Radiative correction to spin polarization in hydrodynamic QGP

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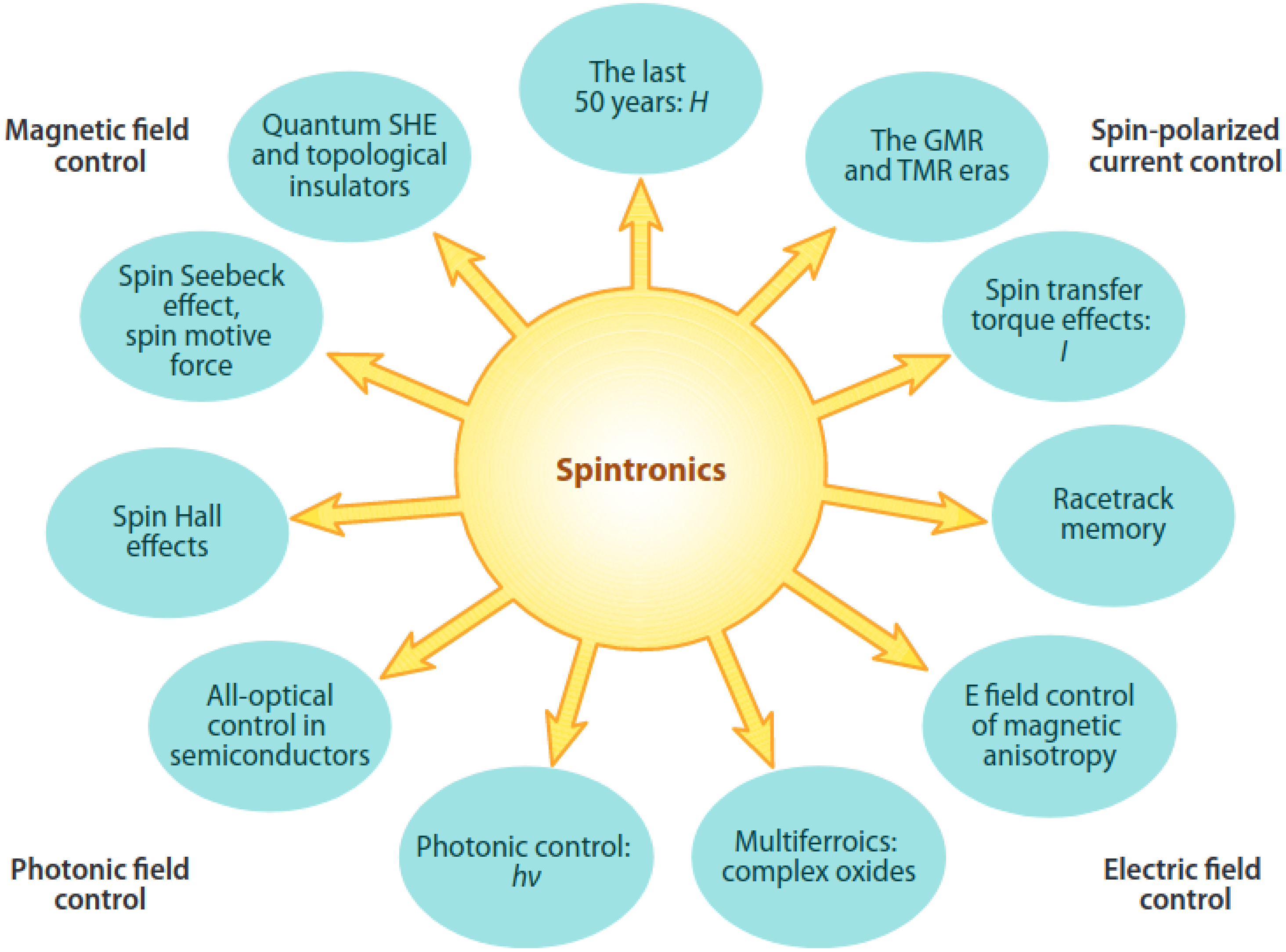
Outline

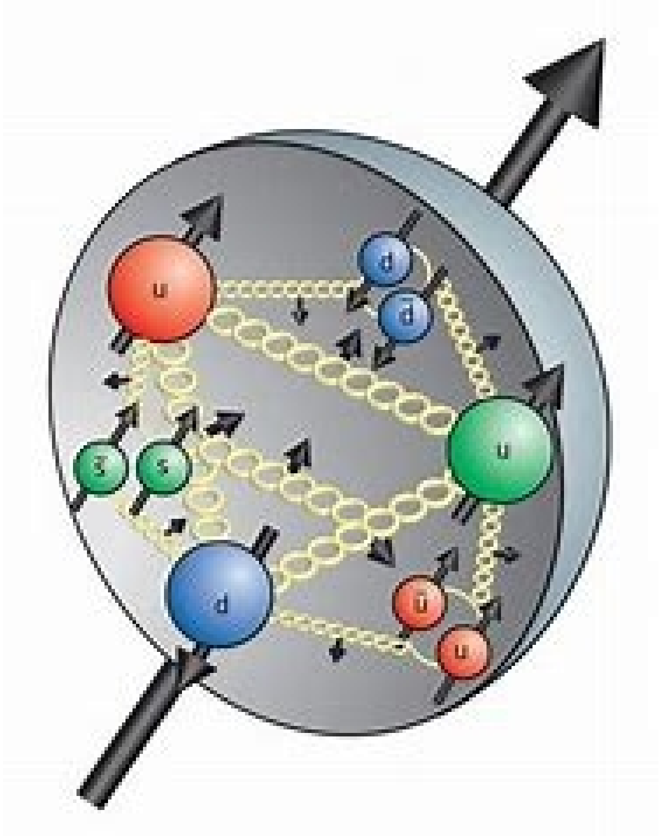
- Uniqueness of spin physics in heavy ion collisions
- Spin polarization in EM fields from chiral kinetic theory
- Radiative corrections to polarization in EM fields
- Spin polarization in hydrodynamic state from chiral kinetic theory
- Classification of contributions to polarization
- Radiative corrections to polarization in hydrodynamic state
- Conclusion and outlook

Spintronics in condensed matter physics



Bader+Parkin
ARCMP 2010

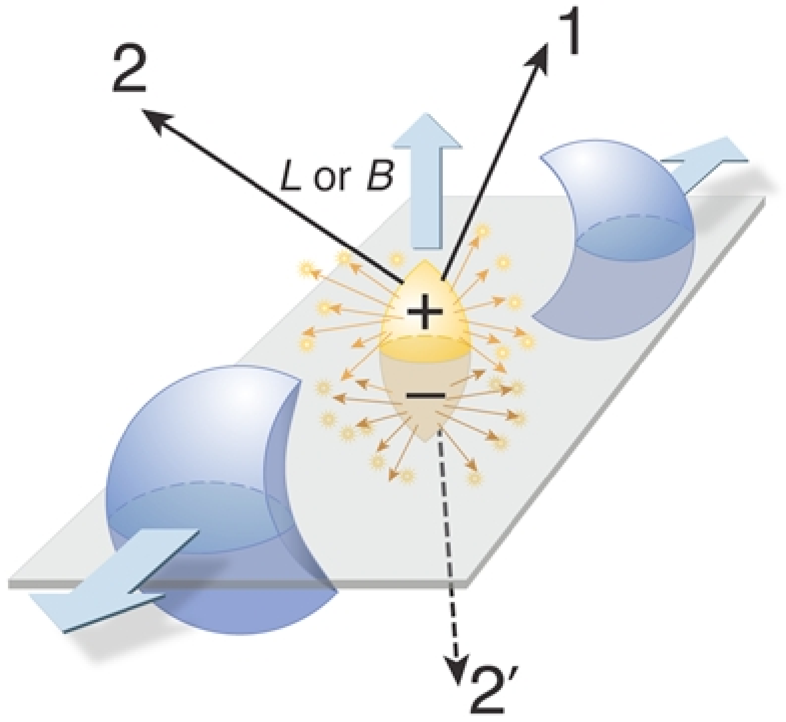
Spin in particle physics



Proton spin puzzle
(1988-now)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

