# Experimental study of fragmentation functions at BESIII and STCF

Yateng Zhang (张亚腾)
Zheng Zhou University (郑州大学)

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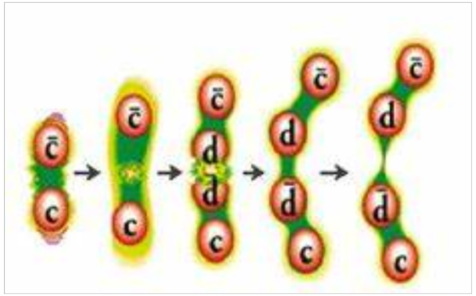
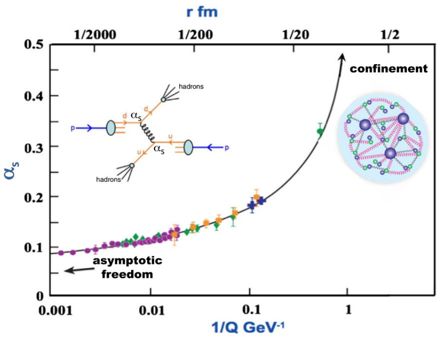
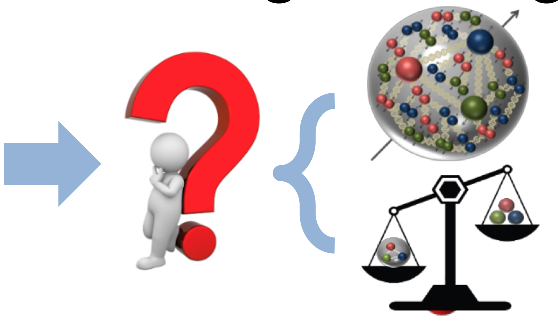
# Several open questions about QCD

- **Confinement**, no existing isolated quarks or gluons



- **Nucleon structure**, what is the origin of nucleon spin and mass in terms of quarks and gluons degree of freedom

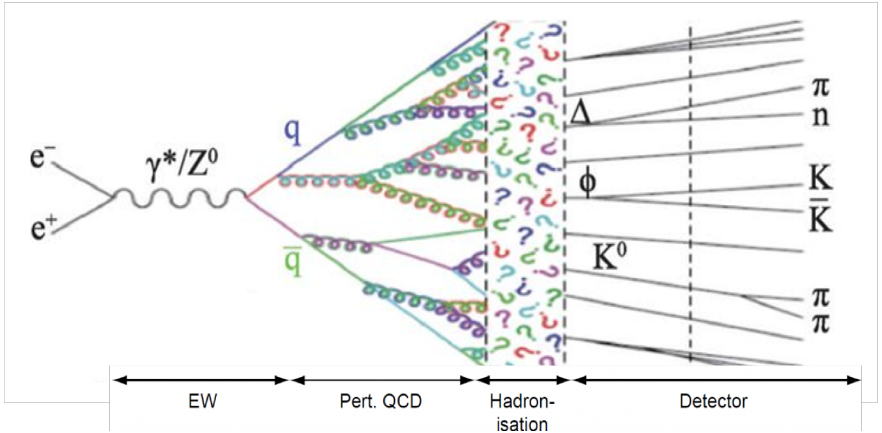


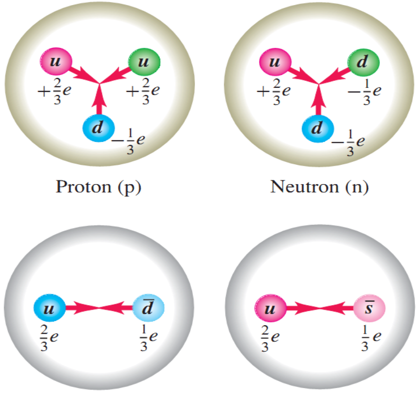



Spin:
How does nucleon spin emerge

Mass:
Higgs mechanism gives only ~few%

# Fragmentation Functions (FFs)



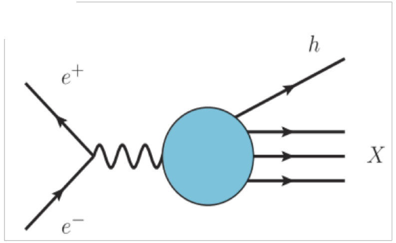




Proton (p) Neutron (n)

Positive pion ($\pi^+$) Positive kaon ($K^+$)

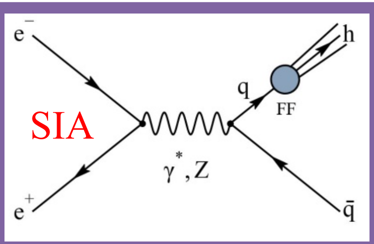
$D_q^h(z)$

- $D_q^h(z)$: describe the fragmentation of an quark into an hadron, where the hadron carries a fraction $z = 2E_h/\sqrt{s}$ of parton's momentum



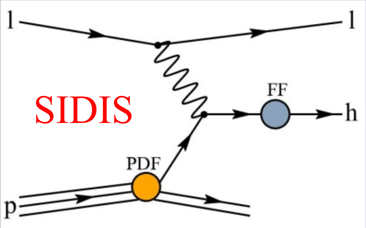

$$E_p \frac{d\sigma_{e^+e^-\to hX}}{d^3p}(s,p) = \sum_f \int \frac{dz}{z^2} D_{h/f}(z,\mu^2) \times E_k \frac{d\hat{\sigma}_{e^+e^-\to \hat{k}X}}{d^3\hat{k}}(s,\hat{k},\mu^2) + \mathcal{O}\left[\frac{\Lambda_{\rm QCD}^2}{Q^2}\right]$$

# Access FFs with QCD factorization



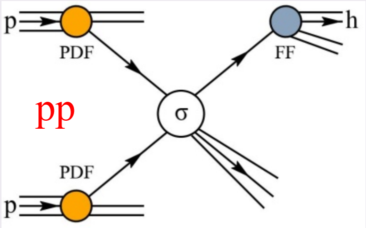

$e^+e^-$: $\sigma = \sum_q \sigma(e^+e^- \to q\bar{q}) \otimes FF$

- No PDFs necessary
- Calculations know at NNLO
- Flavor structure not directly accessible




SIDIS: $\sigma = \sum_q PDF \otimes \sigma(eq \to e'q') \otimes FF$

- Depend on unpolarized PDFs
- Flavor structure directly accessible
- FFs and PDFs

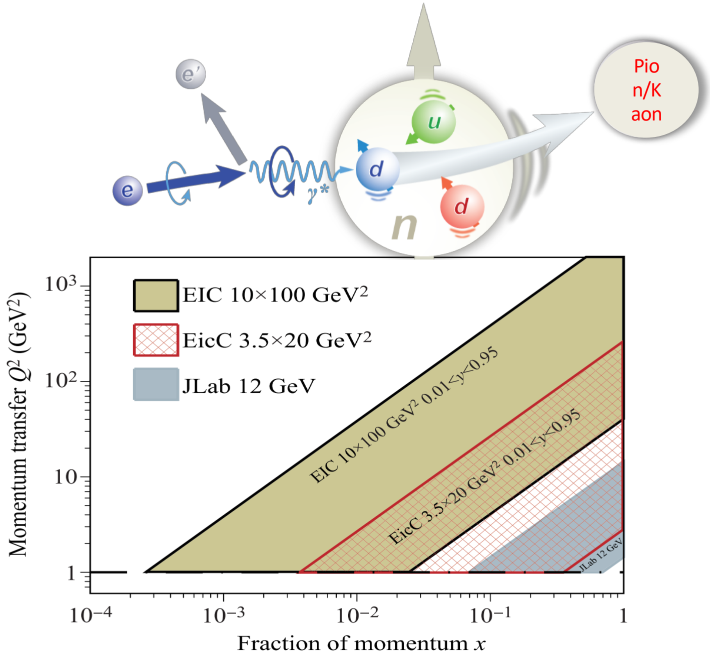



pp: $\sigma = \sum_q PDF \otimes PDF \otimes \sigma(q_1 q_1 \to q_1' q_2') \otimes FF$

- Depend on unpolarized PDFs
- Leading access to gluon FF
- Parton momenta not directly known

