Origin of hadron spin based on Lattice QCD study of the charmed hadrons

Yi-Bo Yang

With Fangcheng He and Jian Liang

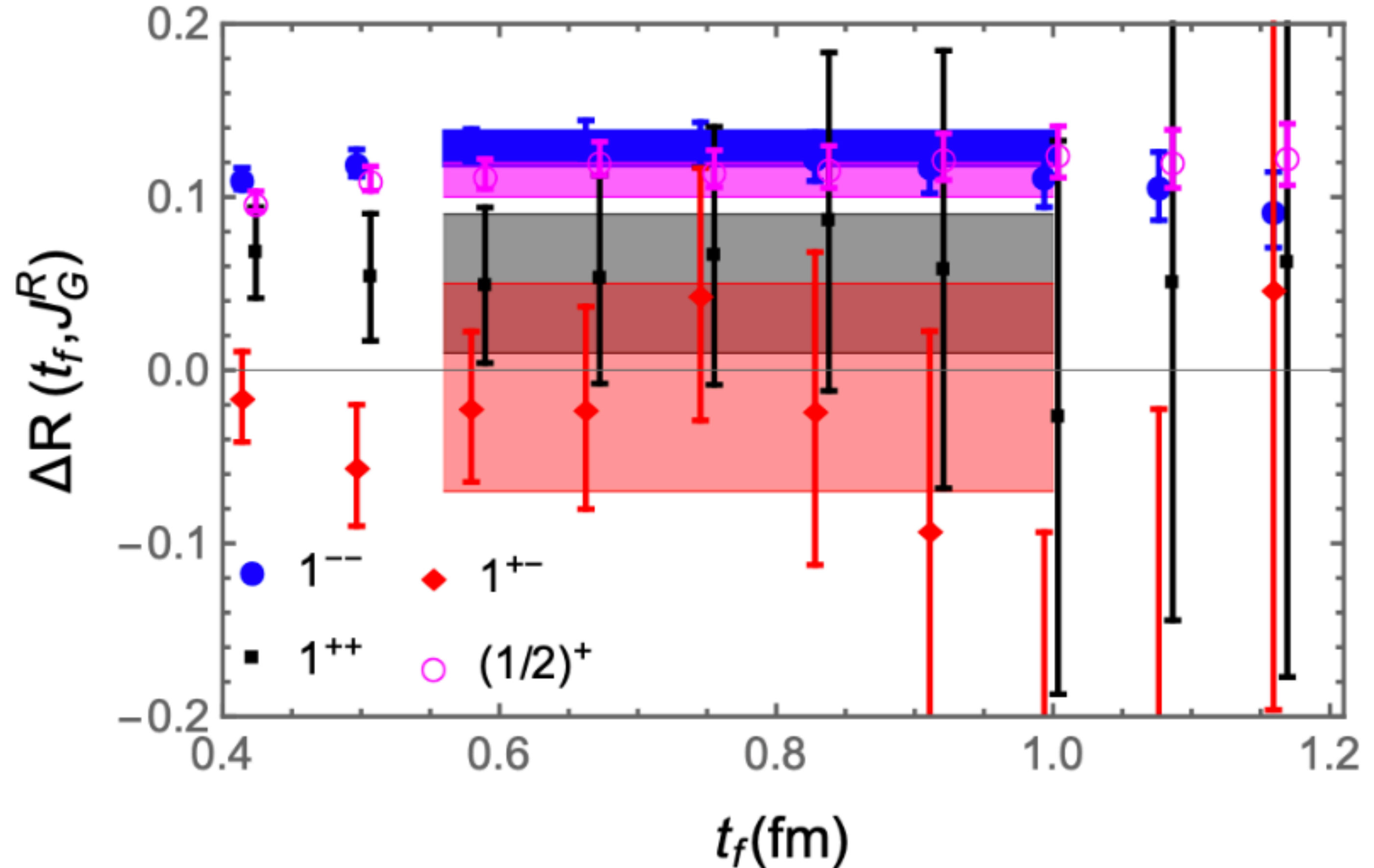
based on arXiv: 2410.08046

Outline

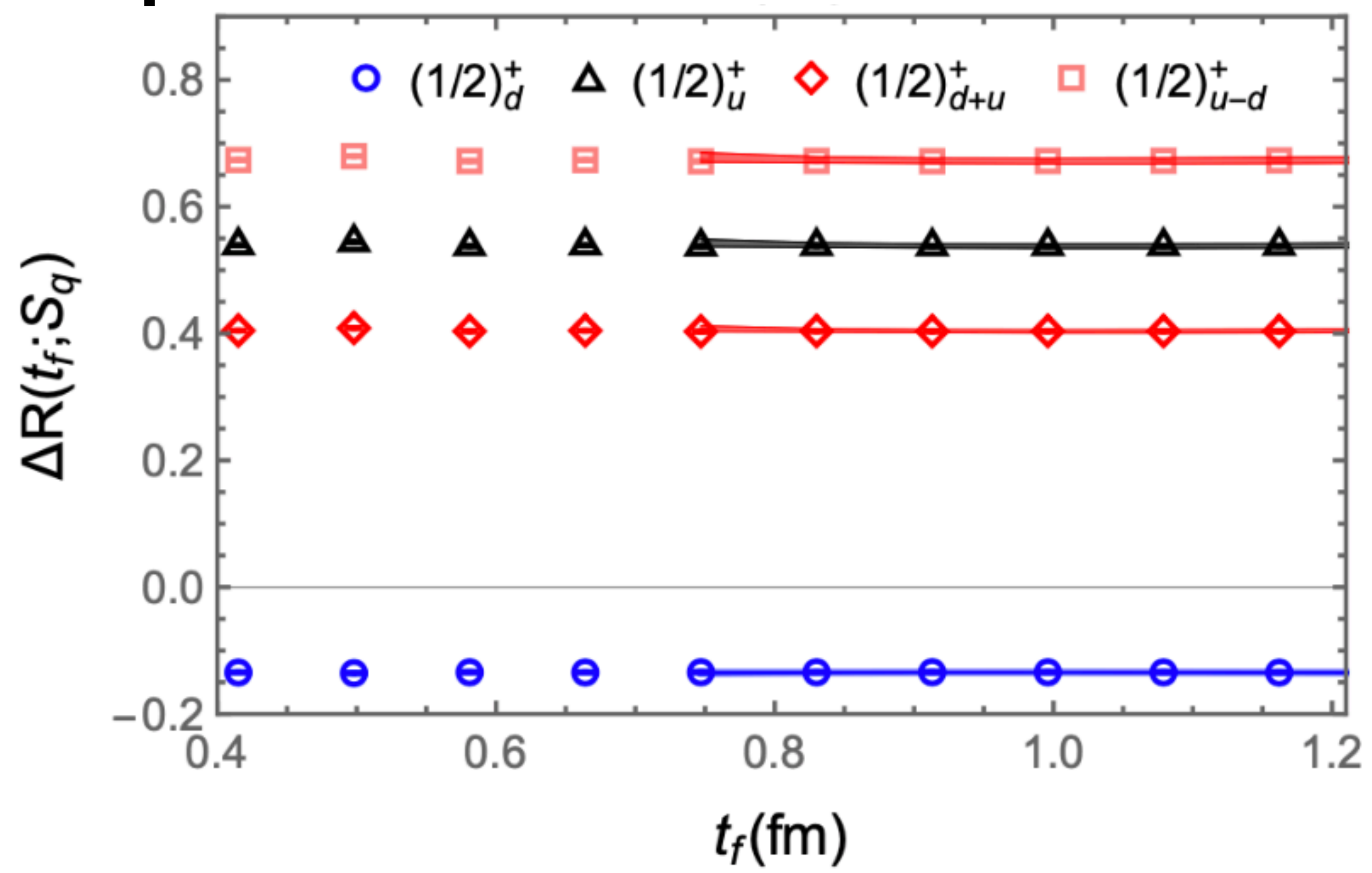
- **Hadron spin and quark model**

$$J = \frac{1}{2}\Delta q + L_q + J_G,$$

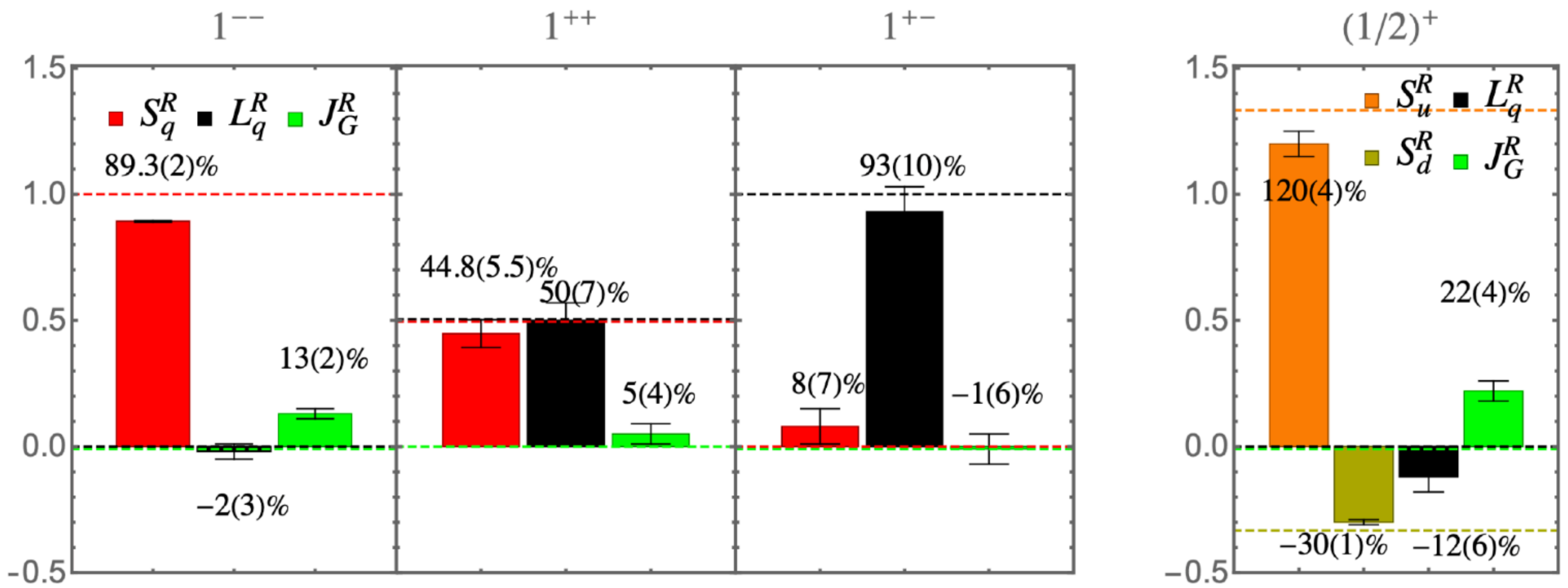
- Gluon AM and spin



- Quark spin



- Discussion



Hadron spin

Connections between decompositions

X. Ji, PRL78 (1997) 610

$$\vec{J} = \int d^3x\, \frac{1}{2}\,\overline{\psi}\,\vec{\gamma}\gamma^5\,\psi + \int d^3x \psi^\dagger \{\vec{x}\times(i\vec{D})\}\,\psi + \int d^3x 2\{\vec{x}\times \mathrm{Tr}[\vec{E}\times\vec{B}]\}$$

$$\int d^3x \psi^\dagger \{\vec{x}\times\vec{A}\}\,\psi$$

quark spin

Different definitions of the quark OAM

Further decomposition of the glue AM

$$\vec{J} = \int d^3x\, \frac{1}{2}\,\overline{\psi}\,\vec{\gamma}\gamma^5\,\psi + \int d^3x \psi^\dagger \{\vec{x}\times(i\vec{\nabla})\}\,\psi$$

$$+\int d^3x 2\mathrm{Tr}[\vec{E}\times\vec{A}] + \int d^3x 2\mathrm{Tr}[E^i\vec{x}\times\vec{\nabla}A^i]$$

glue spin

glue OAM

R. L. Jaffe and A. V. Manohar, NPB337(1990)509

Proton spin

Decomposition from experiment

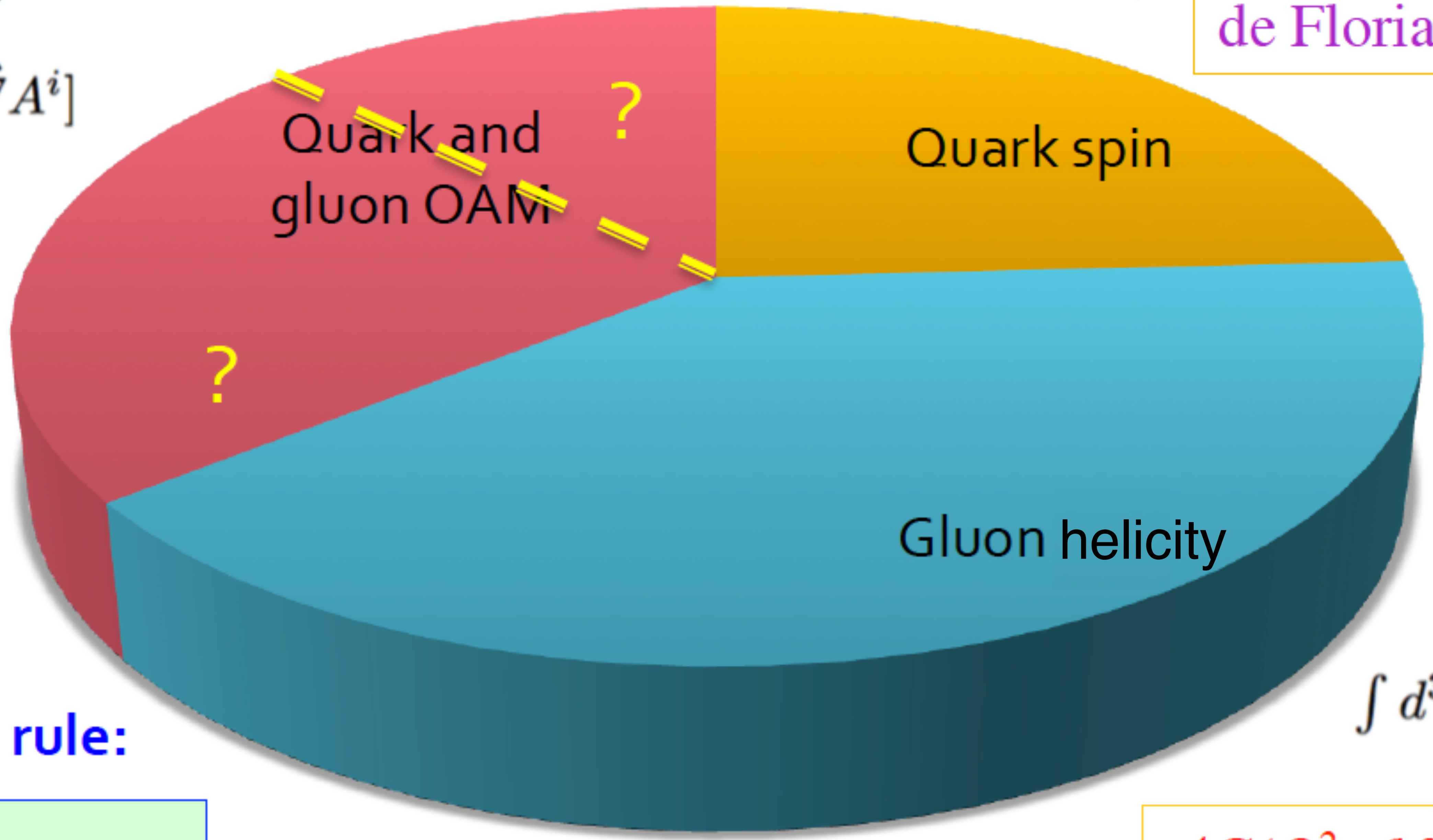
Longitudinal proton spin structure

$\int d^3x \psi^\dagger \{\vec{x}\times(i\vec{\nabla})\}\psi$

$+ \quad \int d^3x 2\mathrm{Tr}[E^i \vec{x}\times\vec{\nabla}A^i]$

$\int d^3x \frac{1}{2}\overline{\psi}\vec{\gamma}\gamma^5\psi$

ΔΣ(Q^2=10 GeV^2*) = 0.242,*
de Florian et al., 2009



SLAC
HERMES (DESY)
COMPASS (CERN)
JLab
RHIC

$\int d^3x 2\mathrm{Tr}[\vec{E}\times\vec{A}]$

ΔG(Q^2=10 GeV^2*) ~ 0.2,*
de Florian et al., 2014

Naïve spin sum rule:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + l_q^z + l_q^z$$

Proton spin

Decomposition of quark polarizations

	u-quark		d-quark		s-quark		gluon
J_z	**1/2**	**-1/2**	**1/2**	**-1/2**	**1/2**	**-1/2**	
Quark model	5/3	1/3	1/3	2/3	0	0	0
Exp.	1.43(2)	0.57(2)	0.29(2)	0.71(2)	-0.02(2)	0.02(2)	~0.4

$$q^{1/2} + q^{-1/2} = g_V, \quad q^{1/2} - q^{-1/2} = g_A \equiv \Delta q$$

- The SU(6) quark model: $\Delta u \to 4/3$, $\Delta d \to -1/3$, $\Delta s \to 0$, $\Delta g \to 0$

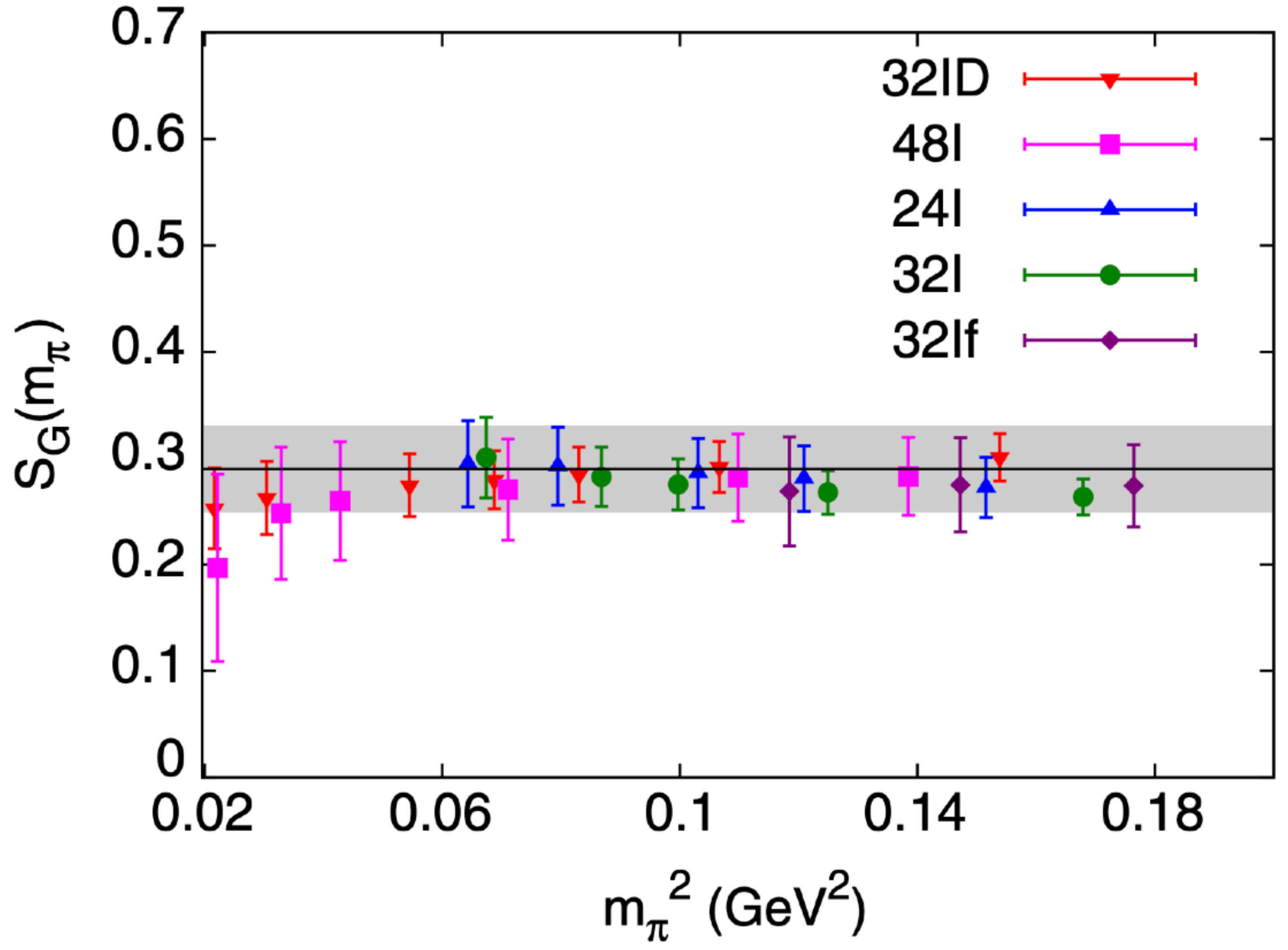
  See e,g, B.Q. Ma, et.al., EPJA 12(2001)353

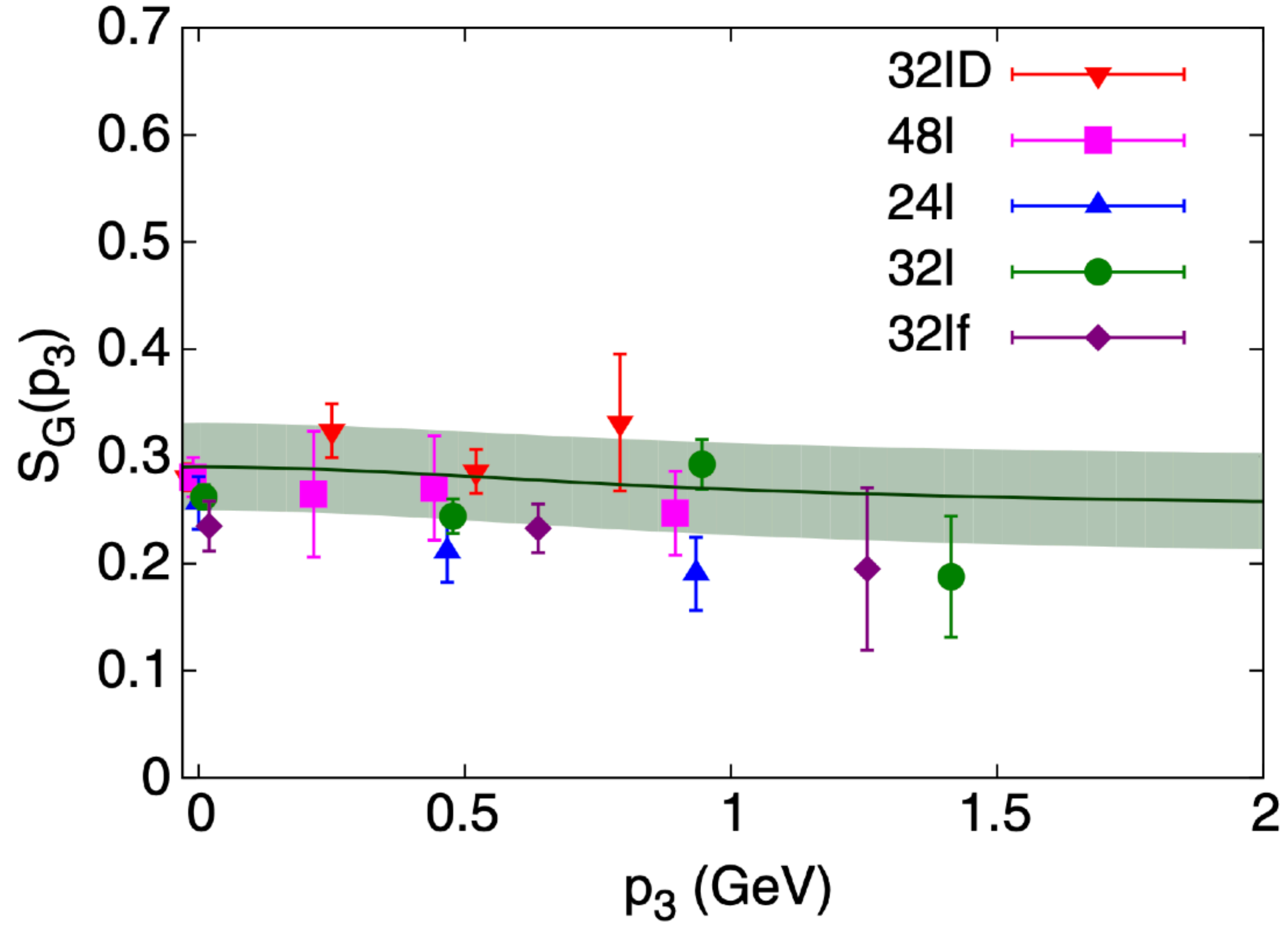
- The polarized neutron decay: $\Delta u - \Delta d$ **= 1.2723(23);**

- Lattice and experimental PDF fit: $\Delta u \sim 0.86$, $\Delta d \sim -0.41$, $\Delta s \sim -0.04$, $\Delta g \sim 0.4$.

