Transverse momentum dependent helicity distributions

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Outlook

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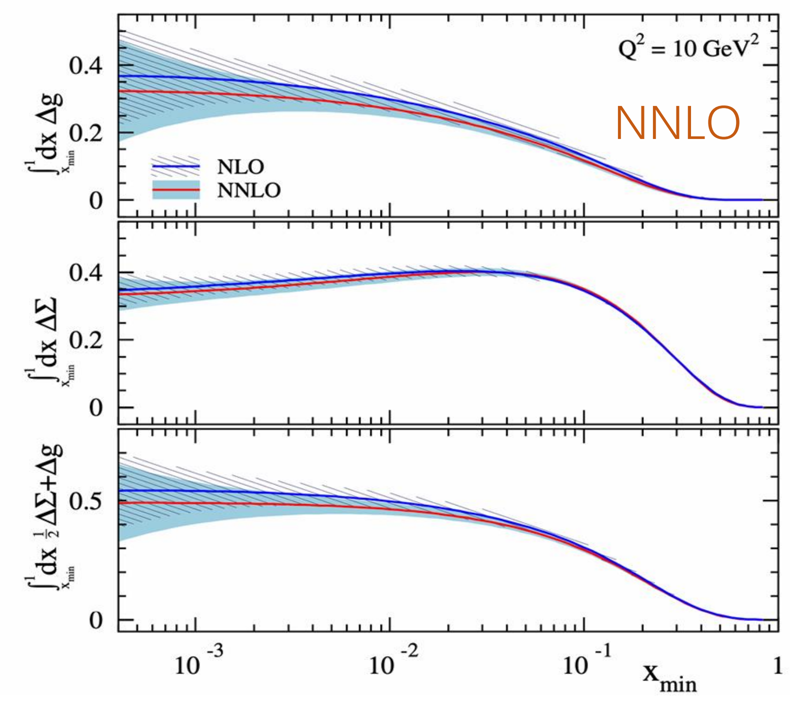
Research Background

Motivation

1, Neutrons and protons dominate the visible world.

2, Proton spin crisis $\Delta u_L + \Delta d_L + \Delta s_L = 1.$

J. Ashman et al. (European Muon Collaboration), Nucl. Phys. B 328, 1 (1989)



I. Borsa, M. Stratmann, W. Vogelsang, D. de Florian, R. Sassot, Phys.Rev.Lett. 133, 151901 (2024)

$$J = \frac{1}{2}\Delta\Sigma + L_q^{JM} + \Delta G + L_g$$

$$\Delta\Sigma = \int_0^1 g_1(x)dx$$

R. Jaffe and A. Manohar, Nucl.Phys. B337 (1990) 509

E. P. Wigner, Annals Math. 40, 149 (1939)
H. J. Melosh, Phys. Rev. D 9, 1095 (1974)

$$\Delta q = \int \mathrm{d}^3\mathbf{p} M_q \left[q^{\uparrow}(p) - q^{\downarrow}(p)\right] = \langle M_q \rangle \Delta q_L$$

B.-Q. Ma, J. Phys. G 17, L53-L58 (1991)
B.-Q. Ma, Z.Phys. C 58, 479 (1993)

Motivation

1, Neutrons and protons dominate the visible world.

2, Proton spin crisis $$\Delta u_L + \Delta d_L + \Delta s_L \; = \; 1.$$

J. Ashman et al. (European Muon Collaboration), Nucl. Phys. B 328, 1 (1989)

3, Melosh Wigner rotation

$$\Delta q = \int \mathrm{d}^3\mathbf{p} M_q \left[q^{\uparrow}(p) - q^{\downarrow}(p)\right] = \langle M_q \rangle \Delta q_L$$

$$M_q = \left[(p_0 + p_3 + m)^2 - \mathbf{p}_{\perp}^2\right] / [2(p_0 + p_3)(m + p_0)]$$

$$g_1(x) \rightarrow g_{1L}(x, p_T)$$

E. P. Wigner, Annals Math. 40, 149 (1939)
H. J. Melosh, Phys. Rev. D 9, 1095 (1974)

B.-Q. Ma, J. Phys. G 17, L53-L58 (1991)
B.-Q. Ma, Z.Phys. C 58, 479 (1993)

4, SIDIS experiment and measurement: $A_{LL} \propto \frac{g_{1L}}{f_1}$ from CLAS and HERMES

CLAS, Phys. Lett. B 782, 662 (2018); HERMES, Phys. Rev. D 99, 112001 (2019)

5, Well-developed factorization theory

J. C. Collins and D. E. Soper, Nucl. Phys. B 193, 381 (1981)
X. d. Ji, J. P. Ma, and F. Yuan, Phys. Lett. B 597, 299 (2004)
X. d. Ji, J. p. Ma, and F. Yuan, Phys. Rev. D 71, 034005 (2005)
S. M. Aybat and T. C. Rogers, Phys. Rev. D 83, 114042 (2011)
I. Scimemi and A. Vladimirov, J. High Energy Phys. 06 (2020) 137

Definition

$$\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P|\bar{\psi}_j(0)\,\mathcal{U}^{n_-}_{(0,+\infty)}\,\mathcal{U}^{n_-}_{(+\infty,\xi)}\,\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

TMDs		Quark polarization: Unpolarized (U)	Quark polarization: Longitudinally polarized (L)	Quark polarization: Transversely polarized (T)
Nucleon polarization	U	f_1 Unpolarized		$h_1^\perp$ Boer–Mulders
Nucleon polarization	L		g_{1L} Helicity	$h_{1L}^\perp$ Longi-transversity
Nucleon polarization	T	$f_{1T}^\perp$ Sivers	g_{1T} Trans-helicity	h_1 Transversity; $h_{1T}^\perp$ Pretzelosity

Nucleon spin   Quark spin

2

Theoretical formalism

Framework

SIDIS process

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\right]$$

$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

$$+|S_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

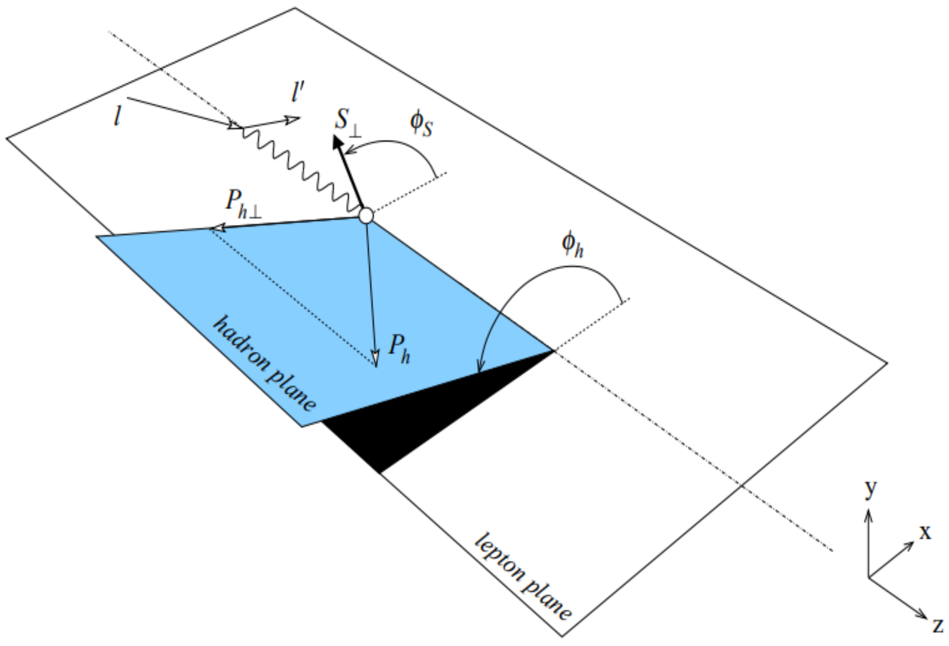
$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|S_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},$$