- SIA @ $e^+e^-$: the cleanest input for FFs fitting

# FFS VS Nucleon structure study




$$\sigma^{\ell N \to \ell h X} = \hat{\sigma} \otimes PDF \otimes FF$$

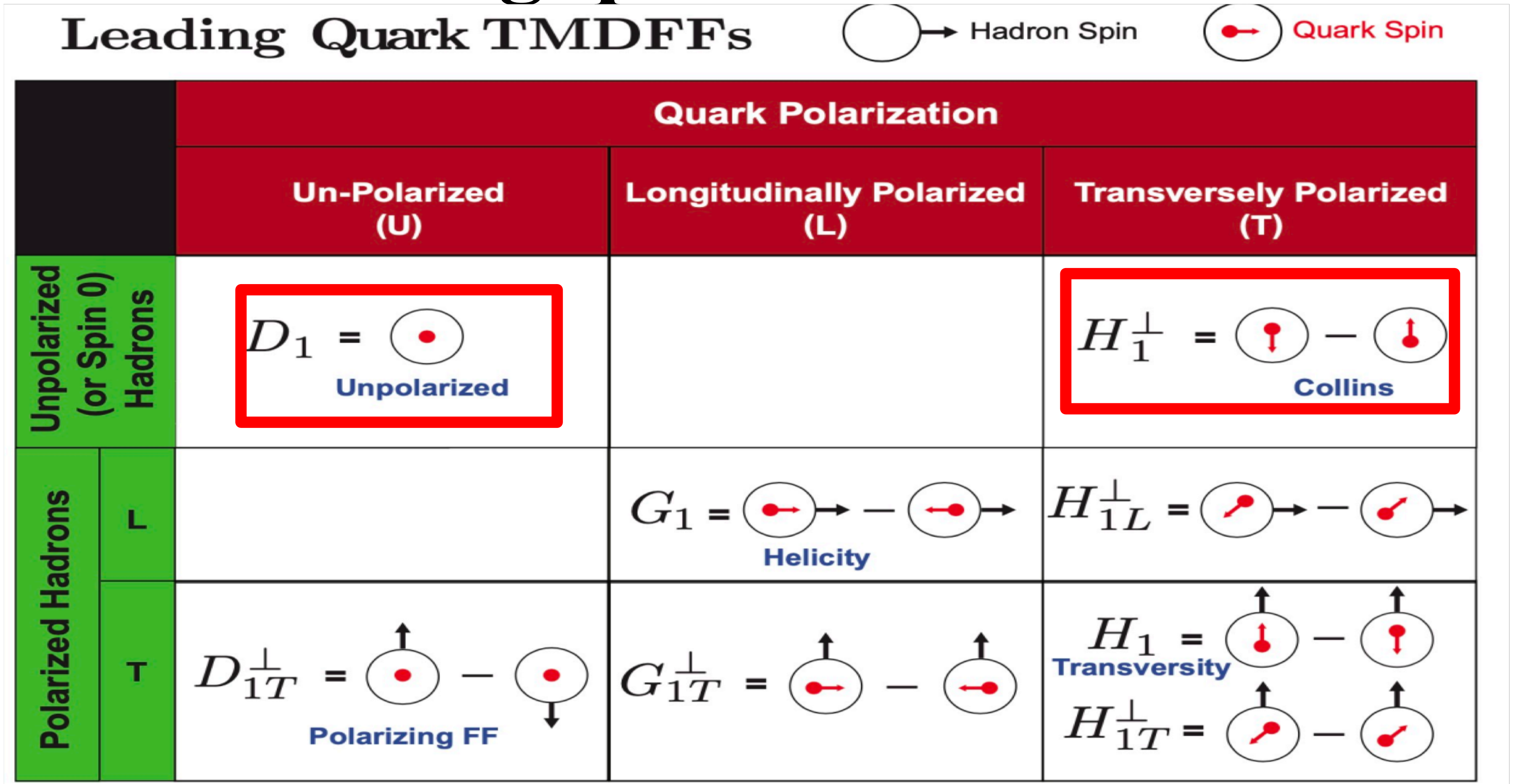
**Belle, BaBar, TASSO…**

**Almost no precision data on FF**

**QCD evolution: from low to high**

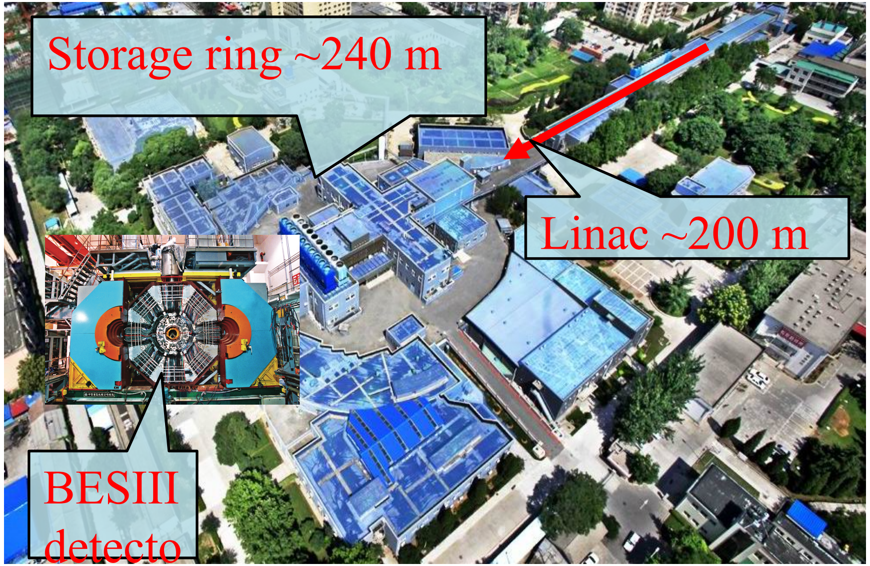
- To accurately extract Parton Distribution Functions (PDFs), more precise FFs are required.

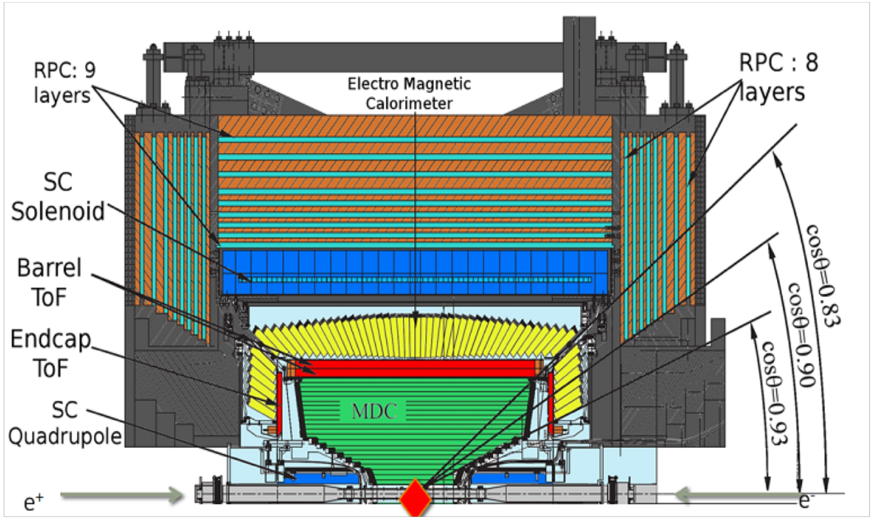
# Leading quark TMDFFS




**Two types of fragmentation functions can be studied at an unpolarized $e^+e^-$ collider: $D$ and $H_1^\perp$**

# BEPCII/BESIII




Double-ring, symmetry, multi-bunch $e^+ e^-$ collider

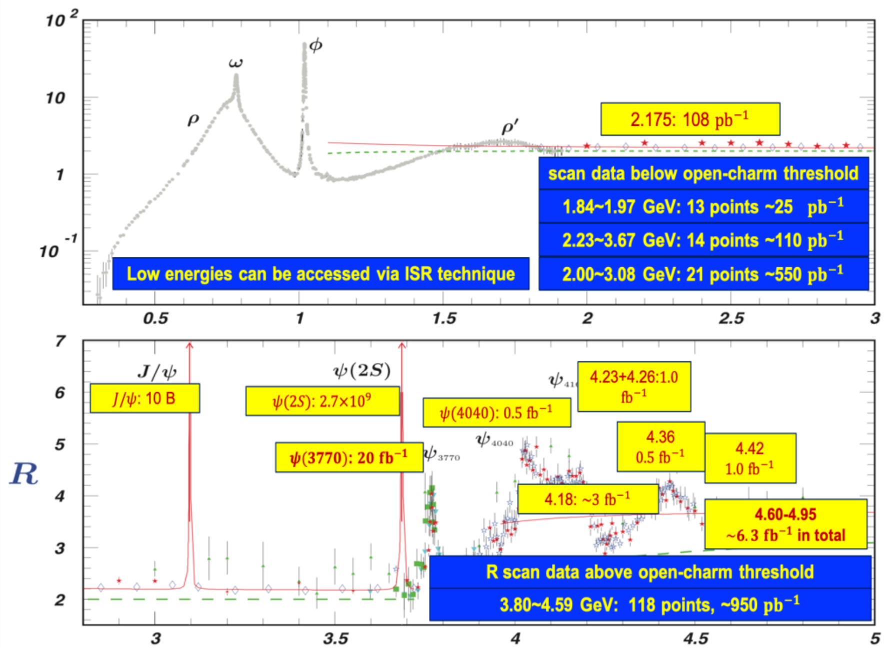
$E_{cm}$= 1.84 to 4.95 GeV

Energy spread: $\Delta E \approx 5\times10^{-4}$

Peak luminosity in continuously operation @$E_{cm}$= 3.77 GeV: $1.1\times10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

| Main Drift Chamber | Time Of Flight | Electromagnetic Calorimeter | Muon Counter |
|---|---|---|---|
| Small cell, 43 layer | Plastic scintillator | CsI(Tl): L=28 cm | RPC |
| $\sigma_{xy}$=130 $\mu m$ | $\sigma_T$(barrel): 65 ps | Barrel $\sigma_E$=2.5% | Barrel: 9 layers |
| dE/dx~6% | $\sigma_T$(endcap): 110 ps | Endcap $\sigma_E$=5.0% | Endcap: 8 layers |
| $\sigma_p/p$= 0.5% at 1 GeV | (update to 60 ps with MRPC) | | $\sigma_{spatial}$: 1.48 cm |

# BESIII data samples




Data sets collected so far include

- $10 \times 10^9$ $J/\psi$ events
- $2.7 \times 10^9$ $\psi(3686)$ events
- 20 $fb^{-1}$ $\psi(3770)$
- Scan data [1.84, 3.08] GeV; [3.735, 4.600] GeV, 143 energy points, ~ 2.0 $fb^{-1}$
- Large data sets for XYZ study ~22 $fb^{-1}$
- Entangled hadron pair-productions near thresholds