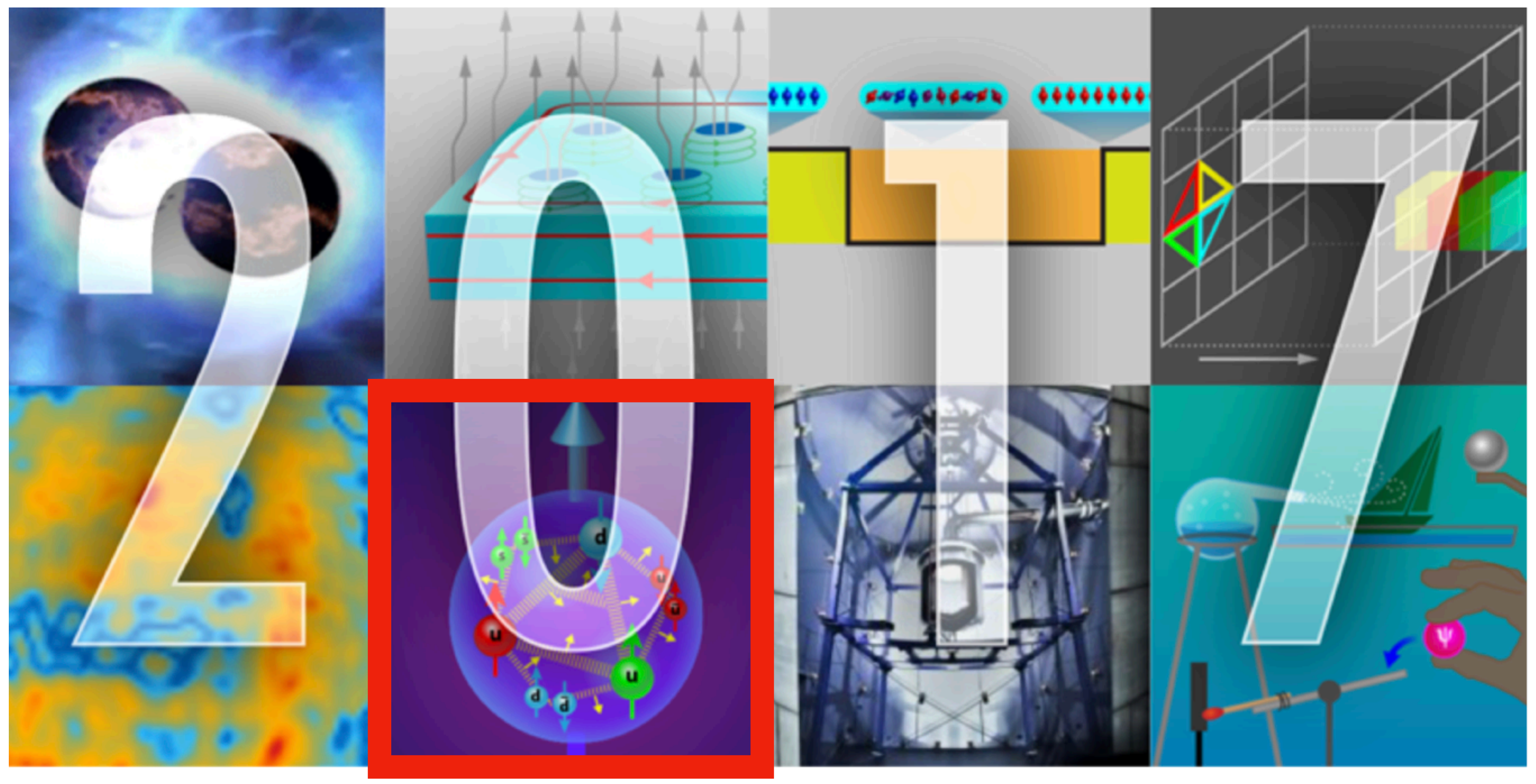
Proton spin

gluon polarizations





- Define the gluon spin ExA under the Coulomb gauge;
- Boost it to the large momentum limit to estimate the gluon helicity.



Selected for a Viewpoint in *Physics*
PRL **118,** 102001 (2017) PHYSICAL REVIEW LETTERS week ending 10 MARCH 2017

Glue Spin and Helicity in the Proton from Lattice QCD

Yi-Bo Yang,[1] Raza Sabbir Sufian,[1] Andrei Alexandru,[2] Terrence Draper,[1] Michael J. Glatzmaier,[1] Keh-Fei Liu,[1] and Yong Zhao[3,4]

(χQCD Collaboration)

YBY, R. Sufian, et. al., χQCD collaboration, PRL118(2017) 042001,
ViewPoint and Editor's suggestion

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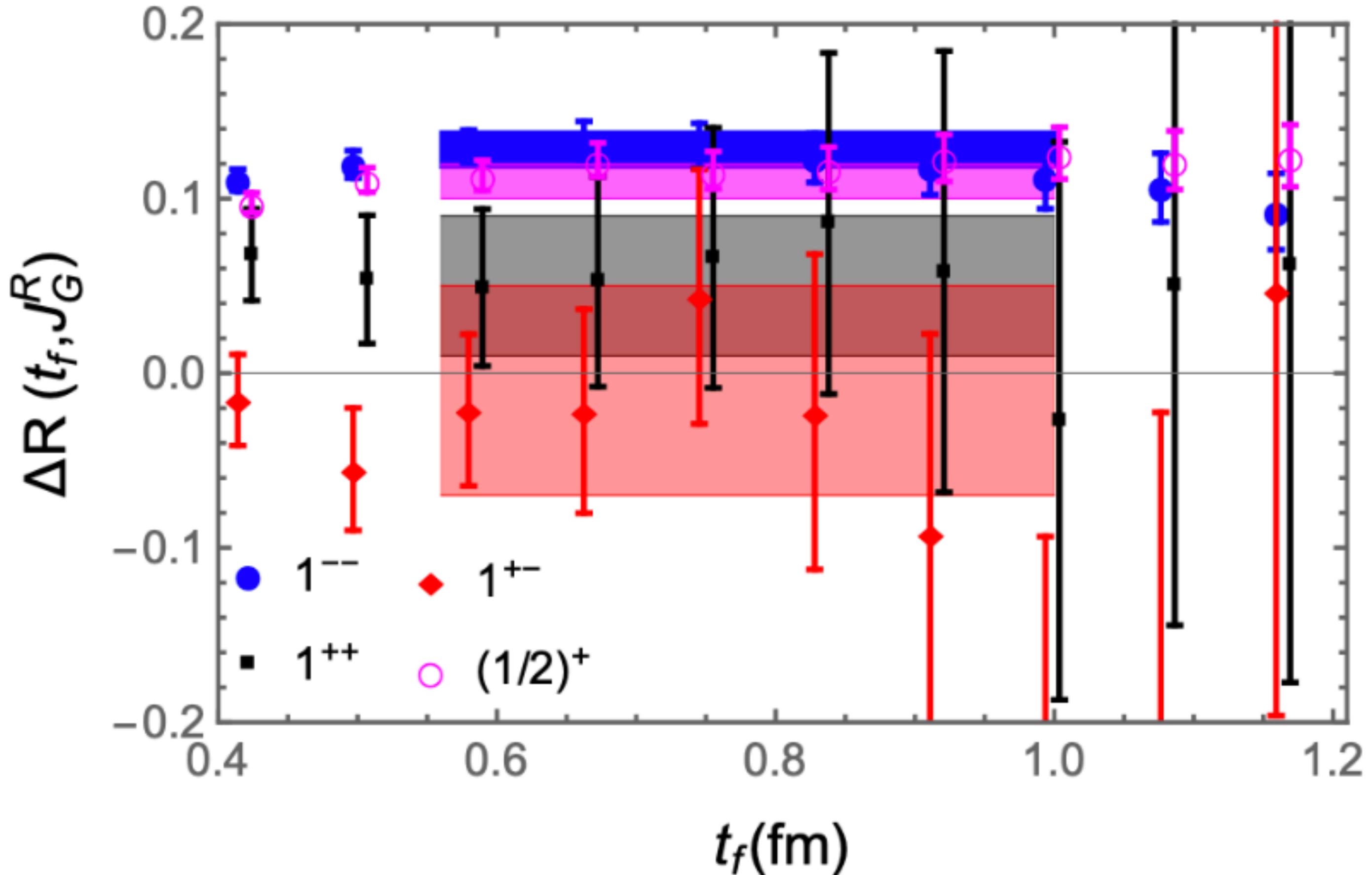
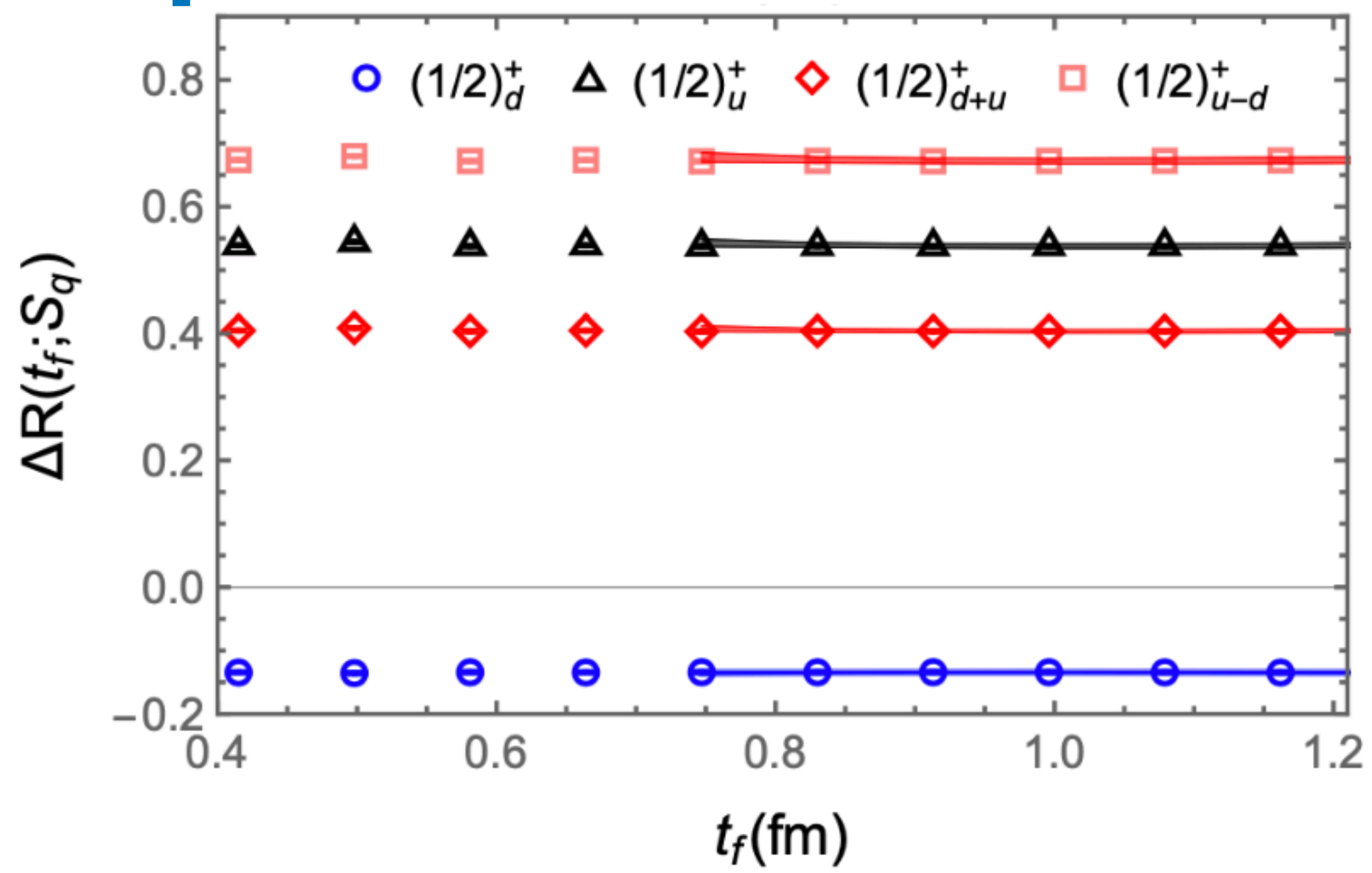
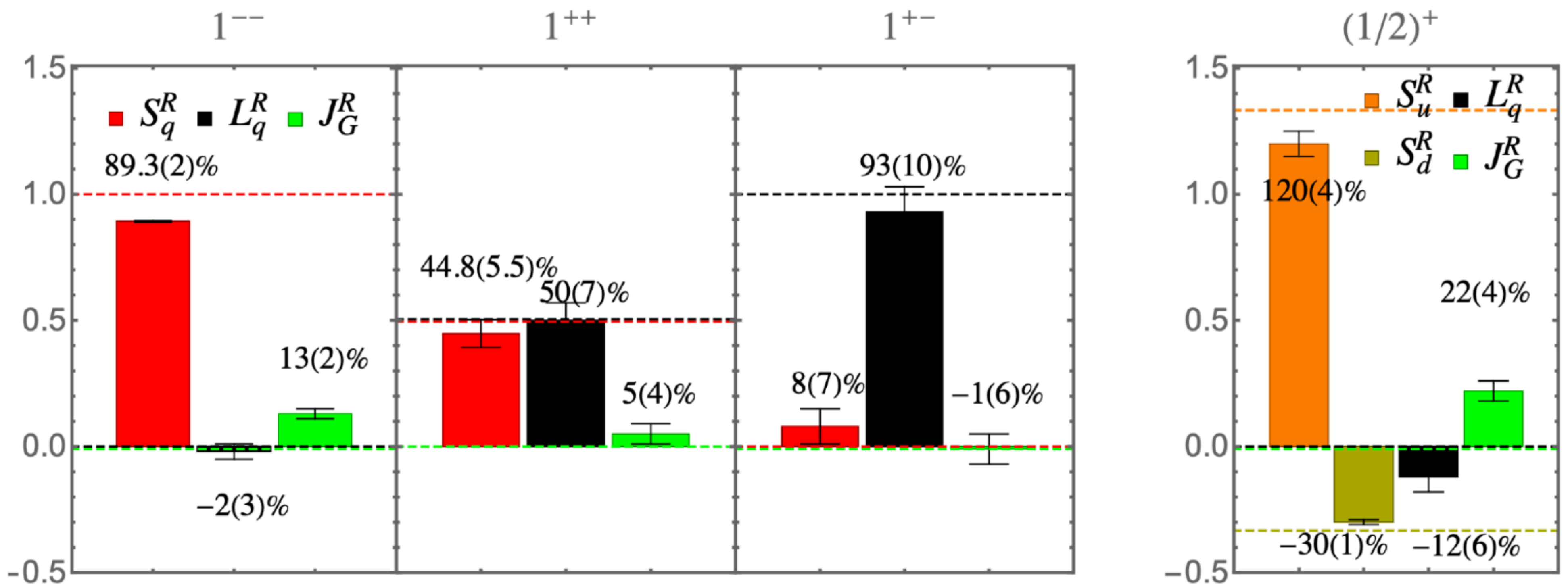


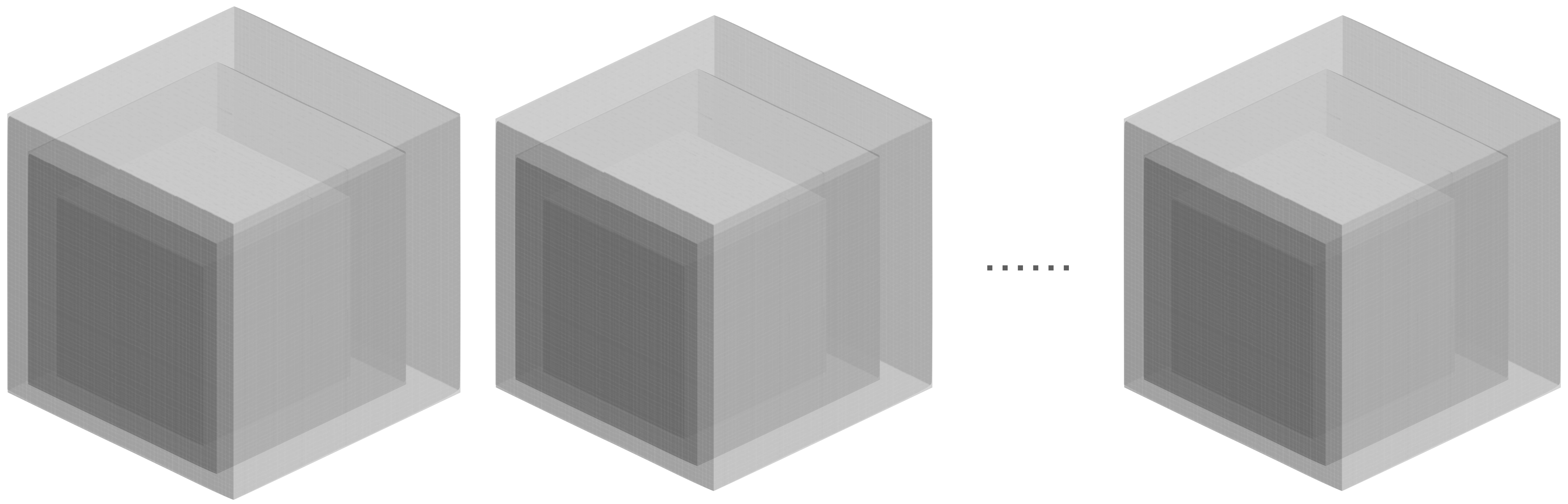
- **Quark spin**

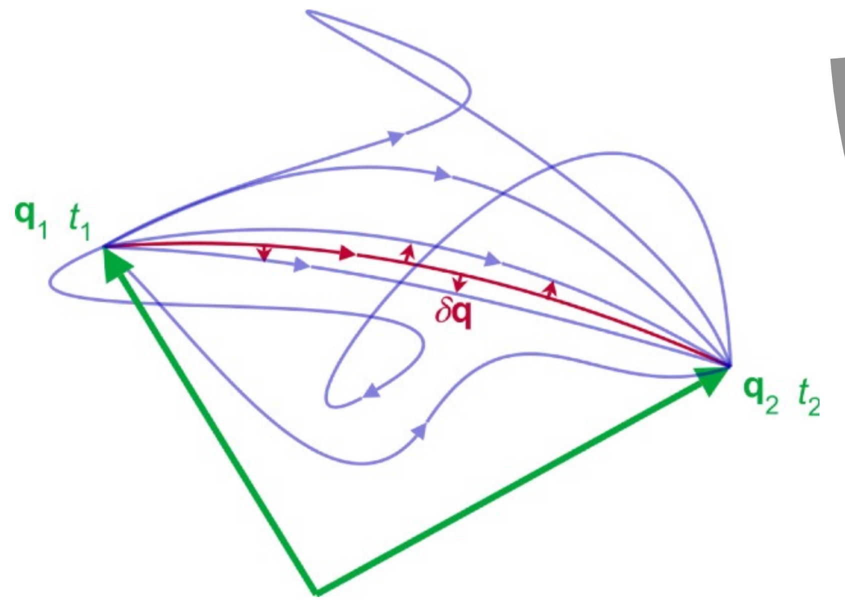


- Discussion



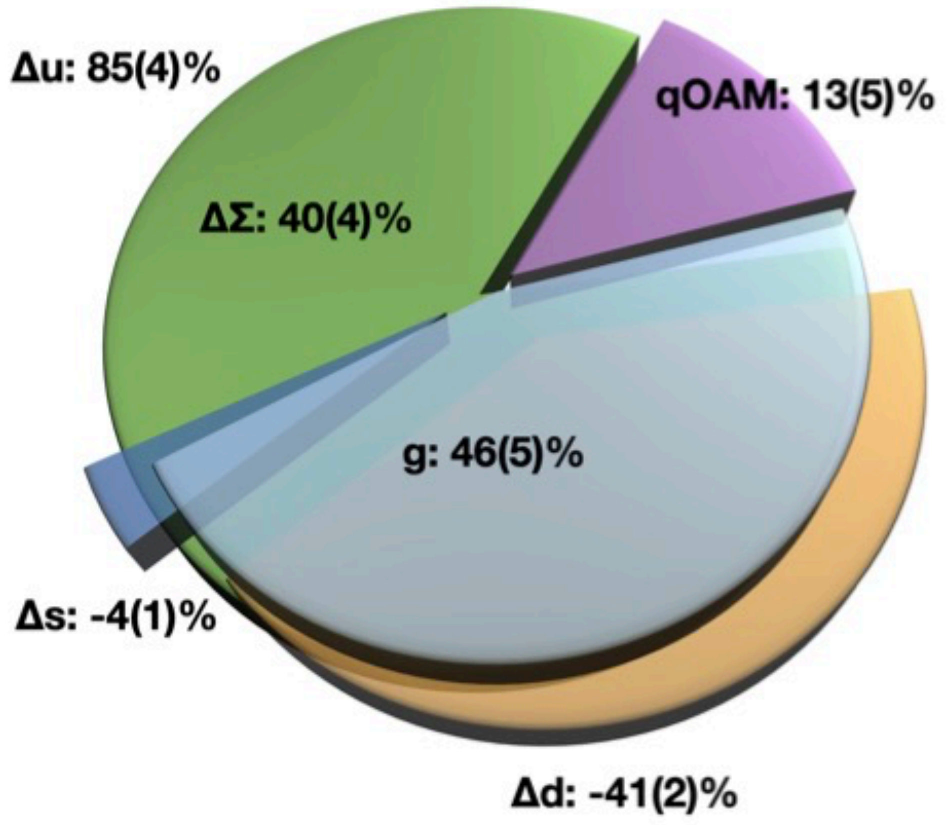
Lattice QCD





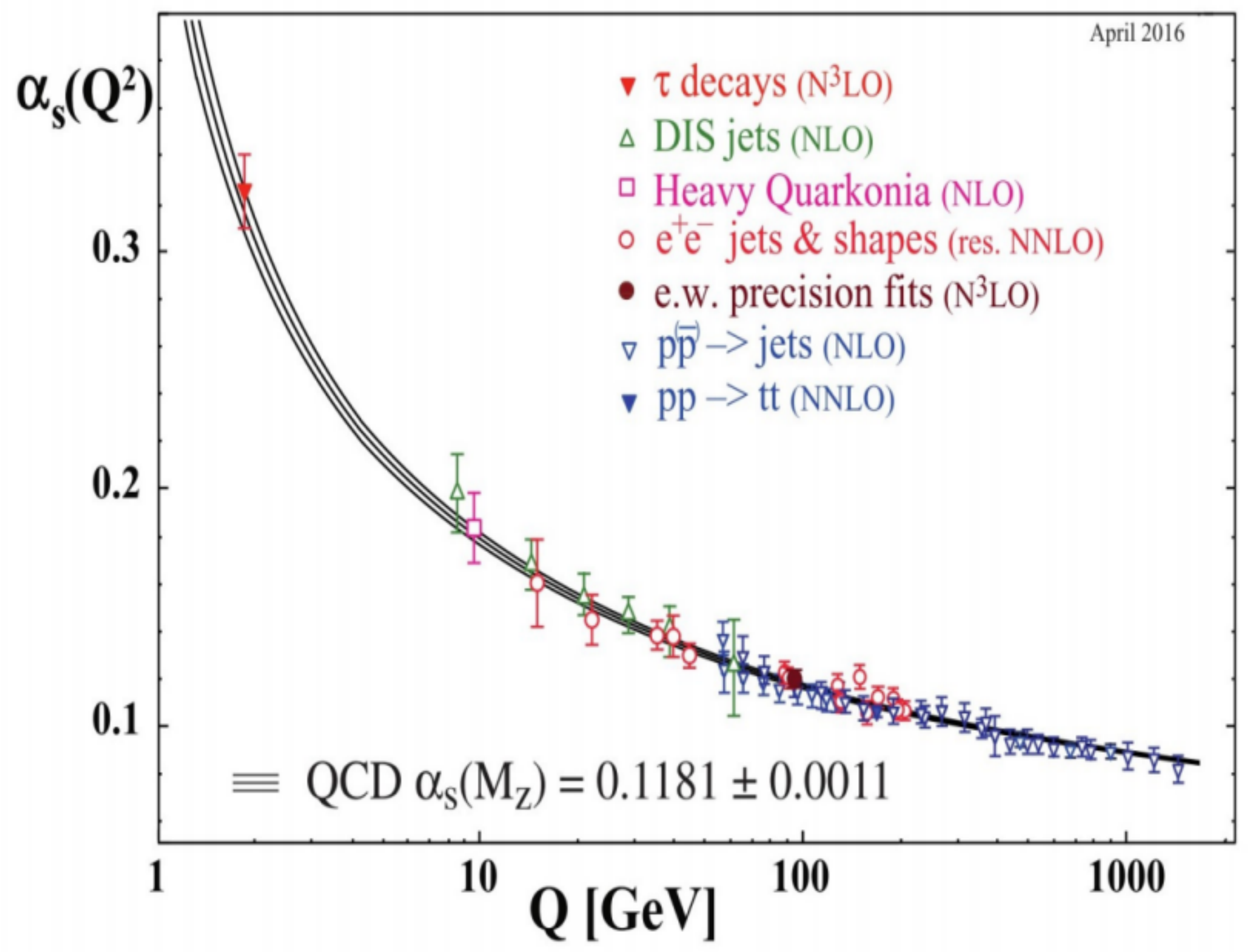
$$\langle \mathcal{O} \rangle = \frac{\int [\mathrm{d}A\mathrm{d}\psi]\mathcal{O}(A,\psi)e^{-\int \mathrm{d}^4x\mathcal{L}(A(x),\psi(x))}}{\int [\mathrm{d}A\mathrm{d}\psi]e^{-\int \mathrm{d}^4x\mathcal{L}(A(x),\psi(x))}}$$

