

- Impact parameter dependent valence quark distribution

$$f_q(x_q, z_\perp) = \sum_{u,d} \left\{ \mathcal{H}(x_q, z_\perp^2) - \frac{1}{2M} \epsilon_\perp^{ij} S_{\perp i} \frac{\partial \mathcal{E}(x_q, z_\perp^2)}{\partial z_\perp^j} \right\}$$



M. Burkardt 2000, 2003

Spin dependent Odderon

$$\int d^2R_\perp \theta(R_0 - |R_\perp|) < \hat{O}(R_\perp, r_\perp) >$$

$$= -\frac{c_0 \alpha_s^3 \pi}{8MR_0^2} e^{-\frac{1}{4} r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j} \int dx_q d^2 z_\perp \sum_{u,d} \mathcal{E}(x_q, z_\perp^2)$$

$$= -\frac{c_0 \alpha_s^3 \pi}{8MR_0^2} e^{-\frac{1}{4} r_\perp^2 Q_s^2} r_\perp^2 \epsilon_\perp^{ij} S_{\perp i} r_{\perp j} \left(\kappa_p^u + \kappa_p^d\right)$$

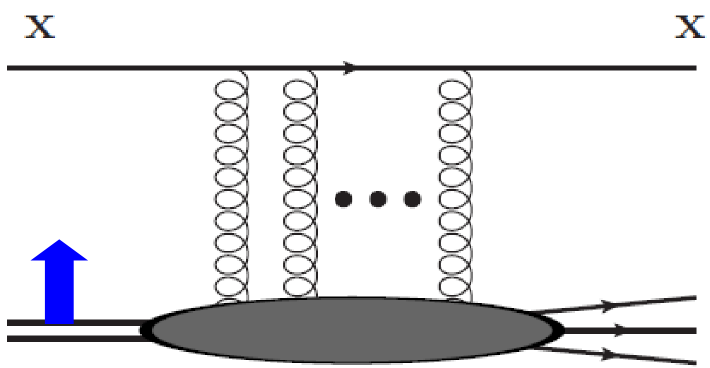
ZJ 2013

- An earlier attempt to connect SSA phenomena to GPD E has been made.

M. Burkardt 2003

- Potential relation to the orbital angular momentum !
- Connection with 3 T-odd gluon TMDs. **D. Boer, M. Echevarria, P. Mulders, ZJ, 2017**

SSA in jet production in the backward region of pp collisions



$$\frac{d\sigma^{pA\to qX}}{d^2k_\perp dY} = \sum_f xq_f(x)\int\frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}\int d^2R_\perp < \hat{D}(R_\perp, r_\perp) >_{x_g}$$
$$= \sum_f xq_f(x)\left\{F_{x_g}(k_\perp^2) + \frac{1}{M}\epsilon_\perp^{ij}S_{\perp i}k_{\perp j}O_{1T,x_g}^{\perp}(k_\perp^2)\right\}$$

- F_{xg} is just the Dipole type unpolarized gluon distribution.

- Spin dependent odderon

$$O_{1T,x_g}^{\perp}(k_\perp^2) = \frac{-c_0\alpha_s^3\left(\kappa_p^u + \kappa_p^d\right)}{4R_0^4}\left[\frac{\partial}{\partial k_\perp^2}\frac{\partial}{\partial k_\perp^i}\frac{\partial}{\partial k_{\perp i}}F_{x_g}(k_\perp^2)\right]$$

- The asymptotic behavior(according to the BLV solution)

$$A_{UT} \sim k_\perp O_{1T,x_g}^{\perp}/F_{x_g}$$ BLV solution → $$A_{UT} \sim (x_g)^{0.3}$$

T-odd gluon TMDs

Identify 6 leading power gluon TMDs for a transversely polarized target (8 in total). Among them, 3 gluon TMDs are T-odd distributions.

$$\frac{1}{xP^+}\int \frac{dy^- d^2y_T}{(2\pi)^3} e^{ik\cdot y}\langle P,S_T|2\mathrm{Tr}\left[F^{\mu}_{+T}(0)UF^{\nu}_{+T}(y)U'\right]|P,S_T\rangle$$

$$= \delta_T^{\mu\nu} f_1^g + \left(\frac{2k_T^\mu k_T^\nu}{k_\perp^2} - \delta_T^{\mu\nu}\right) h_1^{\perp g} - \delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M} f_{1T}^{\perp g}$$

$$-i\epsilon_T^{\mu\nu}\frac{k_T\cdot S_T}{M} g_{1T}^g - \frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M} h_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}} + \tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M} h_{1T}^g$$

Mulders, Rodrigues, 2001

Are the T-odd gluon TMDs relevant at small x?

The common origin of three T-odd TMDs

Equaling two parmetrizations:

$$\frac{k_T^\mu k_T^\nu N_c}{2\pi^2\alpha_s x}\frac{\epsilon_T^{\alpha\beta}S_{T\alpha}k_{T\beta}}{M}O_{1T,x}^{\perp}(k_\perp^2) = -\delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M}f_{1T}^{\perp g}$$
$$-\frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M}h_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}}+\tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M}h_{1T}^g$$

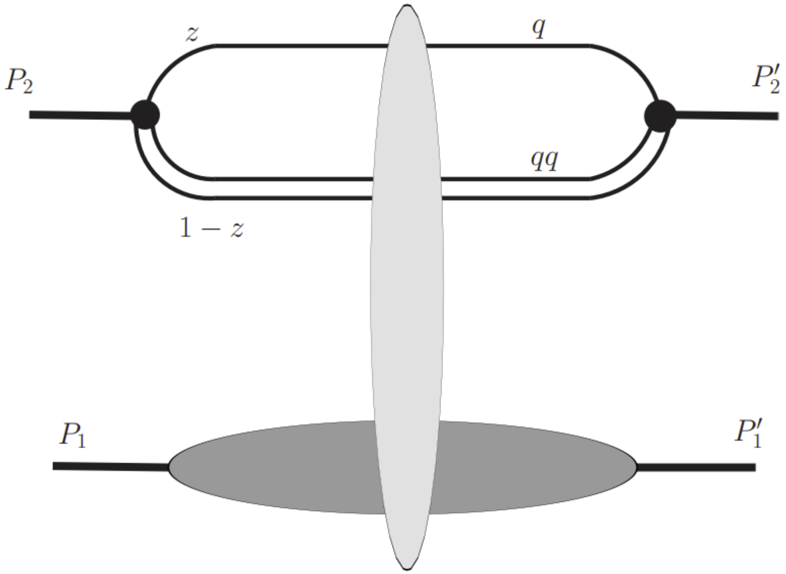
Simple algebra leads to

$$x f_{1T}^{\perp g} = x h_{1T}^g = x h_{1T}^{\perp g} = \frac{k_\perp^2 N_c}{4\pi^2\alpha_s}O_{1T,x}^{\perp}(k_\perp^2)$$

Boer, Echevarria, Mulders, ZJ; PRL，2017

All of three dipole type T-odd gluon TMDs become identical at small x!

