2024, November 9
PacificSpin2024, Hefei

This talk consists of two parts:

1. Evolution of the PacificSpin Symposium

2. Evolution of Studies of the Spin Structure of the Proton

Toshi-Aki Shibata
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1. Evolution of the PacificSpin Symposium

Circum-Pan-Pacific Symposium on High Energy Spin Physics

1st: 1996 in Kobe in Japan by Prof. Morii, 28 years ago

EMC spin experiment, 1988, 1989

RHIC experiments in preparation

2nd: 1999

1st	1996	Kobe University,	Toshiyuki Morii
2nd	1999	RIKEN,	Koichi Yazaki
3rd	2001	Peking University,	Bo-Qiang Ma
4th	2003	University of Washington,	Xiang-Dong Ji
5th	2005	Tokyo Tech,	Toshi-Aki Shibata
6th	2007	Vancouver,	Andy Miller
7th	2009	Yamagata University,	Takahiro Iwata
8th	2011	Cairns,	Anthony W. Thomas
9th	2013	Shandong University,	Zuo-Tang Liang
10th	2015	Academia Sinica,	Hai-Yang Cheng, Wen-Chen Chang
11th	2019	Miyazaki	Takahiro Iwata, Tatsuro Matsuda, Toshi-Aki Shibata
12th	2024	Hefei	Xiao-Rui Lyu, Qun Wang, Yang Li, Xinyang Wang

- In this symposium, we consider that discussions among the participants are most important.

Talks (presentations) are inputs for these discusssions.

- We also think it important to encourage young physicists to participate.

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Evolution of Studies of the Spin Structure of the Proton

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1. Spinors —— Spin 1/2 particles

Schrödinger equation, 1926

$$i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{(p - e\,A)^2}{2m} + e\,\phi \right] \psi. \qquad \text{scalar}$$

Pauli equation, 1927, ‘100 years of spin physics’

$$i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{1}{2m}(\,\sigma \cdot (p - e\,A))^2 + e\,\phi \right] \psi, \qquad \psi = \begin{cases} \psi(r, t\,;\uparrow) \\ \psi(r, t\,;\downarrow) \end{cases} \tag{1}$$

Coefficients $\sigma' s$ were introduced. spinors

$$\hat{H} = \frac{1}{2m}(\,\sigma \cdot (p - e\,A))^2 + e\,\phi.$$

$$i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{1}{2m}(\sigma \cdot (p - e\,A))^2 + e\,\phi \right] \psi, \qquad \psi = \left\{ \begin{array}{l} \psi(r,t\,;\uparrow) \\ \psi(r,t\,;\downarrow) \end{array} \right. \tag{1}$$

$$i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{(p - e\,A)^2}{2m} - \frac{e\hbar}{2m} \sigma \cdot B + e\,\phi \right]. \tag{2}$$

From Eq.(1) to Eq.(2),
the coefficients $\sigma's$ have to satisfy the following conditions.

$$\sigma_i^2 = 1 \ \ (i = 1,2,3), \qquad \sigma_i\,\sigma_j = i\,\varepsilon_{ijk}\,\sigma_k \quad (i \neq j)$$

$$(\sigma \cdot a)(\sigma \cdot b) = (a \cdot b) + i\sigma\,(a \times b),$$
$$a = b = p - e\,A$$

For example,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Dirac equation, 1928

$$(i\hbar\,\gamma^\mu\,\partial_\mu - m)\,\psi = 0, \qquad \text{free particle}$$

With α_i and β matrices,

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\,\psi, \qquad\qquad \hat{H} = \sum_{i=1}^{3}\alpha_i\,\hat{p}_i + m\beta$$

This equation has to be consistent with Klein-Gordon equation which is based on $E^2 = p^2 + m^2$:

The conditions on the coefficients $\alpha_i,\ \beta$ are then

$$\alpha_i^2 = 1, \quad \beta^2 = 1, \tag{1}$$
$$\alpha_i\,\alpha_j + \alpha_j\,\alpha_i = 0 \quad (i \neq j), \qquad \alpha_i\,\beta + \beta\,\alpha_i = 0. \tag{2}$$

1) Eq.2: α_i and β are not just numbers, but matrices.
2) Eq.1: The eigenvalues of the matrices α_i, β are $+1$ and -1.
3) Eqs.1 & 2: α_i and β are traceless. $\mathrm{tr}\,(\alpha_i)$=$\mathrm{tr}\,(\beta) = 0$:
 the sum of the diagonal elements is 0.
4) Four independent matrices $\alpha_i,\ \beta$ are needed.

$\longrightarrow$ The size of matrices is 4x4, 6x6, or ....

The 2x2 matrix such as Pauli matrix has only three independent matrices, and does not satisfy the condition.

We adopt 4x4 matrices for α_i and β. The Hamiltonian becomes a 4x4 matrix. An example is

$$\hat{H} = \alpha \cdot \hat{p} + m\beta = \begin{bmatrix} m1 & \sigma \cdot \hat{p} \\ \sigma \cdot \hat{p} & -m1 \end{bmatrix}$$

Pauli-Dirac representation.
σ is the Pauli matrix.