$$= \frac{\sum_{i=1}^{N} \mathcal{O}(A_i, S_\psi(A_i))}{N} + \mathcal{O}(\frac{1}{\sqrt{N}})$$



G. Wang, et.al., χQCD, PRD106(2022) 014512

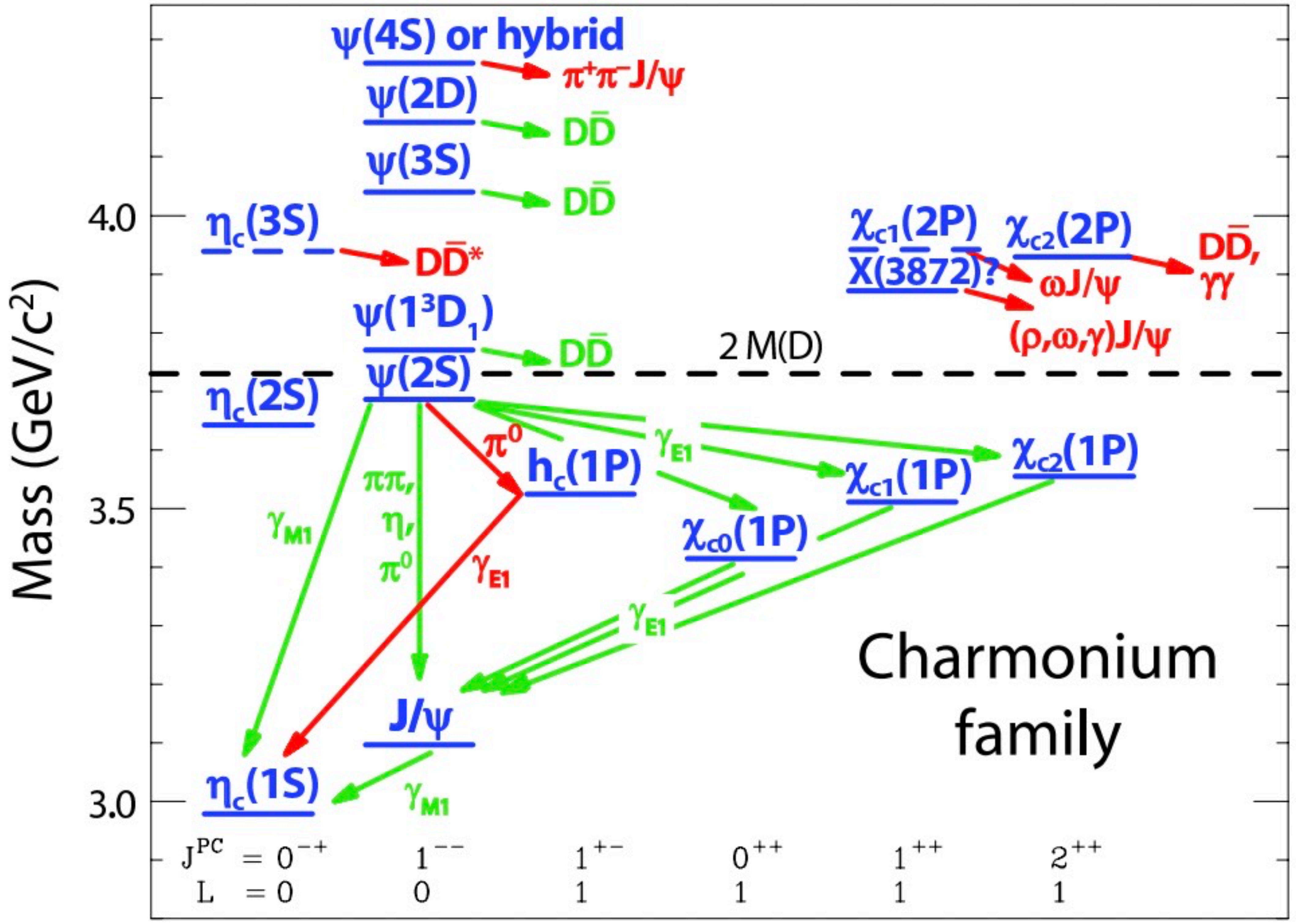
and QCD



- QCD is non-perturbative at the hadron scale;
- Lattice QCD can provide first principle predictions on the hadron spin decomposition, **as functions of quark mass**.

Quark spin

charmonium system



Quantum numbers					Mass	Width
n	L	J^{PC}	$n^{2S+1}L_J$	Name	(MeV)	(MeV[a])
1	0	0^{-+}	1^1S_0	$\eta_c(1S)$	2980.4 ± 1.2	25.5 ± 3.4
1	0	1^{--}	1^3S_1	J/ψ	3096.916 ± 0.011	93.4 ± 2.1 keV
1	1	0^{++}	1^3P_0	$\chi_{c0}(1P)$	3414.76 ± 0.35	10.4 ± 0.7
1	1	1^{++}	1^3P_1	$\chi_{c1}(1P)$	3510.66 ± 0.07	0.89 ± 0.05
1	1	2^{++}	1^3P_2	$\chi_{c2}(1P)$	3556.20 ± 0.09	2.06 ± 0.12
1	1	1^{+-}	1^1P_1	$h_c(1P)$	3525.93 ± 0.27	<1

Charmonium under DD threshold can be treated as stable particles:

- Quark model gives $J_G = 0$, $J = \frac{1}{2}\Delta q + L_q$.
- Both quark and anti-quark contribute equally to the Δq and canceled the factor 1/2;
- The $J = 0$ case can not be decomposed since one can not make the quark to be polarized along "that of hadron".

$$\Delta q_H = \langle H(\uparrow) | \mathscr{A}_z | H(\uparrow) \rangle$$

Quark spin

charmonium spin decomposition

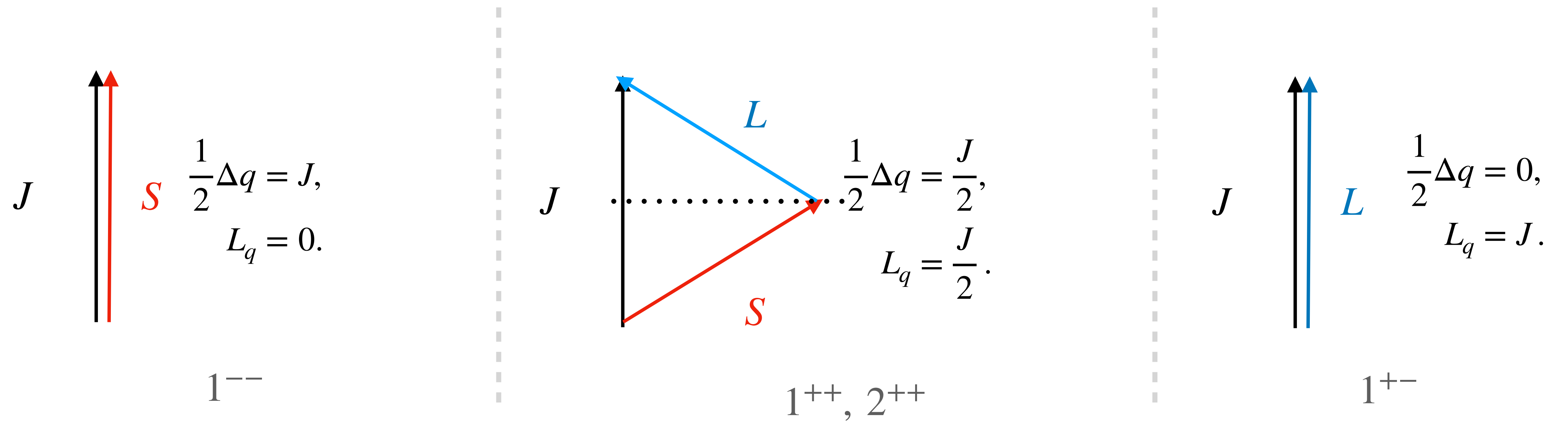
Contribution from quark spin/orbital angular momentum should be understood as the weighted average of quantized values:

- The 1^{--} case: $L_q = 0$, and then $J_H = \langle S_q \rangle_H = \frac{1}{2} \langle \Delta q \rangle_H$;
- The J^{++} case: $L_q = S_q$, and then $\langle S_q \rangle_H = \langle L_q \rangle_H = \frac{1}{2} J$;
- The 1^{+-} case: $S_q = 0$, and then $J = \langle L_q \rangle$.

1^{--}	S=1	L=0
$0^{++}/1^{++}/2^{++}$	S=1	L=1
1^{+-}	S=0	L=1

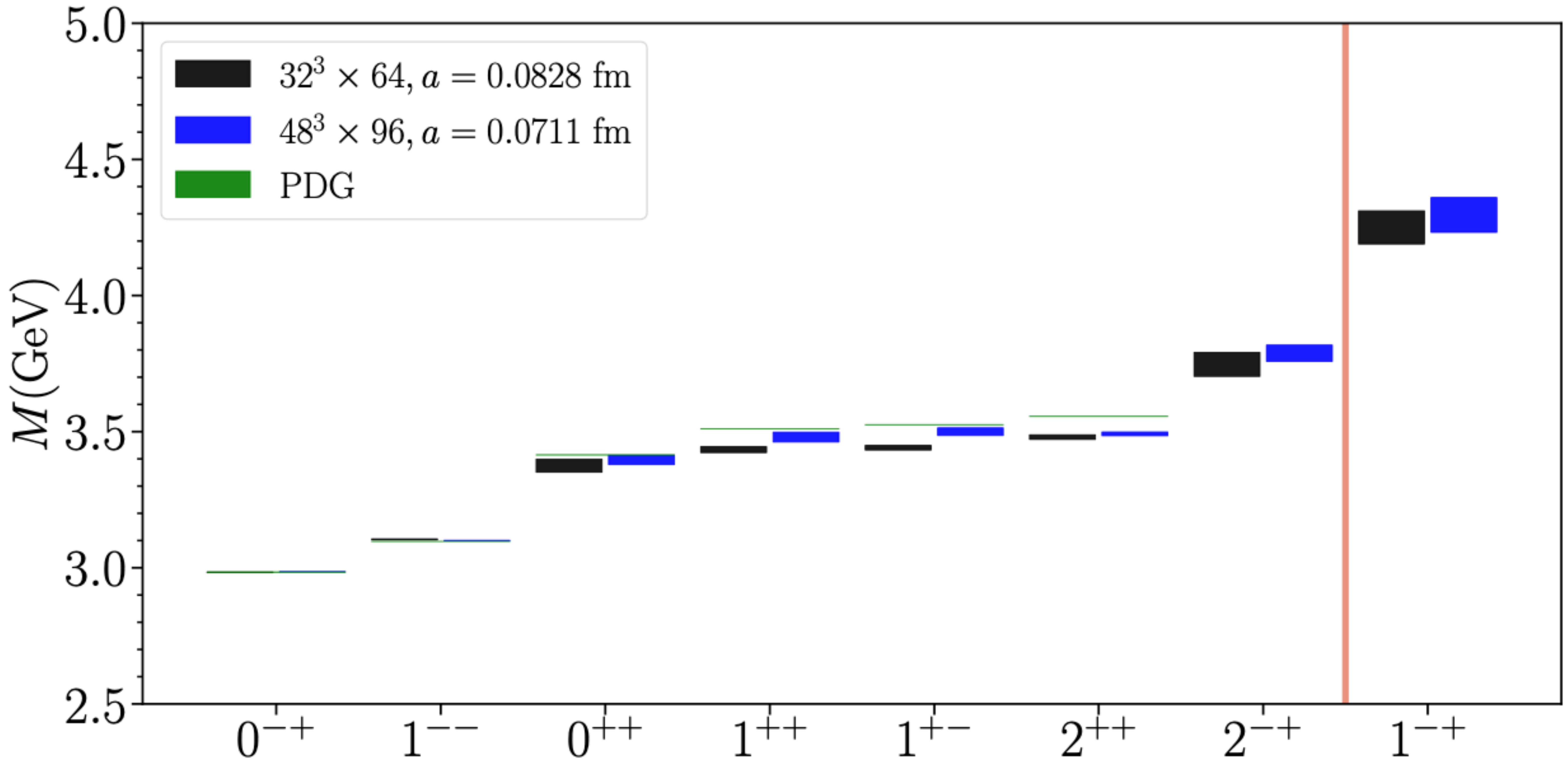
$P = (-1)^{L+1}$

$C = (-1)^{L+S}$



Quark spin

ensemble	$L^3 \times T$	a (fm)	m_π (MeV)	$m_c a$	$N_{\rm cfg}$
32I	$32^3 \times 64$	0.0828(3)	300	0.493	305



Simulation setup

Overlap fermion on 2+1 flavor DWF+Iwasaki configuration from RBC collaboration:

- Chiral fermion which avoid the systematic uncertainty from additive chiral symmetry breaking;
- Tune the charm quark mass using the physical J/ψ mass;
- Predictions of P-wave charmonium masses agree with PDG with in 2%.

W. Sun, et.al., χQCD, PRD103(2021) 094503

Quark spin

and charmonium spin with J=1

Taking the quark spin $S_q = \sum_x \bar{q}(x)\gamma_z\gamma_5 q(x)$ along the z-direction as example, the correlation functions of the hadron with given J_z can be rewritten into those using different Lorentz components:

$$V_+ = \frac{1}{\sqrt{2}}(V_x + iV_y)$$
$$V_0 = V_z$$
$$V_- = \frac{1}{\sqrt{2}}(V_x - iV_y)$$

$$\langle V_+ | S_q | V_+ \rangle = -\langle V_- | S_q | V_- \rangle \neq 0$$
$$\langle V_0 | S_q | V_0 \rangle = 0$$

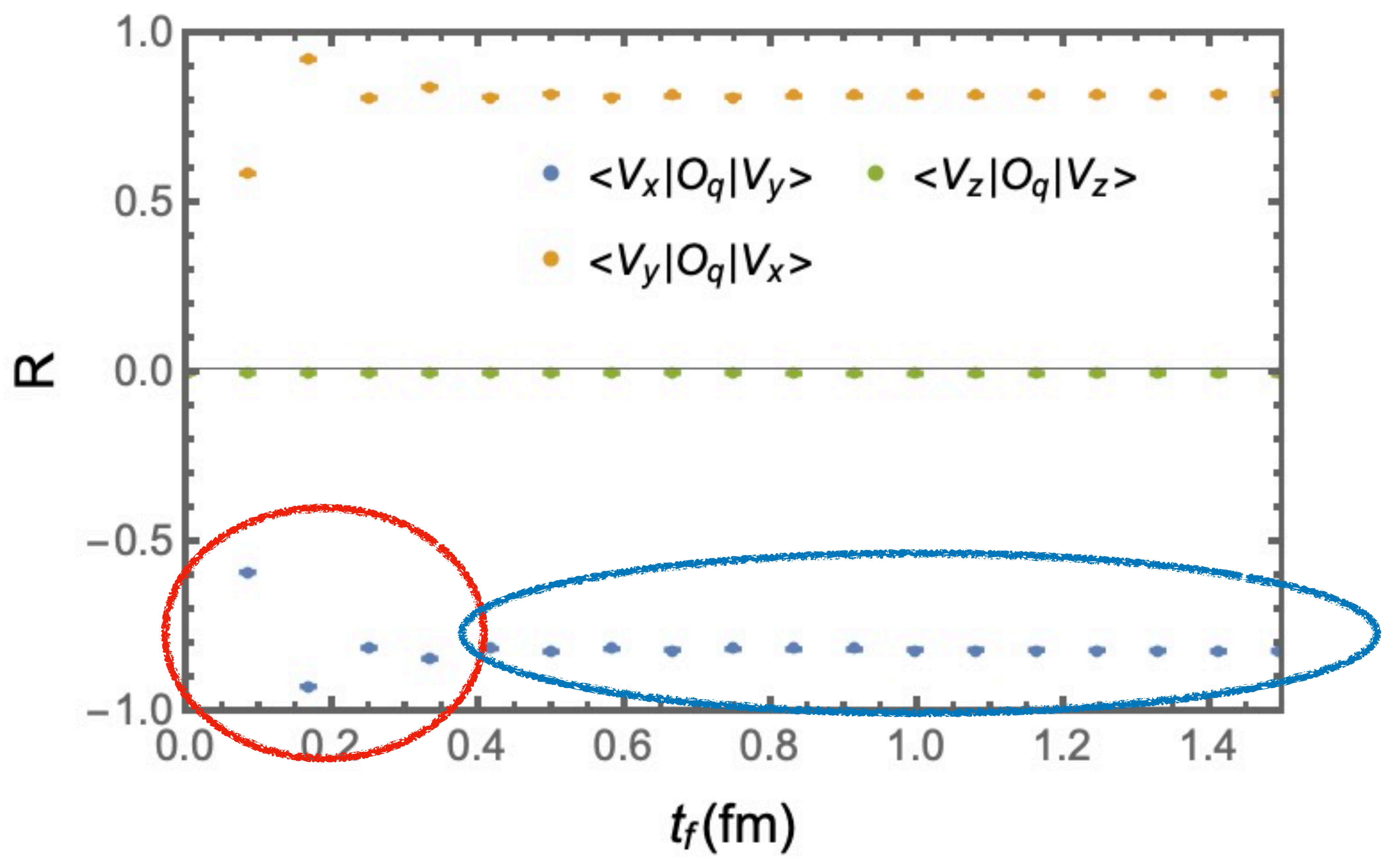
$$\langle V_x | S_q | V_y \rangle = -\langle V_y | S_q | V_x \rangle \neq 0$$
$$\langle V_z | S_q | V_z \rangle = 0$$

The numerical results suggest that the excited state contaminations are highly suppressed at $t_f \geq 0.5$ fm.

$$R(t_f, O) = \frac{\langle SC_3(t_f, O)\rangle}{\langle C_2(t_f)\rangle} - \frac{\langle SC_3(t_f-1, O)\rangle}{\langle C_2(t_f-1)\rangle} = \langle H|O|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Matrix element at the ground state

The contamination of excited state



Quark spin

and charmonium spin with J=2

The $J = 2$ case includes more combinations, while most of them vanish except:

- $\langle T_2^y | S_q | T_2^x \rangle$ needed by $\langle T_{J_z=1} | S_q | T_{J_z=1} \rangle$;
- $\langle E^a | S_q | T_2^z \rangle$ needed by $\langle T_{J_z=2} | S_q | T_{J_z=2} \rangle$.