Spin in heavy ion physics



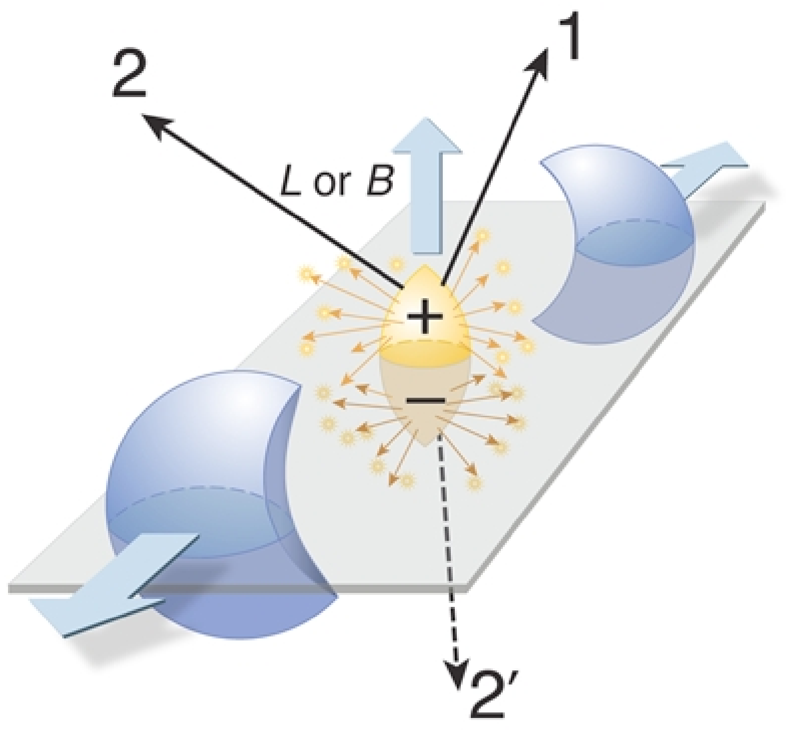
$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

- Spin not conserved, spin from initial orbital angular momentum
- Spin coupling to external field such as magnetic, vorticity etc

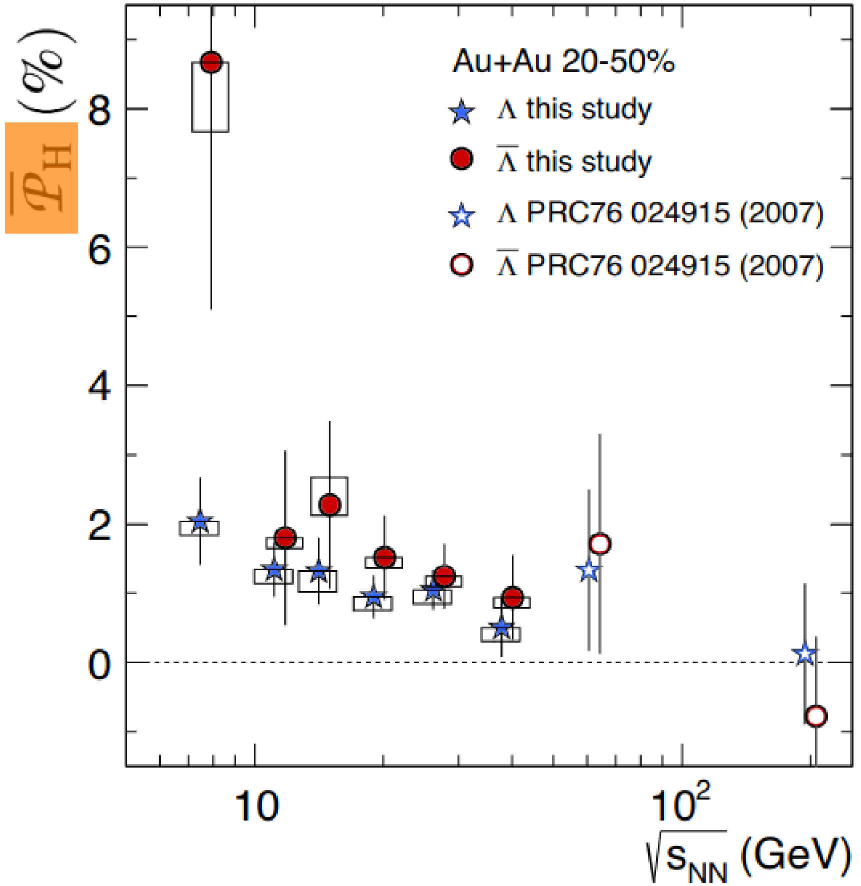
Offer a unique probe of spin property of QGP