$$\ell(l)+N(P)\to\ell(l')+h(P_h)+X,$$



18 structure functions

Double spin asymmetry (DSA)

$$\begin{aligned}A_{LL}(\phi_h) &= \frac{1}{|S_\perp||\lambda_e|}\frac{[d\sigma_{LL}(+,+)-d\sigma_{LL}(-,+)]-[d\sigma_{LL}(+,-)-d\sigma_{LL}(-,-)]}{d\sigma_{LL}(+,+)+d\sigma_{LL}(-,+)+d\sigma_{LL}(+,-)+d\sigma_{LL}(-,-)}\\ &= \sqrt{1-\varepsilon^2}F_{LL}/\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\\ &\quad+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}/\left(F_{UU,T}+\varepsilon F_{UU,L}\right).\end{aligned}$$

$$A_{LL}|_{CSA} = \sqrt{1-\varepsilon^2}\frac{F_{LL}}{F_{UU,T}+\varepsilon F_{UU,L}}.$$

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2},$$

$$\begin{aligned}A_{LL}^{\cos\phi_h}|_{CSA} &= \langle 2\cos\phi_h\rangle_{LL}\\ &= \sqrt{2\varepsilon(1-\varepsilon)}\frac{F_{LL}^{\cos\phi_h}}{F_{UU,T}+\varepsilon F_{UU,L}}\end{aligned}$$

Structure functions

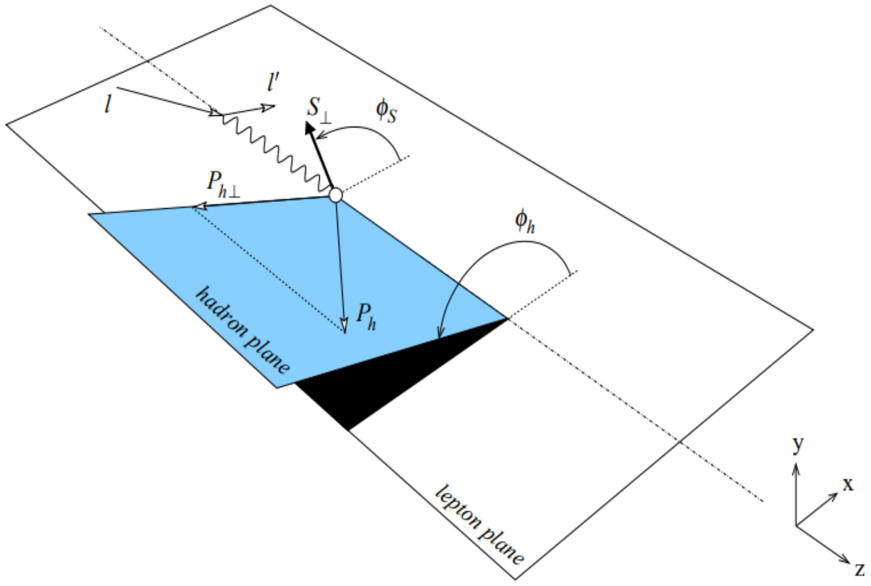
$$F_{UU,T} = \mathcal{C}[f_1 D_1]$$

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X,$$

$$F_{LL} = \mathcal{C}[g_{1L} D_1]$$

$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe_L H_1^{\perp} - \frac{M_h}{M} g_{1L}\frac{\tilde{D}^{\perp}}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_L^{\perp} D_1 + \frac{M_h}{M} h_{1L}^{\perp}\frac{\tilde{E}}{z}\right)\right]$$

$$xg_L^{\perp} = x\tilde{g}_L^{\perp} + g_{1L} + \frac{m}{M} h_{1L}^{\perp}$$



Structure functions

$$
\begin{aligned}
F_{UU,T} &= \mathcal{C}[f_1 D_1] \\
&= x \sum_q e_q^2 \int \mathrm{d}^2 \boldsymbol{p}_T \, \mathrm{d}^2 \boldsymbol{k}_T \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - P_{h\perp}/z) f_1(x, p_T^2) D_1(z, k_T^2) \\
&= x \sum_q e_q^2 \int_0^\infty \frac{|b| \mathrm{d}|b|}{2\pi} J_0\left(\frac{|b||P_{h\perp}|}{z}\right) f_1(x,b) D_1(z,b).
\end{aligned}
$$

$$
\begin{aligned}
F_{LL} =& \mathcal{C}\left[g_{1L} D_1\right] \\
=& x \sum_q e_q^2 \int_0^\infty \frac{b_T db_T}{2\pi} J_0\left(\frac{b_T P_{hT}}{z}\right) g_{1L,q \leftarrow H}(x, b_T) D_{1,q \rightarrow h}(z, b_T).
\end{aligned}
$$

$$
\begin{aligned}
F_{LL}^{\cos\phi} =& \frac{2M}{Q} \mathcal{C}\left[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M_h}\left(x e_L H_1^\perp - \frac{M_h}{M} g_{1L} \frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T}{M}\left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z}\right)\right] \\
\approx& -\frac{2M}{Q} \mathcal{C}\left[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T}{M}\left(g_{1L} D_1\right)\right] \\
=& -\frac{2M^2}{Q} x \sum_q e_q^2 \int_0^\infty \frac{b_T^2 \, \mathrm{d} b_T}{2\pi} J_1\left(\frac{b_T P_{hT}}{z}\right) g_{1L,q \leftarrow H}^{(1)}(x, b_T) D_{1,q \rightarrow h}(z, b_T).
\end{aligned}
$$

TMD PDF:

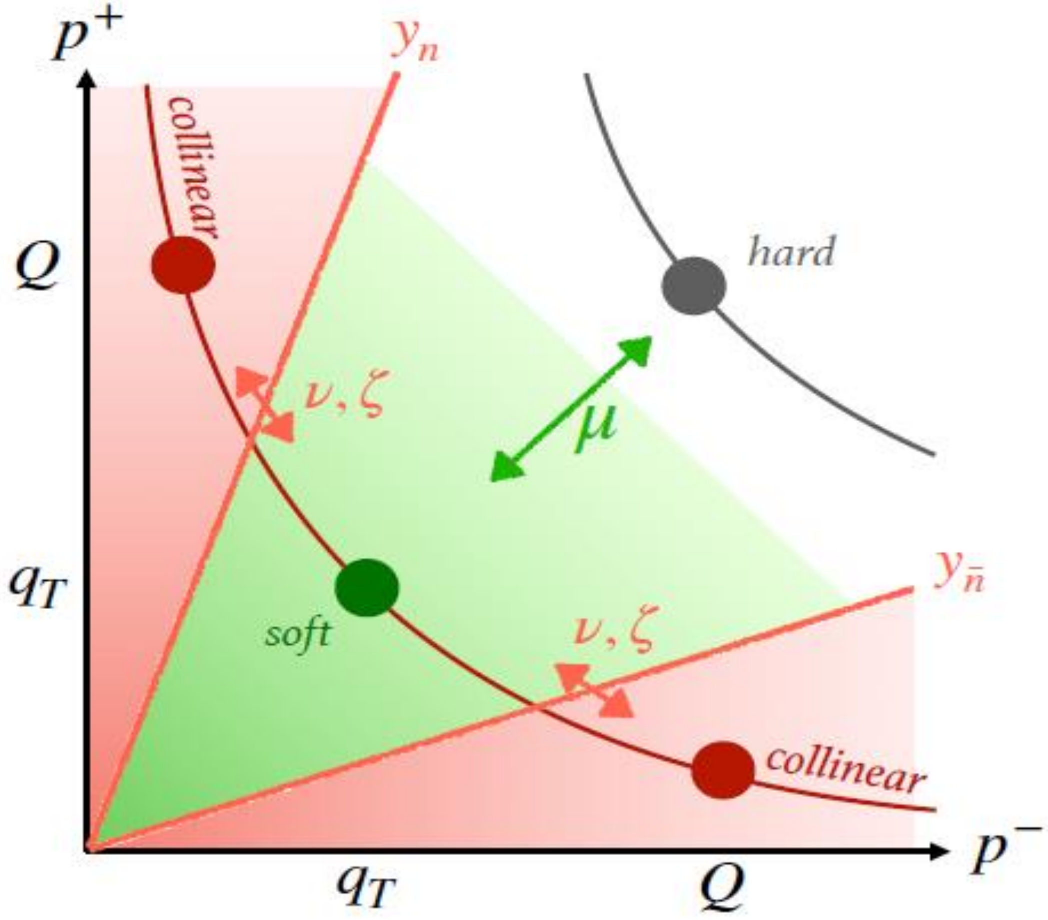
$f_1(x, k_T)$

$g_{1L}(x, k_T)$

TMD FF:

$D_1(x, p_T)$

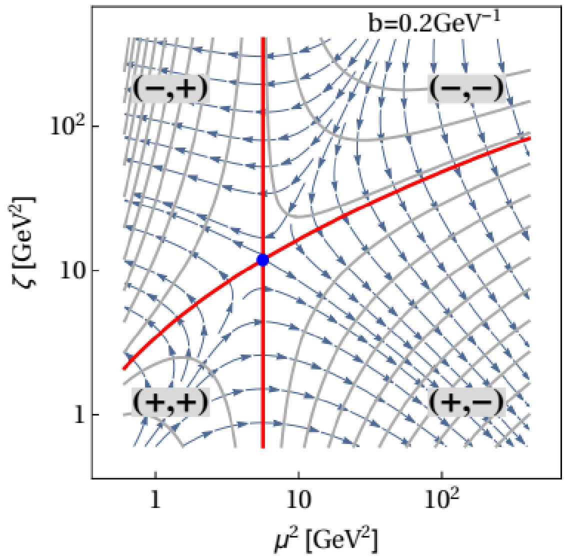
Energy evolution

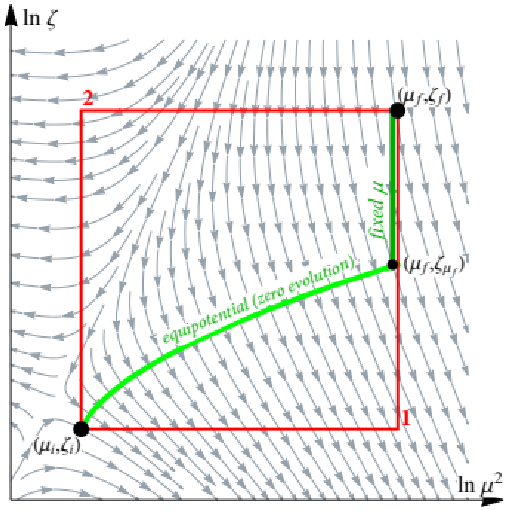


S. M. Aybat and T. C. Rogers, Phys. Rev. D 83, 114042 (2011)
I. Scimemi and A. Vladimirov, J. High Energy Phys. 08 (2018) 003.

$$\mu \frac{d\mathcal{F}(x,b;\mu,\zeta)}{d\mu} = \gamma_F(\mu,\zeta)\mathcal{F}(x,b;\mu,\zeta)$$

$$\zeta \frac{d\mathcal{F}(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,b)\mathcal{F}(x,b;\mu,\zeta)$$





$$R\left[b;(\mu_i,\zeta_i) \to (Q,Q^2)\right] = \left[\frac{Q^2}{\zeta_\mu(Q,b)}\right]^{-\mathcal{D}(Q,b)}$$

I. Scimemia and A. Vladimirovb, JHEP06, 137(2020)

3

World SIDIS data

Data from experiments

World SIDIS data

TABLE II. World SIDIS data that reported by HERMES and CLAS.

Data set	Run	Hadron beam	Lepton beam	point number	Process	Measurement
HERMES [1]	1996-2000	H_2	27.6 GeV $e^{\pm}$	80,30	$e^{\pm}p \to e^{\pm}\pi^{+}X$	A_{LL},$A_{LL}^{\cos\phi}$
HERMES [1]	1996-2000	H_2	27.6 GeV $e^{\pm}$	80,30	$e^{\pm}p \to e^{\pm}\pi^{-}X$	A_{LL},$A_{LL}^{\cos\phi}$
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	80,30	$e^{\pm}d \to e^{\pm}\pi^{+}X$	A_{LL},$A_{LL}^{\cos\phi}$
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	80,30	$e^{\pm}d \to e^{\pm}\pi^{-}X$	A_{LL},$A_{LL}^{\cos\phi}$
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	79,30	$e^{\pm}d \to e^{\pm}K^{+}X$	A_{LL},$A_{LL}^{\cos\phi}$
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	78,30	$e^{\pm}d \to e^{\pm}K^{-}X$	A_{LL},$A_{LL}^{\cos\phi}$
CLAS [4]	2009	$^{14}NH_3$	6 GeV e^{-}	21,21	$e^{-}p \to e^{-}\pi^{0}X$	A_{LL},$A_{LL}^{\cos\phi}$
Total				498,201		

HERMES, Phys. Rev. D 99, 112001 (2019),
CLAS, Phys. Lett. B 782, 662 (2018).