Proton-proton elastic scattering



- quark-diquark dipole scattering off a proton target by exchanging pomeron and odderon in the t-channel

Y. Hagiwara, Y. Hatta, R. Pasechnik, JZ, 2020

$$\frac{d\sigma}{dt} = \frac{2\pi}{s(s-4M^2)}\left(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2\right)$$

- Helicity amplitudes:

$$\langle\lambda_1'\lambda_2'|T|\lambda_1\lambda_2\rangle \equiv e^{\frac{i}{2}(\lambda_1-\lambda_2-\lambda_1'+\lambda_2')\varphi}\langle\lambda_1'\lambda_2'|\tilde{T}|\lambda_1\lambda_2\rangle$$

Small x

GPD

Spin dependent odderon

SSAs

T-odd gluon TMDs

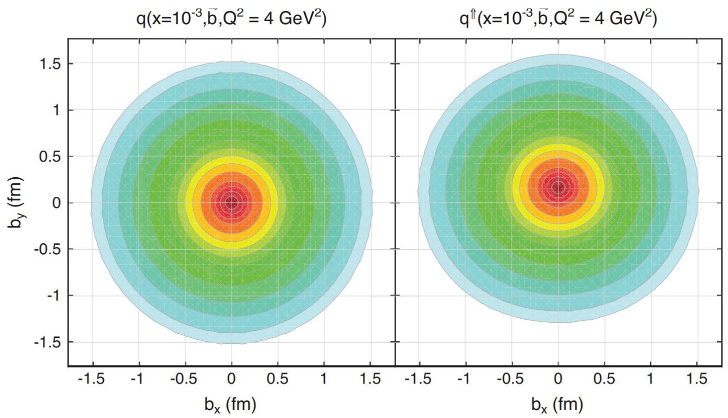
Gluon OAM at small x

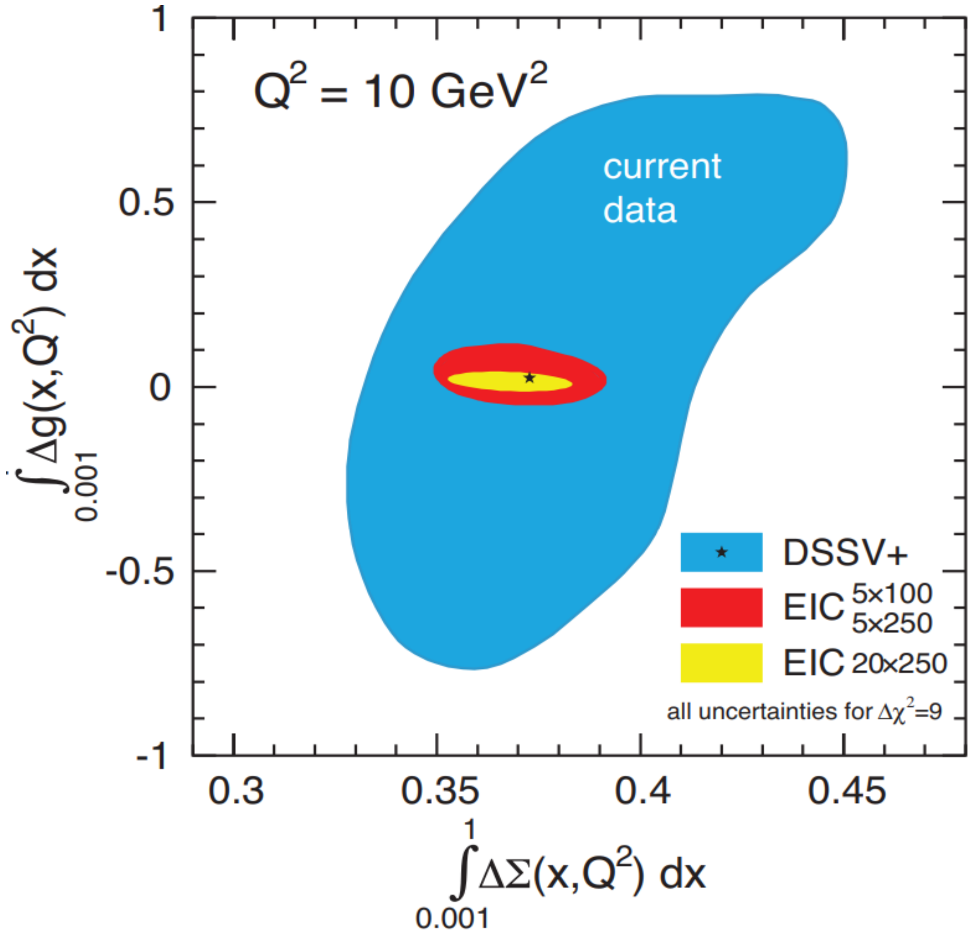
Proton spin decomposition

- **Ji's sum rule**

Ji, 1997

$$J_g = \frac{1}{2}\int_0^1 dx\; x\left[H_g(x,\xi) + E_g(x,\xi)\right]$$





Small x evolution equations

◆BK equation in impact parameter space:

$$\partial_Y \mathcal{N}(\mathbf{x},\mathbf{y}) = \frac{\bar{\alpha}}{2\pi}\int d^2z \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2}\left[\mathcal{N}(\mathbf{x},\mathbf{z})+\mathcal{N}(\mathbf{z},\mathbf{y})-\mathcal{N}(\mathbf{x},\mathbf{y})-\mathcal{N}(\mathbf{x},\mathbf{z})\mathcal{N}(\mathbf{z},\mathbf{y})\right]$$

$$\frac{1}{N_c}\mathrm{Tr}U(b_\perp + r_\perp/2)U^\dagger(b_\perp - r_\perp/2)$$

Balitsky, 1996
Kovchegov, 1997

◆BK equation in momentum space:

$$\partial_Y \mathcal{N}(k_\perp,\Delta_\perp) = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left\{\mathcal{N}(k'_\perp,\Delta_\perp) - \frac{1}{4}\left[\frac{(\frac{\Delta_\perp}{2}+k_\perp)^2}{(\frac{\Delta_\perp}{2}+k'_\perp)^2} + \frac{(\frac{\Delta_\perp}{2}-k_\perp)^2}{(\frac{\Delta_\perp}{2}-k'_\perp)^2}\right]\mathcal{N}(k_\perp,\Delta_\perp)\right\}$$

$$-\frac{\bar{\alpha}_s}{2\pi}\int d^2\Delta'_\perp \mathcal{N}(k_\perp + \frac{\Delta'_\perp}{2},\Delta_\perp - \Delta'_\perp)\mathcal{N}(k_\perp + \frac{\Delta'_\perp - \Delta_\perp}{2},\Delta'_\perp)$$

Forward limit

- **Typical nucleon recoiled transverse momentum is reversely proportional to the radius of nucleon,**

$$\mathcal{F}_{1,1}(k_\perp, \Delta_\perp) = \overline{\mathcal{F}}_{1,1}(k)(2\pi)^2\delta^{(2)}(\Delta_\perp)$$

- **The forward BK(for the unpolarized gluon TMD) reads,**

$$\partial_Y \overline{\mathcal{F}}_{1,1}(k_\perp) = \frac{\bar{\alpha}_s}{\pi}\int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2}\left\{\overline{\mathcal{F}}_{1,1}(k'_\perp) - \frac{1}{2}\frac{k_\perp^2}{k'^2_\perp}\overline{\mathcal{F}}_{1,1}(k_\perp)\right\} - 4\pi^2\alpha_s^2\left[\overline{\mathcal{F}}_{1,1}(k_\perp)\right]^2$$

Spin-dependent small x evolution equation

◆ **Project to the different spin correlation structures,**

$$\partial_Y\left(k_\perp\times S_\perp\frac{k_\perp^i}{M^2}\mathcal{F}_{12}(k_\perp)+\epsilon^{ij}S_\perp^j\left(\mathcal{F}_{13}(k_\perp)-\frac{1}{2}\mathcal{F}_{11}(k_\perp)\right)\right)=\frac{\bar{\alpha}_s}{\pi}\int\frac{d^2k'_\perp}{(k_\perp-k'_\perp)^2}\left[k'_\perp\times S_\perp\frac{k'^i_\perp}{M^2}\mathcal{F}_{12}(k'_\perp)\right.$$
$$+\frac{\epsilon^{ij}S_\perp^j}{2}\left(2\mathcal{F}_{13}(k'_\perp)-\mathcal{F}_{11}(k'_\perp)\right)-\frac{k_\perp^2}{2k'^2_\perp}\left.\left(k_\perp\times S_\perp\frac{k_\perp^i}{M^2}\mathcal{F}_{12}(k_\perp)+\frac{\epsilon^{ij}S_\perp^j}{2}\left(2\mathcal{F}_{13}(k_\perp)-\mathcal{F}_{11}(k_\perp)\right)\right)\right]$$
$$-4\pi^2\alpha_s^2\left(k_\perp\times S_\perp\frac{k_\perp^i}{M^2}\mathcal{F}_{1,2}(k_\perp)+\frac{\epsilon^{ij}S_\perp^j}{2}\left(2\mathcal{F}_{1,3}(k_\perp)-\mathcal{F}_{1,1}(k_\perp)\right)\right)\overline{\mathcal{F}}_{1,1}(k_\perp),$$

◆ **Read off the coefficients of** $k_\perp\times S_\perp$ $\quad\frac{\epsilon^{ij}S_\perp^j}{2}$

$$\partial_Y\mathcal{F}_{1,2}(k_\perp)=\frac{\bar{\alpha}_s}{\pi}\int\frac{d^2k'_\perp}{(k_\perp-k'_\perp)^2}\left[-\frac{k_\perp^2}{2k'^2_\perp}\mathcal{F}_{1,2}(k_\perp)+\frac{2(k_\perp\cdot k'_\perp)^2-k_\perp^2k'^2_\perp}{(k_\perp^2)^2}\mathcal{F}_{1,2}(k'_\perp)\right]-4\pi^2\alpha_s^2\overline{\mathcal{F}}_{1,1}(k_\perp)\mathcal{F}_{1,2}(k_\perp)$$

and,

$$\partial_Y\mathcal{F}_{1,3}(k_\perp)=\frac{\bar{\alpha}_s}{\pi}\int\frac{d^2k'_\perp}{(k_\perp-k'_\perp)^2}\left[-\frac{k_\perp^2}{2k'^2_\perp}\mathcal{F}_{1,3}(k_\perp)+\frac{k_\perp^2k'^2_\perp-(k_\perp\cdot k'_\perp)^2}{k_\perp^2}\frac{\mathcal{F}_{1,2}(k'_\perp)}{M^2}+\mathcal{F}_{1,3}(k'_\perp)\right]-4\pi^2\alpha_s^2\overline{\mathcal{F}}_{1,1}(k_\perp)\mathcal{F}_{1,3}(k_\perp)$$

Small x evolution of Eg

➢ Small x evolution equation for kt dependent Eg,

$$\partial_Y \mathcal{E}(k_\perp) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2 k'_\perp}{(k_\perp - k'_\perp)^2} \left[\mathcal{E}(k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp} \mathcal{E}(k_\perp) \right] - 4\pi^2 \alpha_s^2 \overline{\mathcal{F}}_{1,1}(k_\perp) \mathcal{E}(k_\perp)$$

Hatta, ZJ, PRL, 2022

◆In the dilute limit:

$$x E_g(x) \sim x G(x) \propto \left(\frac{1}{x} \right)^{\bar{\alpha}_s 4 \ln 2}$$