global spin polarization in heavy ion collisions



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

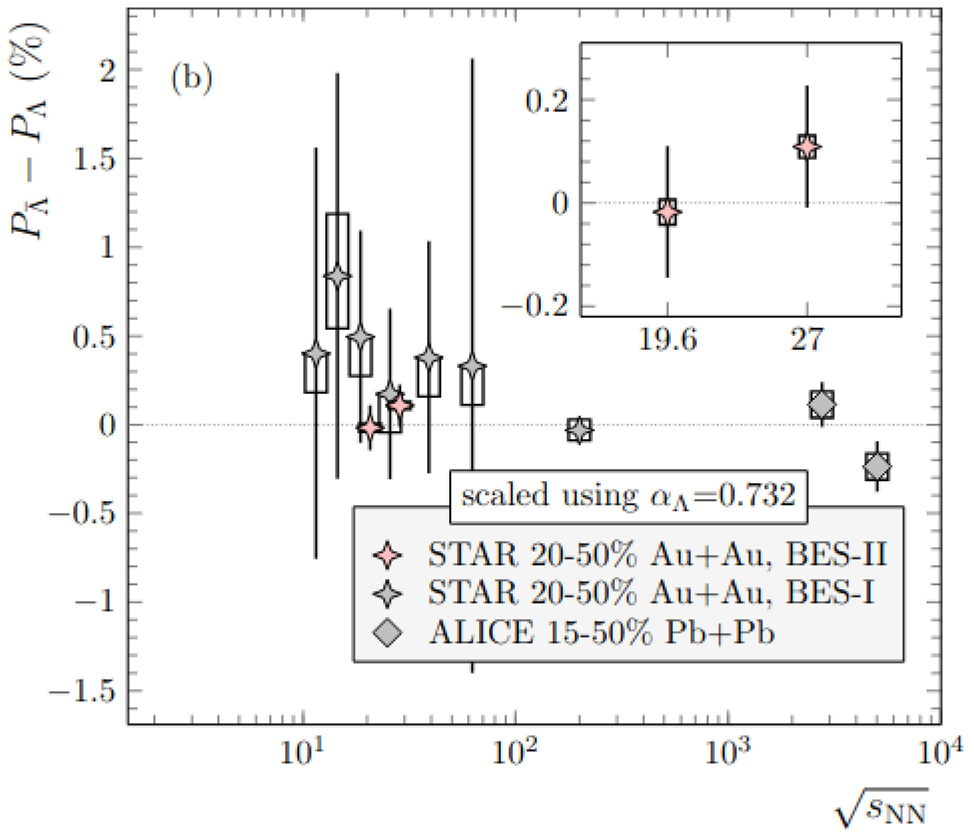


J.-H. Chen's talk

STAR collaboration, Nature 2017

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

Splitting in global spin polarization



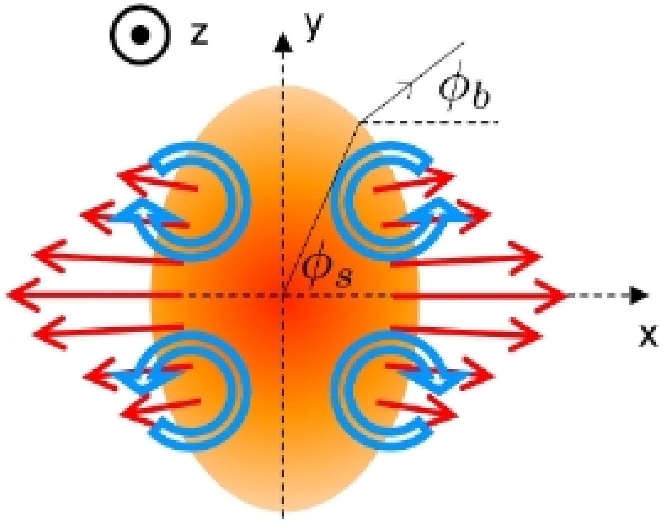
$$e^{-\beta(H_0 - \mathbf{S}\cdot\boldsymbol{\omega} - q\,\mathbf{S}\cdot\mathbf{B})}$$