4

Numerical Analysis

Method and result of global fit

Parametrization for unpolarized TMDs

Ignazio Scimemia and Alexey Vladimirovb, JHEP06, 137(2020)

Initial unpolarized TMD pdf & ff Parametrization：

$$f_{1;f\leftarrow h}(x,b)=\sum_{f'}\int_x^1\frac{dy}{y}C_{f\leftarrow f'}(y,b,\mu_{OPE})f_{1,f'\leftarrow h}\left(\frac{x}{y},\mu_{OPE}\right)f_{NP}(x,b),$$

$$D_{1;f\to h}(z,b)=\frac{1}{z^2}\sum_{f'}\int_z^1\frac{dy}{y}y^2\mathbb{C}_{f\leftarrow f'}(y,b,\mu_{OPE})d_{1,f'\to h}\left(\frac{z}{y},\mu_{OPE}\right)D_{NP}(z,b),$$

$$\mu_{OPE}^{PDF}=\frac{2e^{-\gamma_E}}{b}+2GeV,\qquad \mu_{OPE}^{FF}=\frac{2e^{-\gamma_E}z}{b}+2GeV,$$

$$f_{NP}(x,b)=\exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x+x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right),$$

$$D_{NP}(x,b)=\exp\left(-\frac{\eta_1 z+\eta_2(1-z)}{\sqrt{1+\eta_3(\boldsymbol{b}/z)^2}}\frac{\boldsymbol{b}^2}{z^2}\right)\left(1+\eta_4\frac{\boldsymbol{b}^2}{z^2}\right),$$

evolution：

$$F(x,b;Q,Q^2)=R(b,Q)f_{q\leftarrow h_1}(x,b),\qquad R(b,Q)=\left(\frac{Q^2}{\zeta_Q(b)}\right)^{-\mathcal{D}(b,Q)},$$

Ignazio Scimemia and Alexey Vladimirovb JHEP08, 003(2018)

Parametrization for TMD helicity distribution

Initial TMD helicity distribution:

$$g_{1L}(x,b) = \sum_{f'} \int_x^1 \frac{d\xi}{\xi} \Delta C_{f \leftarrow f'}(\xi, b, \mu_{\rm OPE}) \times g_{1L}^{f'}\left(\frac{x}{\xi}\right) g_{\rm NP}(x,b),$$

$$g_{1L}^f(x) = N_f \frac{(1-x)^{\alpha_f} x^{\beta_f}(1+\epsilon_f x)}{n(\alpha_f, \beta_f, \epsilon_f)} g_1^f(x, \mu_0),$$

$$g_{\rm NP}(x,b) = \exp\left[-\frac{\lambda_1(1-x)+\lambda_2 x+\lambda_5 x(1-x)}{\sqrt{1+\lambda_3 x^{\lambda_4} b^2}} b^2\right],$$

q	N_q	α_q	β_q	ε_q	λ_1	λ_2	λ_3	λ_4	λ_5
u	N_u	α_u	β_u	ε_u	λ_1	λ_2	e^{ℓ_3}	λ_4	λ_5
d	N_d	α_d	β_d	ε_d	λ_1	λ_2	e^{ℓ_3}	λ_4	λ_5
$\bar{u}$	$N_{\bar{u}}$	0	0	0	λ_1	λ_2	e^{ℓ_3}	λ_4	λ_5
$\bar{d}$	$N_{\bar{d}}$	0	0	0	λ_1	λ_2	e^{ℓ_3}	λ_4	λ_5
$s, \bar{s}$	N_s	0	0	0	λ_1	λ_2	e^{ℓ_3}	λ_4	λ_5
g	N_g	0	0	0	λ_1	λ_2	e^{ℓ_3}	λ_4	λ_5

evolution:

$$F(x,b;Q,Q^2) = R(b,Q) f_{q \leftarrow h_1}(x,b), \qquad R(b,Q) = \left(\frac{Q^2}{\zeta_Q(b)}\right)^{-\mathcal{D}(b,Q)},$$

Ignazio Scimemia and Alexey Vladimirovb JHEP08, 003(2018)

Kinematics regions

TABLE III. World SIDIS data that reported by HERMES and CLAS satisfy $\delta < 0.5$.

Data set	Run	Hadron beam	Lepton beam	point number	Process	Measurement
HERMES [1]	1996-2000	H_2	27.6 GeV $e^{\pm}$	42	$e^{\pm}p \to e^{\pm}\pi^{+}X$	A_{LL}
HERMES [1]	1996-2000	H_2	27.6 GeV $e^{\pm}$	42	$e^{\pm}p \to e^{\pm}\pi^{-}X$	A_{LL}
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	41	$e^{\pm}d \to e^{\pm}\pi^{+}X$	A_{LL}
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	40	$e^{\pm}d \to e^{\pm}\pi^{-}X$	A_{LL}
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	40	$e^{\pm}d \to e^{\pm}K^{+}X$	A_{LL}
HERMES [1]	1996-2000	D_2	27.6 GeV $e^{\pm}$	39	$e^{\pm}d \to e^{\pm}K^{-}X$	A_{LL}
CLAS [4]	2009	$^{14}NH_3$	6 GeV e^{-}	9	$e^{-}p \to e^{-}\pi^{0}X$	A_{LL}
Total				253		

$$Q^2 > 1\,\mathrm{GeV}^2.$$
$$\delta = \frac{P_T}{zQ} < 0.5.$$

HERMES, Phys. Rev. D 99, 112001 (2019),
CLAS, Phys. Lett. B 782, 662 (2018).

Analysis method

1, world data (HERMES, CLAS)

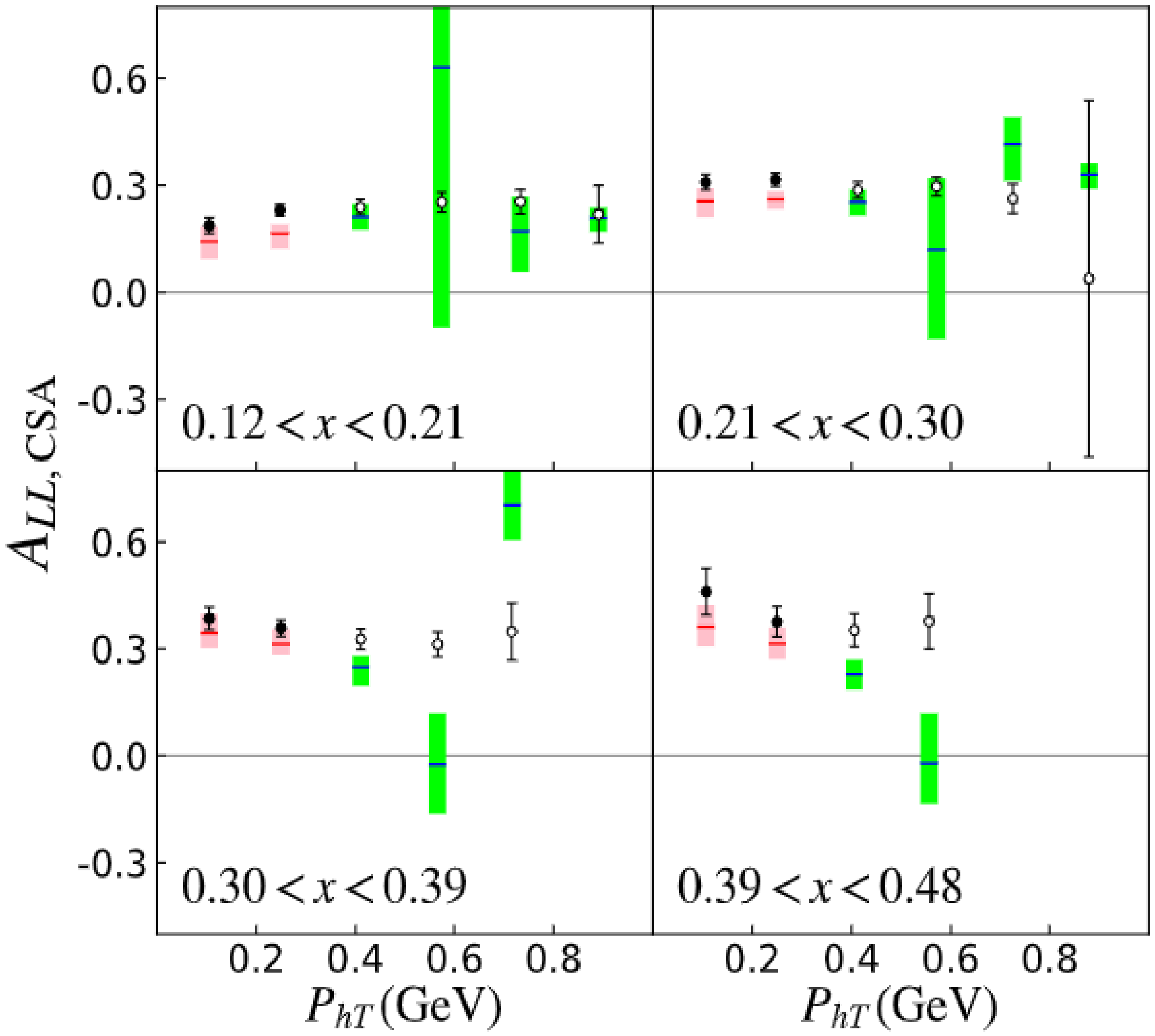
2, replicas generated using central values & uncertainties of world data