As a result, the wave function has four components.

The plane wave is expressed as

$$\psi = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} e^{i(k \cdot x - \omega t)}, \qquad u_A = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \; u_B = \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$

spinors and space-time wave function

After operating $\hat{p}_i = \frac{\hbar}{i}\frac{\partial}{\partial x_i}$ and $\hat{E} = i\hbar\frac{\partial}{\partial t}$,

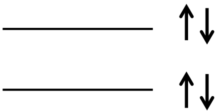
$$E \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} m\,1 & \sigma \cdot p \\ \sigma \cdot p & -m\,1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}.$$

The Hamiltonian can have four eigenvalues.

The eigenvalue-equation gives

$$E = E_+ = \sqrt{p^2 + m^2}, \qquad E = E_- = -\sqrt{p^2 + m^2}.$$

Two quantum states are degenerate at $E = E_+$, and also at $E = E_-$.



When $p = 0$, then $E = m$ and $E = -m$. When $E = m$, $u_A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $u_A = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for example.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

When $E = -m$, $u_B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $u_B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for example.

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

This is a new degree of quantum states: spin.

With the electromagnetic potentials (ϕ, A_x, A_y, A_z),
the operators $\hat{p} \to \hat{p} - qA$, $\hat{E} \to \hat{E} - q\phi$.

In the low energy limit, the Dirac equation is reduced to

$$T\, u_A = \left\{ \frac{(p - qA)^2}{2m} + q\phi - \frac{q\hbar}{2m}\sigma \cdot B \right\} u_A$$

u_A : upper two components,
T : kinetic energy, $E = m + T$.

$$\mu = \frac{q\,\hbar}{2m}\sigma = g\,\frac{q}{2m} \cdot \left(\frac{\hbar}{2}\sigma\right) = g\frac{q}{2m}\,s, \qquad s = \frac{\hbar}{2}\,\sigma, \qquad g = 2.$$

The $-\mu \cdot B$ term causes the energy split.

The Dirac equation describes the spinors — spin $1/2$ particles.

$$J = L + \frac{1}{2}\Sigma, \qquad \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}.$$

$$[\Sigma, \hat{H}] \neq 0, \quad [L, \hat{H}] \neq 0, \qquad [J, \hat{H}] = 0.$$

2. Proton spin, spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L^{q,\bar{q}} + \Delta G + L^{G}$$

$$\Delta\Sigma = \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}$$

$$\Delta u = \int_0^1 dx\,(u^{\uparrow}(x) - u^{\downarrow}(x)), \ldots$$

Decomposition of the proton spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L^{q,\bar{q}} + (\mathrm{Gluon})$$

the Jaffe-Manohar decomposition,
R. L. Jaffe and A. Manohar, Nucl. Phys. B 337 (1990) 509.
the Ji decomposition, X. Ji, Phys. Rev. Lett. 78 (1997) 610.
2008∼ X. S. Chen et al., M. Wakamatsu,...

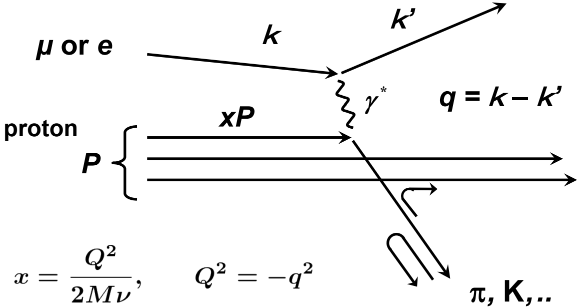
topics at PacificSpin2013 at Ji'nan.

- Gauge invariant decomposition into 4 terms
- The terms can be evaluated from the values of experimental observables.

$$
\begin{aligned}
&\int \psi^\dagger \, \frac{1}{2}\Delta\Sigma \, \psi \, d^3x \\
+&\int \psi^\dagger \; x \times p \; \psi \, d^3x \quad \longrightarrow \int \psi^\dagger x \times (p - g\,A) \, \psi \, d^3x \\
+&\ \text{Gluon terms}
\end{aligned}
$$

Experiments:

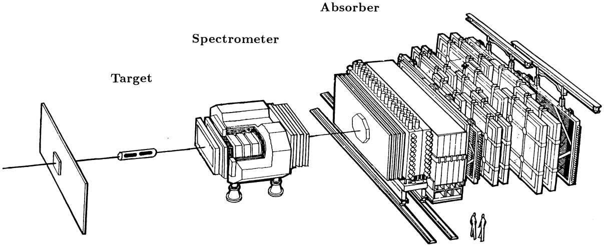
Deep-inelastic Scattering



$$x = \frac{Q^2}{2M\nu}, \qquad Q^2 = -q^2$$

EMC 1988, 1989,
J. Ashman et al., Nucl. Phys. B328 (1989) 1.
$\frac{1}{2}\Delta\Sigma$: contributions of spin of quarks and anti-quarks to the proton spin, $(12 \pm 9 \pm 14)\%$

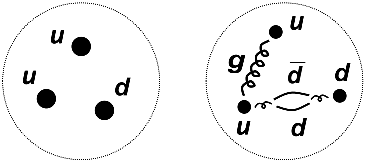
EMC spectrometer

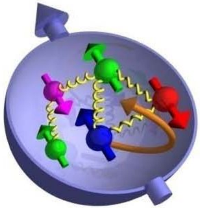


SLAC, SMC, HERMES, COMPASS etc.