# Unpolarized FFs measurements at BESIII

Experimental observable at $e^+e^-$ colliders:

$$\frac{1}{\sigma_{tot}(e^+e^- \to hadrons)} \frac{d\sigma(e^+ e^- \to h + X)}{d P_h}$$

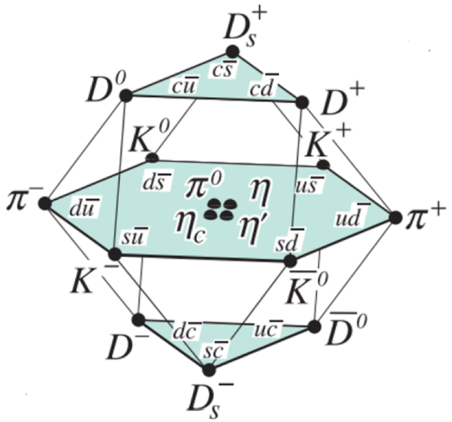
*h* is a particular type of hadron such as $\pi^0$, $\pi^{+/-}$, $K^{+/-}$ …

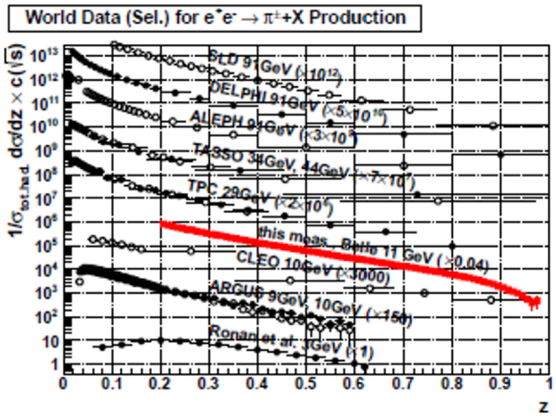
- At Leading order $\sim \sum_q e_q^2 D_1^{h/q}(z)$

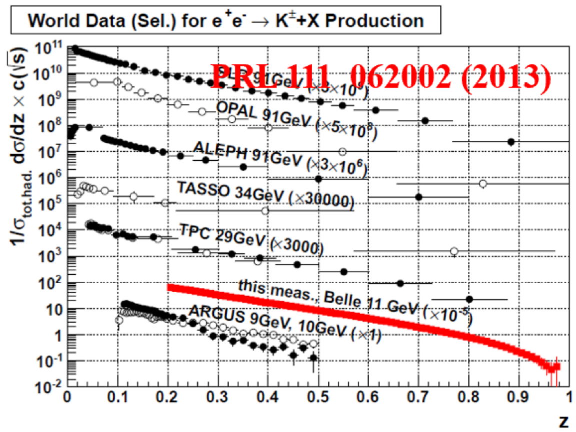
Unpolarized fragmentation function $(D)$

Fractional energy of hadron $z = 2E_h/\sqrt{s}$

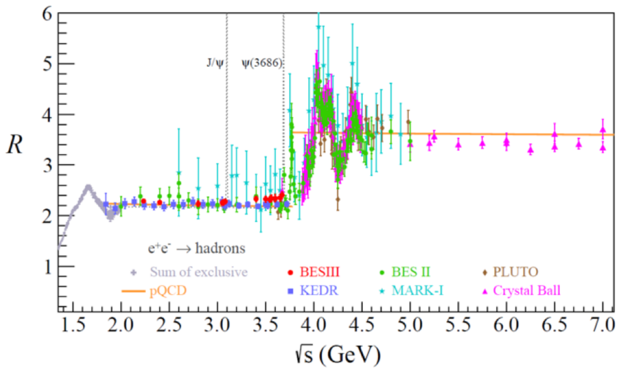
# World π & K data on $e^+e^-$



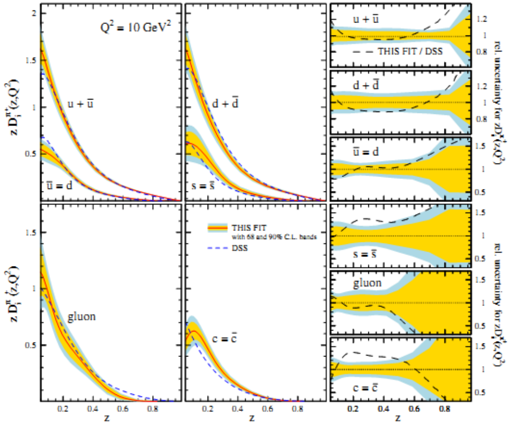



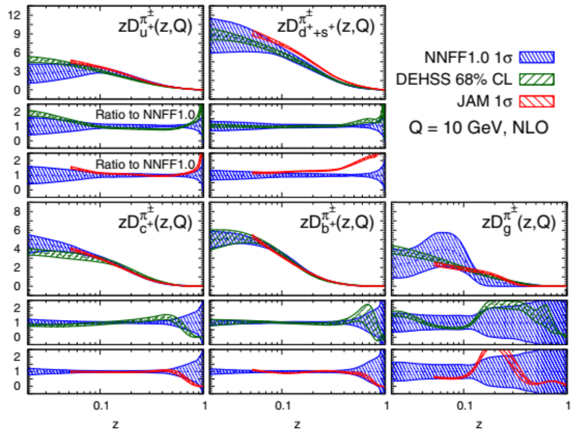

- Precision data includes charged π, K
- Data sets at $\sqrt{s}$ < 10 GeV $e^+e^-$ collision ?
  - high z data sets ?
- R scan data @ BESIII: ~10 $pb^{-1}$ @ each $\sqrt{s}$



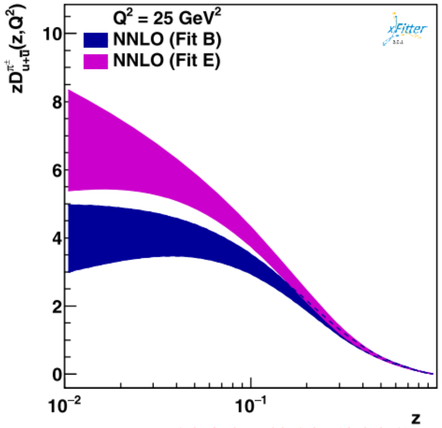
# Pion FF: Best known FF




PRD 91 014015 (2015)

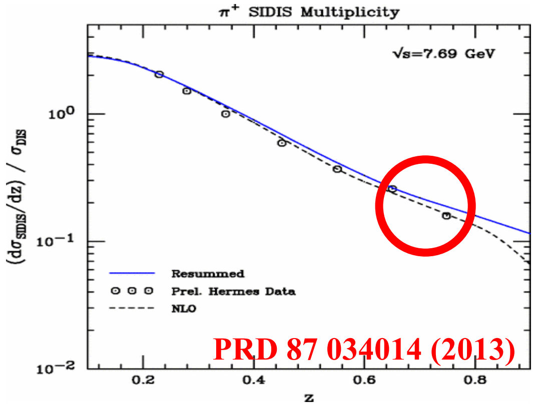



Eur.Phys.J.C77 516 (2017)



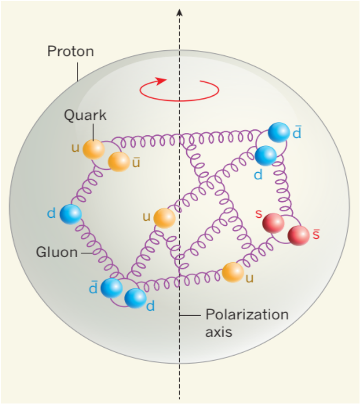

PRD 104 056019 (2021)

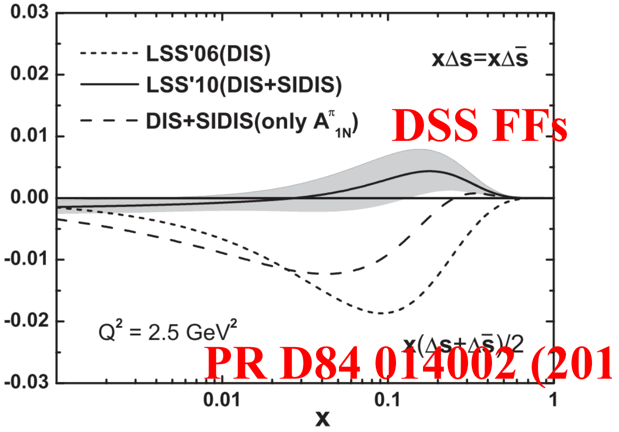
- For $z \geq 0.8$, uncertainty rapidly increase because of the lack of experimental data
- Xfitter: data at $\sqrt{s} > 10$ GeV $e^+e^-$
  - Low $\sqrt{s}$ $e^+e^-$ data ?
- Large $z$ re-summation
  - High $z$ data ?