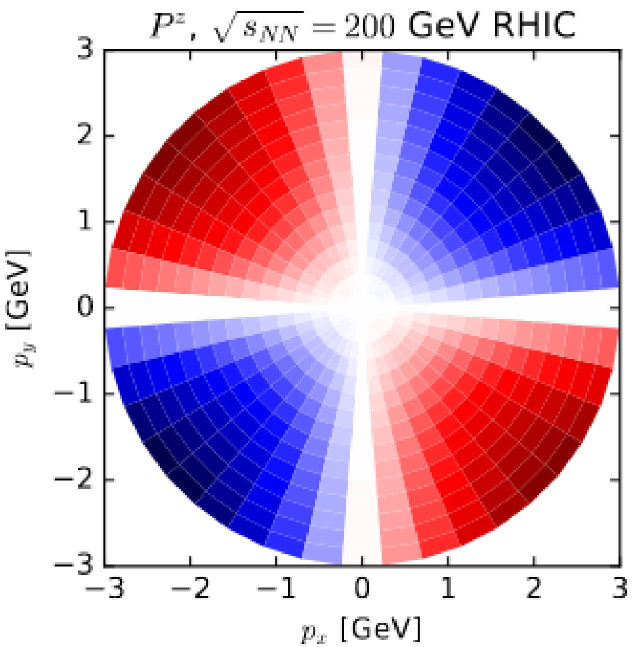
Existence of splitting inconclusive yet

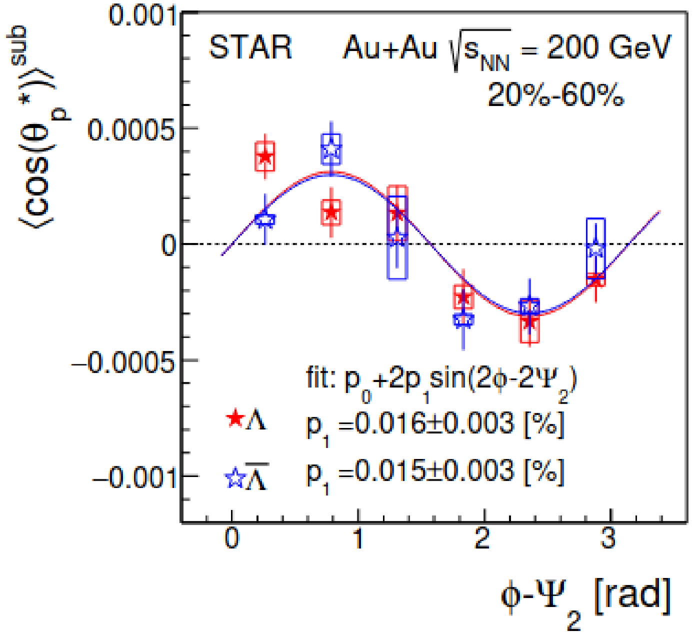
STAR collaboration, PRC 2023

J.-H. Chen's talk

local spin polarization puzzle







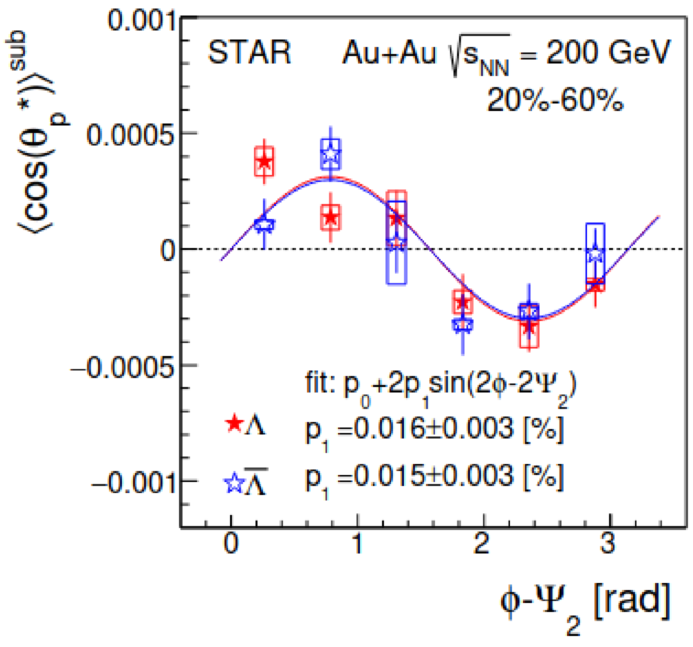
$$\mathcal{P}^i \sim \omega^i$$

Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

STAR collaboration, PRL 2019

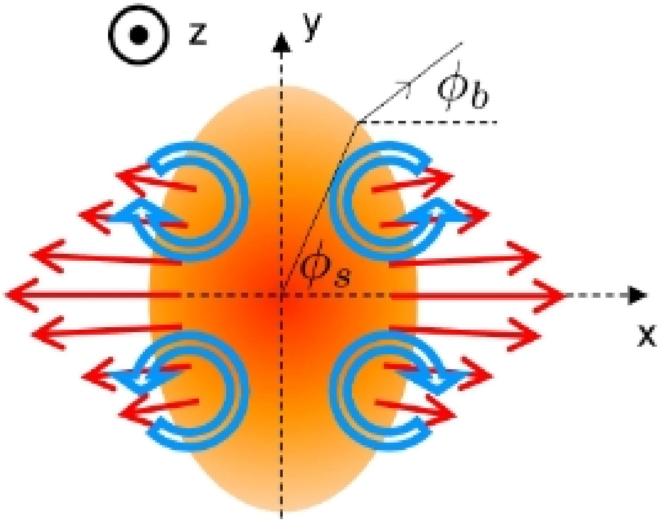
J.-H. Chen's talk

local spin polarization puzzle



STAR collaboration, PRL 2019

J.-H. Chen's talk
X.-G. Huang' talk



Hidaka, Pu, Yang, PRD 2018
Liu, Yin, JHEP 2021
Becattini, et al, PLB 2021

vorticity + shear

$$\mathcal{P}^i \sim \omega^i$$

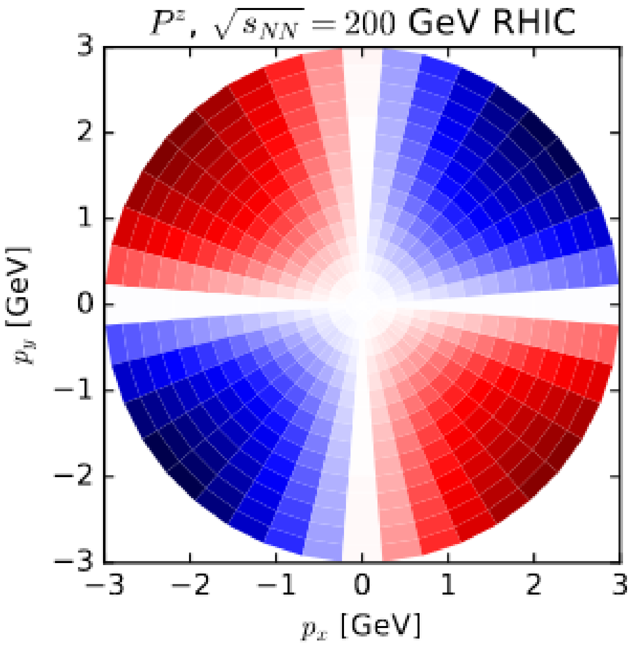
Does spin respond to shear?

$$\mathcal{P}_i \sim \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{lk}$$

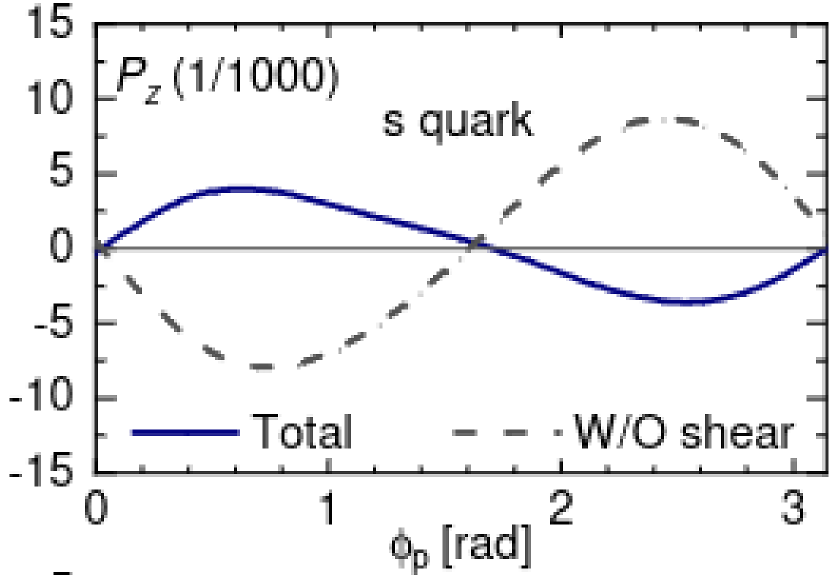
$$\mathcal{P}^z \sim (\langle p_y^2\rangle - \langle p_x^2\rangle)\partial_y u_x$$

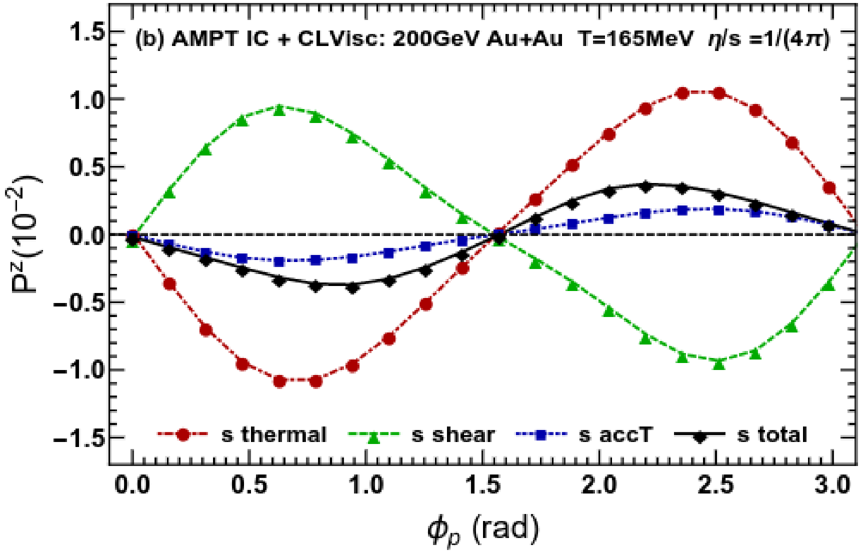
sign correct!

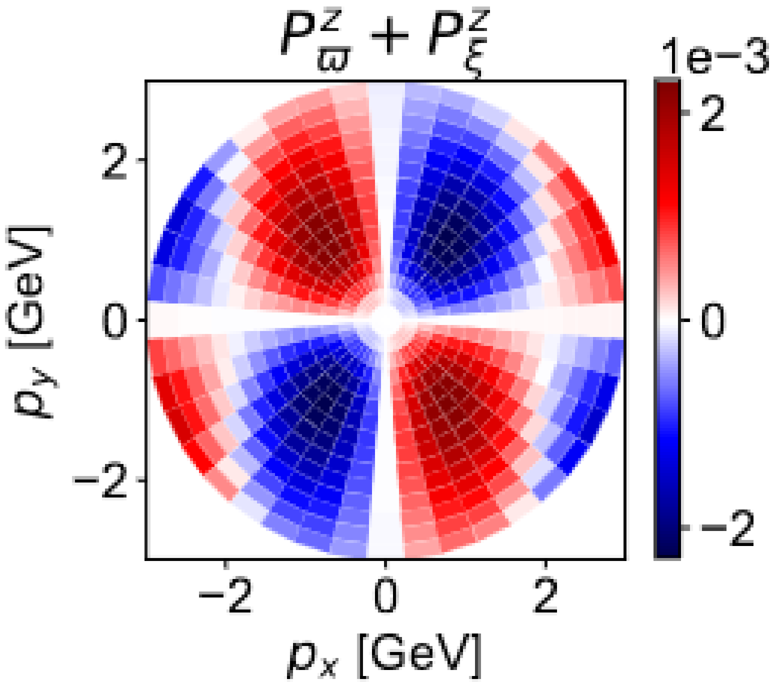
local spin polarization puzzle resolved?



