Prospects on GPDs and gravitational form factors of hadrons

Shunzo Kumano
Japan Women's University
High Energy Accelerator Research Organization (KEK)
http://research.kek.jp/people/kumanos/

recent collaborations

Mainly works based on my collaborators: W.-C. Chang (Academia Sinica), X. Chen (IMP), Y.-B. Dong (IHEP), D. Fu (IMP), M. Hirai (NIT), H. Kawamura (Juntendo), R. Kunitomo (JWU), M. Oka (Riken) J.-C. Peng (Illinois), R. Petti (South Carolina), S. Sawada (KEK), T. Sawada (OMU), Q.-T. Song (Zhengzhou), M. Strikman (Penn State), K. Sudoh (Nishogakusha), K. Tanaka (Juntendo), O. V. Teryaev (JINR), S. Wu (IMP), Y.-P. Xie (IMP), S. Diehl (Giessen/Connecticut) *et al*. on transition GPDs

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https://indico.pnp.ustc.edu.cn/event/1119/

November 9, 2024

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* topics covered by other speakers,
I will skip most slides.

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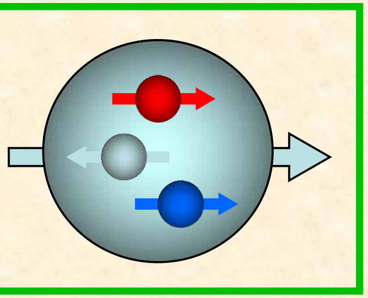
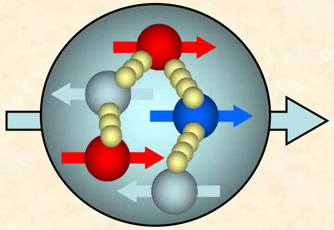
- *s*-channel or timelike GPDs (GDAs) at e^+e^- colliders (KEKB, BES, Tau-Charm, …) P.-J. Lin Y. Zhao
- GPDs at hadron accelerator facilities (J-PARC, LHCspin, NICA, GSI, …) W.-C. Chang S. Sawada L.L. Pappalardo A. Guskov
- GPDs at neutrino facilities (Fermilab, CERN)
- GPDs for exotic hadrons (KEKB, BES, Tau-Charm, …) Y.-B. Dong
  Transition GPDs (JLab, J-PARC, …)
- GPDs for spin-1 and 3/2 hadrons (JLab, KEKB, BES, Tau-Charm, …)
- Future prospects on GPD projects

Motivations for studying gravitational form factors and GPDs

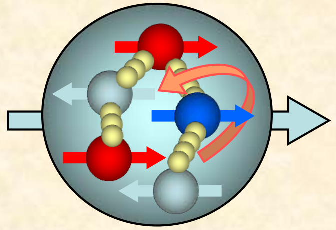
Origin of nucleon spin

"old" standard model





gluon spin

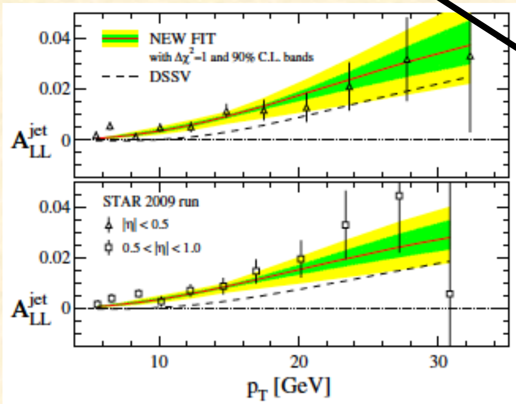


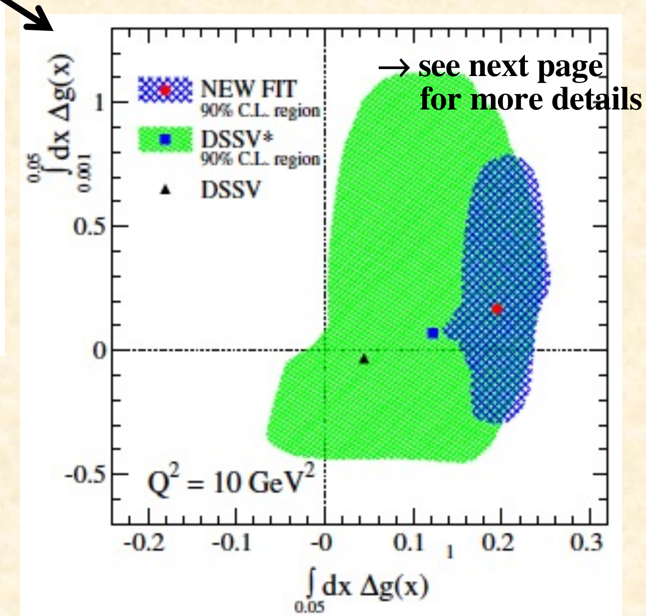
angular momentum

$$p_{\uparrow} = \frac{1}{3\sqrt{2}}\left(uud\left[2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right] + \text{permutations} \right)$$