3, fit each replica independently

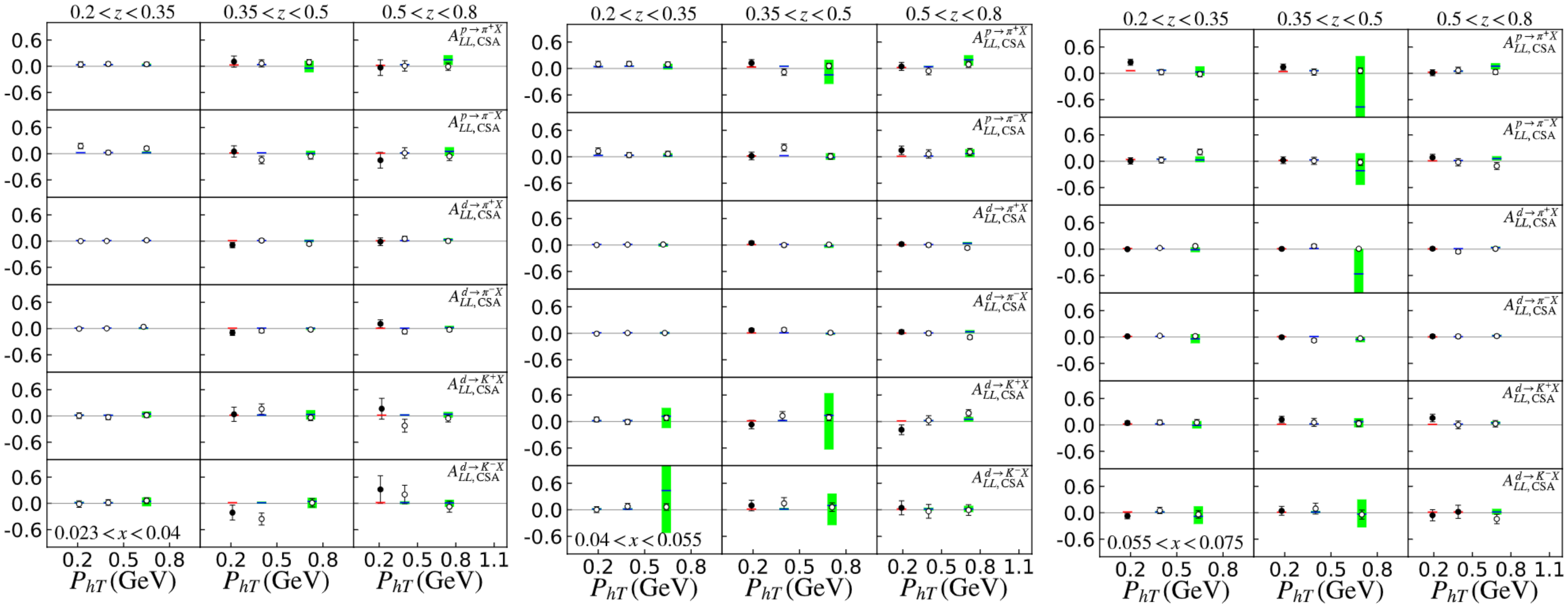
4, every thing we want to know can be described as the central values and standard

deviations of result that from those fits.

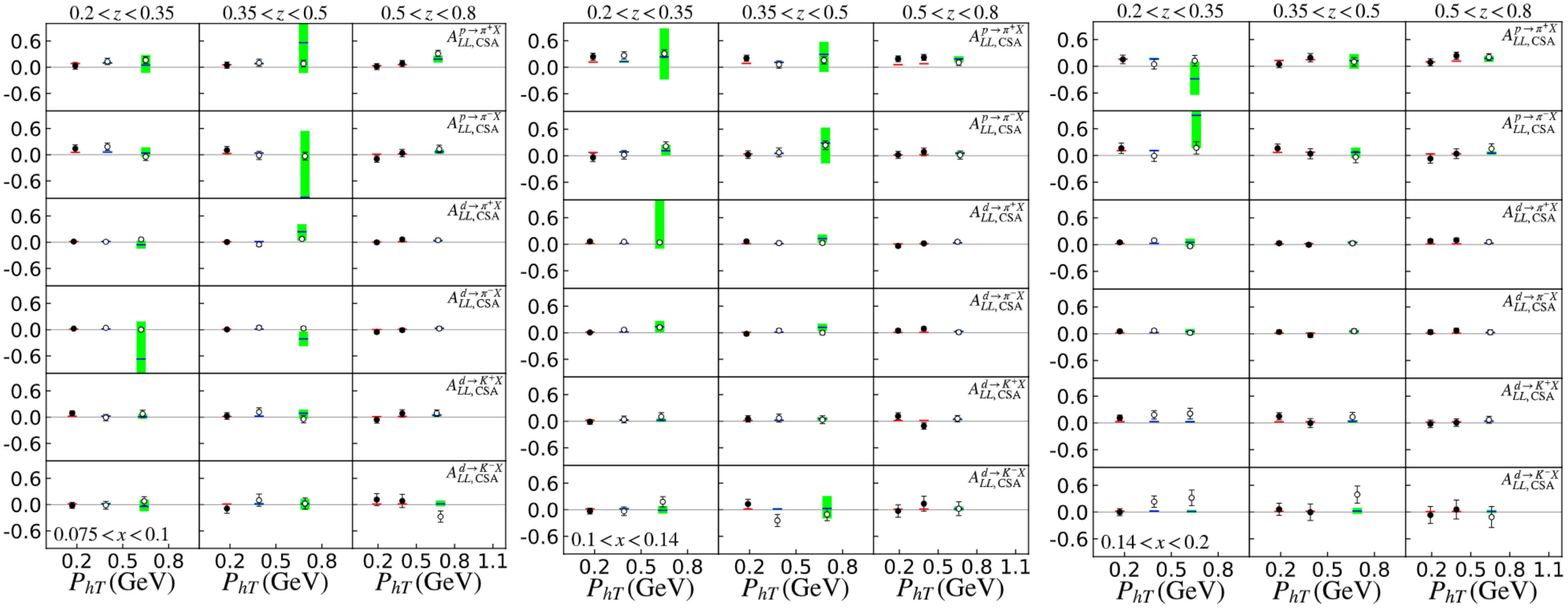
Comparison between extracted result and original CLAS data