vorticity only







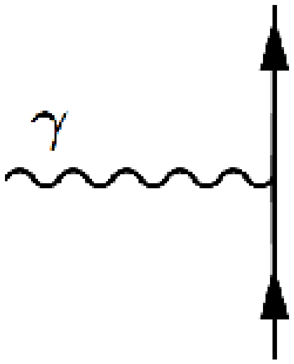
vorticity + shear

Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

Spin responses in heavy ion collisions

$$S^i \sim q\left(B^i + \epsilon^{ijk}\hat{p}_j E^k\right)$$

degenerate couplings to E&B



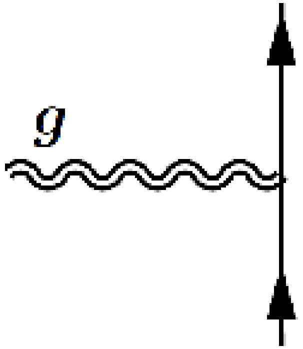
external EM fields

$$E^i \sim -\nabla_i \mu_B$$

Liu, Yin, PRD 2021

$$S^i \sim \left(\beta\omega^i + \epsilon^{ijk}\hat{p}_l\hat{p}_k\beta\sigma_{jl} + \partial_i\beta\right)$$

degenerate couplings
to hydro gradient



hydrodynamic gradient
(mimicked by metric)

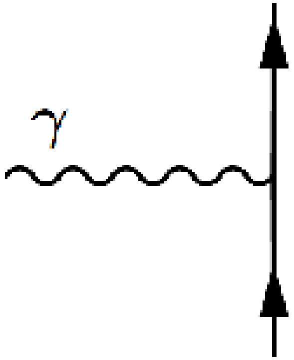
Can radiative correction lift the degeneracy?

Spin polarization from correlation functions

Wigner function

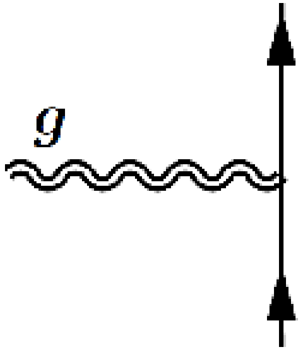
$$S^{<}_{\alpha\beta}(X=\frac{x+y}{2},P)=\int d^4(x-y)e^{iP\cdot(x-y)/\hbar}\left(-\langle\bar{\psi}_\beta(y)\psi_\alpha(x)\rangle\right)$$

- Spin polarization in EM fields



$$\langle S^{<}(X,P)\rangle_{\mathrm{eq},A_\mu}$$

- Spin polarization in hydrodynamic state

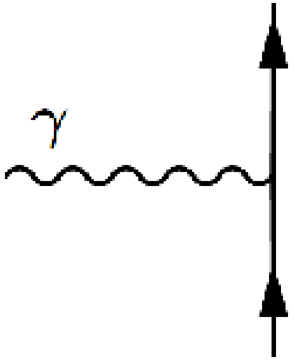


$$\langle S^{<}(X,P)\rangle_{\text{off-eq}}=\langle S^{<}(X,P)\rangle_{\mathrm{eq},h_{\mu\nu}}$$

$A_\mu,\ h_{\mu\nu}$ slow-varying $\partial_X \ll P$

Spin polarization in EM fields

$$\langle S^{<}(X,P)\rangle_{\mathrm{eq},A_\mu}$$



Chiral kinetic theory description

consider
massless quark

$$\gamma_\mu \left(P^\mu + \frac{i}{2}\partial_X^\mu - \frac{i}{2}F^{\mu\nu}\partial_\nu^p \right) S^< = 0$$

Hidaka, Pu, Wang, Yang, PPNP 2022

$$S^< = \frac{1}{4}\left[\left(1+\gamma^5\right)\gamma^\mu R_\mu + \left(1-\gamma^5\right)\gamma^\mu L_\mu \right]$$

Spin polarization $\mathcal{P}^i \sim \delta R^i - \delta L^i = 2\delta R^i$

$$L_\mu = -R_\mu$$

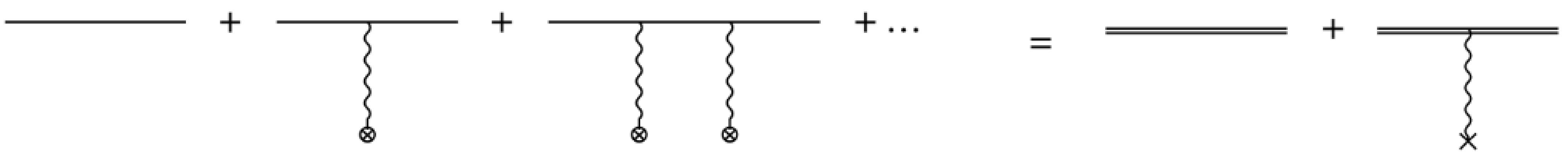
$$\delta R^0 = 2\pi\, \mathbf{p}\cdot\mathbf{B}\, \delta'(P^2) f(p_0)$$

Hidaka, Pu, Yang 2016

$$\delta R^i = 2\pi \left[\epsilon^{ijk} E_j p_k + p_0 B_i \right] \delta'(P^2) f(p_0)$$

degenerate coupling

Equivalent diagrammatic description: EM fields



gauge link

scattering on EM fields

$$\delta R^0 = 2\pi\, \mathbf{p}\cdot\mathbf{B}\,\delta'(P^2)\, f(p_0)$$

$$\delta R^i = 2\pi \left[\epsilon^{ijk} E_j p_k + p_0 B_i\right] \delta'(P^2)\, f(p_0)$$

modified spectral function

equilibrium distribution

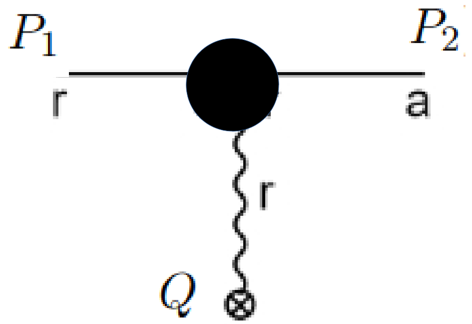
SL, Tian, 2306.14811

Spin polarization = modified spectral function $\times$ equilbrium distibution

KMS relation

In-medium electromagnetic form factors (FF)

$$\Gamma^\mu = F_0 u^\mu + F_1 \hat{p}^\mu + F_2 \frac{i\epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho Q_\sigma}{2(P\cdot u)^2}$$



u^μ QGP frame vector

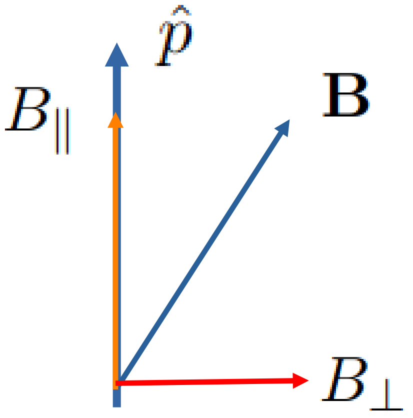
$$S^{<0} = 2\pi F_2 p B_{||} \delta'(P^2) f(p_0)$$

$$S^{<i} = 2\pi \left[F_0 \epsilon^{ijk} E_j p_k + F_1 p_0 B_\perp^i + F_2 B_{||} p^i\right] \delta'(P^2) f(p_0)$$