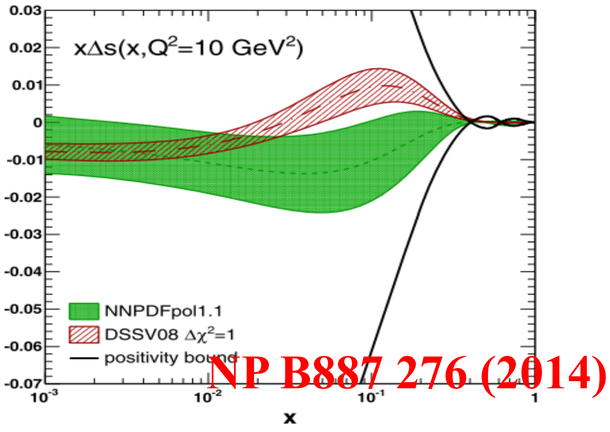



PRD 87 034014 (2013)

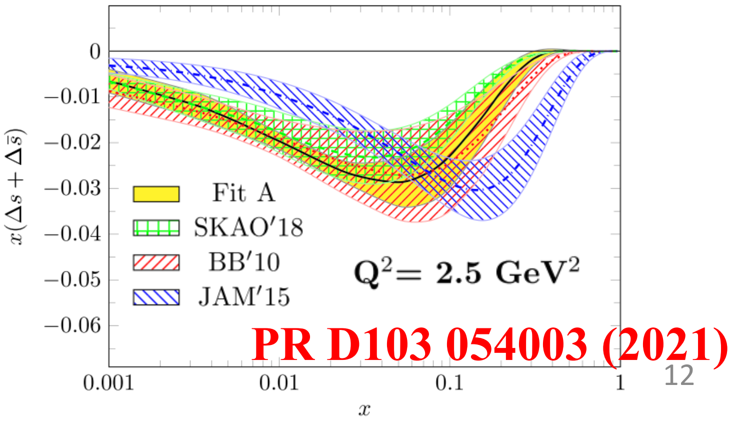
# Strange quark polarization puzzle




- Strange quark density function: $\Delta s(x)+\Delta\bar{s}(x)$
  - Inclusive DIS: only proton PDF
    - negative for all values of x
  - Semi-inclusive DIS: proton PDF & kaon FF
    - DSS FFs: positive for most of measured x
    - HKNS FF: negative
    - JAM FFs: negative
- Reliable FFs knowledge ? Need more efforts

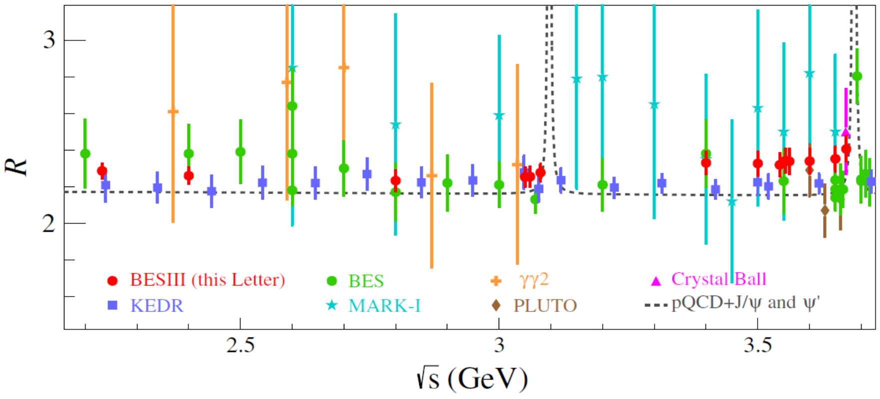


# Analysis at BESIII

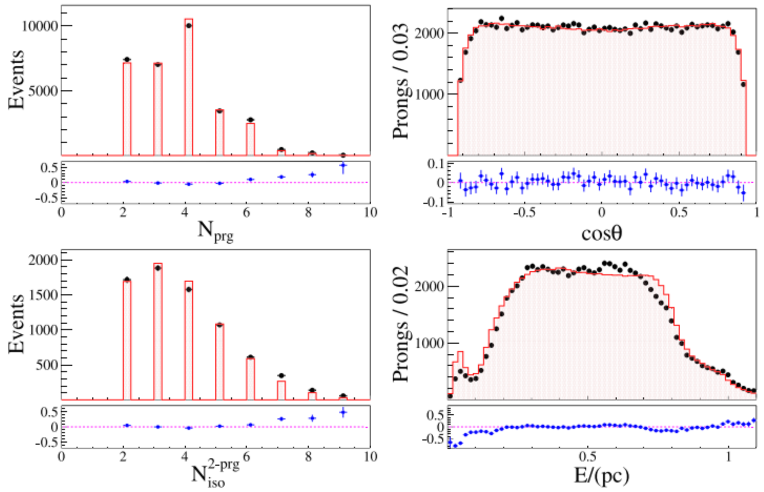
- Normalized differential cross section (take $\pi^0$ as an example):

$$\frac{1}{\sigma_{\mathrm{had}}}\frac{d\sigma_{\pi^0}}{dp_{\pi^0}} = \frac{N_{\pi^0}}{N_{\mathrm{had}}}\frac{1}{\Delta p_{\pi^0}}$$

- Hardronic events $N_{\mathrm{had}}$ : $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$

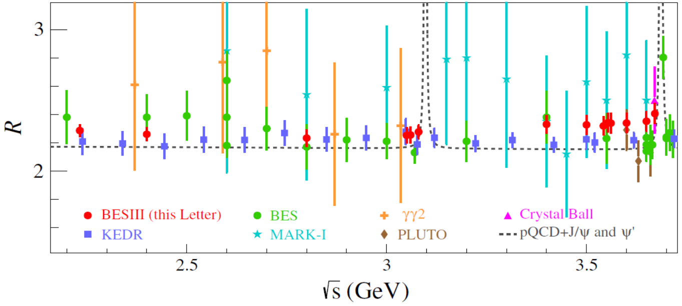



PRL 128 062004(2022)



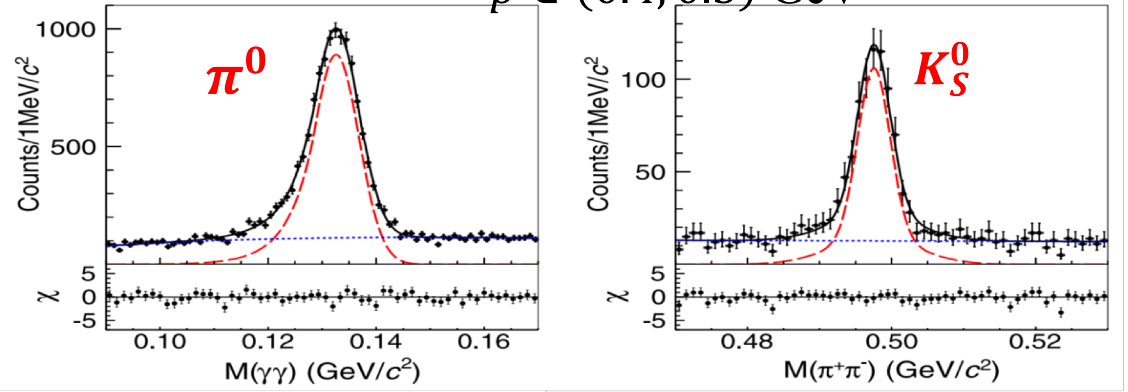

LUARLW MC generator

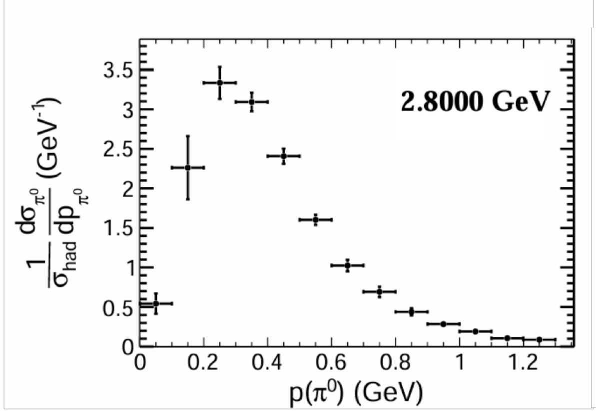
# Inclusive $\pi^0/K_S^0$ production

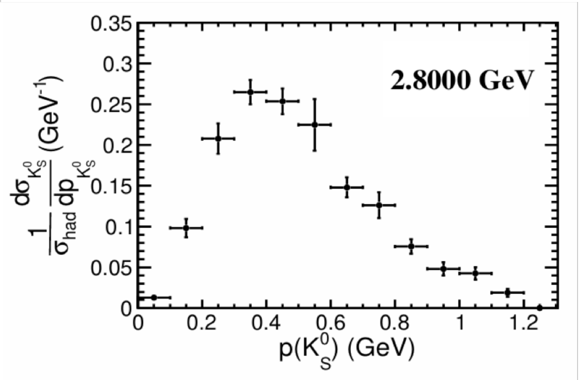



| $\sqrt{s}$ (GeV) | $\mathcal{L}$ ($pb^{-1}$) | $N_{had}^{tot}$ | $N_{bkg}$ |
|---|---|---|---|
| 2.2324 | 2.645 | 83227 | 2041 |
| 2.4000 | 3.415 | 96627 | 2331 |
| 2.8000 | 3.753 | 83802 | 2075 |
| 3.0500 | 14.89 | 283822 | 7719 |
| 3.4000 | 1.733 | 32202 | 843 |
| 3.6710 | 4.628 | 75253 | 6461 |