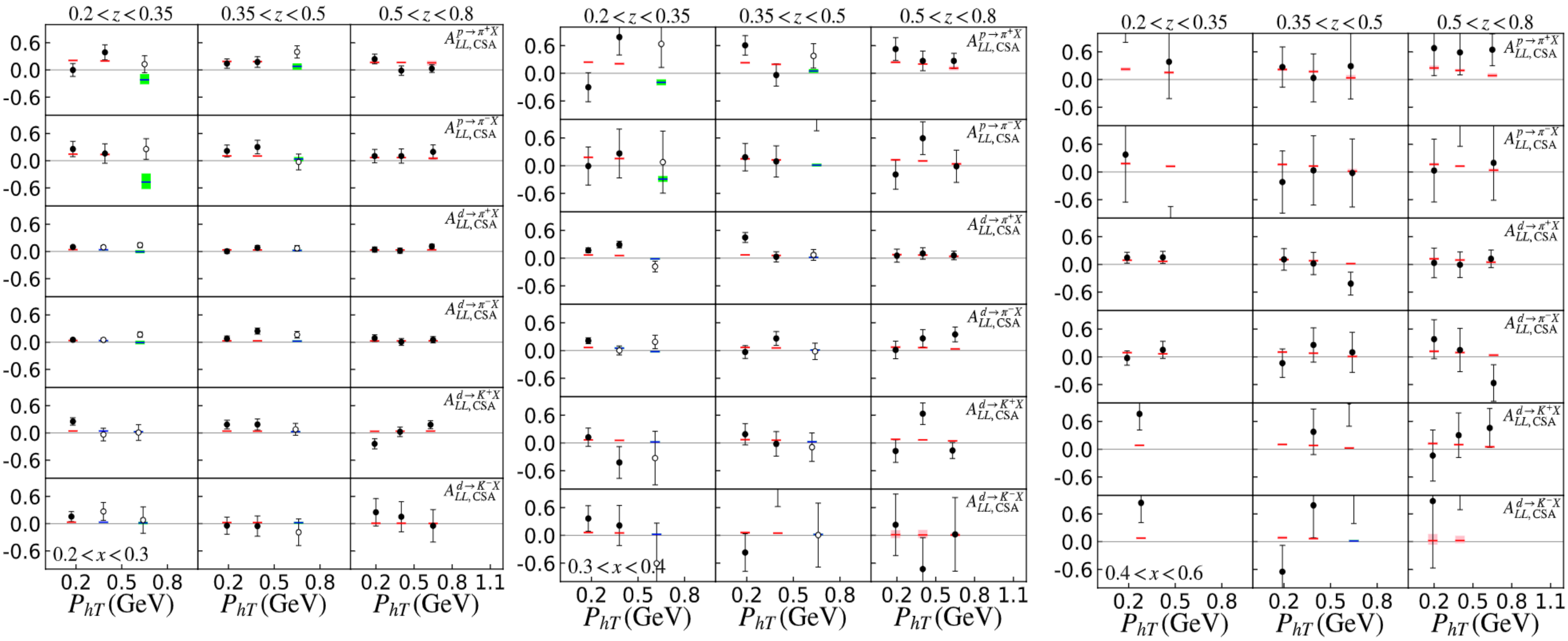
Comparison between extracted result and original HERMES data



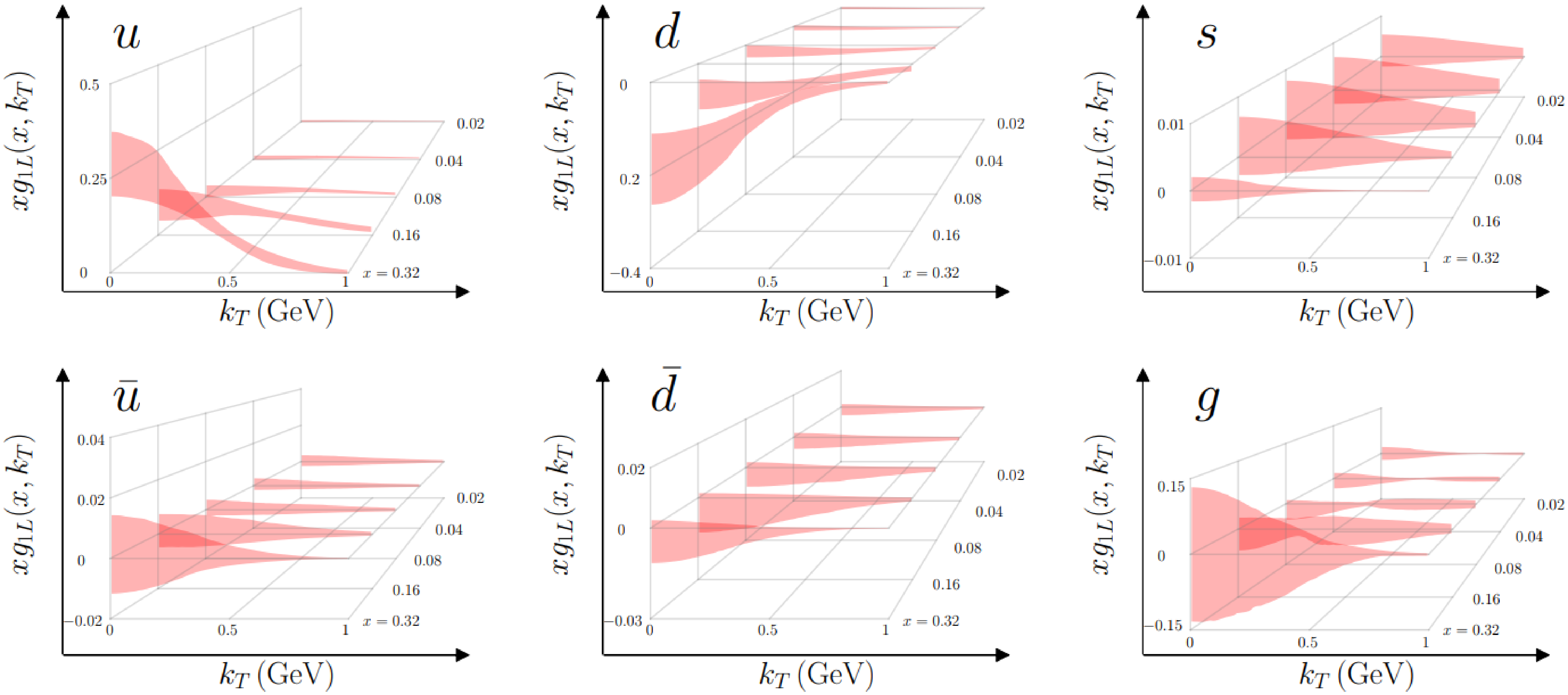
Comparison between extracted result and original HERMES data