spin Hall effect

spin-perpendicular magnetic coupling

spin-parallel magnetic coupling

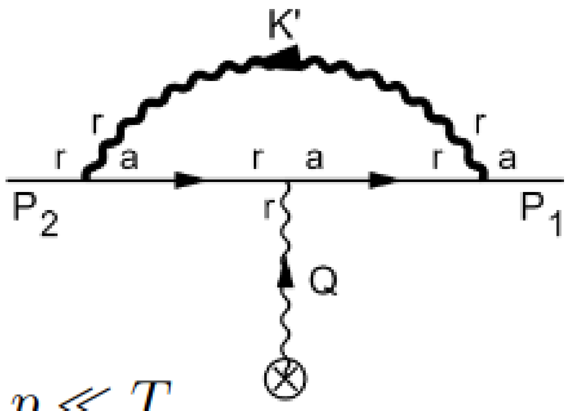


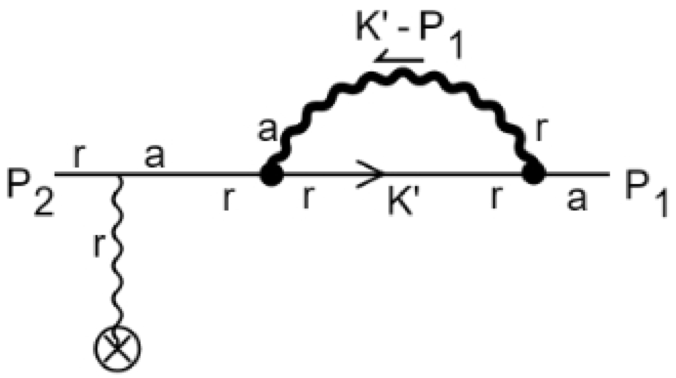
In vacuum $F_0 = F_1 = F_2 = 1$

In medium: lift of degeneracy expected

SL, Tian, 2306.14811

Radiative correction to in-medium electromagnetic FF





$$m_f \sim gT \ll p \ll T$$

$$\delta F_0 = \frac{2m_f^2}{p^2}X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin Hall effect

$$\delta F_1 = \frac{2m_f^2}{p^2}(X-1) + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-perpendicular magnetic coupling

$$\delta F_2 = \frac{2m_f^2}{p^2}X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-parallel magnetic coupling

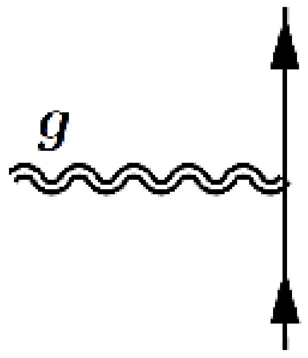
degeneracy partially lifted

$$m_f^2 = \frac{1}{8}C_F g^2 T^2$$ quark thermal mass

SL, Tian, 2306.14811

Spin polarization in hydrodynamic state

$$\langle S^{<}(X,P)\rangle_{\text{off-eq}} = \langle S^{<}(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$



CKT for off-equilibrium state

$$\frac{i}{2}\partial\!\!\!/ S^< + P\!\!\!/ S^< = 0$$

$$S^< = \frac{1}{4}\left[\,(1+\gamma^5)\,\gamma^\mu R_\mu + (1-\gamma^5)\,\gamma^\mu L_\mu\right]$$

Hidaka, Pu, Yang 2016, 2017

$$R^\mu = -2\pi\delta(P^2)\left(P^\mu f_n + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho n_\sigma}{2P\cdot n}\partial_\nu f_n\right)$$

n^μ arbitrary frame vector $\rightarrow u^\mu$

$\partial_i f\left(\frac{P\cdot u(X)}{T(X)}\right)$ modified KMS relation

free theory dispersion + local equilibrium

$f(p_0) \rightarrow f(p_0 - \frac{1}{2}\hat{p}\cdot\omega)$ modified distribution

$\rightarrow$ $S^i \sim \left(\beta\omega^i + \epsilon^{ijk}\hat{p}_l\hat{p}_k\beta\sigma_{jl} + \partial_i\beta\right)$

degenerate couplings to vorticity, shear, T-grad

Classification of contributions to spin polarization

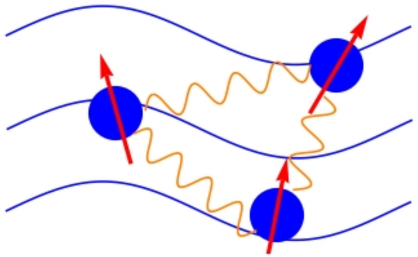
- modified spectral function
- modified distribution function
- modified KMS relation

At tree level, spin polarization in hydrodynamic QGP from
modified KMS + modified distribution

Radiative corrections to spin polarization in hydrodynamic QGP

- modified spectral function
- modified distribution function   in-medium gravitational FF   SL, Tian 2023
- modified KMS relation   Fang, Pu, Yang 2024

Focus on radiative correction to spectral function



Do quasi-particles get modified in hydrodynamic medium?

Off-equilibrium spectral function

$$\rho_{\alpha\beta}(P) = \int d^4x e^{iP\cdot x} \langle \psi_\alpha(x)\bar{\psi}_\beta(0) + \bar{\psi}_\beta(0)\psi_\alpha(x)\rangle$$

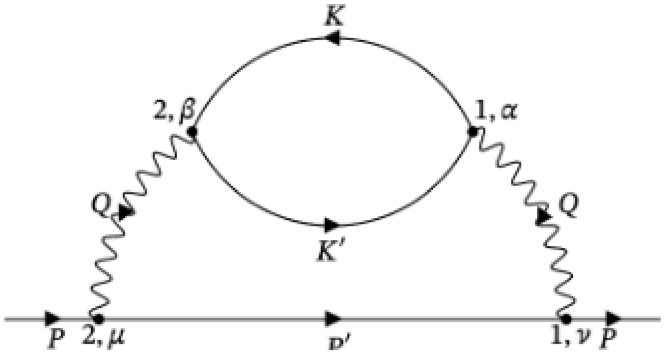
$$S_{ra,\alpha\beta} = \int d^4x e^{iP\cdot x} \theta(x_0) \langle \psi_\alpha(x)\bar{\psi}_\beta(0) + \bar{\psi}_\beta(0)\psi_\alpha(x)\rangle$$

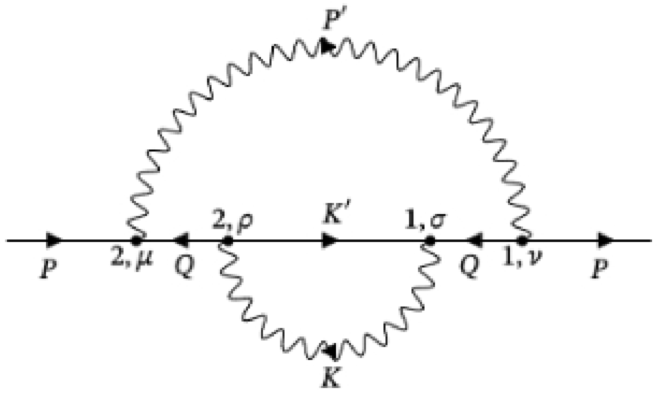
$$\rho(P) = 2\mathrm{Re}[S_{ra}(P)] = 2\mathrm{Im}[S_R]$$