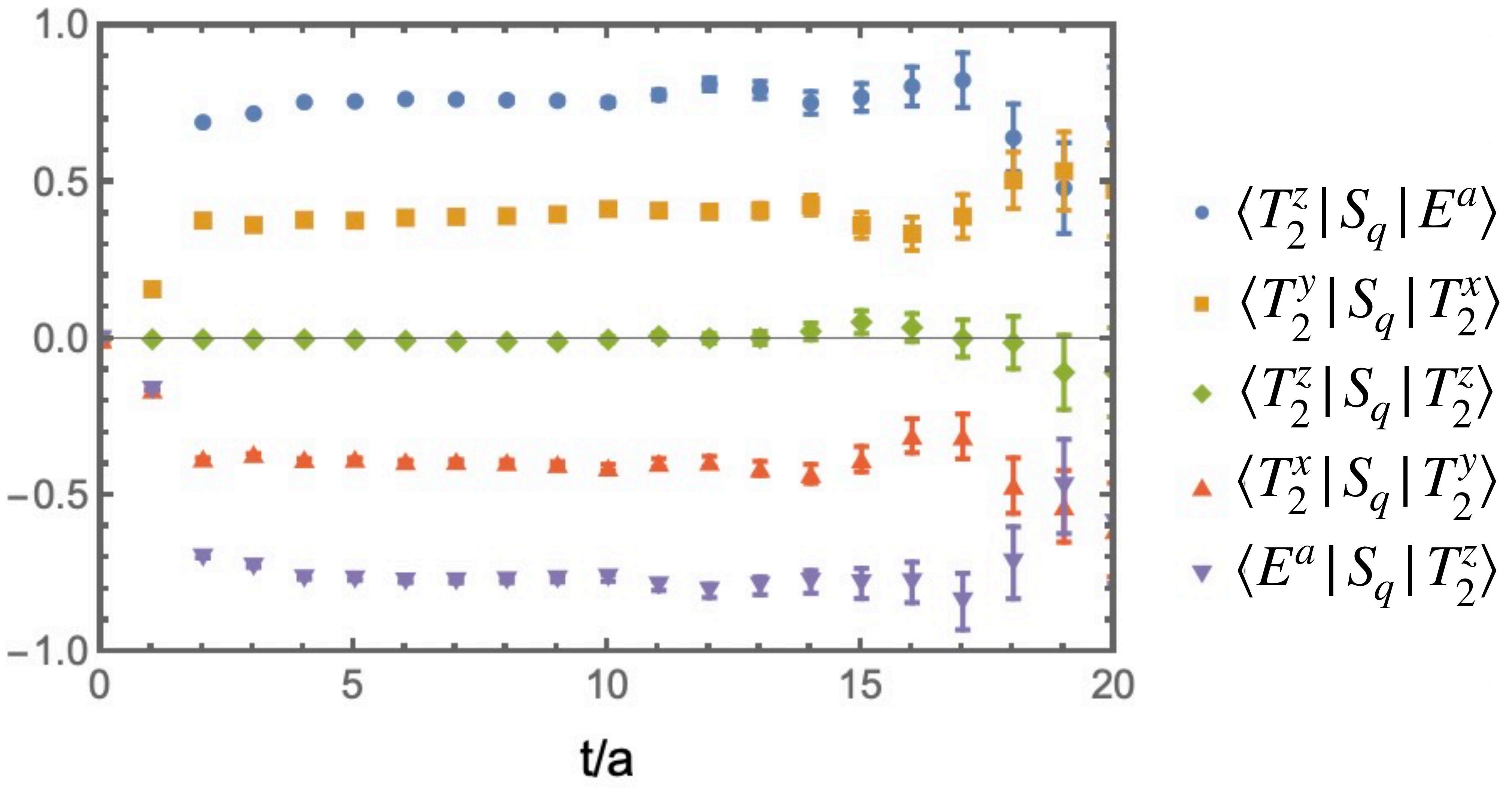
T_2^x	$\lvert\epsilon_{1jk}\rvert \gamma^j D^k/\sqrt{2}$
T_2^y	$\lvert\epsilon_{2jk}\rvert \gamma^j D^k/\sqrt{2}$
T_2^z	$\lvert\epsilon_{3jk}\rvert \gamma^j D^k/\sqrt{2}$
E^a	$Q_{1jk}\gamma^j D^k$
E^b	$Q_{2jk}\gamma^j D^k$

$$\mathbb{Q}_{111} = \frac{1}{\sqrt{2}};\ \mathbb{Q}_{122} = -\frac{1}{\sqrt{2}};\ \mathbb{Q}_{211} = -\frac{1}{\sqrt{6}};$$
$$\mathbb{Q}_{222} = -\frac{1}{\sqrt{6}};\ \mathbb{Q}_{233} = \frac{2}{\sqrt{3}}.$$

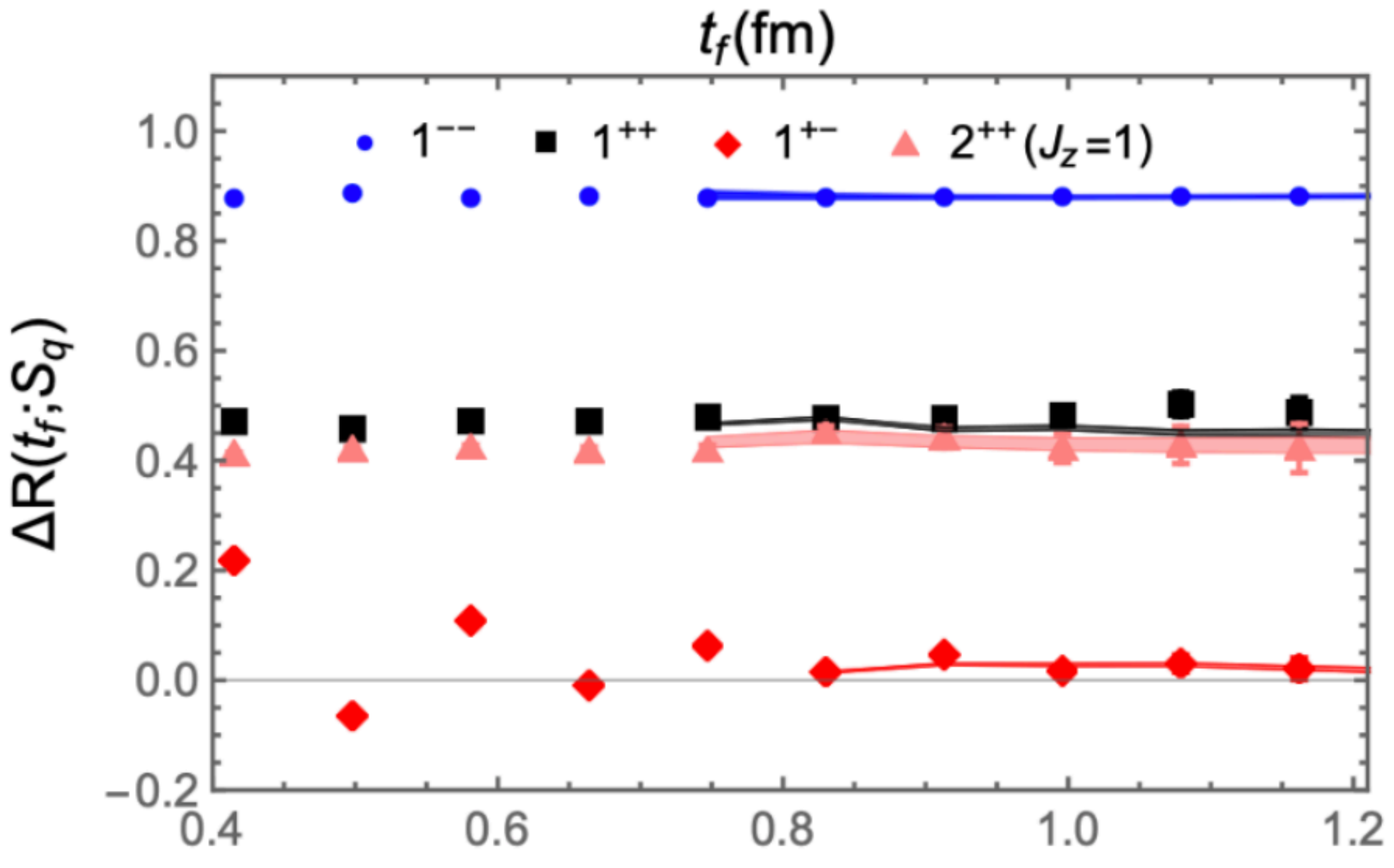
$T_{J_z=2}$	$(iT_2^z + E^a)/\sqrt{2}$
$T_{J_z=1}$	$(iT_2^y + T_2^x)/\sqrt{2}$
$T_{J_z=0}$	E^b
$T_{J_z=-1}$	$(-iT_2^y + T_2^x)/\sqrt{2}$
$T_{J_z=-2}$	$(-iT_2^z + E^a)/\sqrt{2}$

$$\langle T_{J_z=2} | S_q | T_{J_z=2} \rangle = 2\langle T_{J_z=1} | S_q | T_{J_z=1} \rangle$$

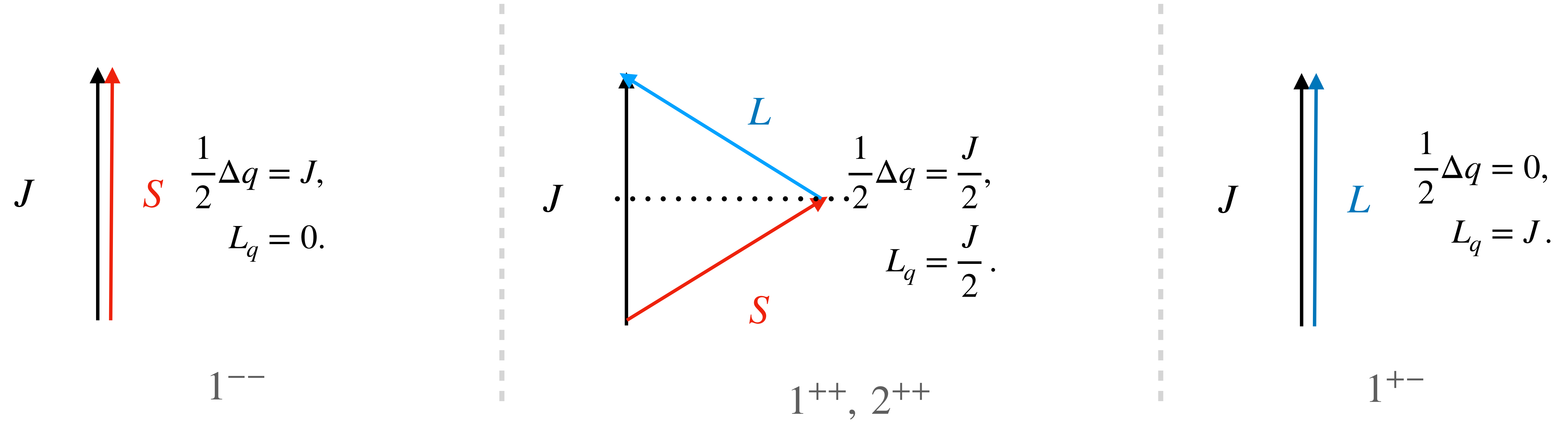
$$\langle E^a | S_q | T_2^z \rangle = 2\langle T_2^x | S_q | T_2^y \rangle$$



Quark spin



contribution to charmonium spin

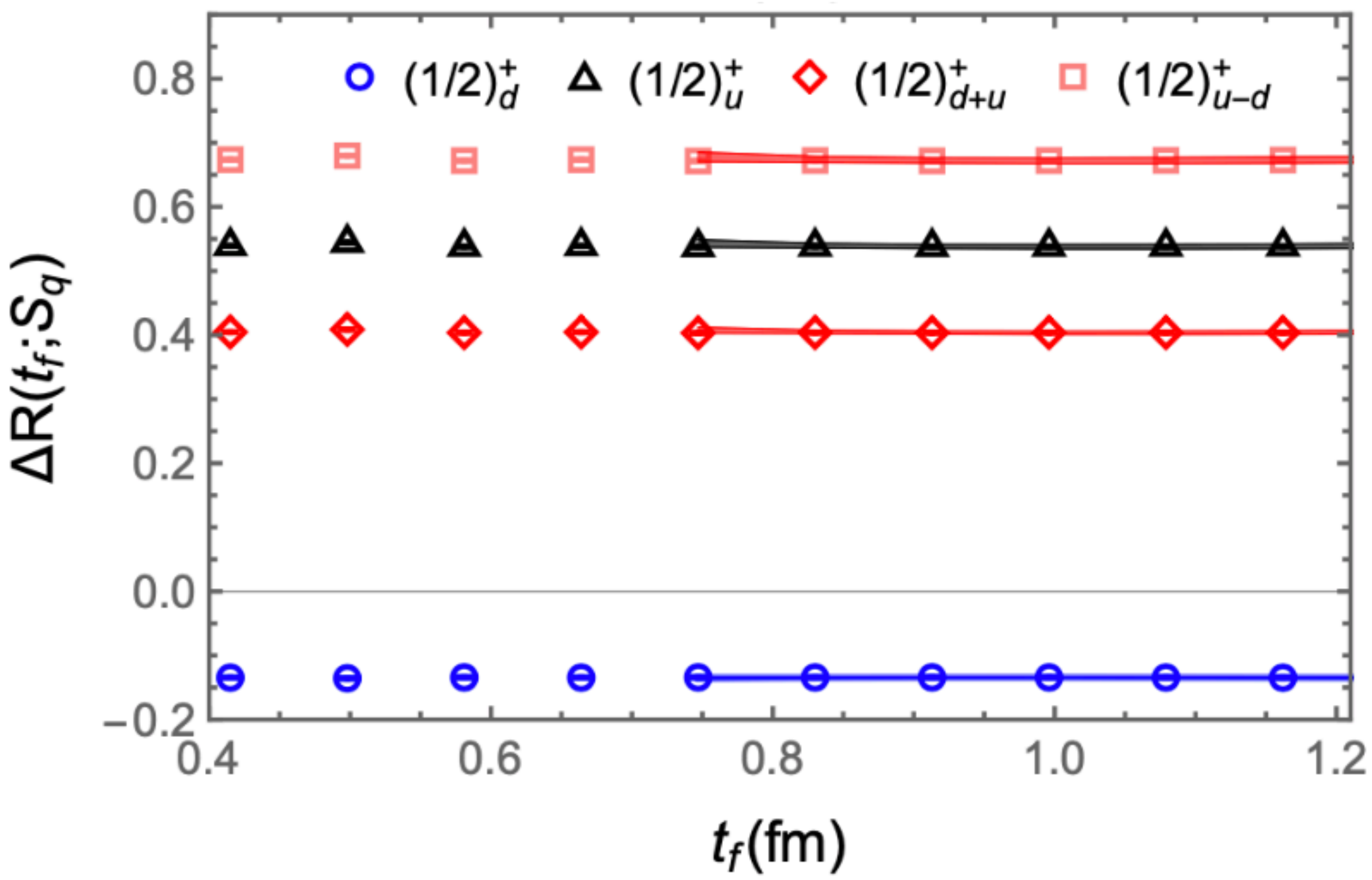


- The 1^{--} case: $\langle S_q \rangle_H = 0.893(03)$;
- The 1^{++} case: $\langle S_q \rangle_H = 0.448(55)$;
- The 2^{++} case: $\langle S_q \rangle_H = 0.436(11)$ for $J_z = 1$;
- The 1^{+-} case: $\langle S_q \rangle_H = 0.080(70)$.

Agree with the quark model prediction at 90% level.

Quark spin

and $(1/2)^+$ triple-heavy quark baryon spin



$\langle S_{u,d}\rangle_N$ also agree with the quark model prediction at 90% level:

- The u-type quark:
  $$\langle S_u\rangle_N = \frac{1}{2}\times 1.20(4) = 0.90(3)\langle S_u\rangle_N^{\text{quark model}};$$
- The d-type quark:
  $$\langle S_d\rangle_N = \frac{1}{2}\times(-)0.30(1) = 0.90(3)\langle S_d\rangle_N^{\text{quark model}};$$

$\langle S_u\rangle_N/\langle S_d\rangle_N = -4.0(1)$ is exactly the same as the quark model prediction!

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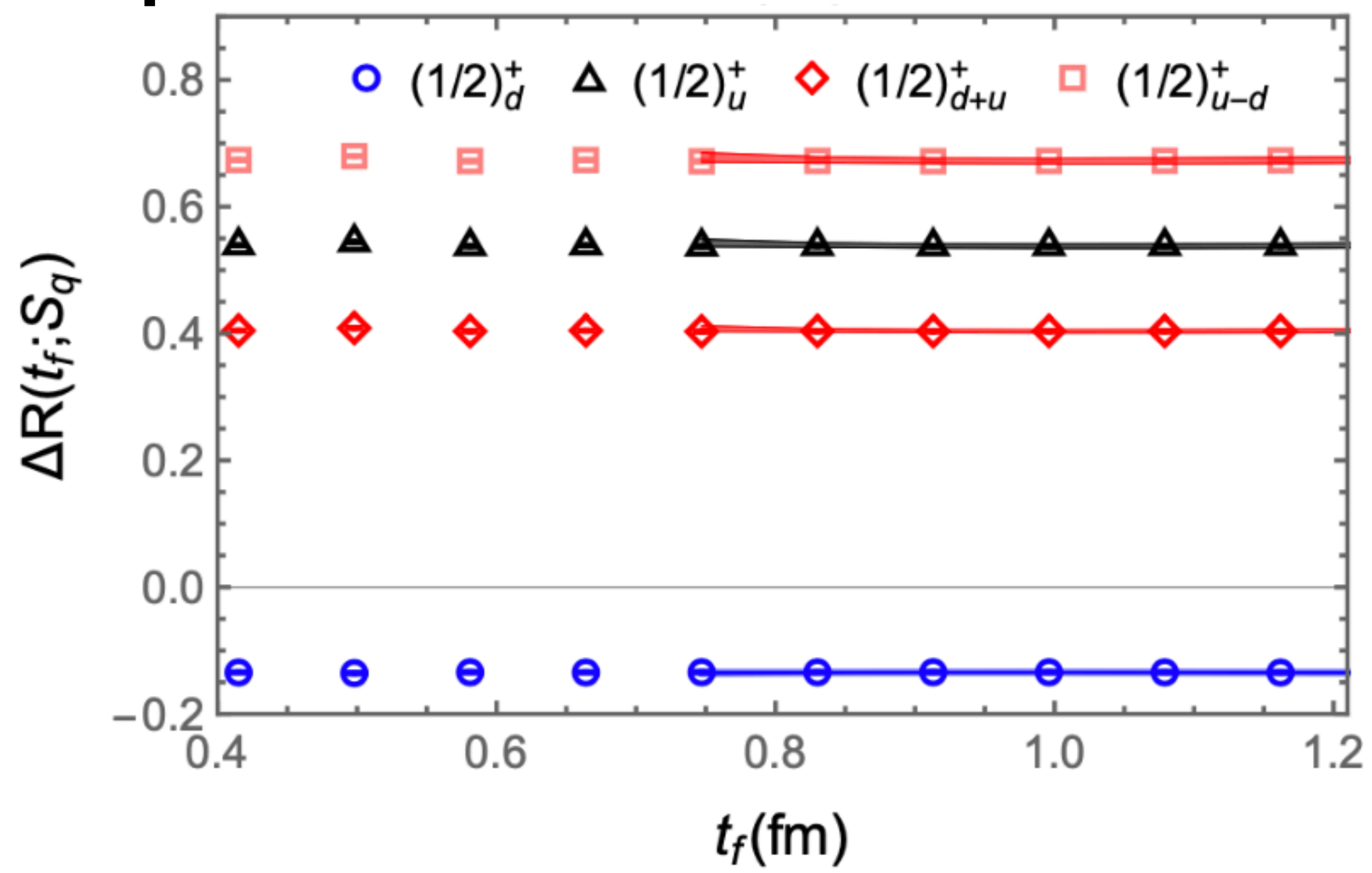
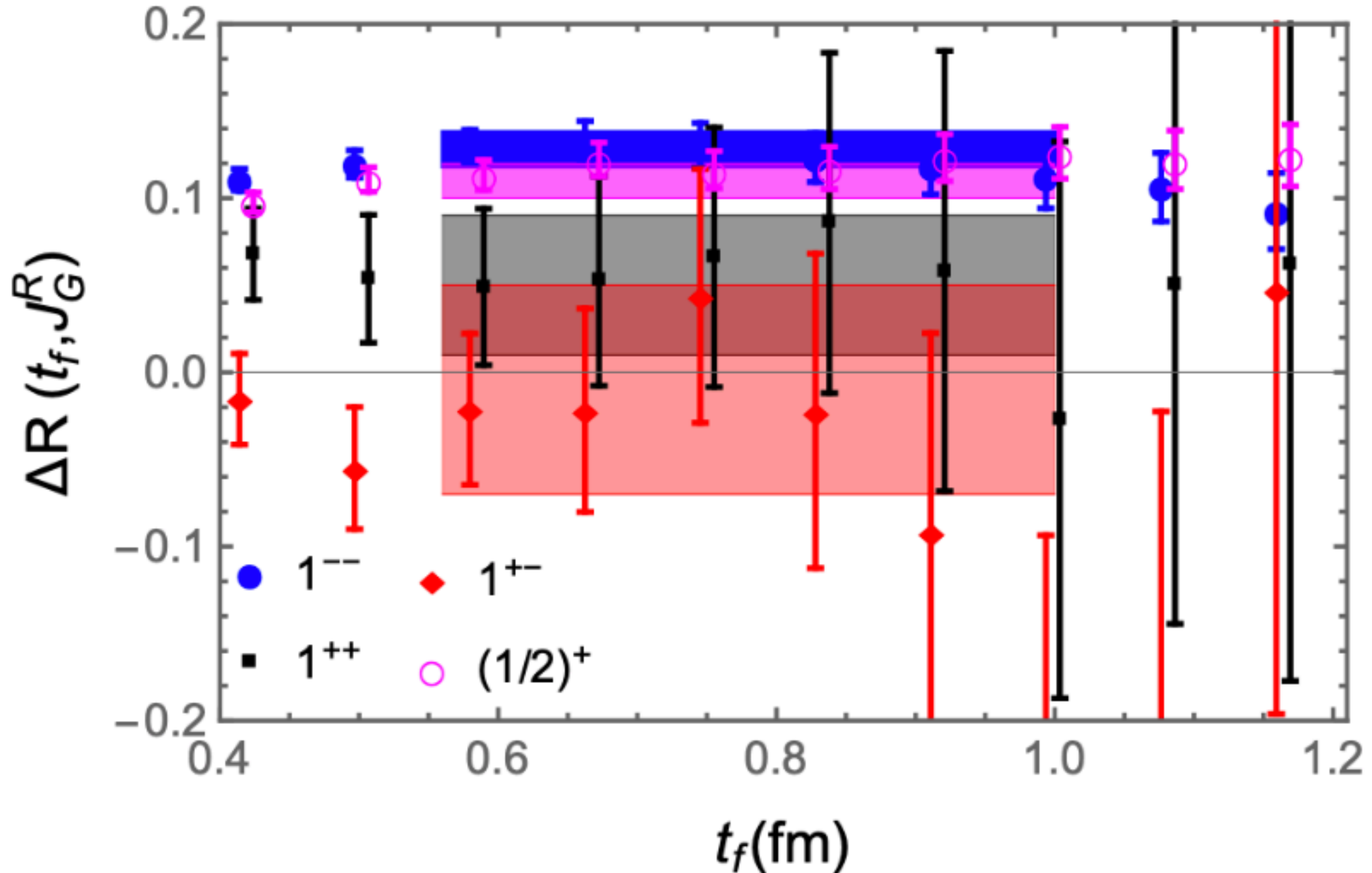


- **Gluon AM and spin**



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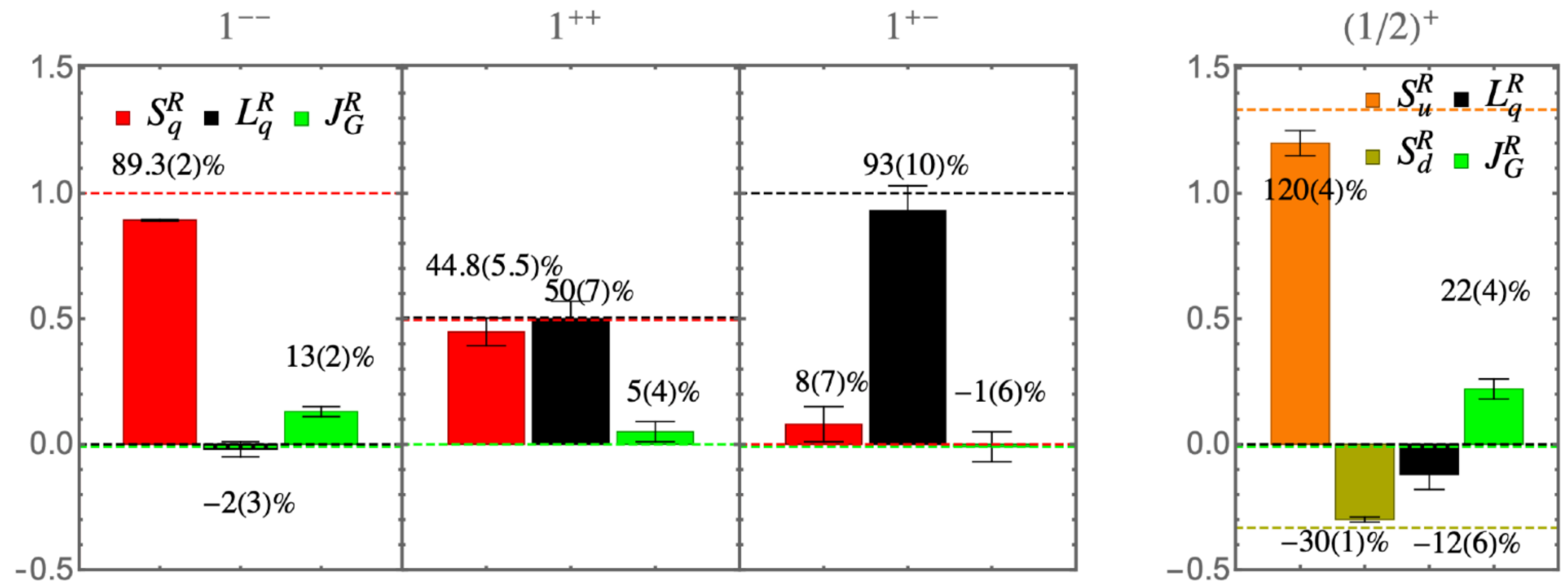