Numerical results

- **The MV model (x_0=0.01)** $Y = \ln \frac{x_0}{x}$

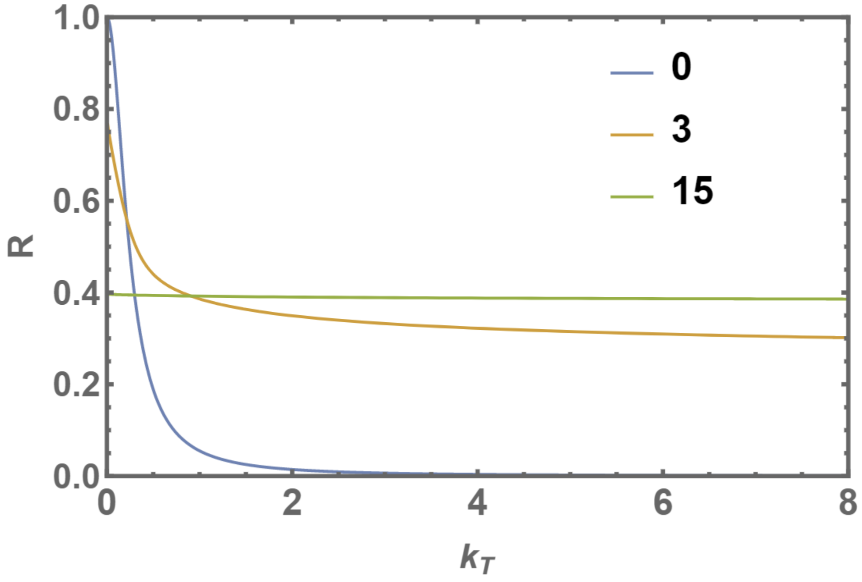
$$\mathcal{F}_{1,1}(Y=0,k_\perp) = \frac{N_c \mathcal{A}_\perp}{2\pi^2 \alpha_s} \int \frac{d^2 r_\perp}{(2\pi)^2 r_\perp^2} e^{-ik_\perp \cdot r_\perp} \left\{ 1 - \exp\left[-\frac{r_\perp^2 Q_{s0}^2}{4} \ln\left(\frac{1}{r_\perp \Lambda_{\rm mv}} + e \right) \right] \right\}$$

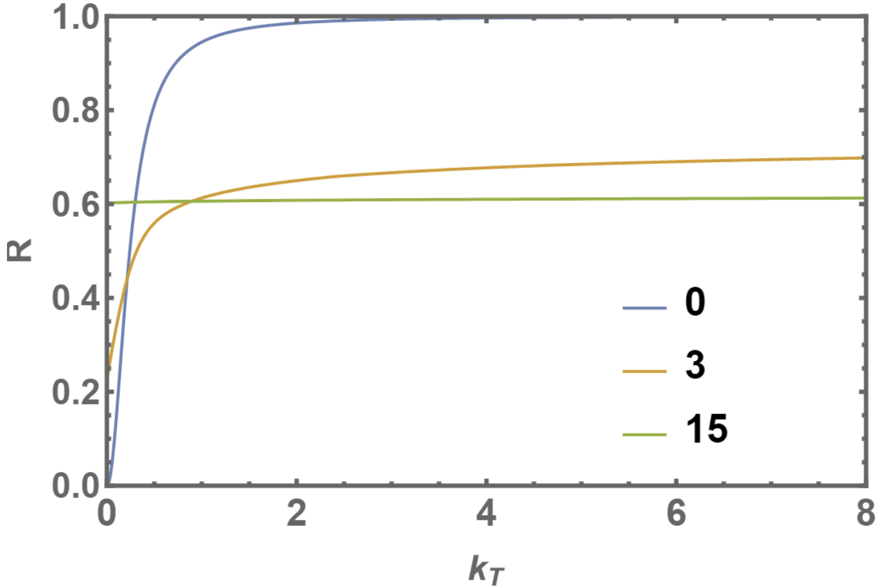
- **Two toy models:**

$$\mathcal{E}(Y=0,k_\perp) = \frac{\Lambda_{\rm mv}^2}{k_\perp^2 + \Lambda_{\rm mv}^2} \mathcal{F}_{1,1}(Y=0,k_\perp)$$

$$\mathcal{E}(Y=0,k_\perp) = \frac{k_\perp^2}{k_\perp^2 + \Lambda_{\rm mv}^2} \mathcal{F}_{1,1}(Y=0,k_\perp)$$

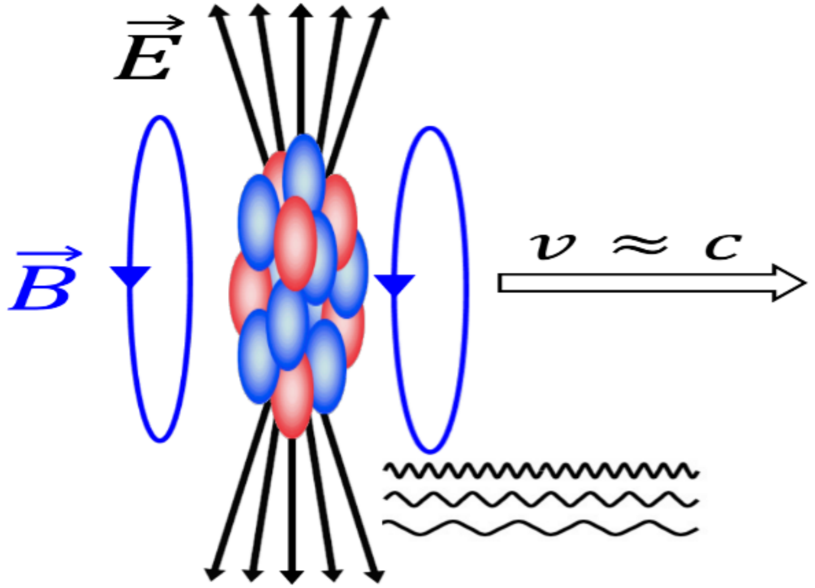
$$R \equiv \frac{\mathcal{E}(x,k_\perp)}{\mathcal{F}_{1,1}(x,k_\perp)}$$