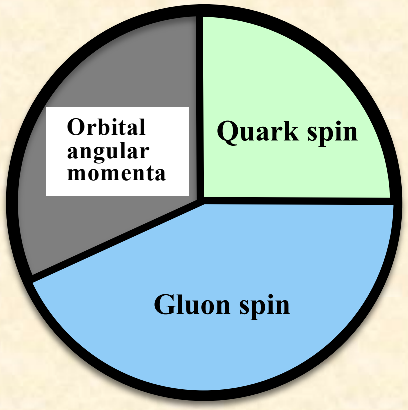
$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

$$\Delta\Sigma = \sum_i \int dx \left[\Delta q_i(x) + \Delta \bar{q}_i(x) \right] \to 1 \ (100\%)$$

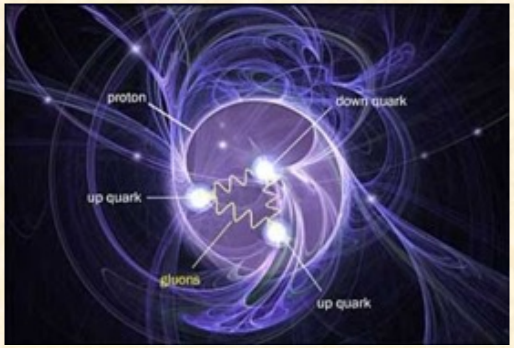




→ see next page for more details



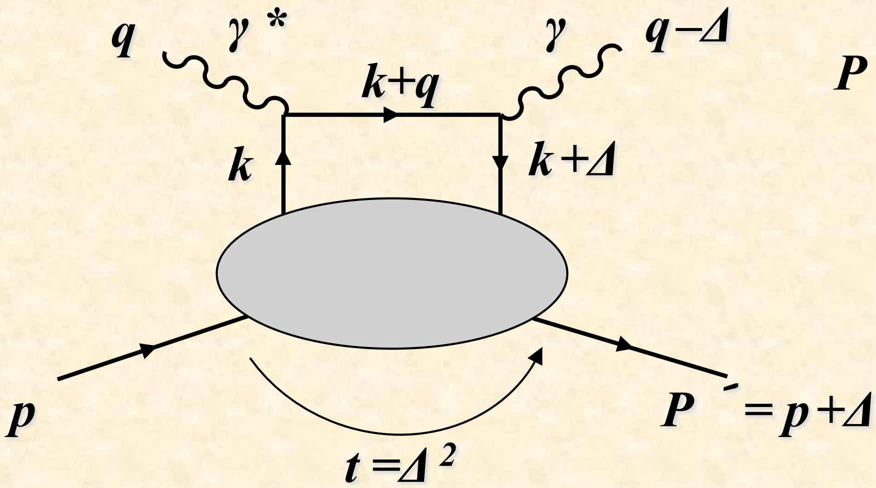
"A possible" spin decmposion



Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_{q,g}$$

Generalized Parton Distributions (GPDs)



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $\quad x = \frac{Q^2}{2p\cdot q}$

Momentum transfer squared $\quad t = \Delta^2$

Skewdness parameter $\quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^+\psi(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p'|\bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2)|p\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

Forward limit: PDFs $\quad H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\quad \int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

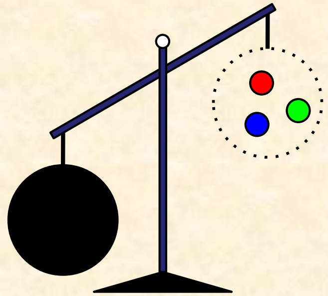
Axial and Pseudoscalar form factors G_A, G_P $\quad \int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \frac{1}{2}\Delta q + L_q$

$\Rightarrow$ probe L_q, key quantity to solve the spin puzzle!