Gluon AM and spin

Form factors of EMT

$$T^g_{\mu\nu} = 2\mathrm{Tr}G_\mu^{\ \rho}G_{\rho\nu} + \frac{1}{2}g_{\mu\nu}\mathrm{Tr}G^{\rho\lambda}G_{\rho\lambda}$$

- The total angular momentum (AM) of gluon (and also quark) can be extracted from the form factors of their energy momentum tensor (EMT) in the hadron,

$$\langle p'|T^a_{\mu\nu}|p\rangle = \bar{u}(p')\Big(A^a(q^2)\gamma^{(\mu}\bar{P}^{\nu)} + B^a(q^2)\frac{i\bar{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha}{2m_N} + C^a(q^2)\frac{q^\mu q^\nu - \eta^{\mu\nu}q^2}{m_N}\Big)u(p),$$

$$J_g^N = \frac{1}{2}(A^g(0) + B^g(0)) \qquad B^q(0) + B^g(0) = 0$$

Spin 1/2 case

$$\begin{aligned}\langle p',\sigma'|\hat{T}^a_{\mu\nu}(0)|p,\sigma\rangle = \Big[&2\bar{P}_\mu\bar{P}_\nu\Big(-\epsilon'^*\cdot\epsilon\,A_0^a(q^2) + \frac{\epsilon'^*\cdot\bar{P}\,\epsilon\cdot\bar{P}}{m^2}A_1^a(q^2)\Big) + g_{\mu\nu}\Big(\epsilon'^*\cdot\epsilon\,m^2\,\bar{c}_0^a(q^2) + \epsilon'^*\cdot\bar{P}\,\epsilon\cdot\bar{P}\,\bar{c}_1^a(q^2)\Big)\Big]\\ &+\frac{1}{2}(q_\mu q_\nu - g_{\mu\nu}q^2)\Big(\epsilon'^*\cdot\epsilon\,D_0^a(q^2) + \frac{\epsilon'^*\cdot\bar{P}\,\epsilon\cdot\bar{P}}{m^2}D_1^a(q^2)\Big)\\ &+2\Big[\bar{P}_\mu(\epsilon'^*_\nu\,\epsilon\cdot\bar{P} + \epsilon_\nu\,\epsilon'^*\cdot\bar{P}) + \bar{P}_\nu(\epsilon'^*_\mu\,\epsilon\cdot\bar{P} + \epsilon_\mu\,\epsilon'^*\cdot\bar{P})\Big]J^a(q^2)\\ &+\Big(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu - \frac{\epsilon'^*\cdot\epsilon}{2}g_{\mu\nu}\Big)m^2\bar{f}^a(q^2)\\ &+\Big[\frac{1}{2}(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu)q^2 - (\epsilon'^*_\mu q_\nu + \epsilon'^*_\nu q_\mu)\,\epsilon\cdot\bar{P} + (\epsilon_\mu q_\nu + \epsilon_\nu q_\mu)\,\epsilon'^*\cdot\bar{P} - 4g_{\mu\nu}\,\epsilon'^*\cdot\bar{P}\,\epsilon\cdot\bar{P}\Big]E^a(q^2),\end{aligned}$$

$$\epsilon^{('*)}\cdot\bar{P}|_{q^2\to 0} = 0$$

$$\bar{f}^q(0) + \bar{f}^g(0) = 0$$

$$J_g^V = J^g(0) + \frac{1}{2}\bar{f}^g(0)$$

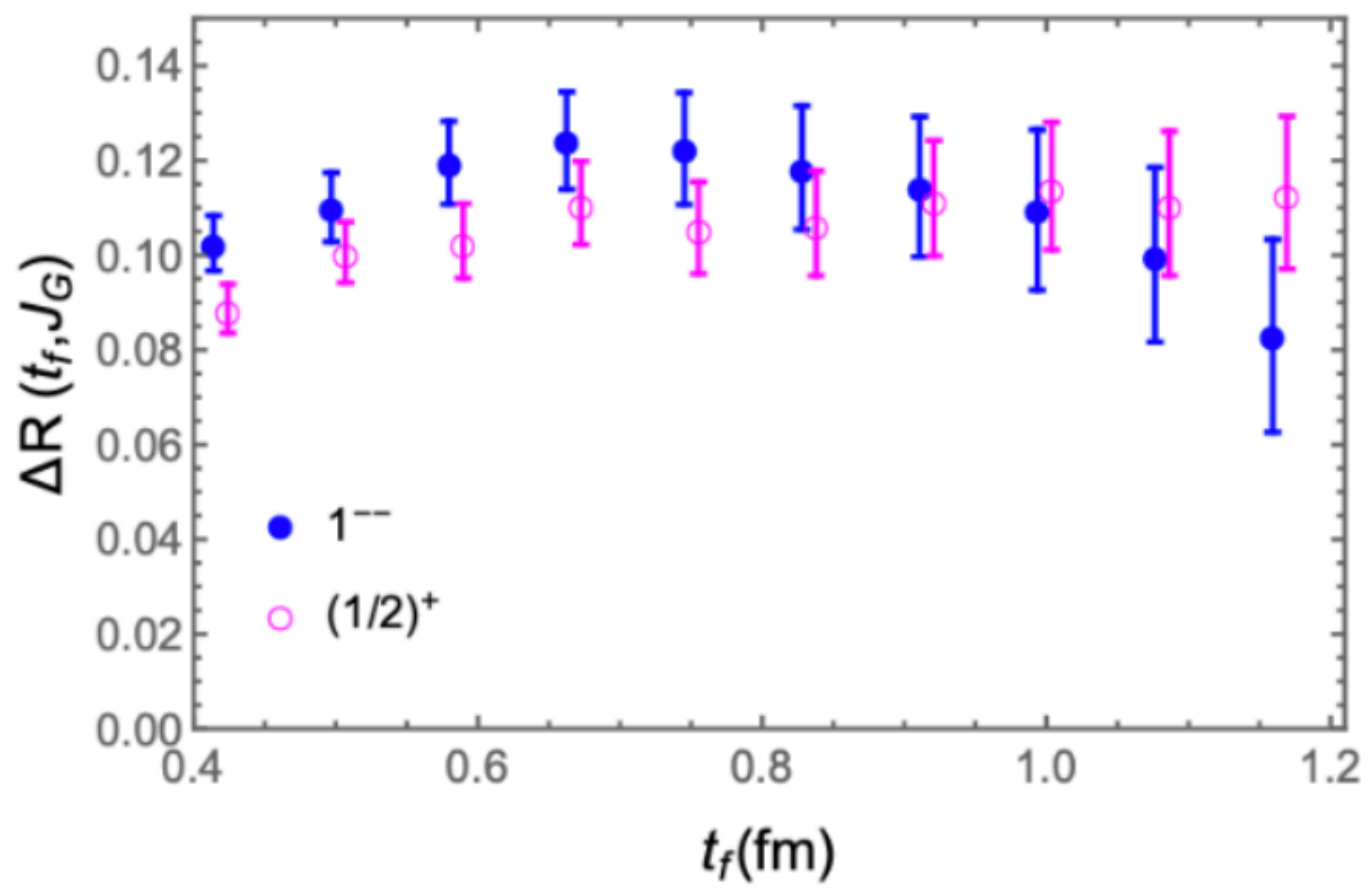
Spin 1 case

- One shall calculate the form factors at finite q^2, and extrapolate to the forward limit. The quark orbital angular momentum can be obtained through the sum rule $L_q = J - S_q - J_g$.

Gluon AM and spin

Baryon and J/ψ

$$J_g^V = J^g(0) + \frac{1}{2}\bar{f}^g(0)$$

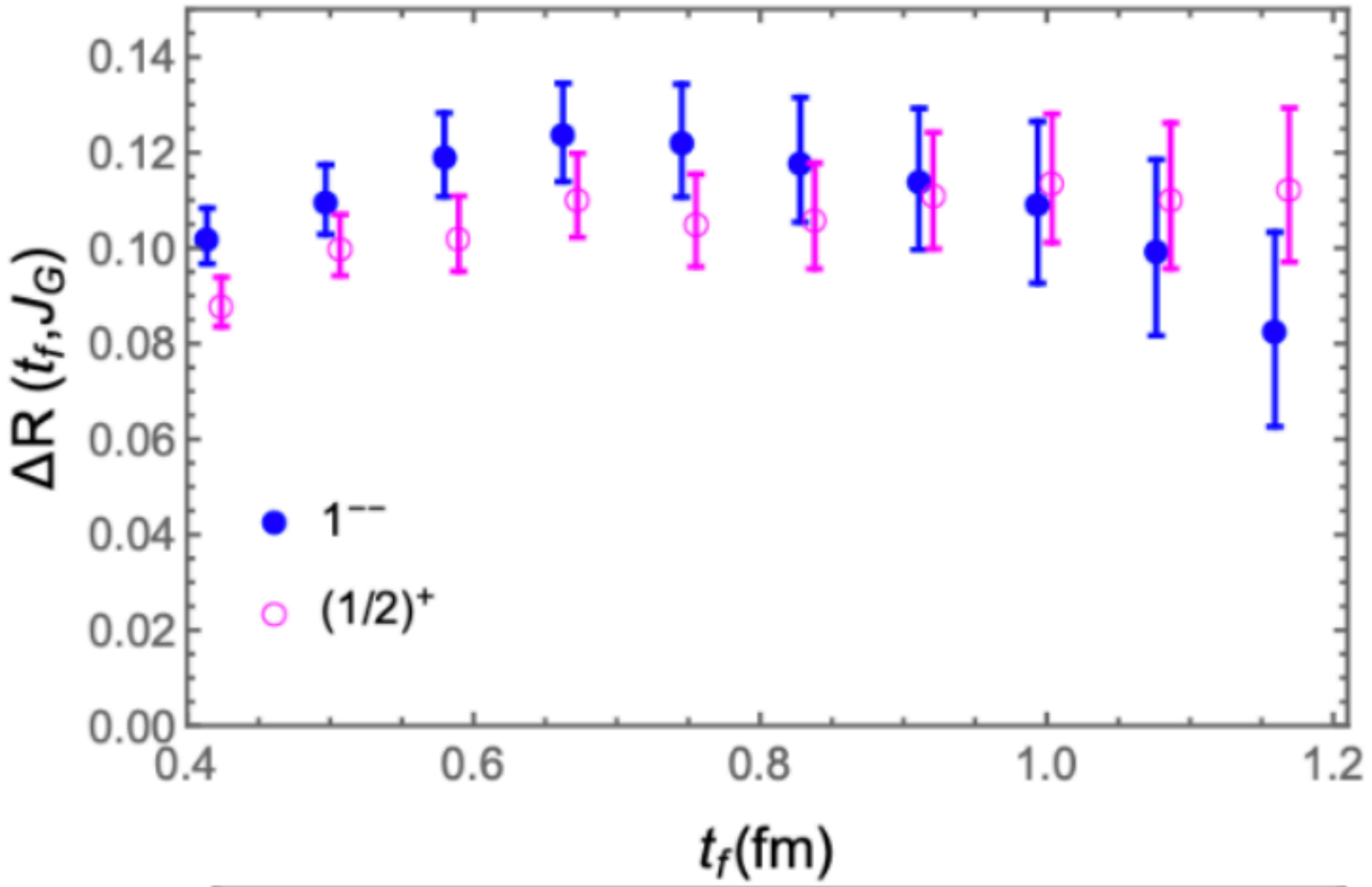


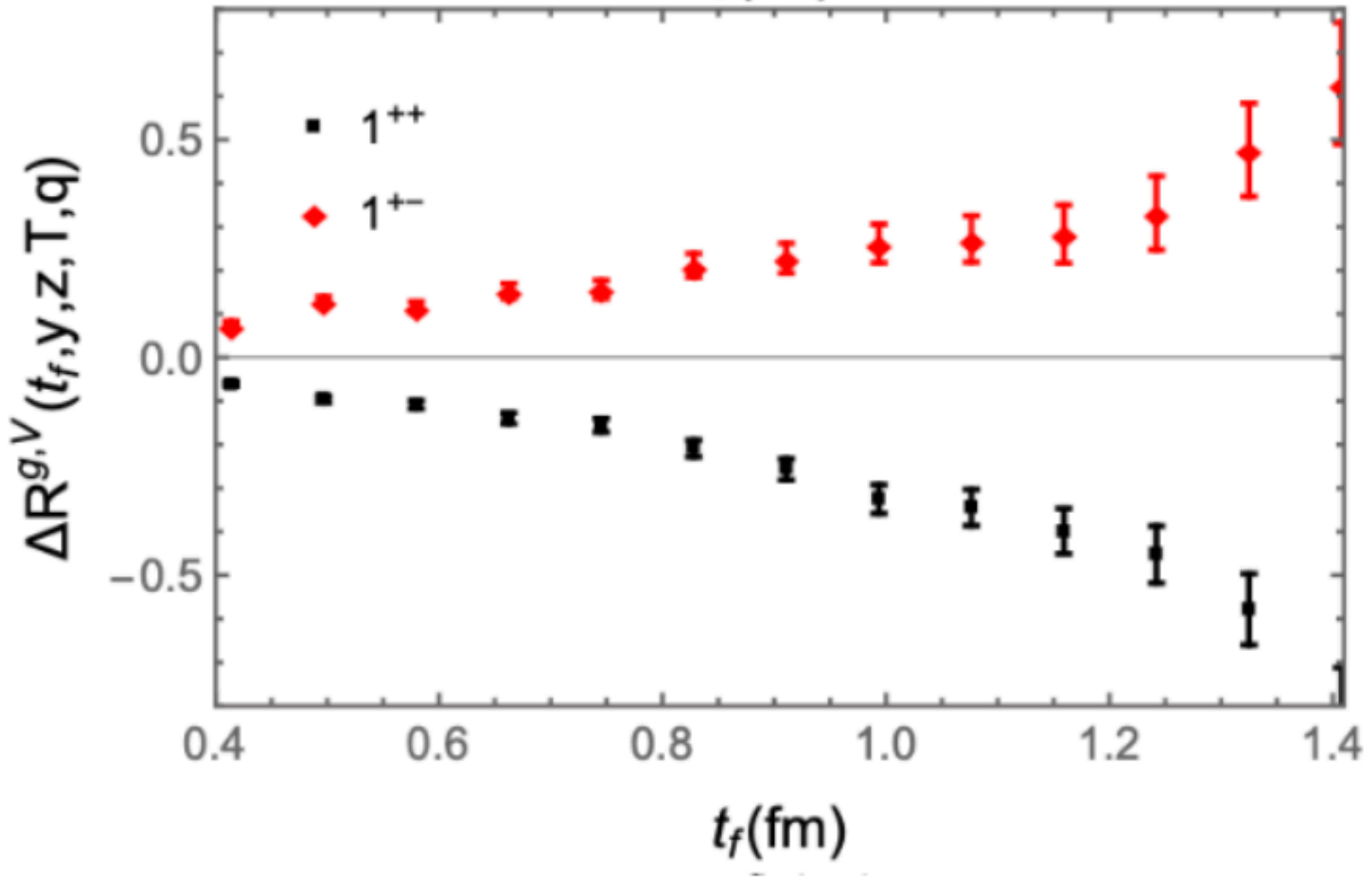
$$J_g^N = \frac{1}{2}(A^g(0) + B^g(0))$$

	Final state $p' = (E,0,0,k)$	Initial state $p = (m,0,0,0)$	Matrix elements
Condition I	$\epsilon' = (0,0,1,0)$	$\epsilon = (0,0,0,1)$	$\langle p', \sigma_y' \vert T_{4y} \vert p, \sigma_z \rangle = -\mathrm{i}\frac{k}{4E_f}\left[(E_f+m)J^g(q^2) - (E_f - m)E^g(q^2)\right]$
Condition II	$\epsilon' = (\frac{q}{m},0,0,\frac{E}{m})$	$\epsilon = (0,0,1,0)$	$\langle p', \sigma_z' \vert T_{4y} \vert p, \sigma_y \rangle = \mathrm{i}\frac{k}{4E_f}\Big[\frac{(E_f+m)(2m^2+k^2)}{2m^2}J^g(q^2) + \Big((E_f - m) - \frac{k^2(E_f+m)}{2m^2}\Big)E^g(q^2) + 2E_f\bar{f}^g(q^2)\Big],$
Condition III	$\epsilon' = (0,1,0,0)$	$\epsilon = (0,0,1,0)$	$\langle p', \sigma_x' \vert T_{xy} \vert p, \sigma_y \rangle = \left[-\frac{k^2}{2}E^g(q^2) + m^2\bar{f}^g(q^2)\right]\frac{1}{2E_f}$

- For the $1^{--}(J/\psi)$ case, one can obtain $\bar{f}^g(0)$ in the rest frame, plus $J^g(0)$ through the approximation $J^g(0) \simeq J^g(q^2)(1 + \frac{q^2}{M_{\rm pole}^2}) = J^g(q^2) + \mathcal{O}(5\%)$ using $J^g(q^2)$ at the smallest non-zero q^2.
- For the $(1/2)^+$ triple-heavy quark baryon, we neglect $B^g(0)$ which is small even in the light quark case, and obtain $A^g(0)$ in the rest frame.

Gluon AM and spin





Operator mixing

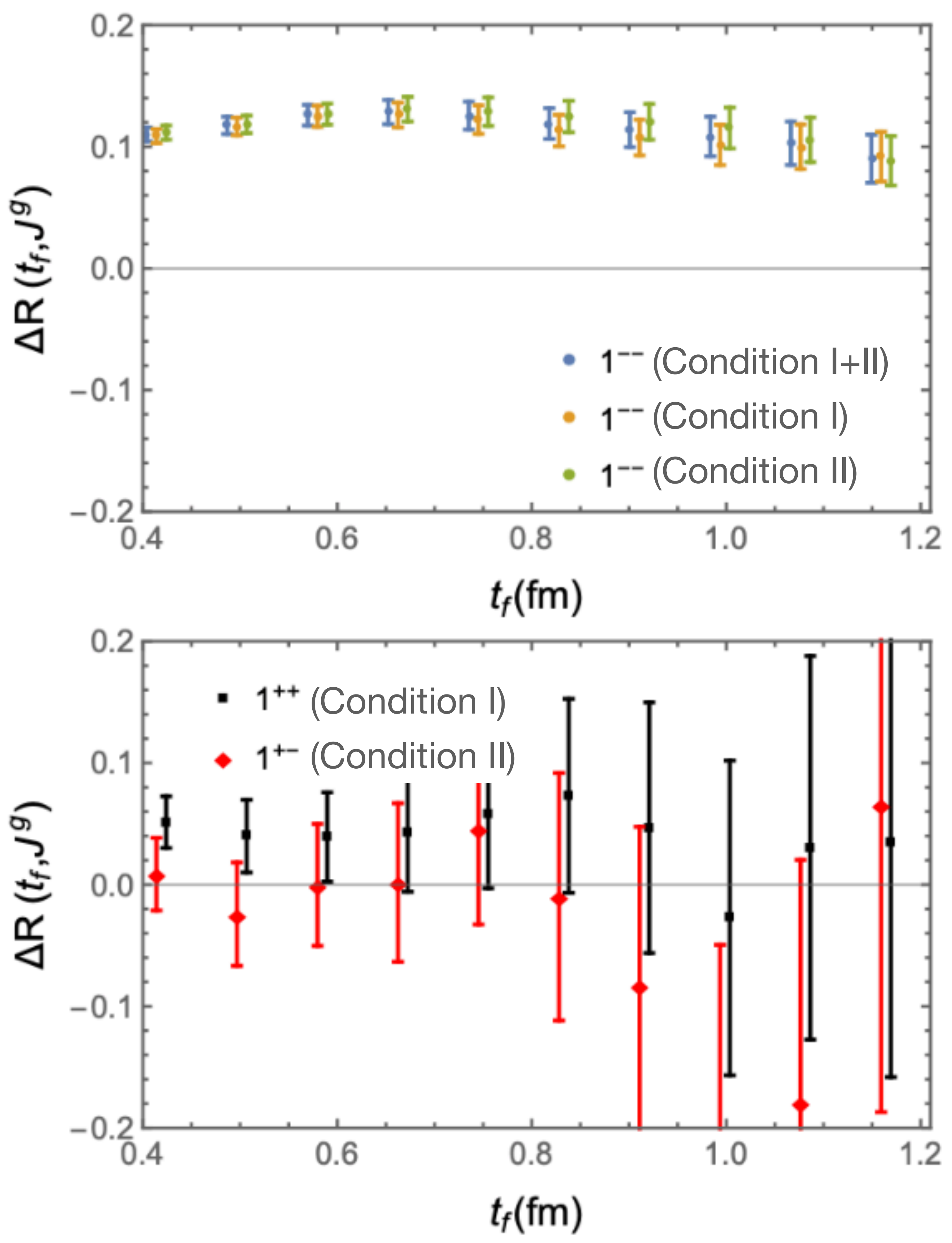
Mix with 1^{--} for the boosted 1^{+-}

	Final state $p' = (E,0,0,k)$	Initial state $p = (m,0,0,0)$	Matrix elements
Condition I	$\epsilon' = (0,0,1,0)$	$\epsilon = (0,0,0,1)$	$\langle p', \sigma'_y \vert T_{4y} \vert p, \sigma_z \rangle = -\mathrm{i}\frac{k}{4E_f}\left[(E_f+m)J^g(q^2) - (E_f-m)E^g(q^2)\right]$
Condition II	$\epsilon' = (\frac{q}{m},0,0,\frac{E}{m})$	$\epsilon = (0,0,1,0)$	$\langle p', \sigma'_z \vert T_{4y} \vert p, \sigma_y \rangle = \mathrm{i}\frac{k}{4E_f}\Big[\frac{(E_f+m)(2m^2+k^2)}{2m^2}J^g(q^2) + \left((E_f-m) - \frac{k^2(E_f+m)}{2m^2}\right)E^g(q^2) + 2E_f\bar{f}^g(q^2)\Big],$
Condition III	$\epsilon' = (0,1,0,0)$	$\epsilon = (0,0,1,0)$	$\langle p', \sigma'_x \vert T_{xy} \vert p, \sigma_y \rangle = \left[-\frac{k^2}{2}E^g(q^2) + m^2\bar{f}^g(q^2)\right]\frac{1}{2E_f}$

Mix with 0^{-+} for the boosted 1^{++}

- But for the $1^{++(-)}$ cases, not all the conditions can be used to solve $J^g(q^2)$, due to the operator mixing with the S-wave charmonium states.