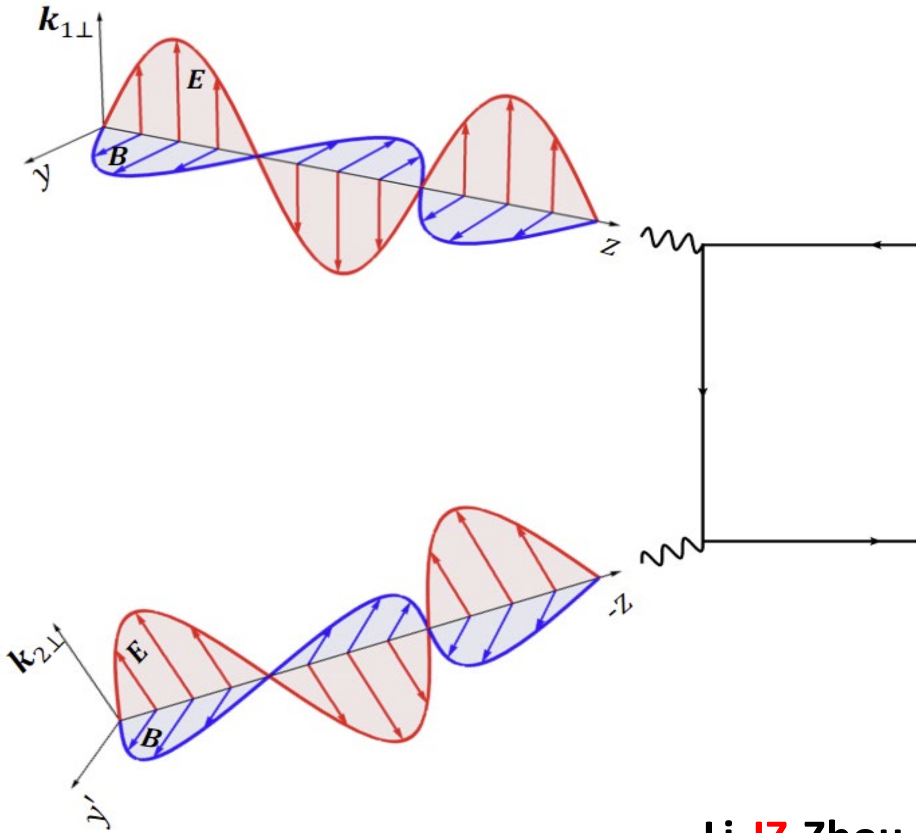




Linearly polarized photons

The boosted Coulomb potential

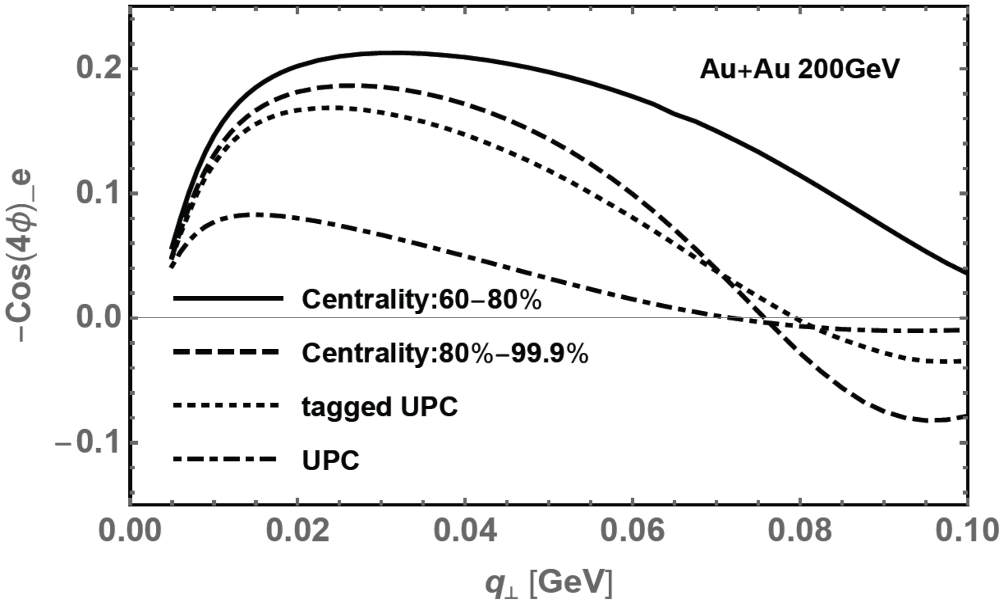




Li-JZ-Zhou, 2019

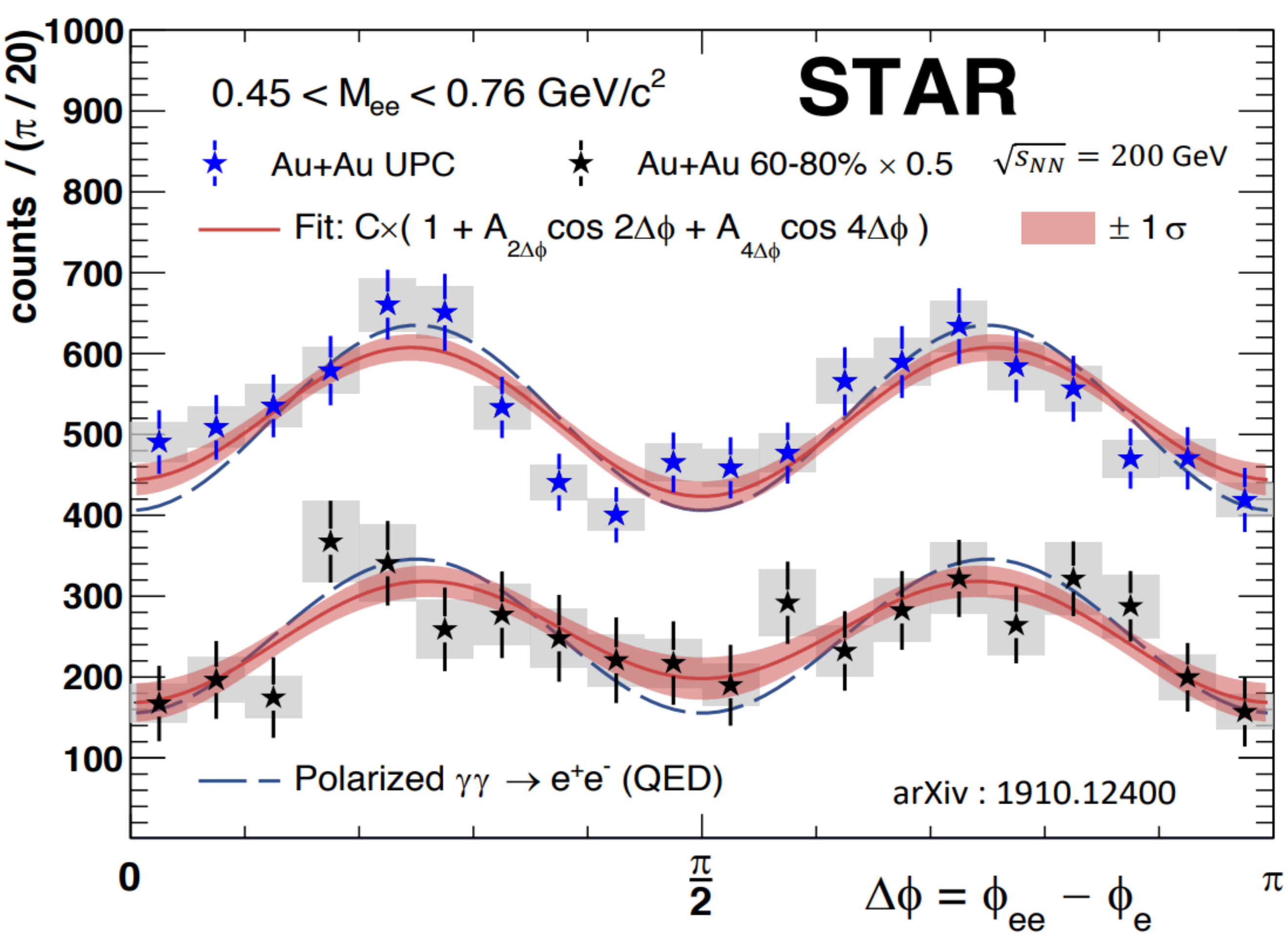
Linear polarization of photons: induce cos4ϕ modulation in di-lepton production.