$p \in (0.4, 0.5)$ GeV



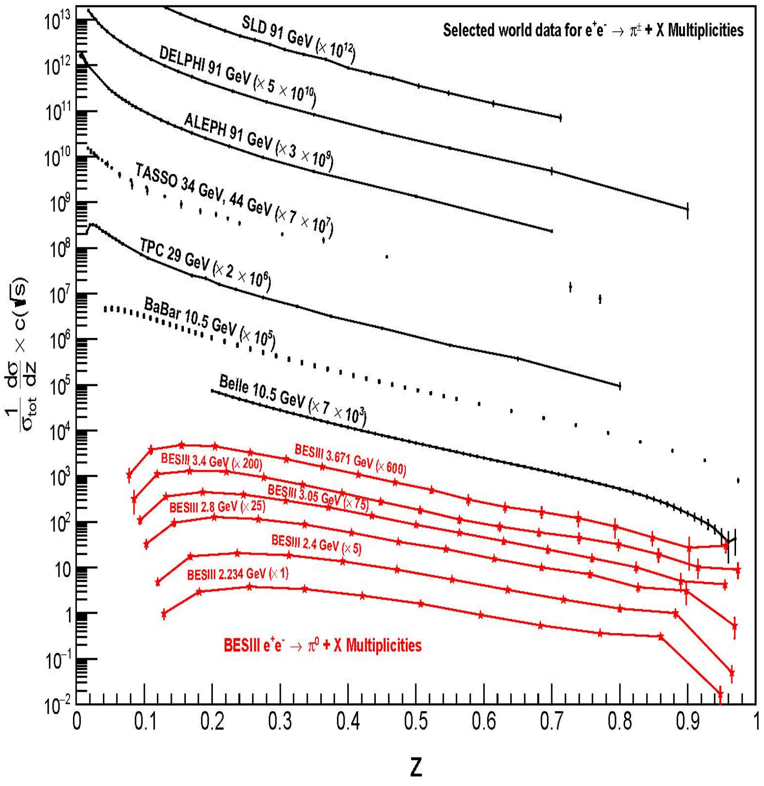



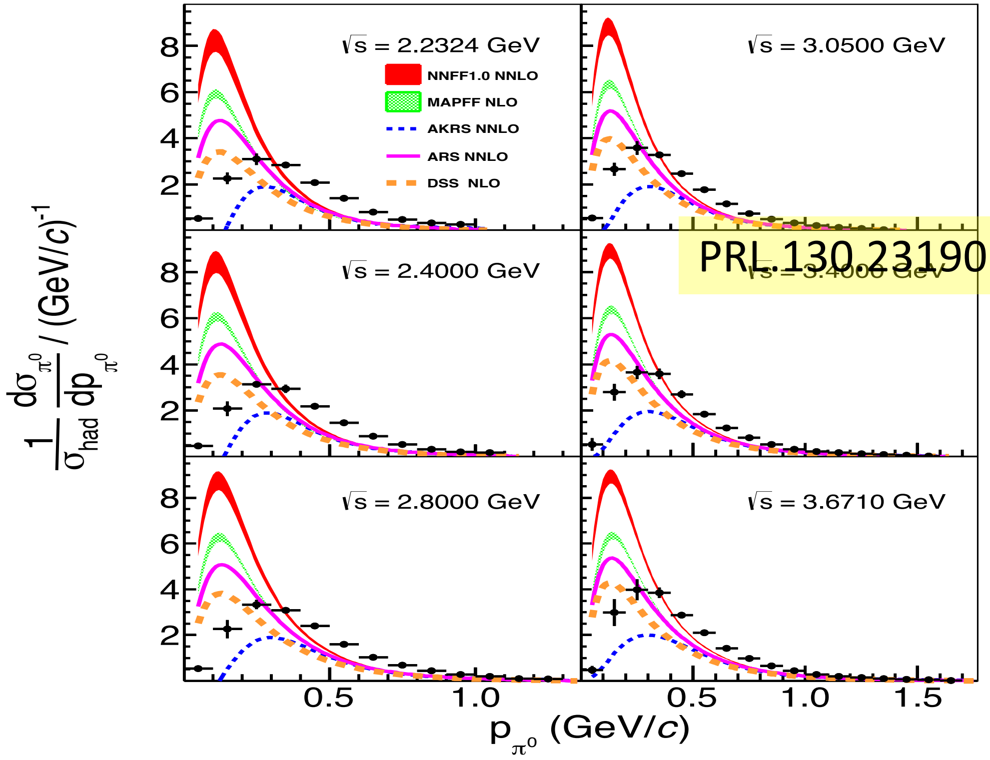

PRL 130 231901(2023)

# Results: inclusive $\pi^0$

Theory support: Hongxi Xing, Daniele Anderle

## Compared with theoretical estimation



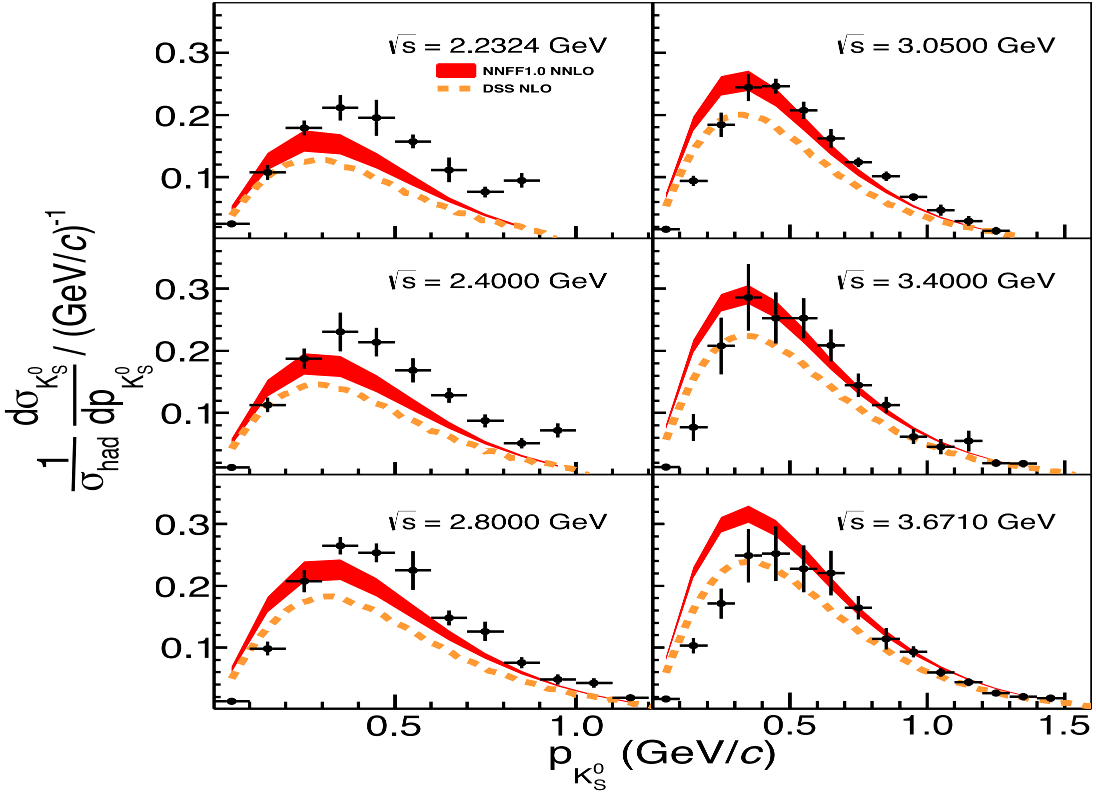

PRL.130.231901

**Uncertainties ~ less 10%**

# Results: Inclusive $K_s^0$

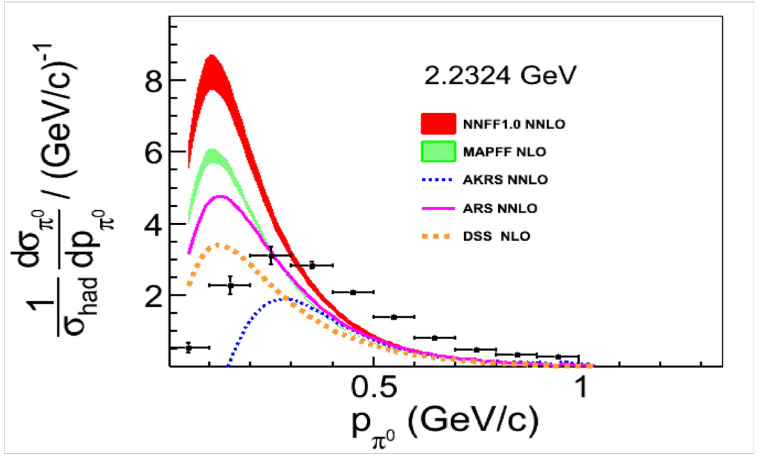
## Compared with theoretical estimation

Theory support: Hongxi Xing, Daniele Anderle




PRL.130.231901

# Results: inclusive $\pi^0/K_s^0$




PRL 130 231901(2023) BESIII

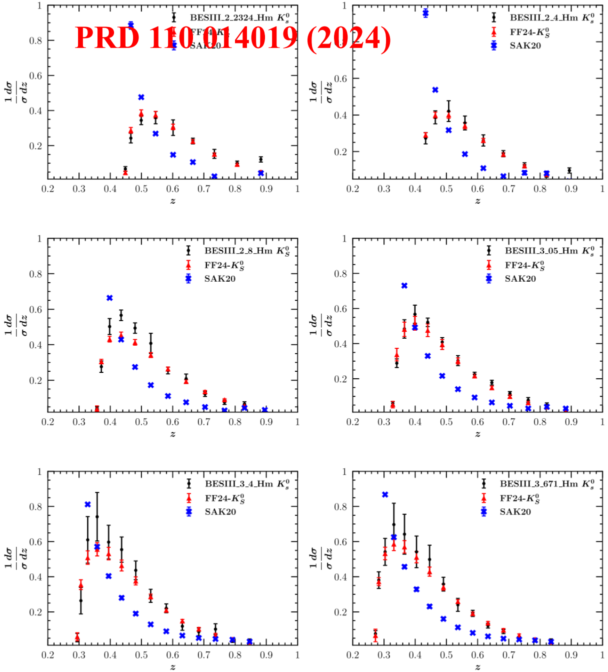
- From theory side: fitting with BESIII data, hadron mass effect, large *z* re-summation, and so on
- From experimental side
  - Primary hadron vs from resonance decay
  - $\Longrightarrow$ measure $e^+ e^- \rightarrow \rho(\omega, \phi) + X$, and so on
  - Contribution of vector states $\rho^*$, $\omega^*$ and $\phi^*$
  - $\Longrightarrow e^+ e^- \rightarrow \rho^*/\omega^*/\phi^* \rightarrow h + X$