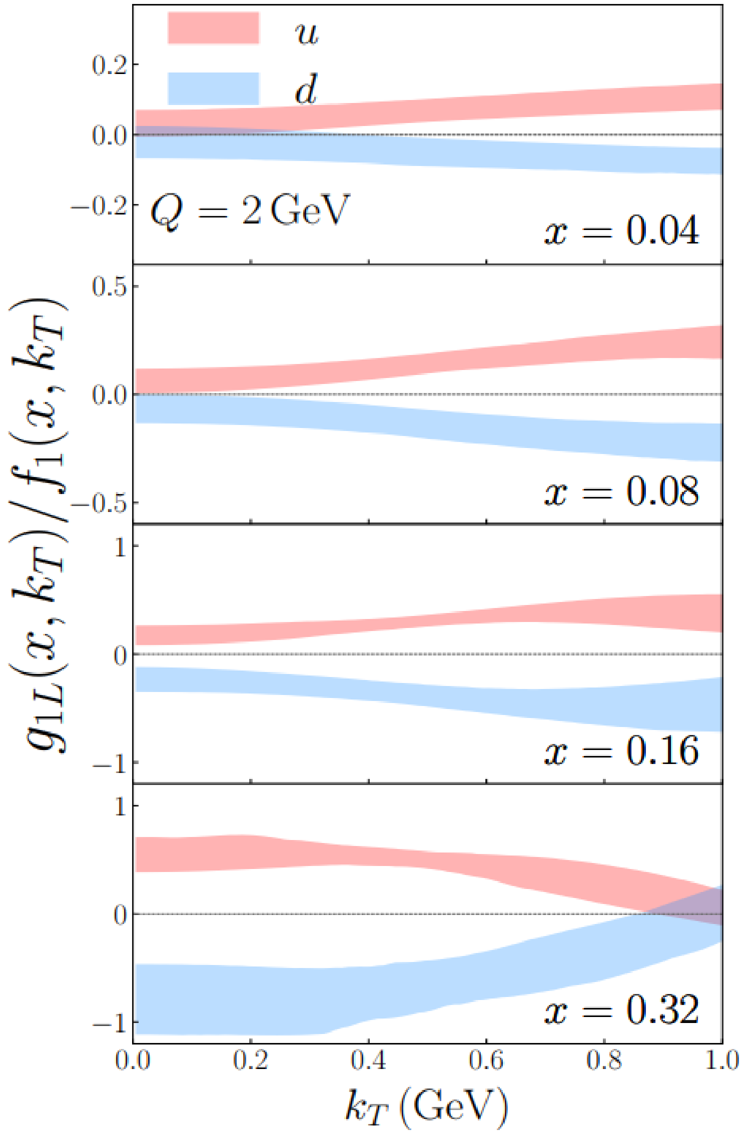
Comparison between extracted result and original HERMES data