Verified by STAR experiment



Li-JZ-Zhou, 2020

STAR collaboration, PRL, 2021

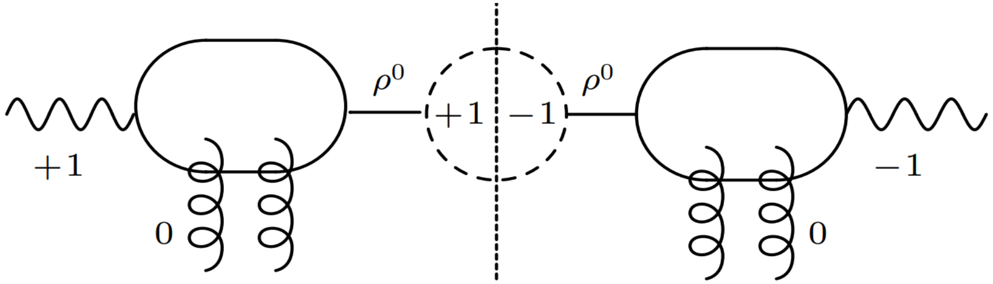


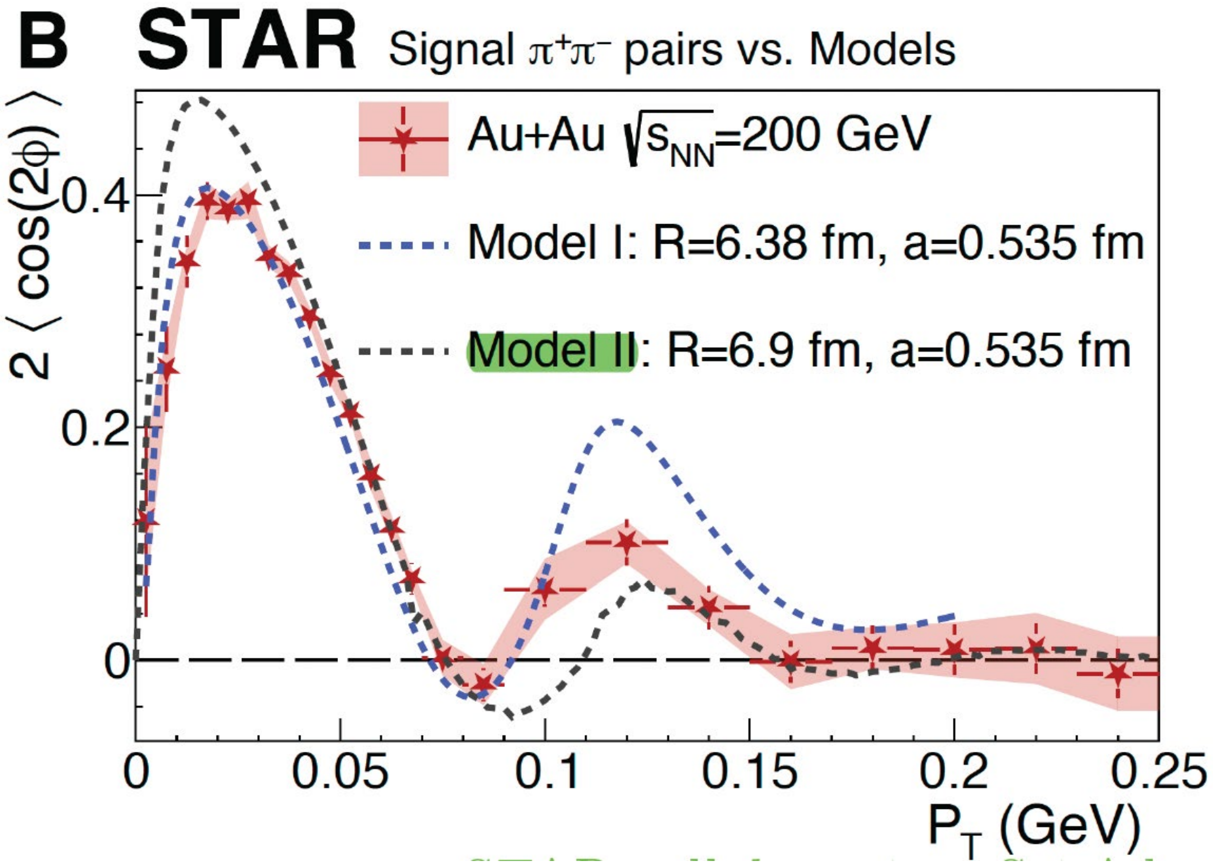
	Measured	QED calculation
Tagged UPC	16.8%±2.5%	16.5%
60%-80%	27%±6%	34.5%