Gluon AM and spin



Consistency check of form factor

Mix with 1^{--} for the boosted 1^{+-}

	Final state $p' = (E,0,0,k)$	Initial state $p = (m,0,0,0)$	Matrix elements
Condition I	$\epsilon' = (0,0,1,0)$	$\epsilon = (0,0,0,1)$	$\langle p', \sigma'_y \lvert T_{4y} \rvert p, \sigma_z \rangle = -\mathrm{i}\frac{k}{4E_f}\left[(E_f+m)J^g(q^2) - (E_f-m)E^g(q^2)\right]$
Condition II	$\epsilon' = (\frac{q}{m},0,0,\frac{E}{m})$	$\epsilon = (0,0,1,0)$	$\langle p', \sigma'_z \lvert T_{4y} \rvert p, \sigma_y \rangle = \mathrm{i}\frac{k}{4E_f}\Big[\frac{(E_f+m)(2m^2+k^2)}{2m^2}J^g(q^2) + \left((E_f-m) - \frac{k^2(E_f+m)}{2m^2}\right)E^g(q^2) + 2E_f\bar{f}^g(q^2)\Big],$
Condition III	$\epsilon' = (0,1,0,0)$	$\epsilon = (0,0,1,0)$	$\langle p', \sigma'_x \lvert T_{xy} \rvert p, \sigma_y \rangle = \left[-\frac{k^2}{2}E^g(q^2) + m^2\bar{f}^g(q^2)\right]\frac{1}{2E_f}$

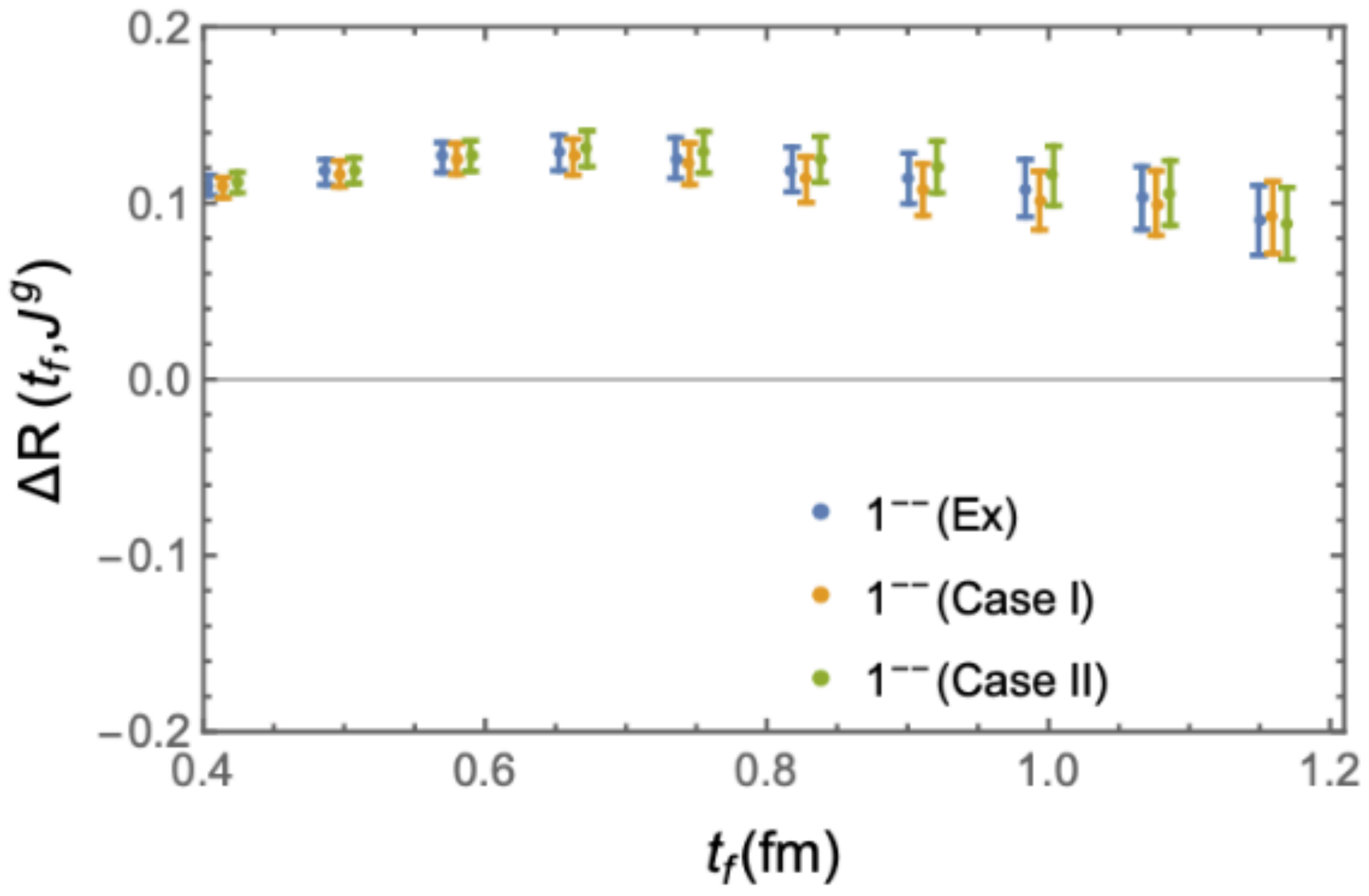
Mix with 0^{-+} for the boosted 1^{++}

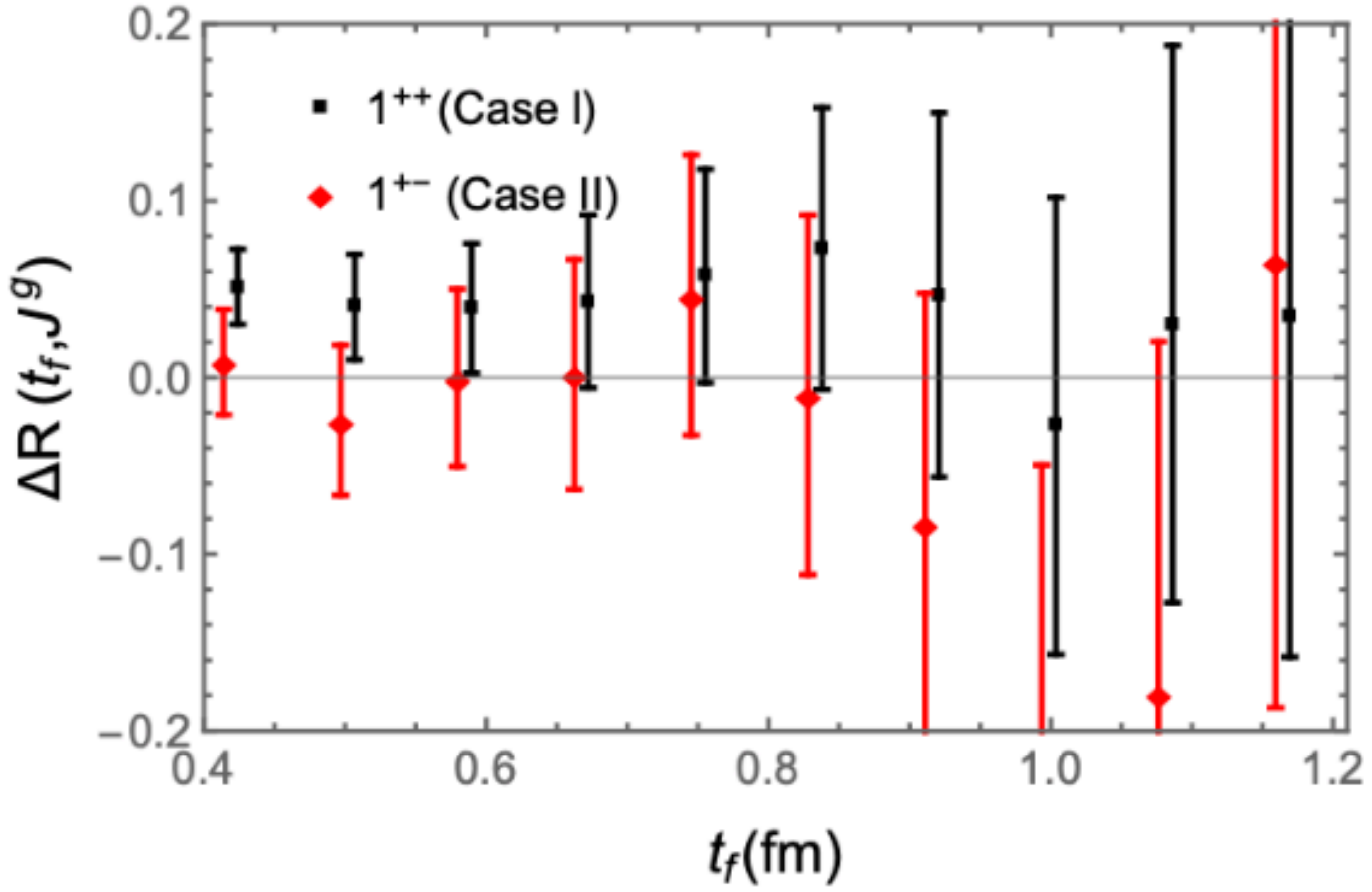
If we approximate $\bar{f}^g(q^2)$ with $\bar{f}^g(0)$:

- 1^{--}: $J^g(q^2)$ can be obtained through Condition I+III or II+III, or I+II+III;
- 1^{++}: $J^g(q^2)$ can be obtained through Condition I+III;
- 1^{+-}: $J^g(q^2)$ can be obtained through Condition II+III.

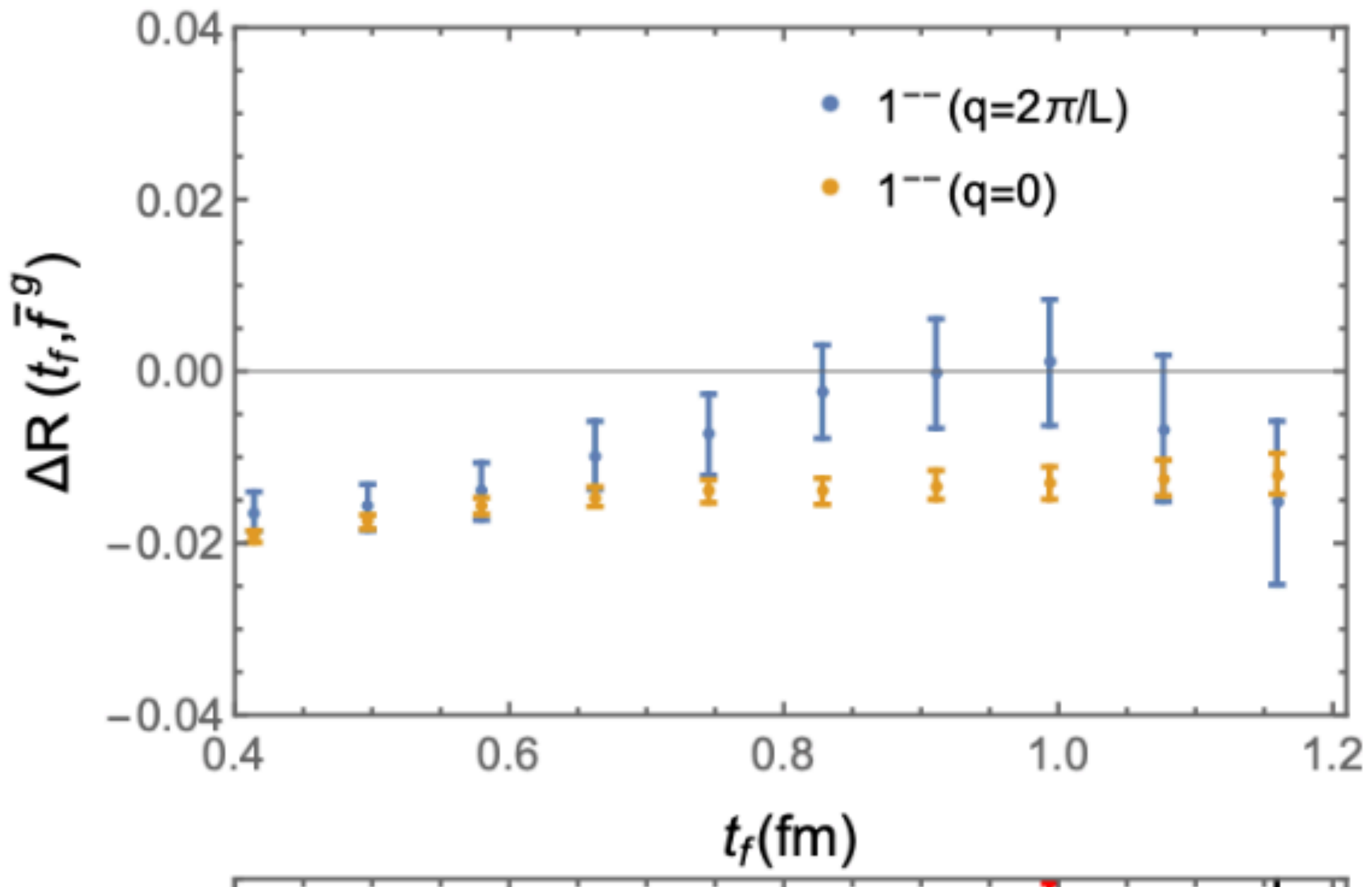
Gluon AM and spin

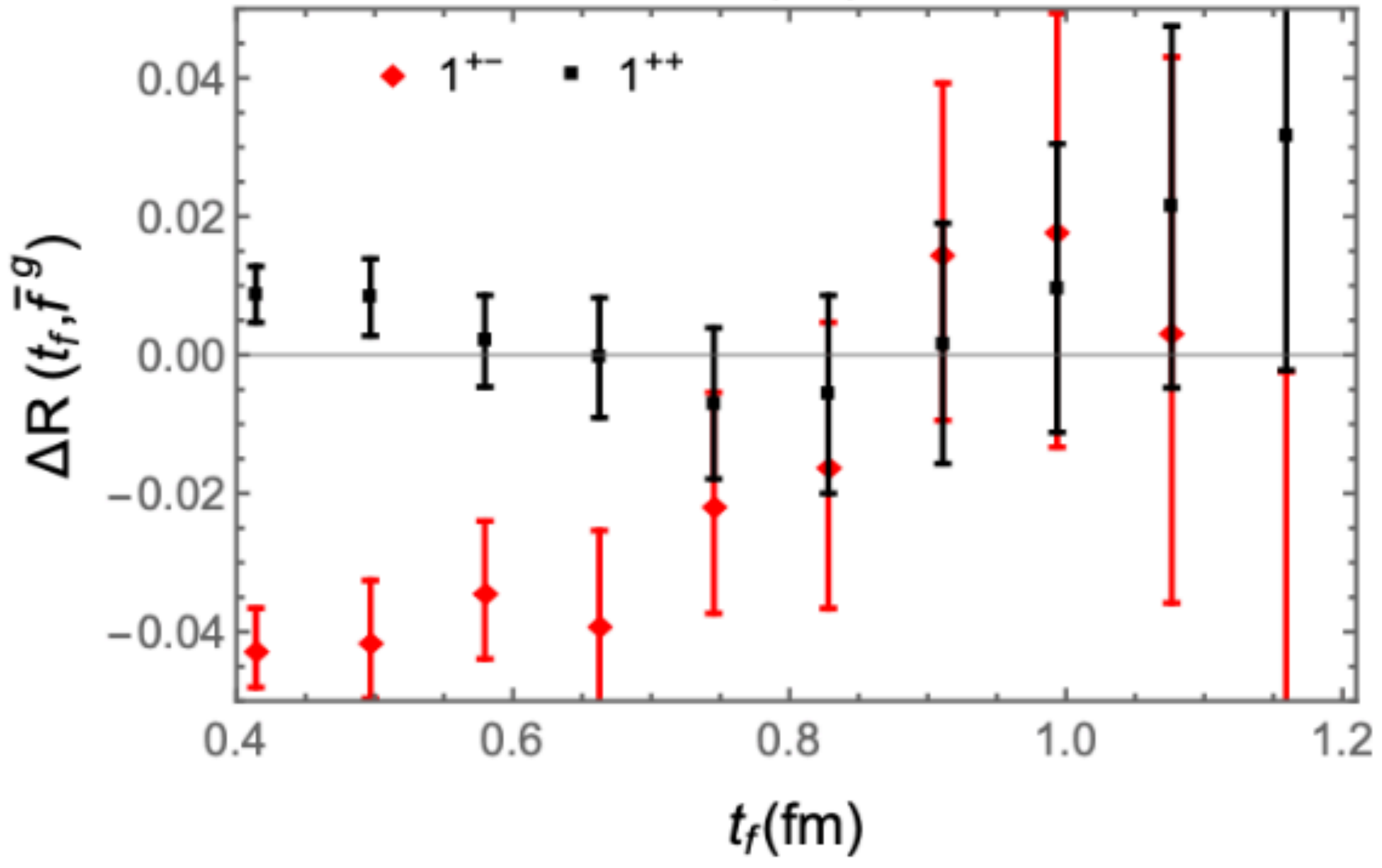
Contribution from different form factors





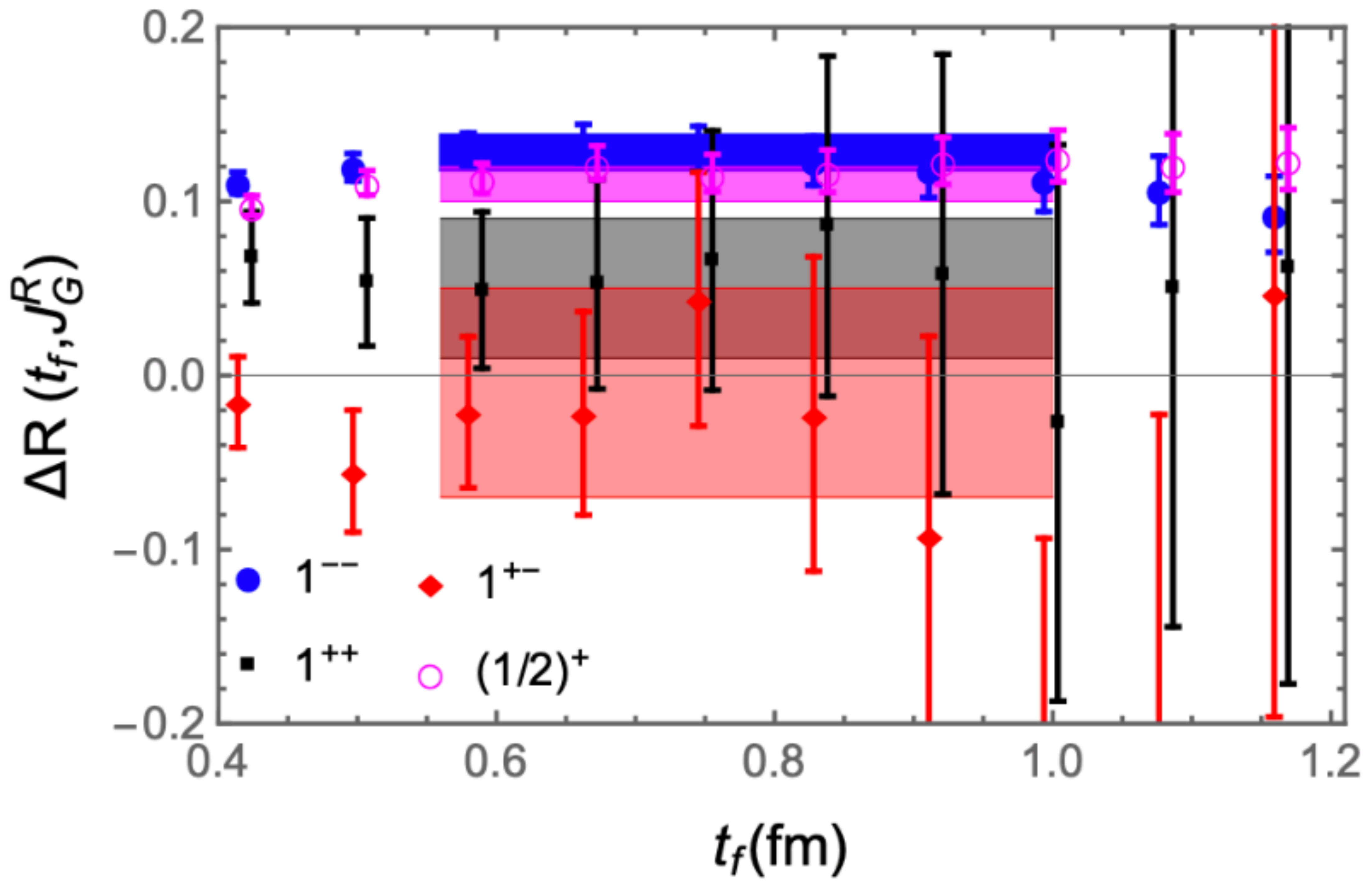
- Comparing with $\bar{f}^g$, J^g is much larger in the 1^{--} case;
- In the $1^{++(-)}$ cases, both J^g and $\bar{f}^g$ are consistent with zero, while J^g has larger uncertainty.
- Thus $J_g^V = J^g(0) + \frac{1}{2}\bar{f}^g(0)$ would be dominant by J^g, while J^g can not be obtained in the rest frame, which is different from the 1/2 baryon case.





Gluon AM and spin

gluon AM in different charmed hadron

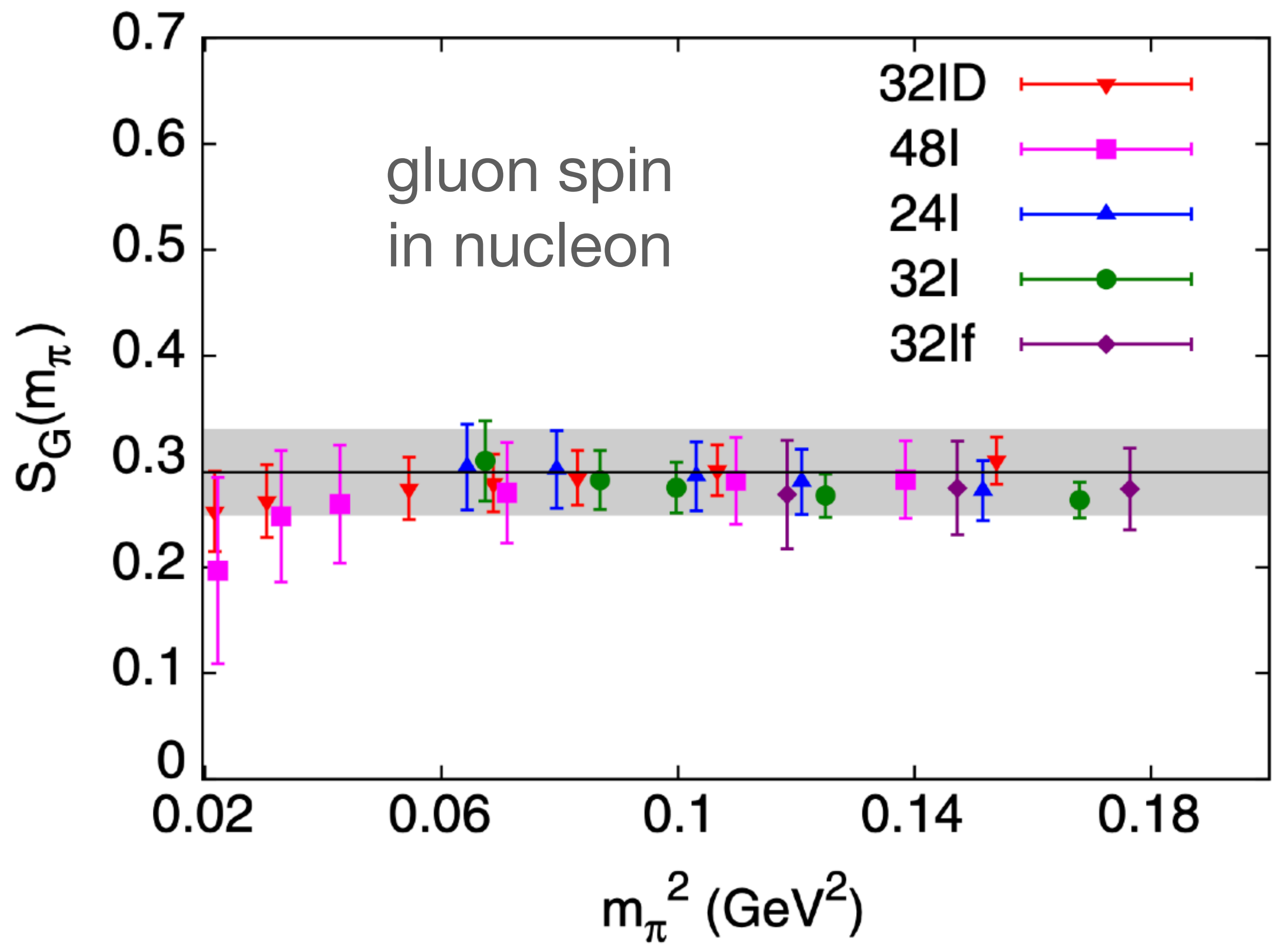


- $J_g^V = J^g(0) + \frac{1}{2}\bar{f}^g(0)$ in all the cases we studied here are small (~0.1);
- Contribution in the $(1/2)^+$ triple-heavy quark baryon case is 0.1/0.5~20% which is approximated by $A(0) = \langle x \rangle_g$;
- $\langle x \rangle_g$ in the charmonium states are also ~20%, but gluon AM is 10% (1^{--}) or even smaller ($1^{++(-)}$);
- Direct calculation of $B^g(q^2)$ should be helpful to provide more accurate prediction on $J_g^N = \frac{1}{2}(A^g(0) + B^g(0))$.