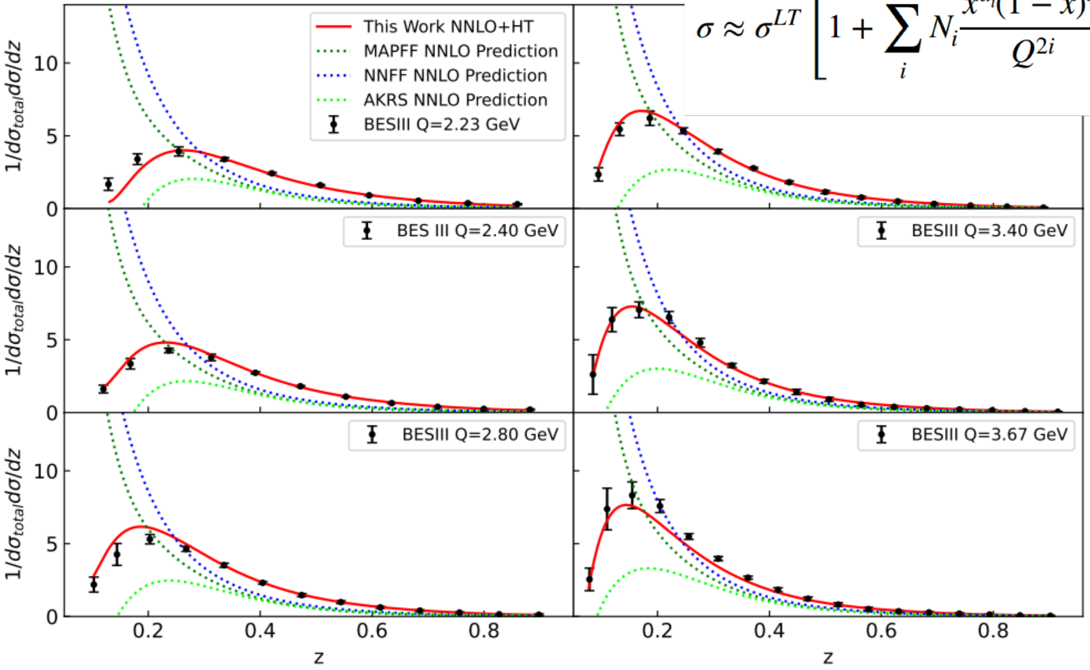
# Results: inclusive $\pi^0/K_s^0$

Theory

$$\sigma \approx \sigma^{LT}\left[1+\sum_i N_i \frac{x^{a_i}(1-x)^{b_i}}{Q^{2i}}\right]$$

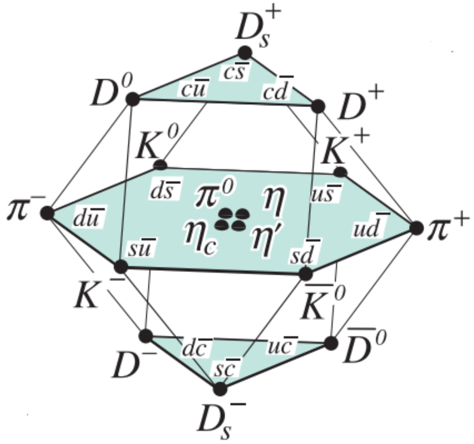
PRD 110 014019 (2024)



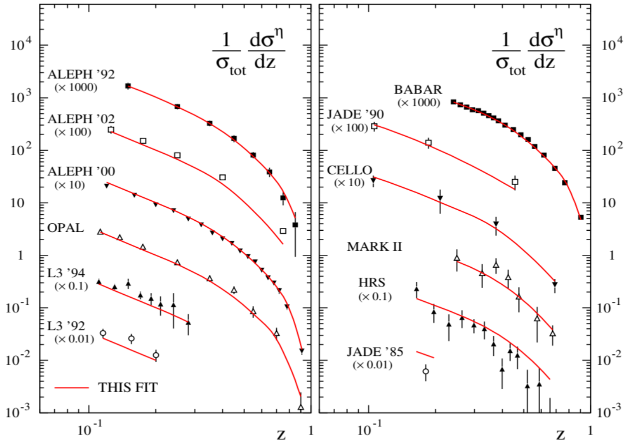




- **PRD 110 014019 (2024): NNLO & hadron mass correction for $K_S$**
- **arXiv:2404.11527: NNLO & higher twist contribution for $\pi^0$**

# World $\eta$ data on $e^+e^-$

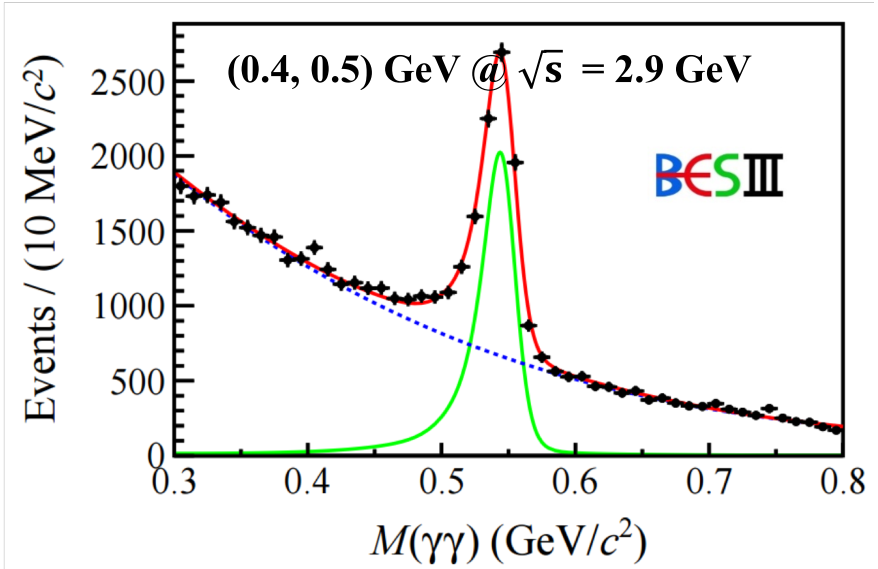


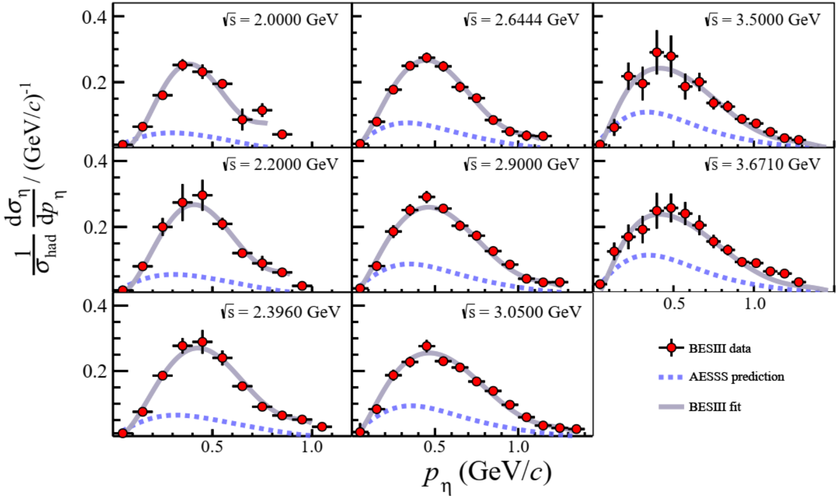

PRD83 (2001) 034002




- $\eta$ FF @ NLO: data at $\sqrt{s} > 10$GeV $e^+e^-$ collision
  - Missing theory uncertainty
- Theory improvement:
  - NNLO accuracy, hadron mass correction & higher twist contributions
- BESIII results and its possible impact ?