Cos2ϕ in ρ^0 production

- **Interference between two p waves**

$$\langle +1 | -1 \rangle \sim \cos 2\phi$$

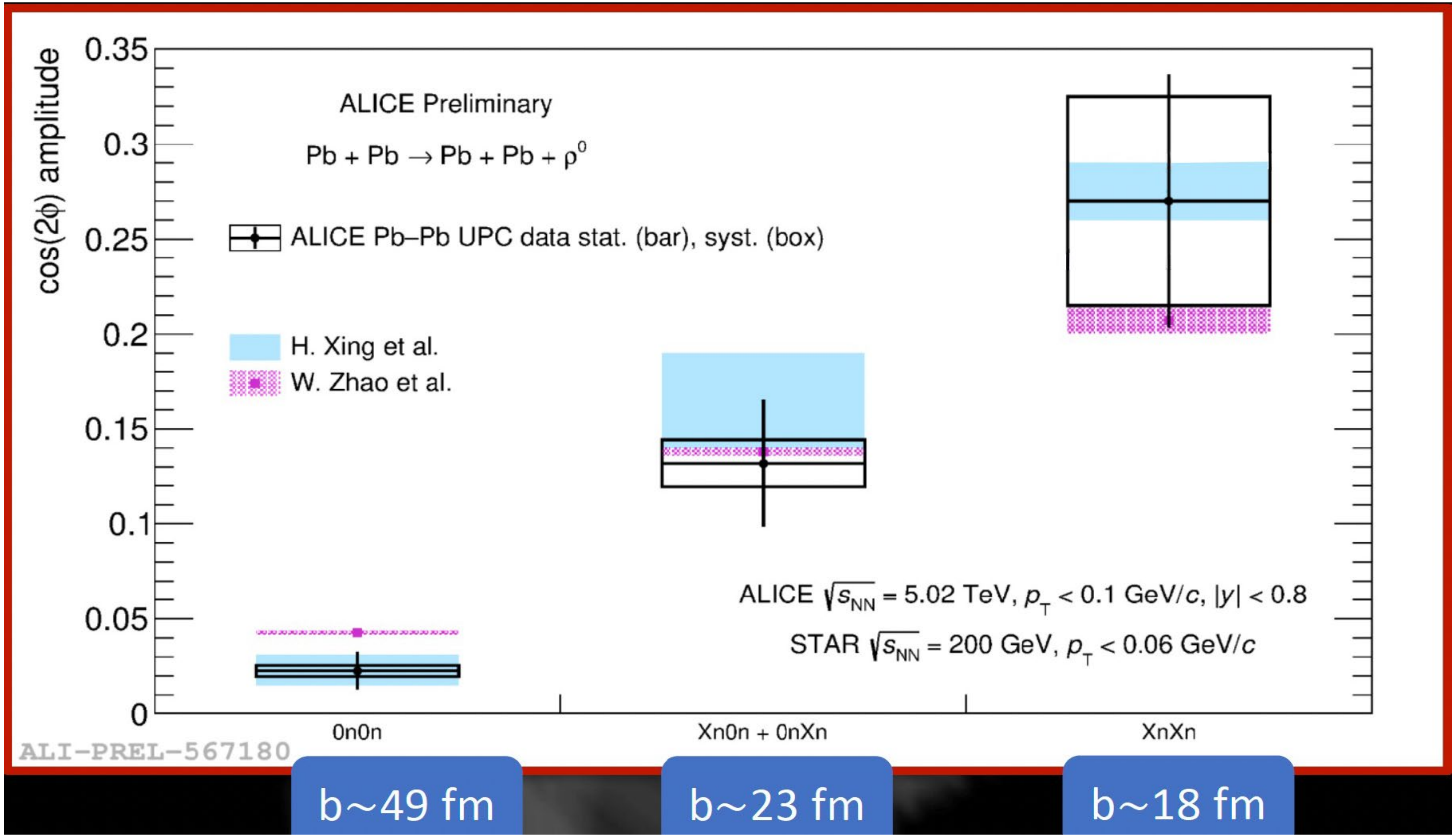




Theory curve II taken from
Xing-Zhang-ZJ-Zhou, 2020

Data points taken from
STAR collaboration, Sci.Adv. 2023

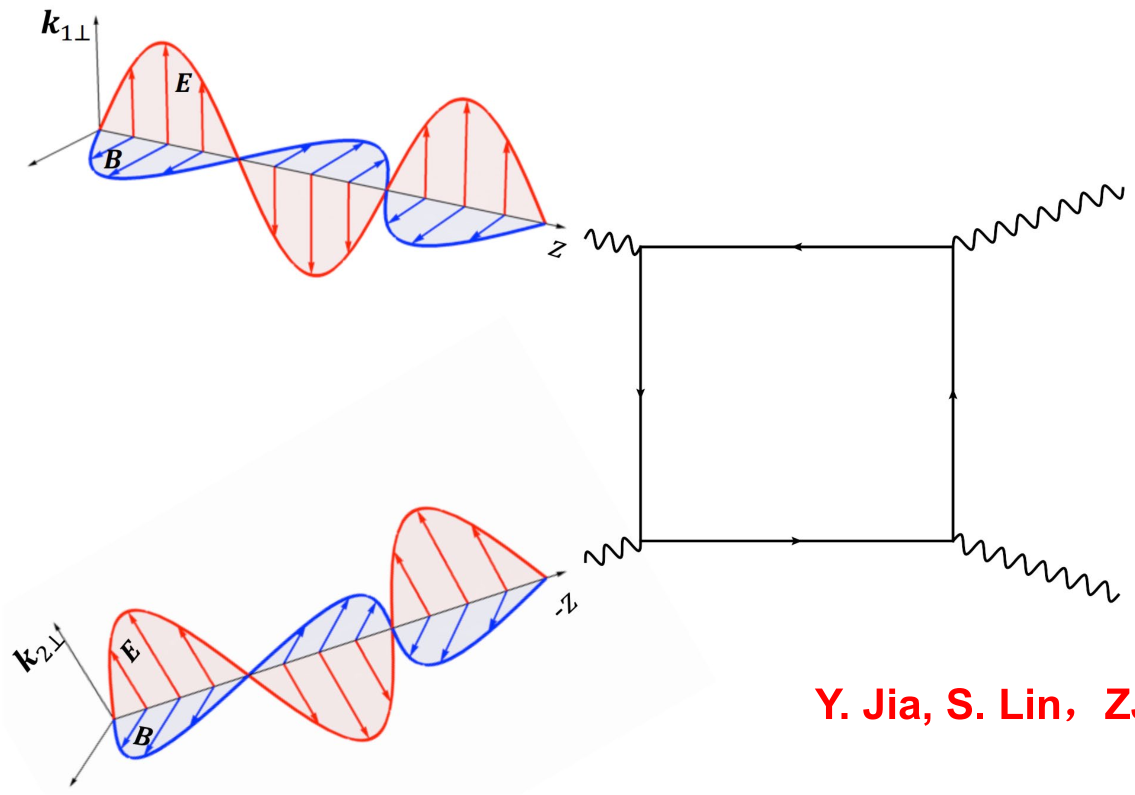
ALICE measurement of Cos2ϕ asymmetry



SDU-SCNU组： Hongxi Xing, Cheng Zhang, Jian Zhou, Ya-Jin Zhou *JHEP* 10 (2020) 064

BNL组: Heikki Mäntysaari, Farid Salazar, Björn Schenke, Chun Shen, Wenbin Zhao *Phys.Rev.C* 109 (2024) 2, 024908

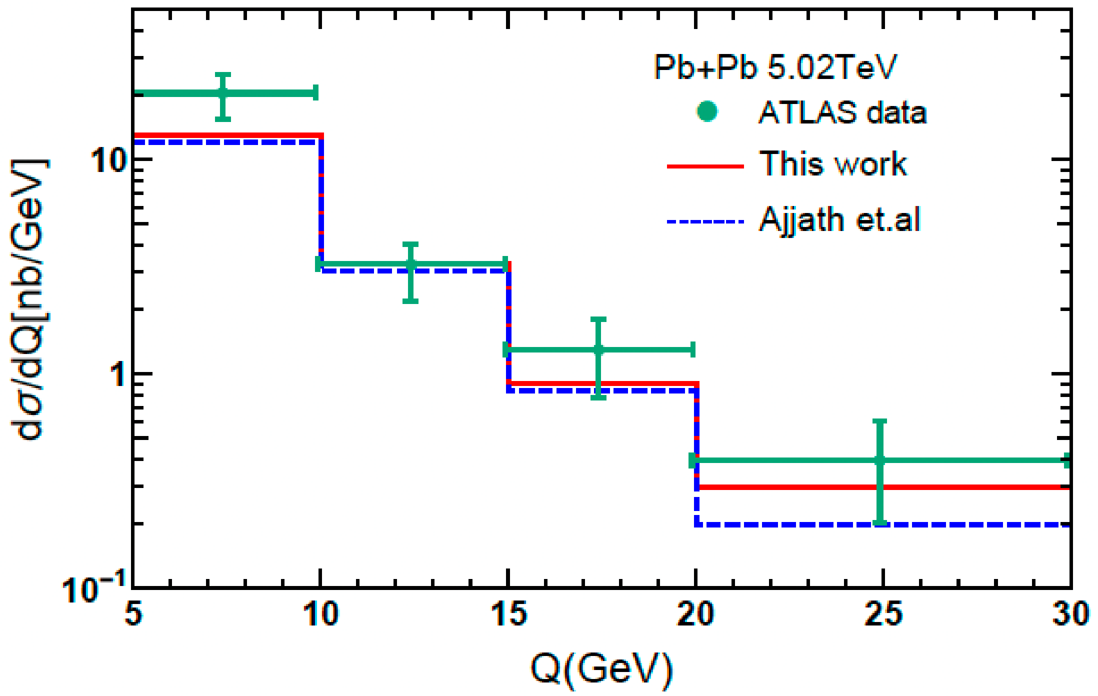