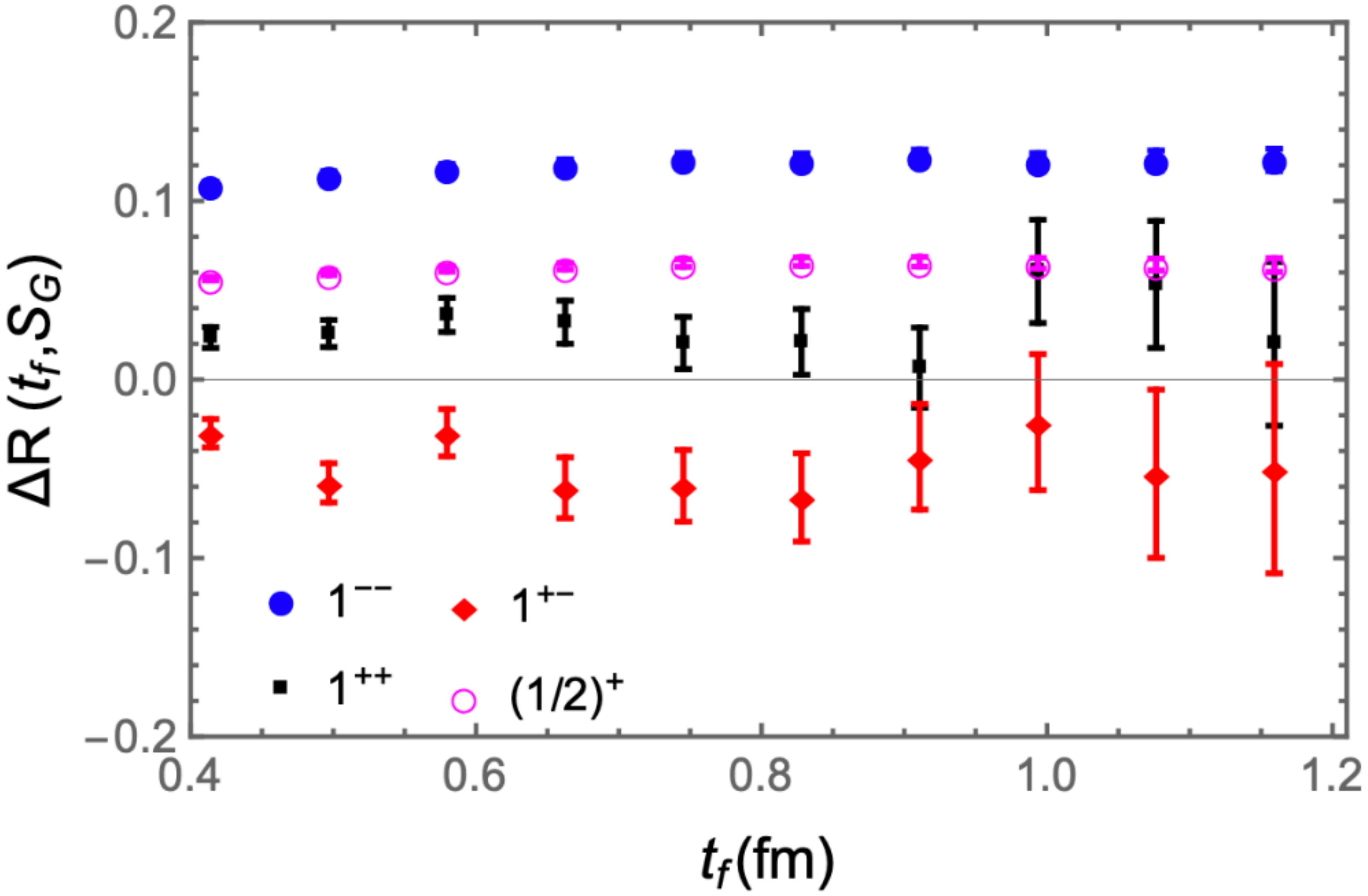
Gluon AM and spin

gluon spin under Coulomb gauge

$$\vec{J} = \int d^3x \frac{1}{2} \overline{\psi} \vec{\gamma} \gamma^5 \psi + \int d^3x \psi^\dagger \{\vec{x} \times (iD^{\vec{pure}})\} \psi$$
$$+ \int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}] + \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]$$



YBY, R. Sufian, et. al., χQCD collaboration, PRL118(2017) 042001



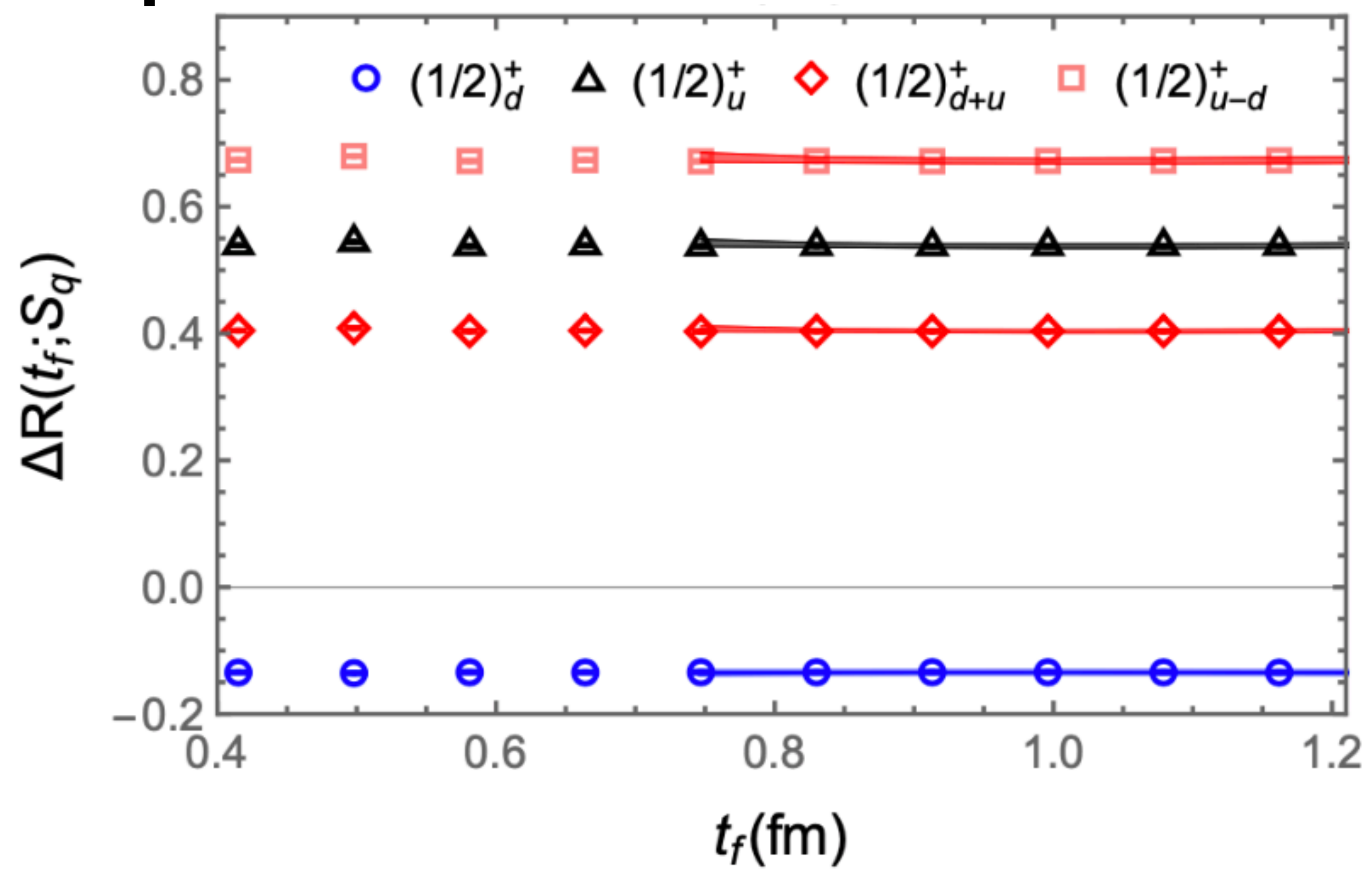
- Gluon spin $E \times A$ under Coulomb gauge can also be calculated for the charmed hadron;
- ~10% for J/ψ and $(1/2)^+$ heavy quark baryon, and even smaller for the $1^{++(-)}$ states;
- More or less similar to the gluon AM.