Origin of hadron masses



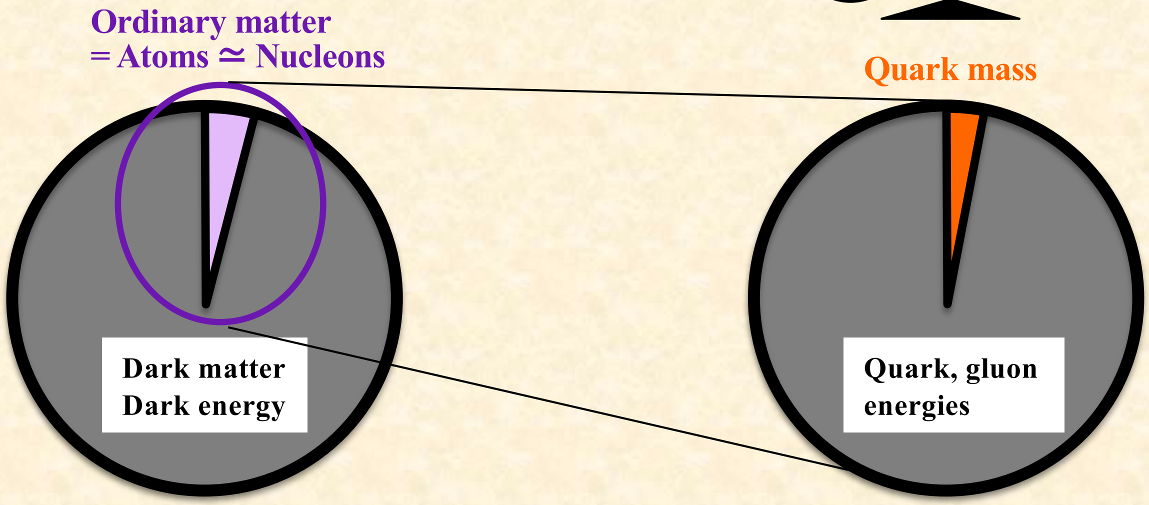
Mass and spin of the nucleon are two of fundamental quantities in physics.

Nucleon mass: $M = \langle p | \int d^3x \, T^{00}(x) | p \rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$

Tanaka's talk

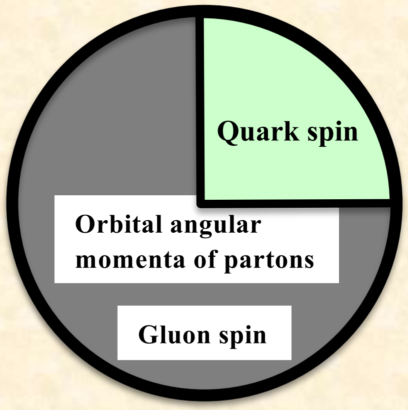


Dark matter | **Origin of nucleon mass**

Nucleon spin: $\frac{1}{2} = \langle p | J^3 | p \rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2} \varepsilon^{3jk} \int d^3x \, M^{3jk}(x)$

Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x) x^{\mu} - T^{\alpha\mu}(x) x^{\nu}$



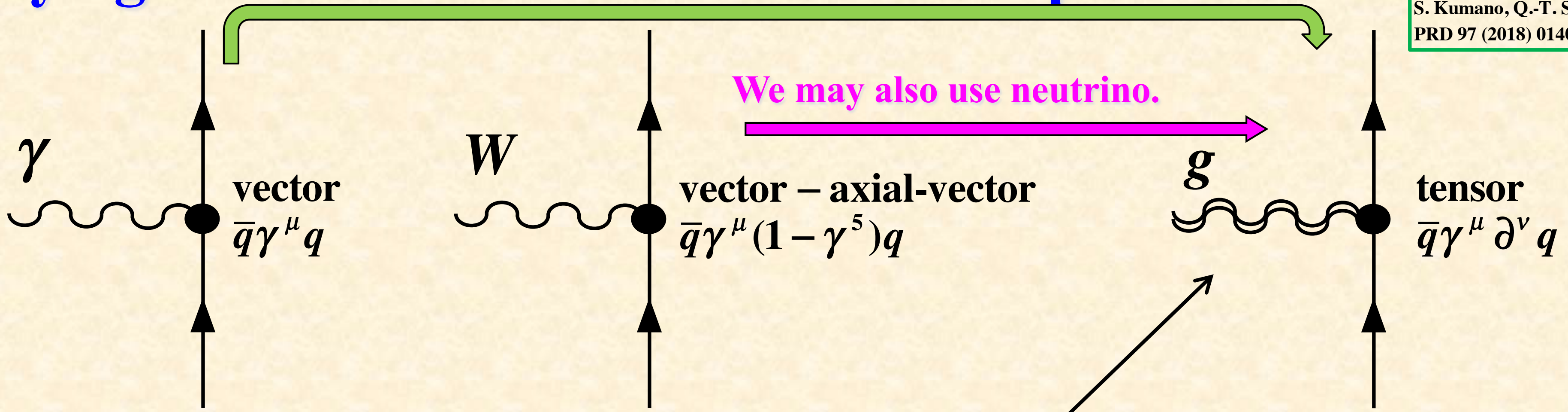
Origin of nucleon spin
("Dark spin")

Why “gravitational” interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020.