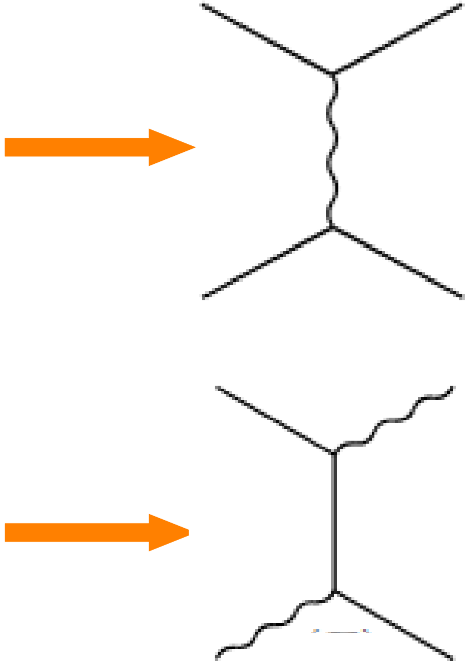
valid for off-equilibrium state invariant under time-reversal

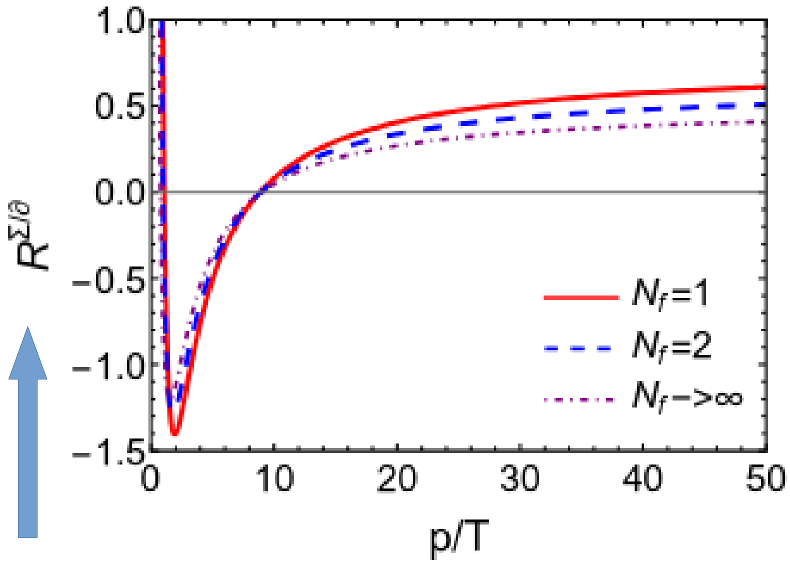
SL, Tian, 2410.22935

Two-loop self-energy correction









collisional/tree level

collisional effect

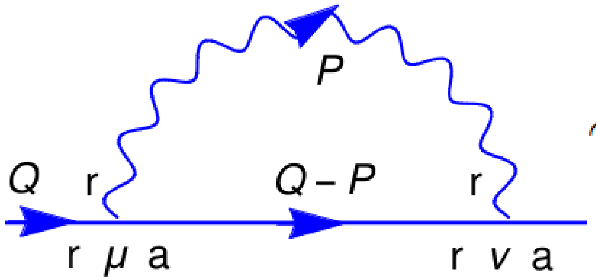
steady state $\delta f \sim O\left(\frac{\partial}{g^4}\right)$

$$g^4 \times \delta f \sim O(\partial)$$

diagramatic resummation Gagnon, Jeon, 2006

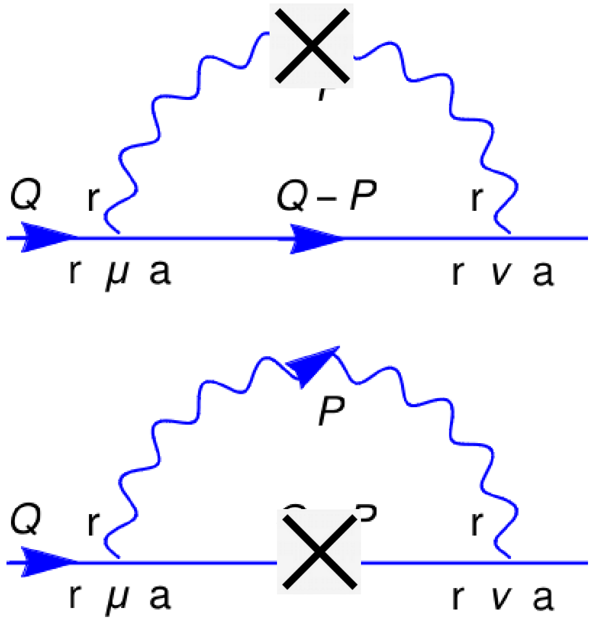
collisional contribution to spin-shear coupling: SL, Wang, 2022, 2024

One-loop self-energy correction



$\sim O(g^2)$

equilibrium correction to spectral function



$\sim O(g^2 \partial)$

off-equilibrium correction to spectral function

both contribute to spin polarization!

Self-energy correction to retarded function

$$\frac{i}{2}\not{\partial} S_R(X,P) + \not{P} S_R(X,P) - \left(\Sigma_R(X,P)S_R(X,P) + \frac{i}{2}\{\Sigma_R(X,P), S_R(X,P)\}_{\mathrm{PB}}\right) = -1$$

$$S_R = S_R^{(0)} + S_R^{(1)} + \cdots,$$

$$\{A,B\}_{\mathrm{PB}} = \partial_P A \cdot \partial_X B - \partial_X A \cdot \partial_P B$$

slow-varying local equilibrium self-energy

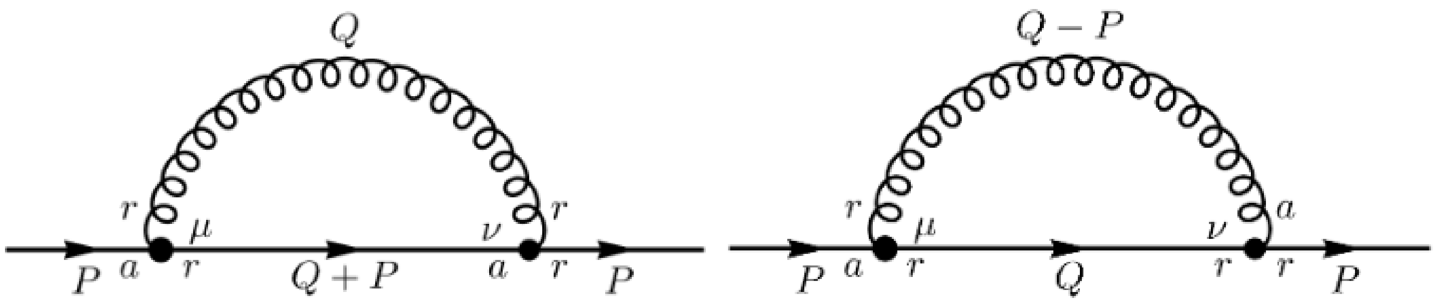
$$S_R^{(0)} = -\frac{1}{\not{P}} - \frac{1}{\not{P}}\Sigma_R\frac{1}{\not{P}}$$

$$S_R^{(1)} = -\frac{1}{\not{P}}\delta\Sigma_R\frac{1}{\not{P}} + \gamma^5\gamma^\beta P^\nu T^{\mu\lambda}\epsilon_{\beta\lambda\mu\nu}\frac{-1}{(P^2)^2} \qquad T_{\mu\lambda} = \partial_{[\mu}\Sigma^R_{\lambda]}$$

equilibrium/off-equilibrium self-energy

SL, Tian, 2410.22935

Equilibrium self-energy



$P \gg T$ $P^2 \ll p/\beta,$

Energetic particle close to mass shell

$$\frac{\Sigma_{ar}}{g^2 C_F} = 2i\not{P}(A+B) + 4ip_0\gamma^0 A.$$

$$\rightarrow \quad p_0 = p(1+8A)$$

$$A = \frac{1}{2(2\pi)^2}\frac{-i\pi}{2p\beta}.$$

modified dispersion:

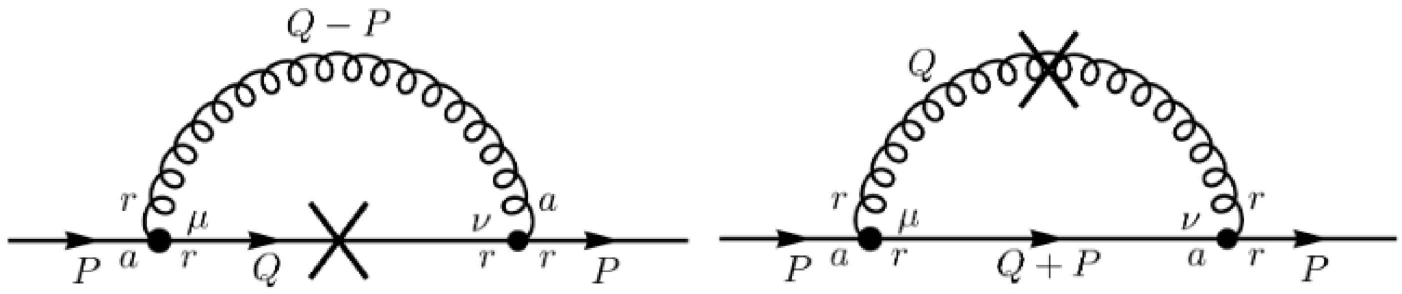
finite damping

equilibrium self-energy linear in T

$$S_R^{(1)} = \gamma^5\gamma_\beta P_\nu \partial_\mu \Sigma_\lambda^R \frac{-1}{(P^2)^2}\epsilon^{\beta\lambda\mu\nu} \sim O(T\partial)$$

SL, Tian, 2410.22935

Off-equilibrium self-energy



off-equilibrium propagators for quark/gluon from CKT

Hidaka, Pu, Yang 2017

Huang et al 2020

Hattori, Hidaka, Yamamoto, Yang 2020

$$\frac{\delta\Sigma_{ar}}{g^2 C_F} = \gamma^5\gamma^\mu \mathcal{A}_\mu,$$

$$P \gg T \quad P^2 \ll p/\beta,$$

Energetic particle close to mass shell

$$\mathcal{A}^0 = i\omega^i p_i \beta(-4\delta A - 2\delta B),$$

$$\mathcal{A}^k = i\omega_\parallel^k \beta p(-4\delta A - 2\delta B) + \omega_\perp^k \beta p(-4\delta A - \delta B) + \epsilon^{ijk}\hat{p}_i\hat{p}_l\sigma_{jl}\beta p(-\delta B) + \epsilon^{ijk}p_i\partial_j\beta(-\delta C)$$

$$\delta C = \frac{1}{4(2\pi)^2}\left(\frac{-4C_a + 2C_b + i\pi C_a - 2C_a \ln\frac{p\beta(-1+a)}{2}}{2p\beta}\right) \qquad a = p_0/p + i\eta.$$

off-equilibrium self-energy T-independent

$$\delta S_R^{(0)} = -\frac{1}{\not{P}}\delta\Sigma_R\frac{1}{\not{P}} \sim O(p\partial)$$

dominate equilibrium contribution

Polarized quasi-particle

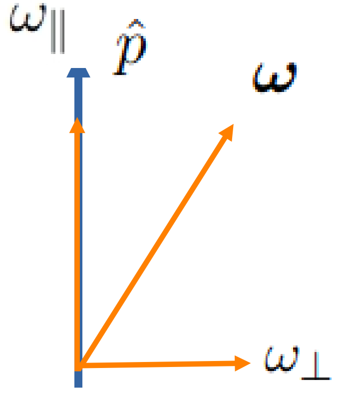
loop

$$\int dp_0 \delta S^<(P) = \int dp_0 \delta\rho(P) f(p_0)$$

$$= \frac{g^2 C_F}{2(2\pi)^2}\frac{\pi}{2p}\gamma^5\gamma_i\left[0.95\omega_{\parallel}^i + 1.48\omega_{\perp}^i - 0.52\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl} - 0.02\epsilon^{ijk}\hat{p}_j\frac{\partial_k\beta}{\beta}\right] f(p),$$

tree

$$\int dp_0 S^<_{(0)}(P) \simeq \gamma^5\gamma_i\frac{2\pi\beta}{2}\left(\omega^i + \epsilon^{ijk}\hat{p}_k\frac{\partial_j\beta}{\beta} + \epsilon^{ijk}\hat{p}_k\hat{p}_l\sigma_{lj}\right) f(p),$$



degeneracy in couplings to
vorticity, shear, T-grad lifted

Polarized quasi-particle

SL, Tian, 2410.22935

Conclusion

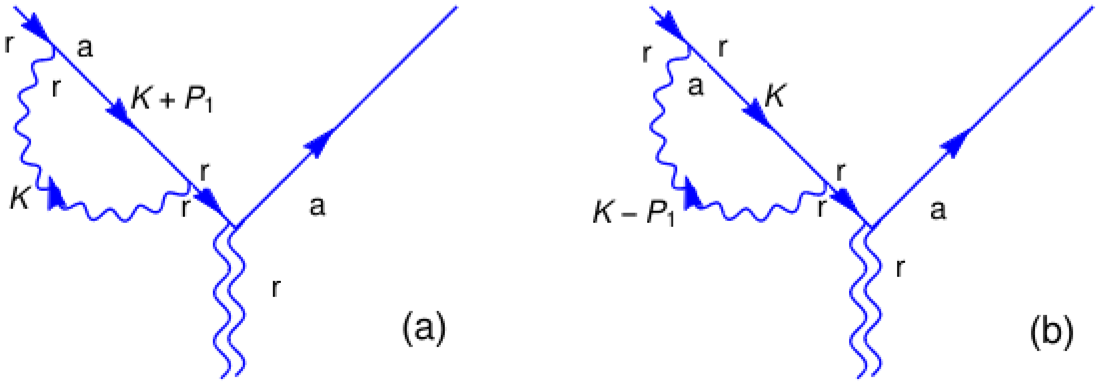
- Radiative correction to spin polarization in EM fields with electromagnetic FFs.
- Classificaiton of controbution to polarization: modified spectral; modified distribution; modified KMS
- Radiative correction to spectral function in hydrodynamic QGP: spin polarized quasi-particles

Outlook

- Radiative correction to distribution function with gravitational FFs
- Radiative correction to modified KMS

Thank you!

In-medium gravitational form factors



$$m_f \sim gT \ll p \ll T$$

loop correction to spin-vorticity coupling

$$\delta\Gamma^{\mu\nu} = \delta Z_+ \gamma^{\{\mu} P^{\nu\}}.$$

$$\delta Z_+ = \frac{m_f^2}{2p^2}\left(1 - \ln\frac{2p^2}{m_f^2}\right)$$

tree level spin-vorticity coupling

$$\Gamma_{\mu\nu} = \gamma^{\{\mu} P^{\nu\}}$$