Physics of Spin

$\longrightarrow$ Physics of Spin and Orbital Angular Momentum





Gluon spin — PHENIX, STAR etc.

Contributions of spins of quarks and anti-quarks to the proton spin: 25-35%

3. Transverse-momentum-dependent parton distributions

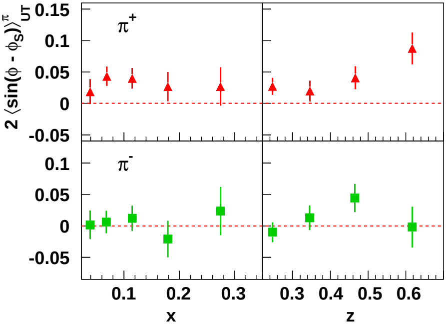
Sivers function

$$f_{q/p^{\uparrow}}(x, k_T) = f_1^q(x, k_T^2) - f_{1T}^{\perp q}(x, k_T^2)\, \frac{(\hat{P} \times k_T) \cdot S}{M}$$

A. Bacchetta et al., Phys. Rev. D70. 117504 (2004)

Single spin asymmetry in hadron reactions
$p + p \to \pi^{\pm,0} + X$, The hard scale is determined by p_T.

DIS The hard scale is determined by Q^2.
p_T can be low. Sivers asymmetry:



A. Airapetian et al., HERMES, Phys. Rev. Lett. 94 012002 (2005).
About 20 years ago.

HERMES, COMPASS, JLab, RHIC, SpinQuest etc.

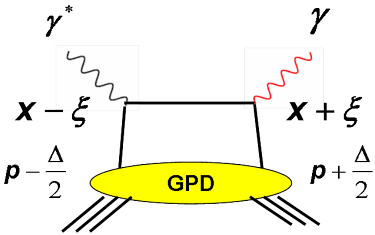
4. Generalized parton distributions

Deeply virtual Compton scattering: $e + N \to e' + \gamma + N$,
Hard exclusive meson production: $e + N \to e' + \text{meson} + N$.

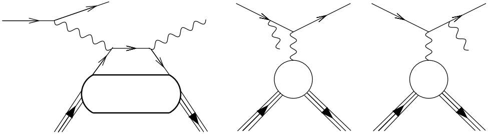
$H(x, \xi, t),\ E(x, \xi, t),\ \tilde{H}(x, \xi, t),\ \tilde{E}(x, \xi, t)$

$$J_{q,G} = \lim_{t \to 0} \int dx\, x \cdot \left[H^{q,G}(x, \xi, t) + E^{q,G}(x, \xi, t) \right]$$

X.D. Ji, Phys. Rev. Lett. 78 610 (1997)

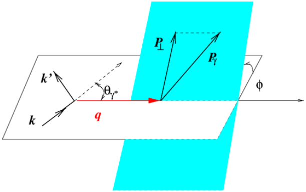


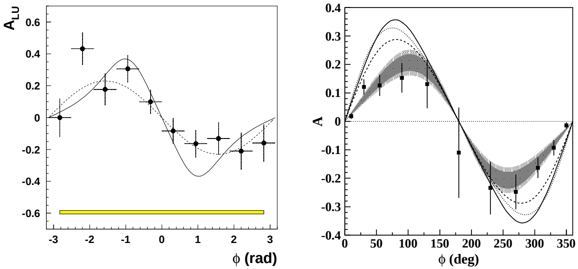
Interference between DVCS and Bethe-Heitler process



(a) (b)

$$A_{LU}(\phi) = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \mathrm{Im}\ \ (F \cdot H) \sin\phi$$





beam-spin asymmetry in DVCS.

A. Airapetian et al., HERMES, Phys. Rev. Lett. 87, 182001 (2001)

S. Stepanyan et al., CLAS, Phys. Rev. Lett. 87, 182002 (2001)

JLab, HERMES, COMPASS etc.

5. Summary

· Spin $1/2$ particles are described as spinors.
  Pauli equation and Dirac equation

· Proton spin – spin sum rule,
  The EMC result motivated further experiments of DIS and hadron reactions.
  Contributions of spins of quarks and anti-quarks to the proton spin: 25-35% from DIS.
  Gluon spin contribution measured by hadron reactions etc.

· Transverse-momentum dependent parton distributions
  Sivers function etc. since 20 years.

· Generalized parton distributions
  Exclusive production of a photon or a meson

· EIC will provide further oppotunities to study the spin structure of the proton.