# Inclusive $\eta$ production at BESIII



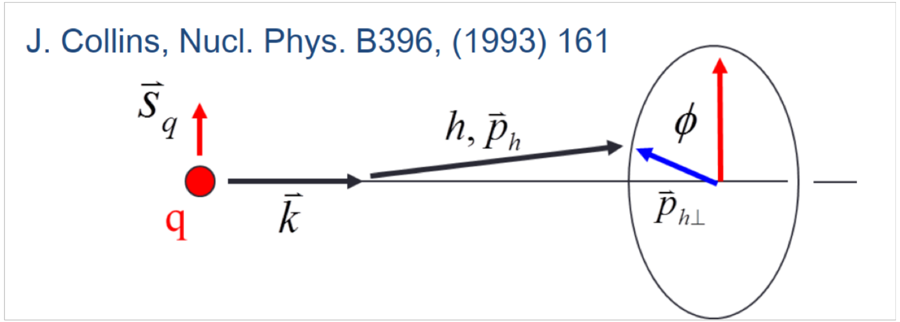




PRL 133, 021901 (2024)

$$\sigma \approx \sigma^{LT}\left[1 + \sum_i N_i \frac{x^{a_i}(1-x)^{b_i}}{Q^{2i}}\right]$$

- PRD83 (2001) 034002 prediction vs. BESIII data: tension !
- BESIII fit: detail @ arXiv:2404.11527
  - $\sqrt{s}$ > 10GeV $e^+e^-$ data + BESIII data
  - NNLO accuracy, hadron mass correction & higher twist contributions

# Collins FFs




- Spin of quark correlates with hadron transverse momentum

➔translates into azimuthal anisotropy of final state hadrons

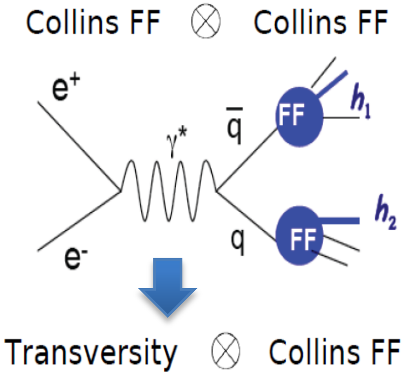
- The possibilities for finding a hadron produced from a transversely polarized quark:

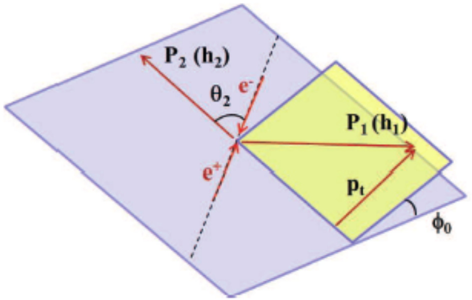
$$D_{hq^\uparrow}(z, P_{h\perp}) = D_1^q(z, P_{h\perp}^2) + H_1^{\perp q}(z, P_{h\perp}^2) \frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{zM_h},$$

- Unpolarized fragmentation function ($D$)
- Collins fragmentation function ($H_1^\perp$)

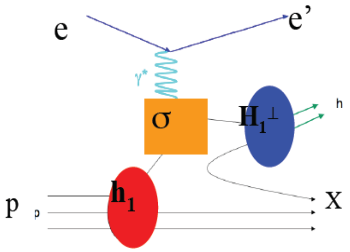
- Fractional energy of hadron $z = 2E_h/\sqrt{s}$
- Transverse momentum of the hadron $P_{h\perp}$

# Collins effects in $e^+e^-$ annihilation




- Normalized ratio R = $N(2\phi_0)/\langle N_0 \rangle$
  - ✓ $N(2\phi_0)$: di-pion yield in each $2\phi_0$ bin
  - ✓ $\langle N_0 \rangle$: averaged bin content
  - ✓ $R^U$: unlike sign $(\pi^\pm\pi^\mp)$;
  - ✓ $R^L$: like sign $(\pi^\pm\pi^\pm)$
  - ✓ $R^C$: all pion pair




- Double ratio: reduce acceptance and radiation effect

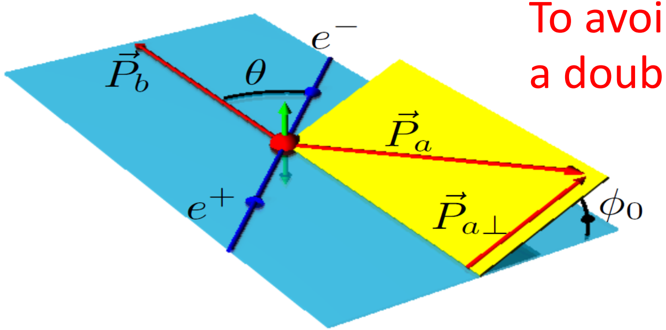
$$\frac{R^U}{R^{L(C)}} = 1 + \cos(2\phi_0) \cdot \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \frac{\mathcal{F}(H_1^\perp(z_1)\bar{H}_1^\perp(z_2)/M_1M_2)}{D_1(z_1)\bar{D}_1(z_2)} = 1 + \cos(2\phi_0) \cdot A^{UL(UC)}$$

**Fit function** $\frac{R^U}{R^{L(C)}} = A\cos(2\phi_0) + B$

$A^{UL/UC}$ mainly contains Collins effect

B should be consistent with unity

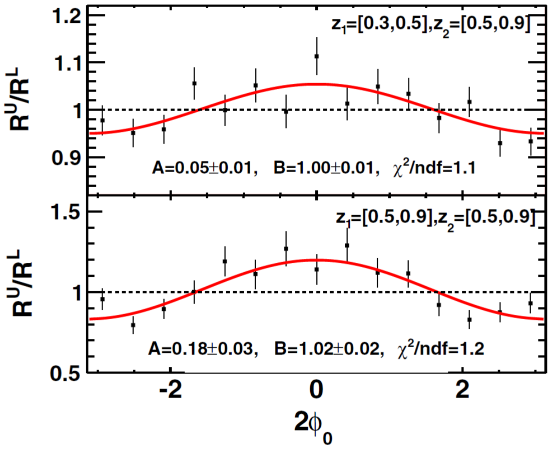
# Collins effects at BESIII

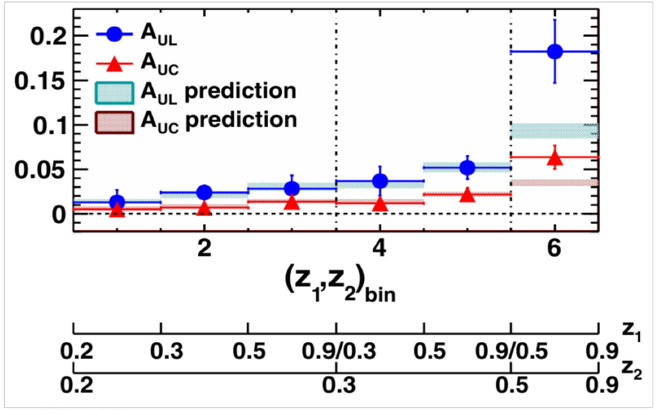



To avoid detection-related effects, experimentally, a double ratio measurement was proposed:

U: pi+&pi- or pi-&pi+
L： pi+&pi+ or pi-&pi-

$$\frac{R^U}{R^{L(C)}} = A\cos(2\phi_0) + B,$$

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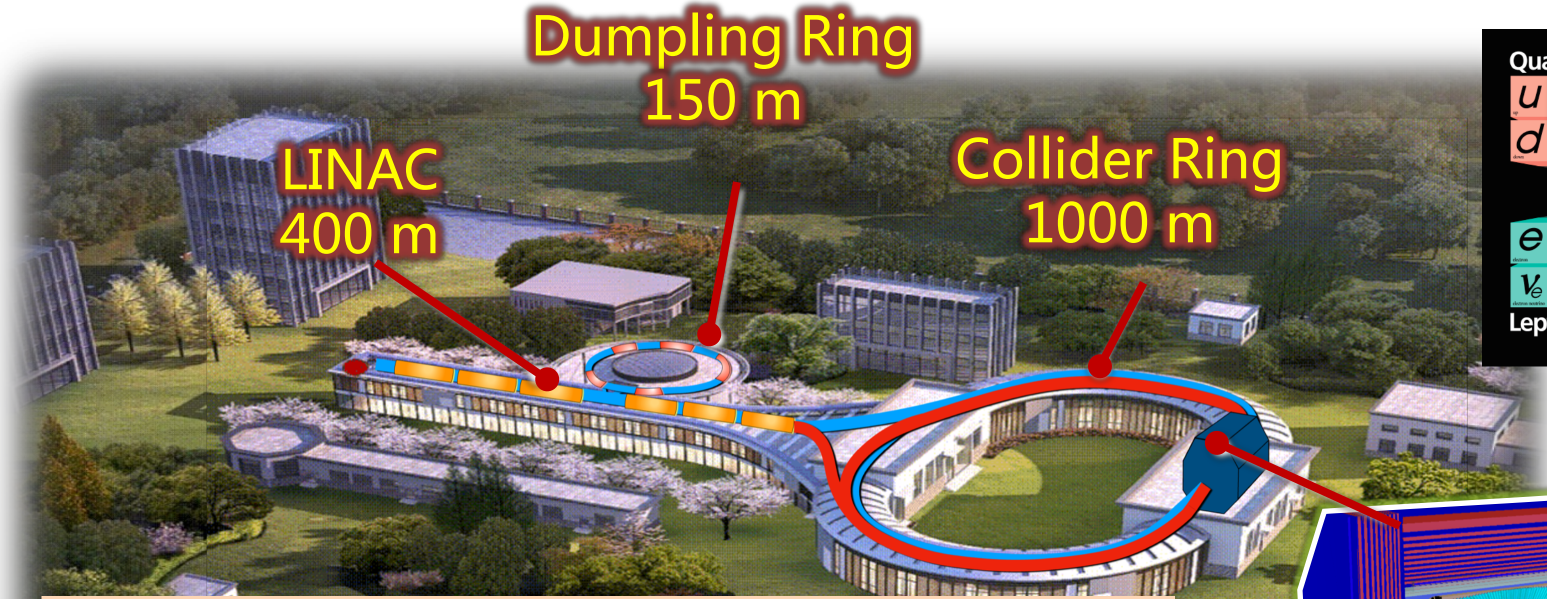
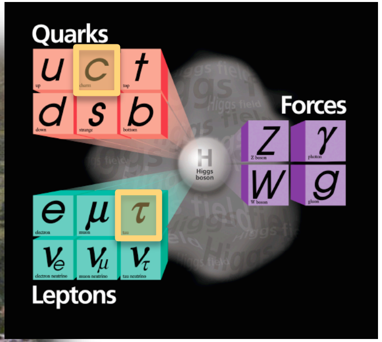
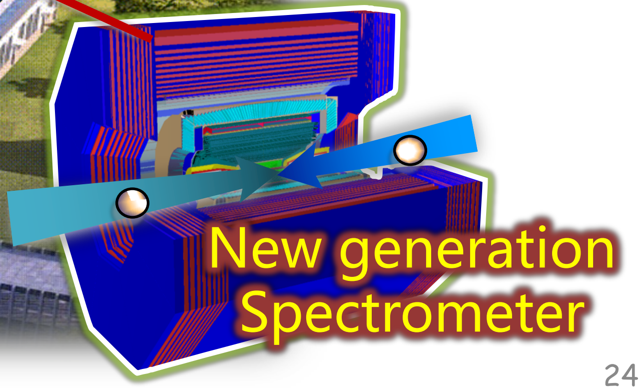
BESIII