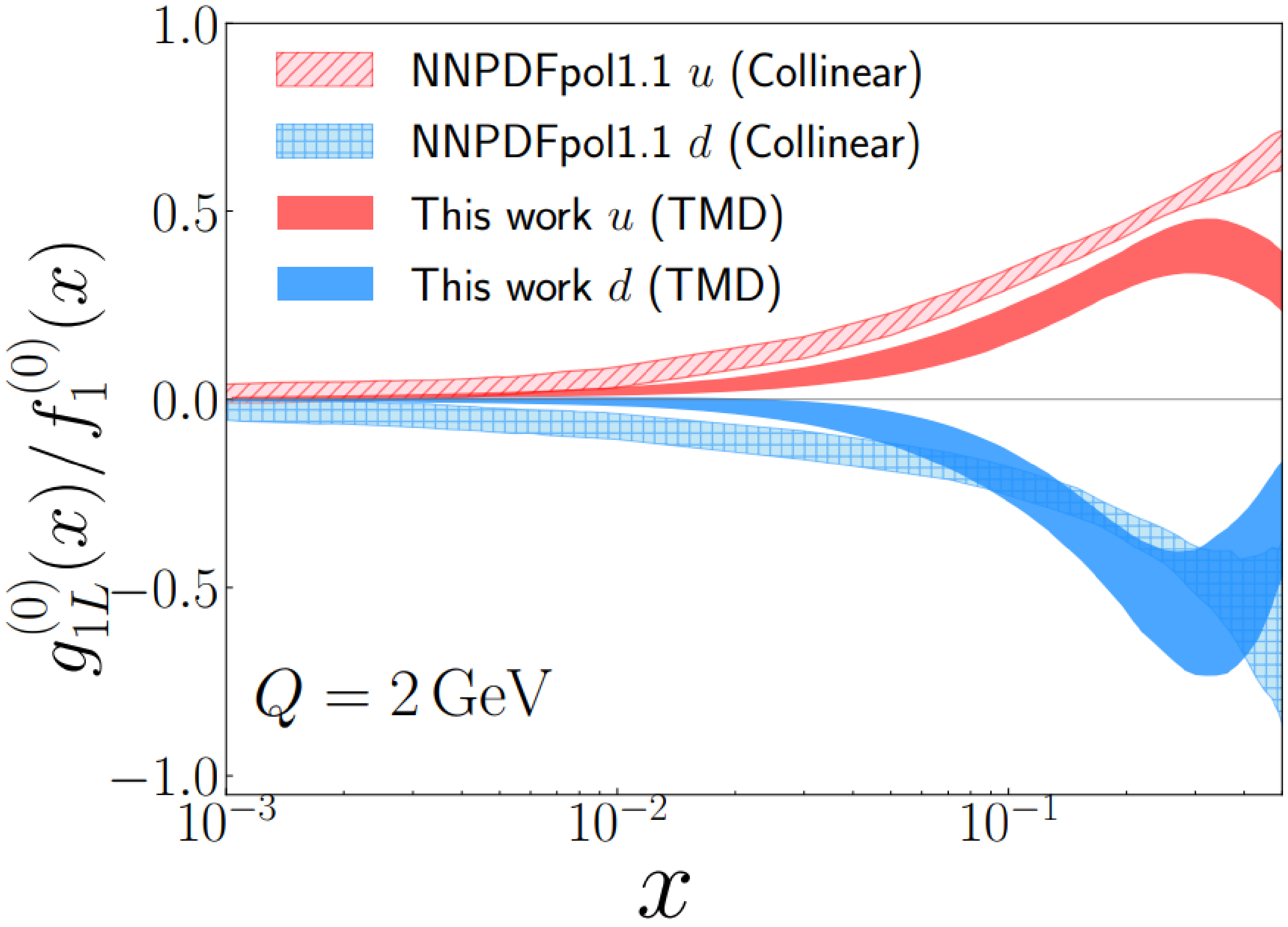


Result of $xg_{1L}(x, k_T)$



Polarization of up quark and down quark





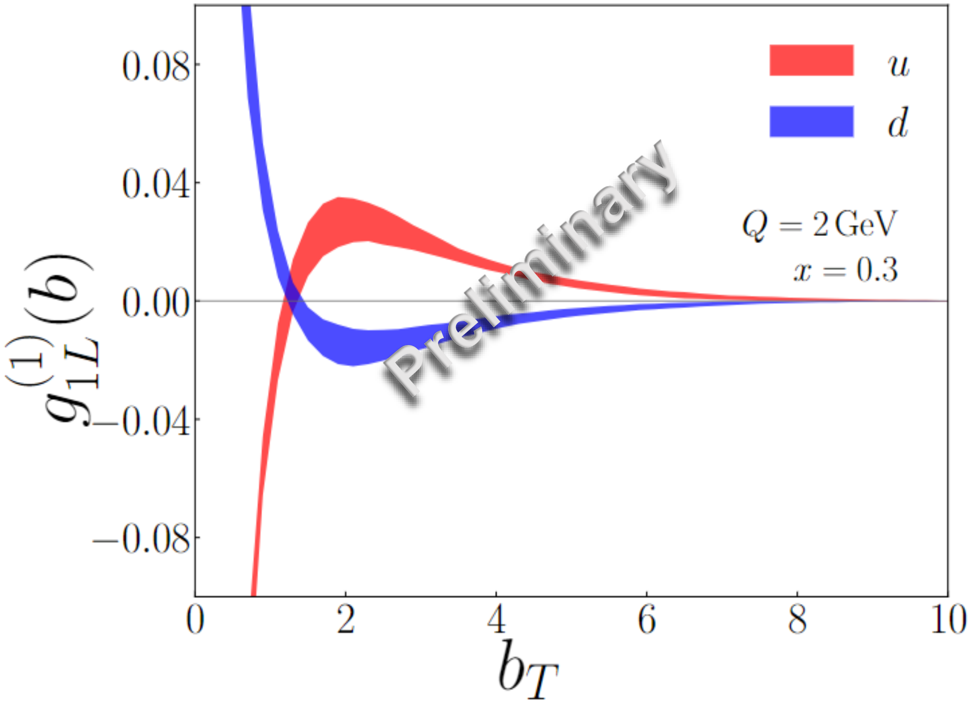
Explanation of cos(ϕ) modulation data

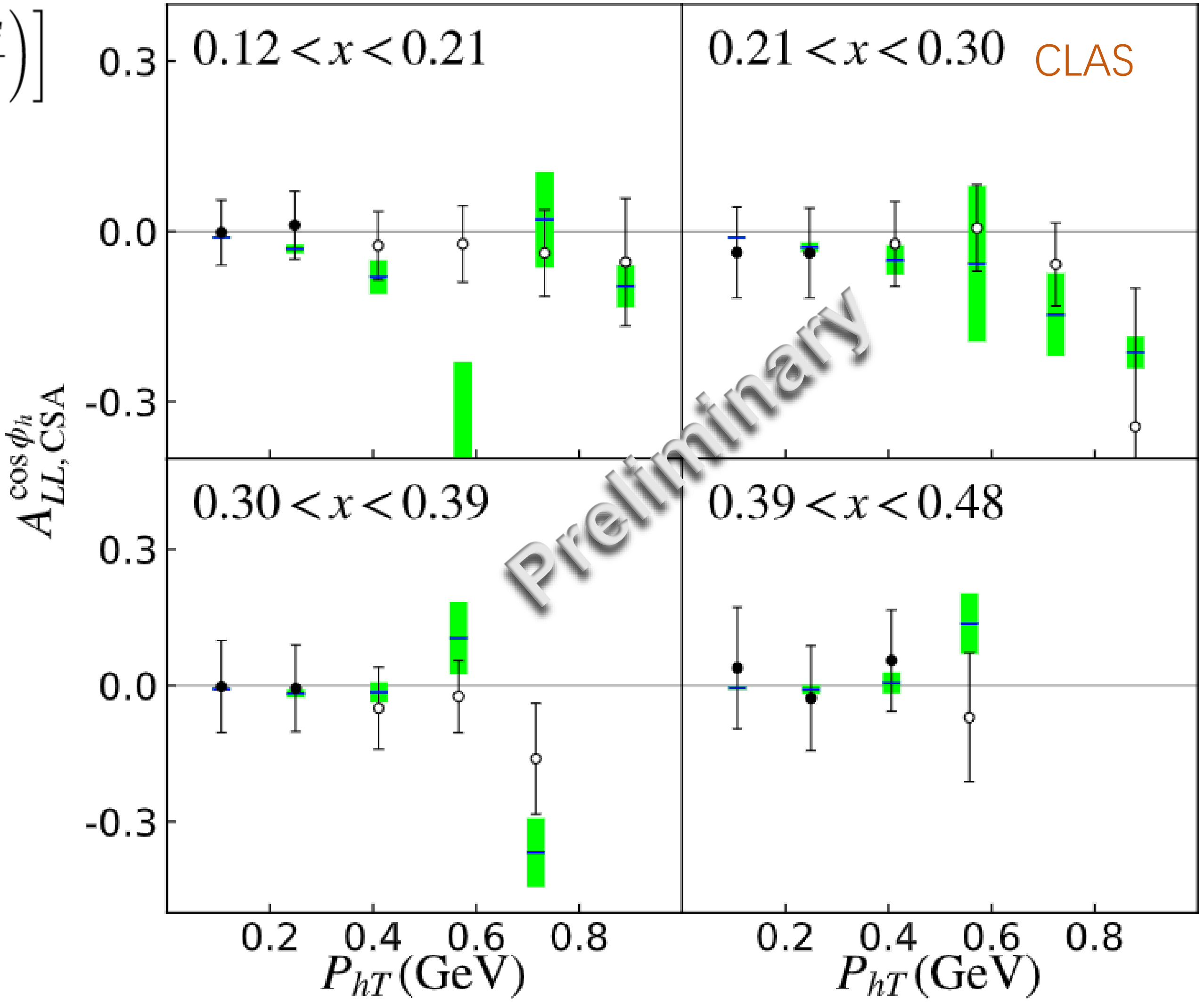
$$F_{LL}^{\cos\phi} = \frac{2M}{Q}\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M_h}\left(xe_L H_1^\perp - \frac{M_h}{M}g_{1L}\frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M}\left(xg_L^\perp D_1 + \frac{M_h}{M}h_{1L}^\perp\frac{\tilde{E}}{z}\right)\right]$$

$$\approx -\frac{2M}{Q}\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M}(g_{1L}D_1)\right]$$

$$= -\frac{2M^2}{Q}x\sum_q e_q^2\int_0^\infty \frac{b_T^2\,\mathrm{d}b_T}{2\pi}J_1\left(\frac{b_T P_{hT}}{z}\right)g_{1L,q\leftarrow H}^{(1)}(x,b_T)D_{1,q\to h}(z,b_T)$$

$$g_{1L}^{(n)}(b_T) = n!\left(\frac{-1}{M^2 b_T}\partial_{b_T}\right)^n g_{1L}(b_T)$$





5

Summary

Conclusions

Conclusions

1. We have extracted the TMD helicity functions with error bands from SIDIS data;

2. The x dependence of polarization is consistent with collinear helicity distribution;

3. Around the peak of x dependence, the polarization is concentrate on the low k_T region, which is consistent with Melosh Wigner rotation;

4. At low x region, we observe slightly increasing polarization, which imply the rich dynamics of QCD;

5. The TMD helicity extracted can also explain the $\cos(\phi)$ modulation of the DSA measurement of the SIDIS.

THANKS FOR LISTENING

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Collaborators: Tianbo Liu, Bo-qiang Ma, Peng Sun, Yuxiang Zhao

Date : 2024.11.11

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