Light-by-Light scattering in UPCs at LHC



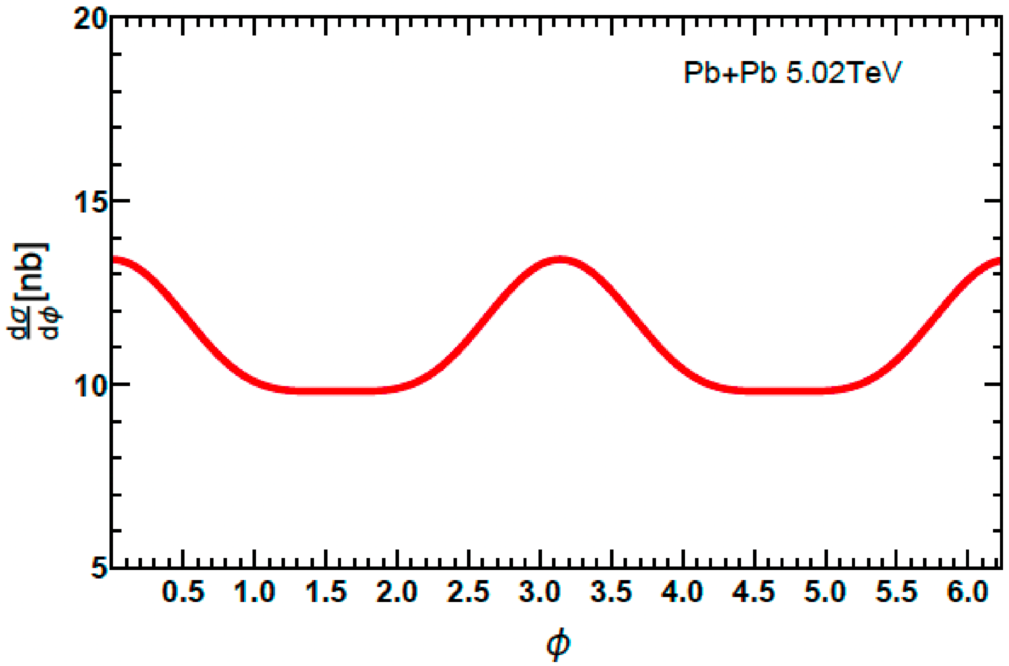
Y. Jia, S. Lin，ZJ，Y.j. Zhou，arXiv:2410.13781

- ☆ Fundamental QED process
- ☆ 2 sigma deviation from SM predications
- ☆ Study azimuthal asymmetries in LbL for the first time

Numerical results



Unpolarized cross section



azimuthal modulation

Summary

- Very rich polarization dependent phenomenology at small x

- A new avenue opened for the strong field QED and QCD phenomenology studies with linearly polarized photons

Thank you !

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SHANDONG UNIVERSITY