# Super Tau-Charm Facility (STCF)



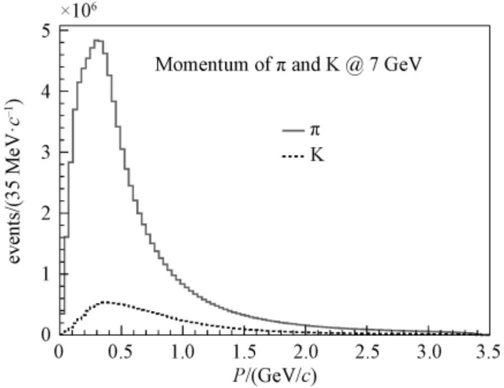



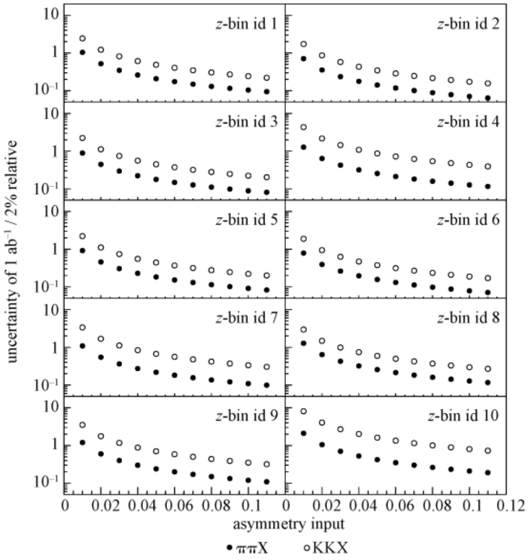

- $E_{cm}$ = 2-7 GeV, peak luminosity > $0.5\times10^{35}$ $cm^{-2}$ $s^{-1}$
- Potential for luminosity upgrade, and a polarized electron beam
- Site: Suburban “Future Big Science City” in Hefei

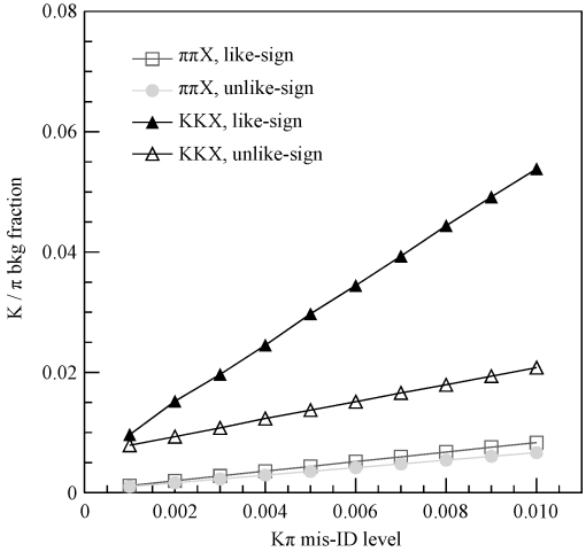


# Collins FFs @ STCF

| Id | $z_1z_2$ 范围 | Id | $z_1z_2$ 范围 |
|---|---|---|---|
| 1 | [0.15,0.2),[0.15,0.2) | 6 | [0.2,0.3),[0.3,0.5) |
| 2 | [0.15,0.2),[0.2,0.3) | 7 | [0.2,0.3),[0.5,0.9] |
| 3 | [0.15,0.2),[0.3,0.5) | 8 | [0.3,0.5),[0.3,0.5) |
| 4 | [0.15,0.2),[0.5,0.9] | 9 | [0.3,0.5),[0.5,0.9] |
| 5 | [0.2,0.3),[0.2,0.3) | 10 | [0.5,0.9],[0.5,0.9] |



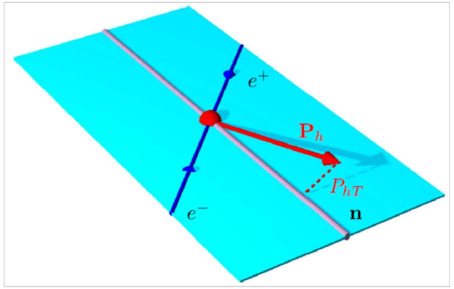




- The statistical uncertainty on asymmetry $A^{UL}$ with $1ab^{-1}$ @ 7 GeV
  - ✓ $(1.4, 4.2)\times10^{-4}$ for $e^+e^- \to \pi\,\pi + X$
  - ✓ $(3.5,\ 20)\times10^{-3}$ for $e^+e^- \to K\,K + X$
- Key process for PID of STCF

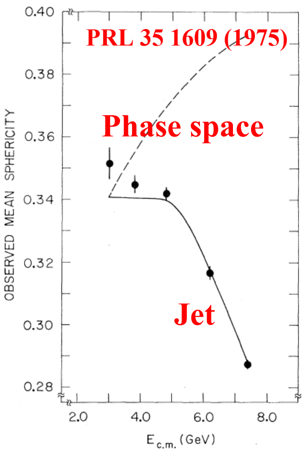
# TMD FFs @ STCF

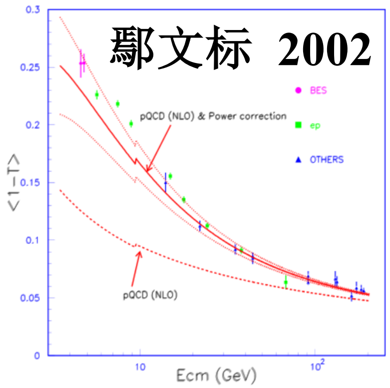
- Theoretically many more, in particular with polarized hadrons in the final state and transverse momentum dependence (TMD)

| TMD FF $D_1(z, k_T)$ | | |
|---|---|---|
| $e^+e^- \to h_a h_b X$ | $\sum_q e_q^2 D_1^{h_a/q}(z_a, k_{aT}) \otimes D_1^{h_b/\bar{q}}(z_b, k_{bT}) + \{q \leftrightarrow \bar{q}\}$ | back-to-back production of hadron pair |
| $e^+e^- \to (h, \text{jet/thrust axis})X$ | $\sum_q e_q^2 D_1^{h/q}(z, k_T)$ | can access $z$, $k_T$ |




- Jet structure at STCF
  - ✓ reconstruct thrust axis correctly ?
- Phase space model vs. Jet model
  - ✓ $\sqrt{s} > 5$ GeV ?
- At higher $\sqrt{s}$: jet @ [5, 7] GeV ?
  - ✓ Evidence for jet structure
- Longitudinally polarized e$^-$ beam @ STCF
  - ✓ Effect on fragmentation function



# Summary

- The knowledge of FFs is an important ingredient in our understanding of non-perturbative QCD dynamics. $e^+e^-$ annihilation experiments provide the cleanest environment to measure FFs.
- Two types of fragmentation functions can be studied at BESIII and STCF
  - Unpolarized fragmentation function
    - ✓ Unique Q<10 GeV data
    - ✓ More results from charged $\pi/K$ and heavy flavor
  - Collins fragmentation function
    - ✓ Essential input in the 3D imaging era of the nucleon structure study
    - ✓ More results from $K\pi + X$ and $KK + X$

***Thanks***

Wigner distribution $W_\Gamma(\vec{r}, k)$ 7 variables

**Integrating out $k^-$**

Reduced Wigner distribution $W_\Gamma(\vec{r}, \vec{k})$ 6 variables

**Integrating out $\vec{k}_\perp$**

**Integrating out $\vec{r}$**

$\tilde{f}_{\gamma^+}(\vec{r}, k^+)$ or $f_{\gamma^+}(\vec{r}, x)$ 4 variables

3D density in the rest frame of the proton

$f(x, \vec{k}_\perp)$ 3 variables

Transverse-momentum dependent parton distributions

**Integrating out $z$**

**Fourier transformation**

$f_{\gamma^+}(\vec{r}_\perp, x)$ 3 variables

Impact parameter space distribution

$F_{\gamma^+}(x, \xi, t)$ 3 variables

Generalized parton distribution

**Integrating out $\vec{k}_\perp$**

**Integrating out $\vec{r}_\perp$**

**Taking $\xi, t = 0$**

$f(x)$ 1 variables

Parton distribution function