Outline

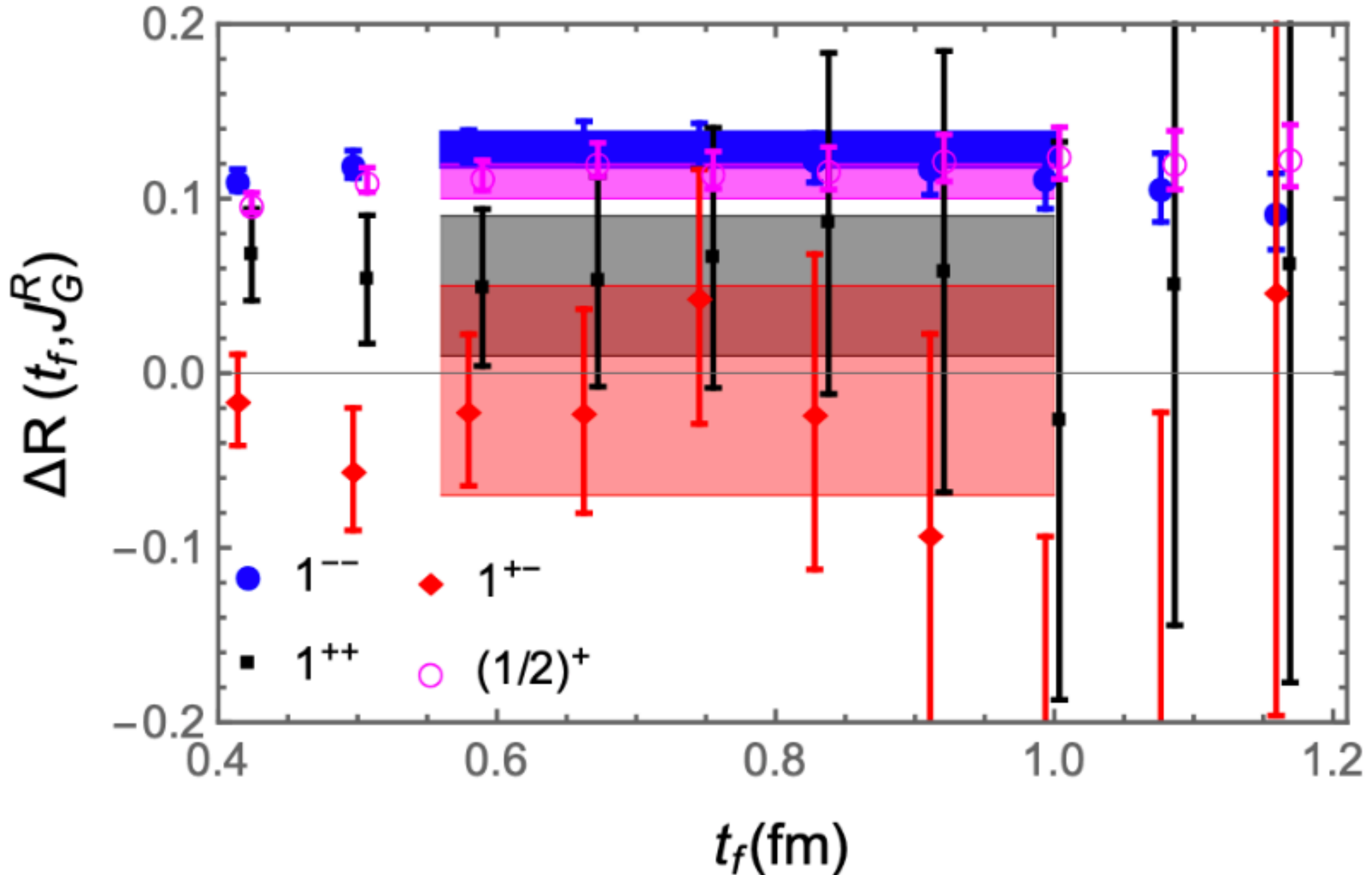
- Hadron spin and quark model

$$J = \frac{1}{2}\Delta q + L_q + J_G,$$

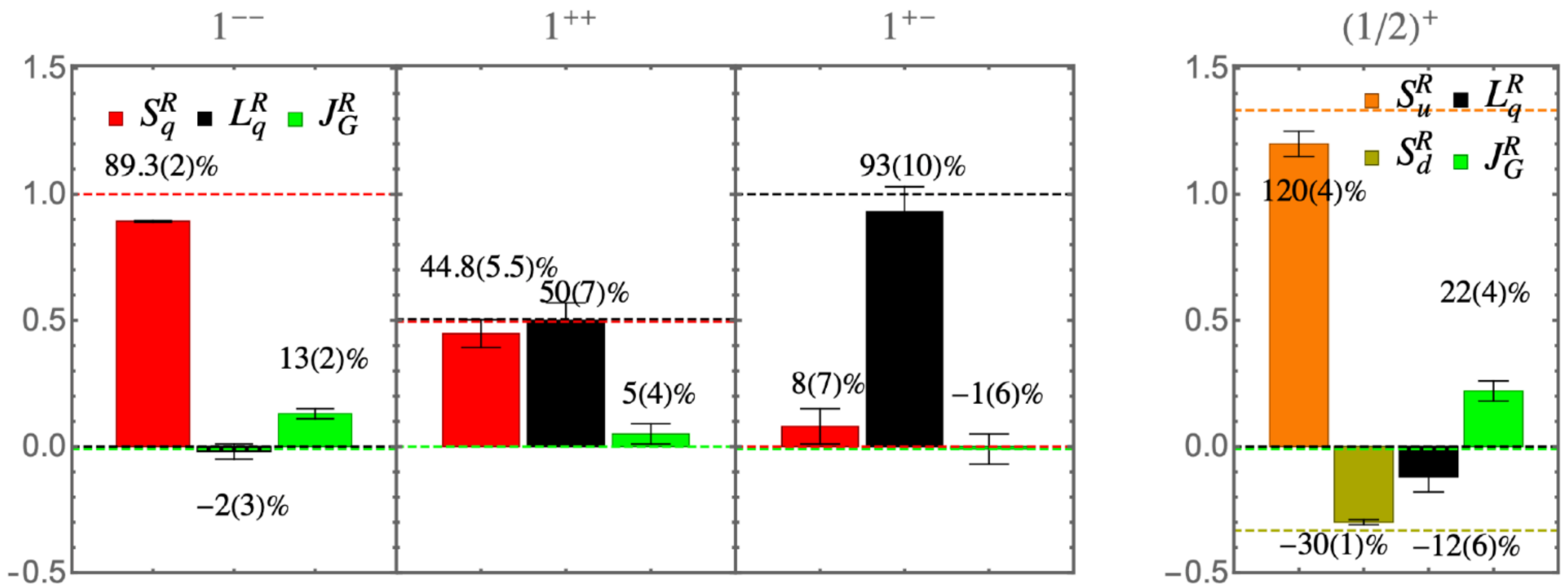
- Quark spin



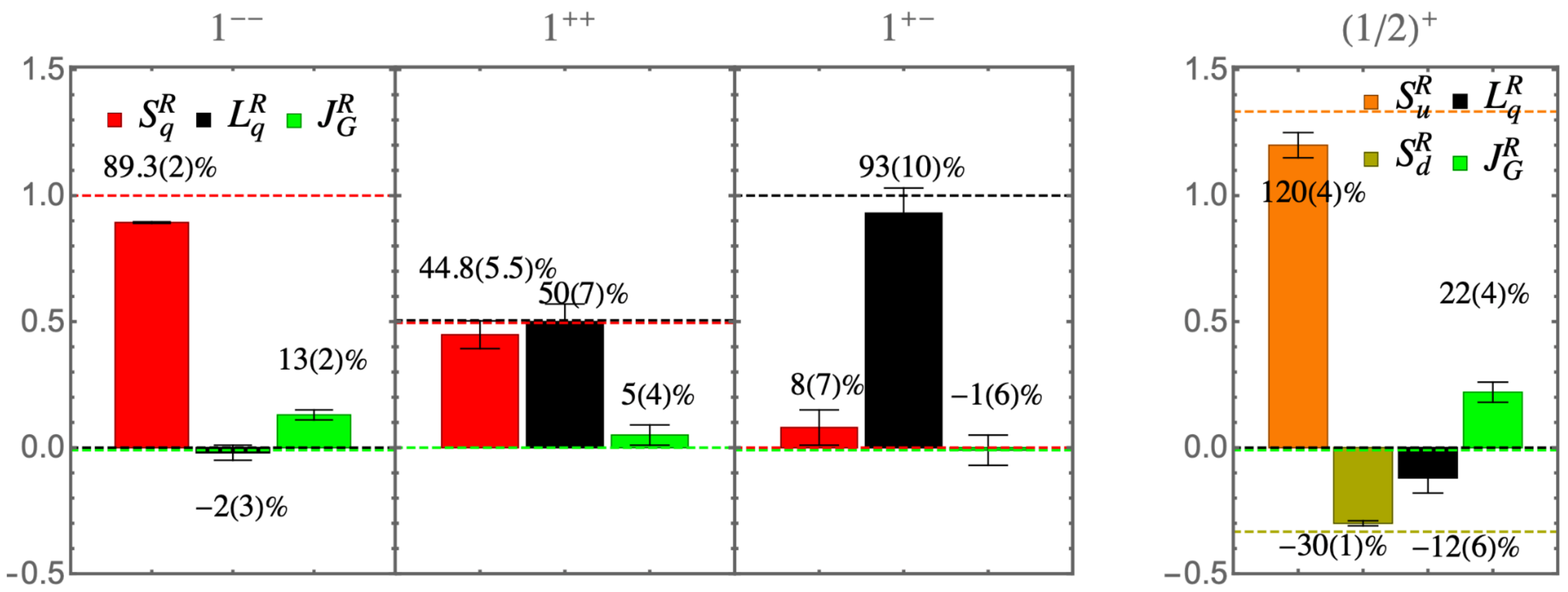
- Gluon AM and spin

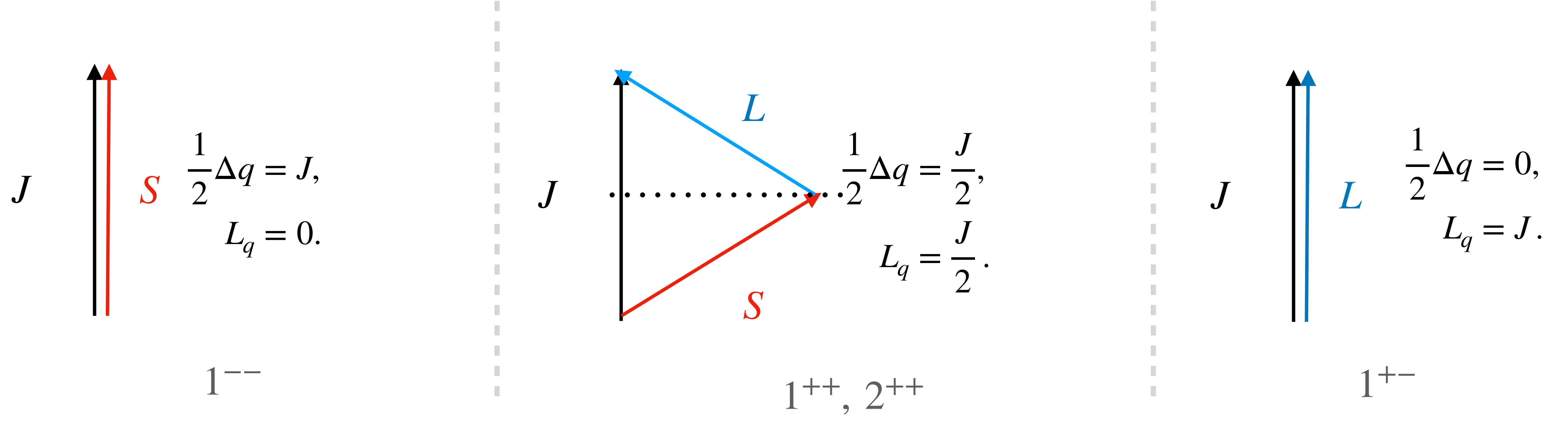


- **Discussion**



Discussion





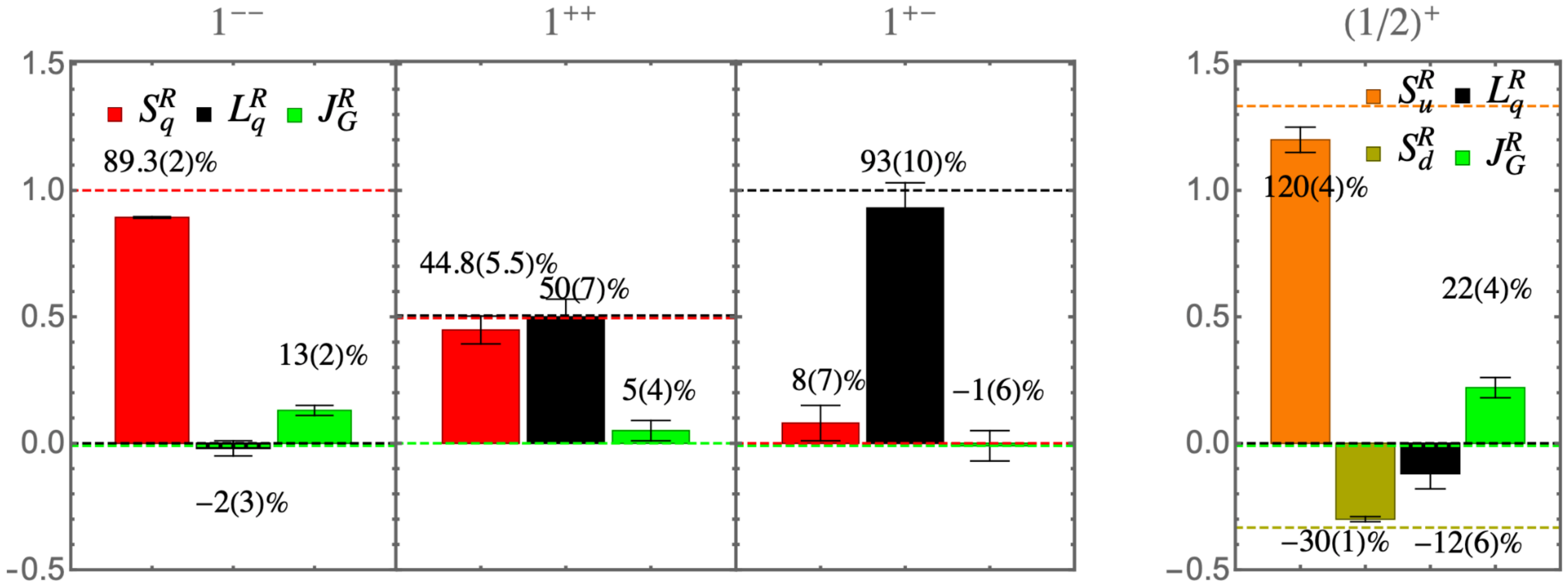
Summary of the results

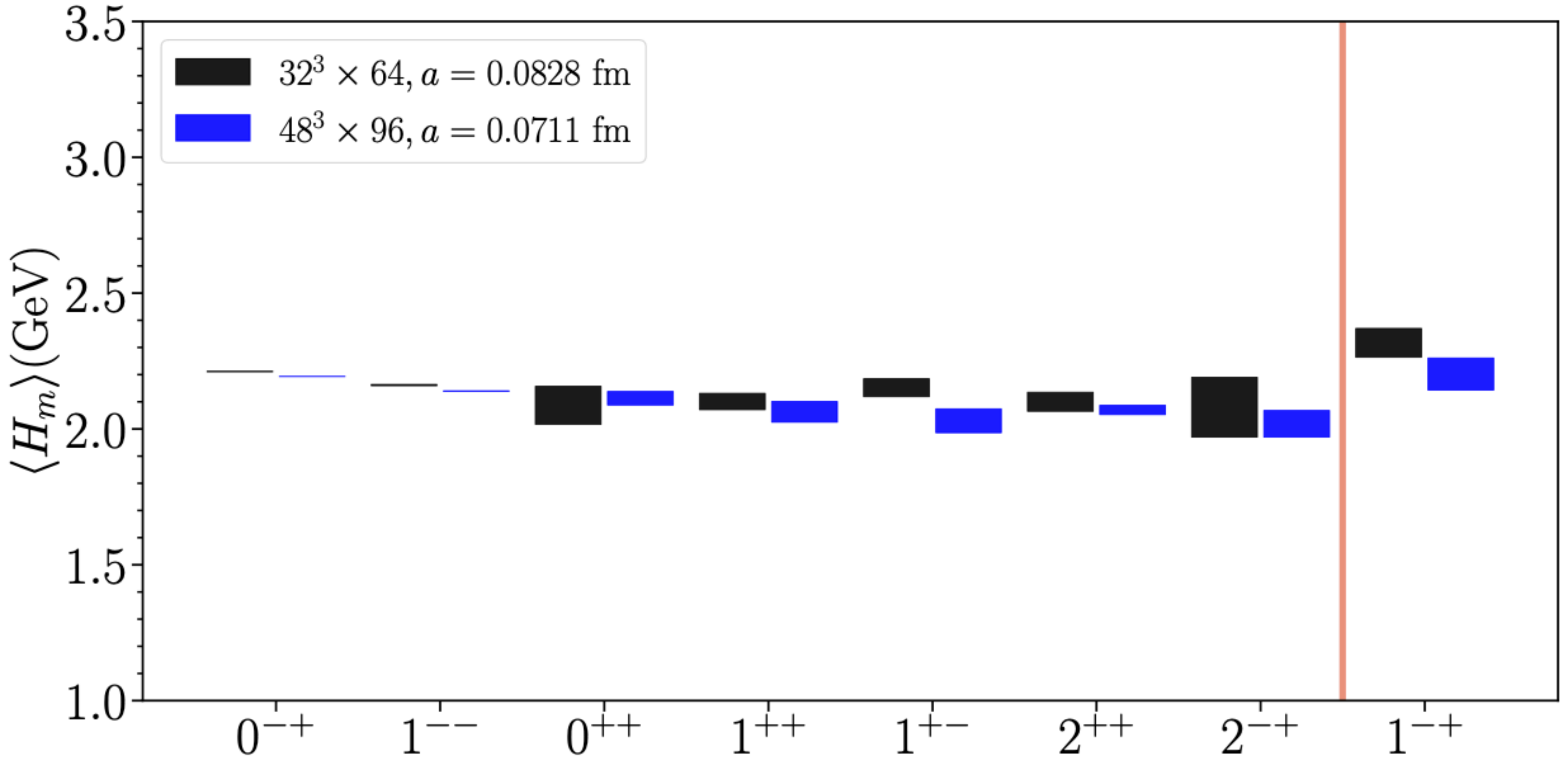
When the quark mass is as heavy as $m_q = m_c \sim 1.2$ GeV:

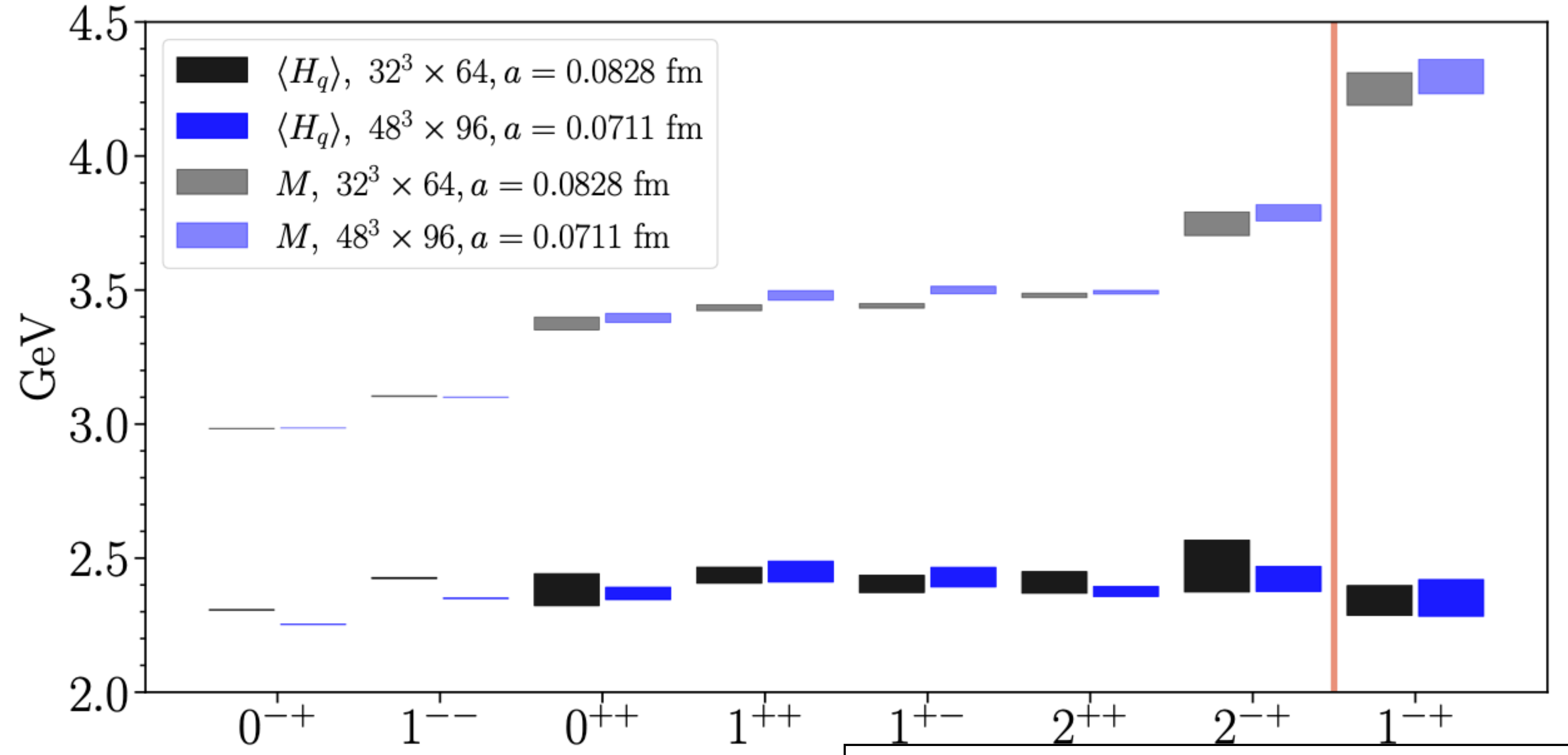
- Quark spin contribution agree with the quark model prediction at 90% level;
- Quark OAM obtained through the sum rule $L_q = J - S_q - J_g$ also consistent with expectation.
- Gluon contributions are not negligible in some cases which suggests that the charm quark is still not heavy enough.

Discussion

Relativity of quark







The charmonium mass decomposition suggests that $1 - v^2 \simeq \langle H_m \rangle_H / \langle H_q \rangle_H \sim 0.9$;

Similar to $\langle S_q \rangle_H / \langle S_q \rangle_H^{\text{Quarkmodel}}$.

$$M_H = T^{00} = H_E + H_m + H_g + \frac{1}{4}(H_a^q + H_a^g)$$

$$H_E = \langle \int d^3x \bar{\psi}(\vec{D} \cdot \vec{\gamma})\psi \rangle_H$$

$$H_m = \langle \int d^3x m \bar{\psi}\psi \rangle_H$$

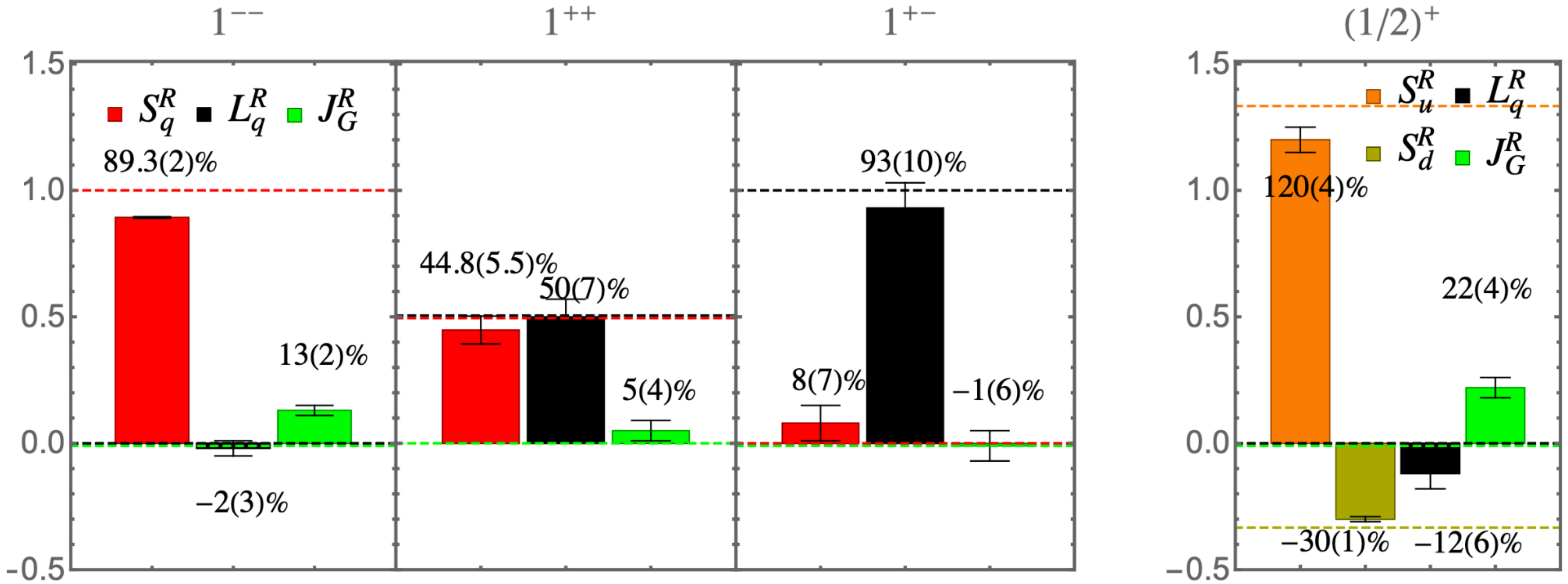
$$H_q = H_E + H_m$$

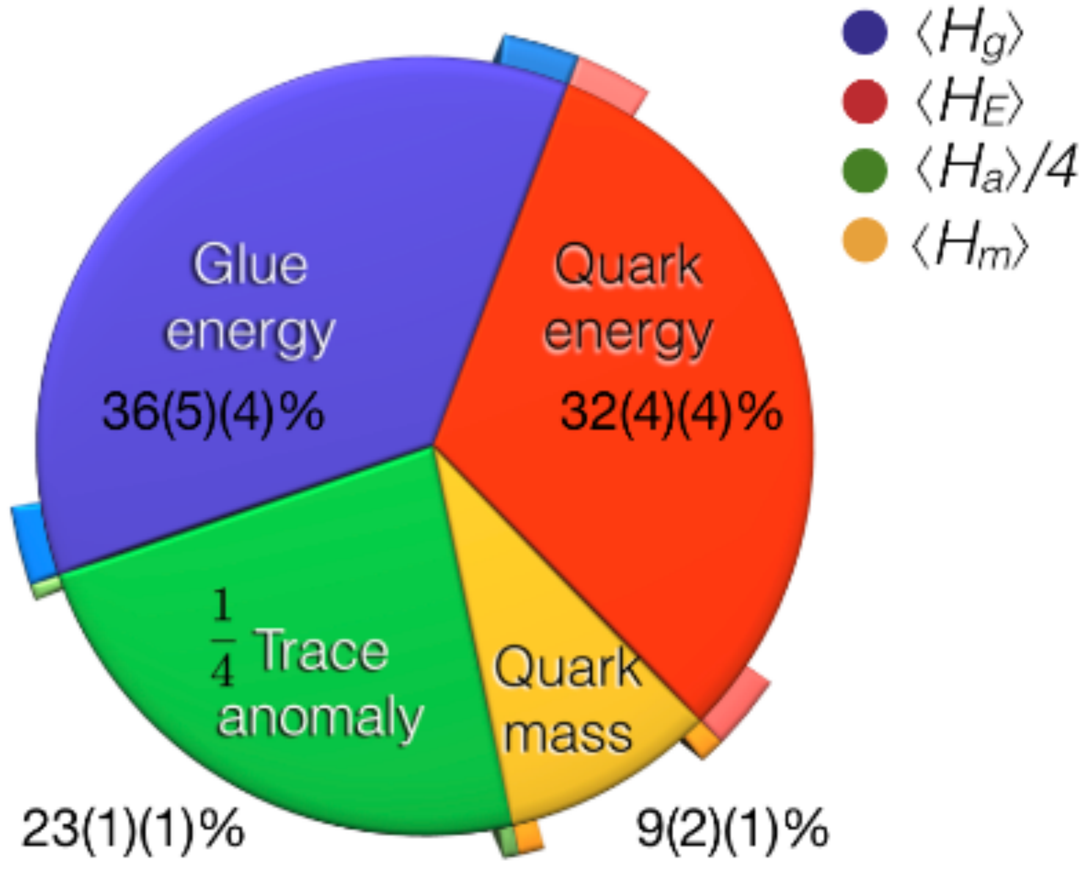
$$= \langle \int d^3x \bar{\psi} D_4 \gamma_4 \psi \rangle_H$$

W. Sun, et.al., χQCD, PRD103(2021) 094503

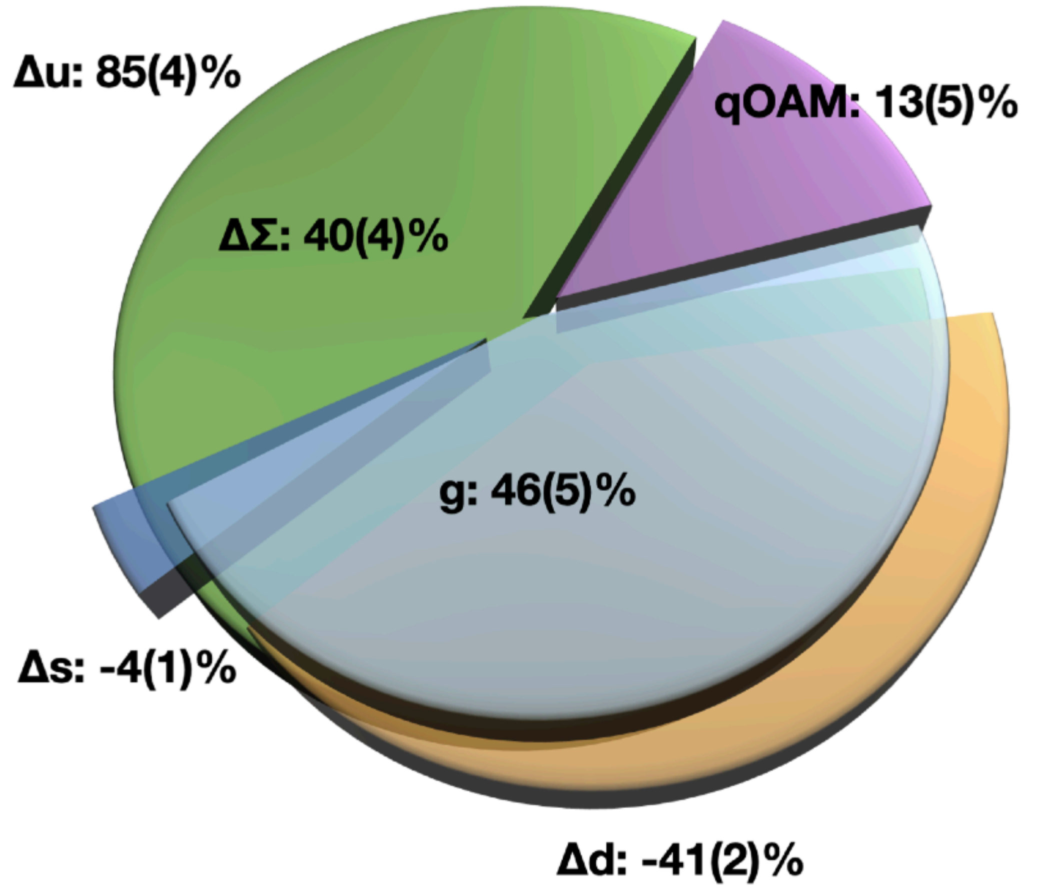
Discussion

Relativity of quark





Y. Yang, et.al., χQCD, PRL121(2018) 212001



G. Wang, et.al., χQCD, PRD106(2022) 014512

The charmonium mass decomposition suggests that $1 - v^2 \simeq \langle H_m \rangle_H / \langle H_q \rangle_H \sim 0.9$;

Similar to $\langle S_q \rangle_H / \langle S_q \rangle_H^{\text{Quarkmodel}}$**.**

$$M_H = T^{00} = H_E + H_m + H_g + \frac{1}{4}(H_a^q + H_a^g)$$

$$H_E = \langle \int \mathrm{d}^3x \bar{\psi}(\vec{D} \cdot \vec{\gamma})\psi \rangle_H$$

$$H_m = \langle \int \mathrm{d}^3x m \bar{\psi}\psi \rangle_H$$

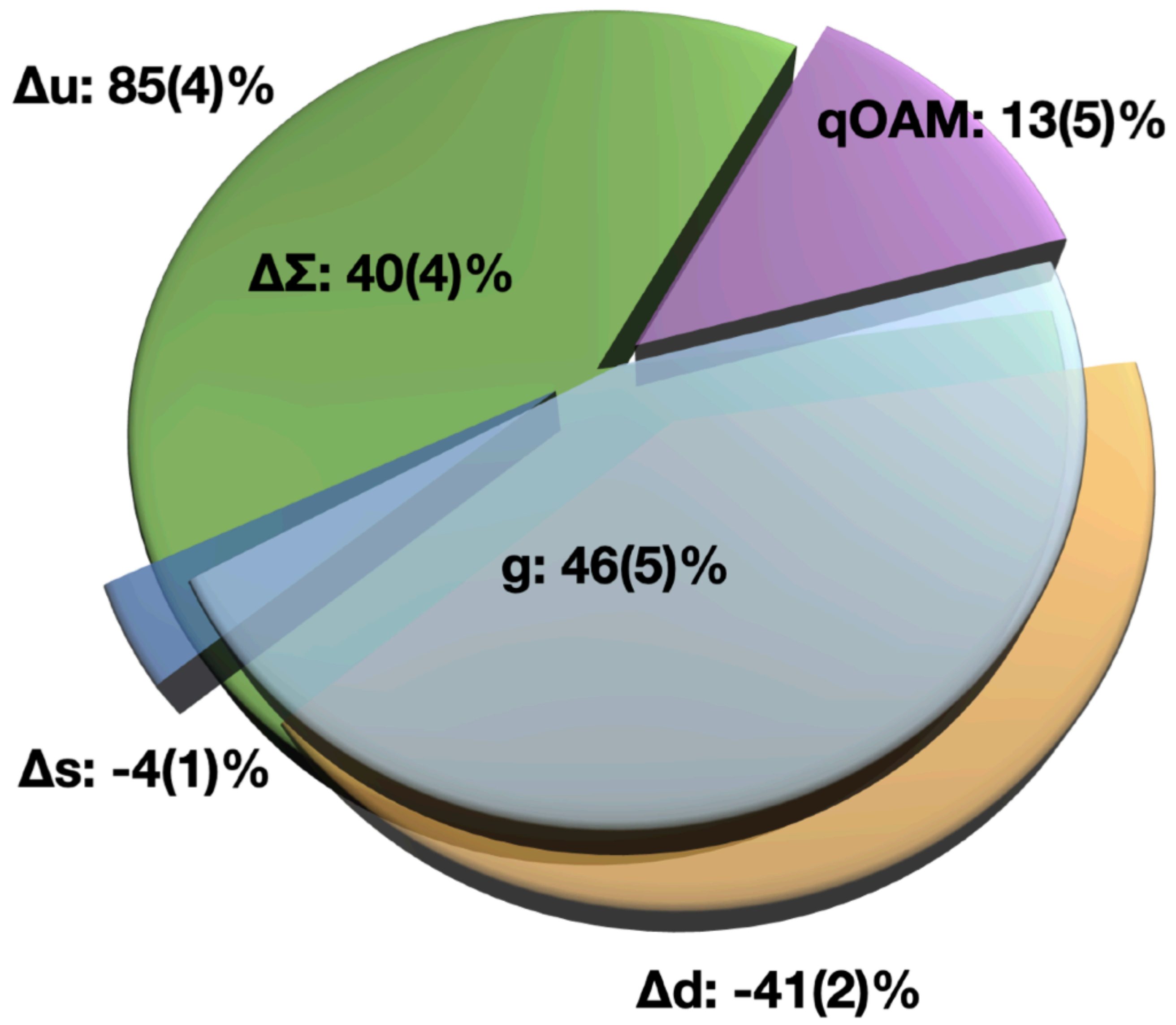
$$H_q = H_E + H_m$$

$$= \langle \int \mathrm{d}^3x \bar{\psi} D_4 \gamma_4 \psi \rangle_H$$

The nucleon mass decomposition suggests that $1 - v^2 \simeq \langle H_m \rangle_H^l / \langle H_q \rangle_H \sim 0.1$;

Relativistic effect makes nucleon to be complicated.

Summary



- Contributions of quark spin and OAM to the charmonium and also proton-like triple heavy quark state are comparable with the expectation of non-relativistic quark model;

- Provides evidence that the non-triviality of proton spin decomposition mainly arises from the relativistic effects of the light quark.

- More systematic study is on going.