We may also use neutrino.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2)\gamma^+ q(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

Non-local operator of GPDs/GDAs:

$$(P^+)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z^+=0,\vec{z}_\perp=0}$$

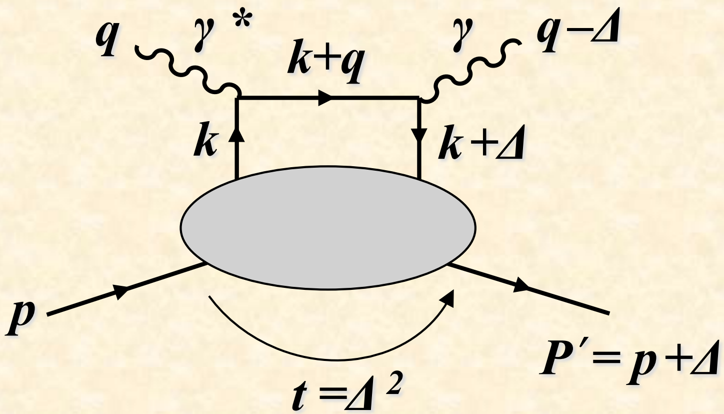
$$= \left(i\frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z=0}$$

$$= \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+ \right)^{n-1} q(0)$$

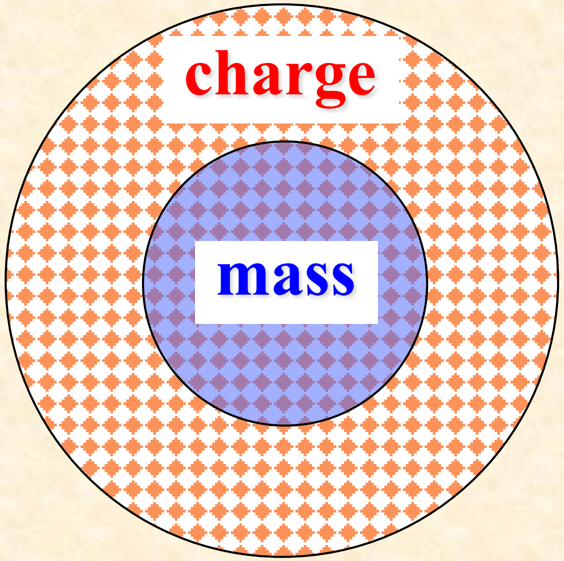
= energy-momentum tensor of a quark for $n=2$
(electromagnetic for $n=1$)

= source of gravitiy

Virtual Compton
or (timelike) two-photon process



Gravitational form factors and radii for pion



This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

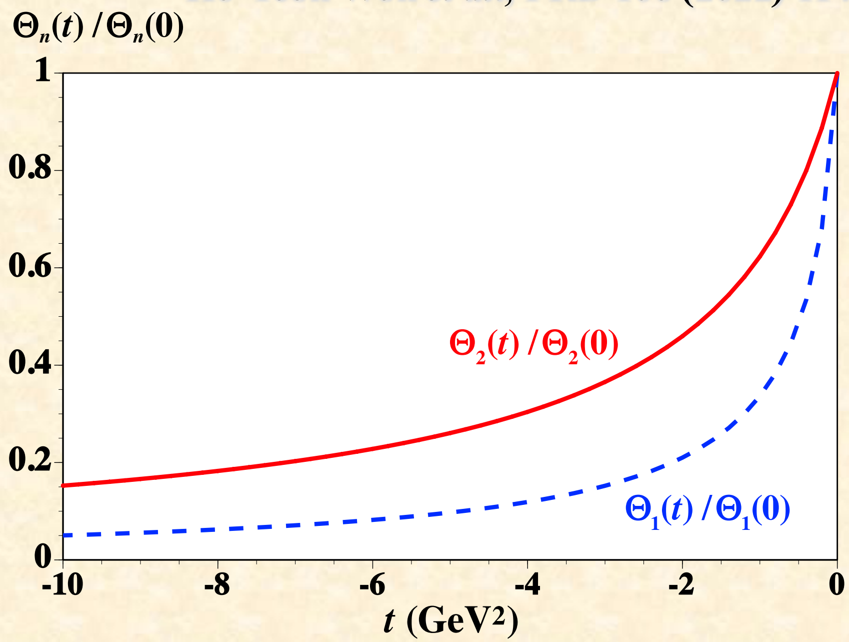
$$\Leftrightarrow \quad \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

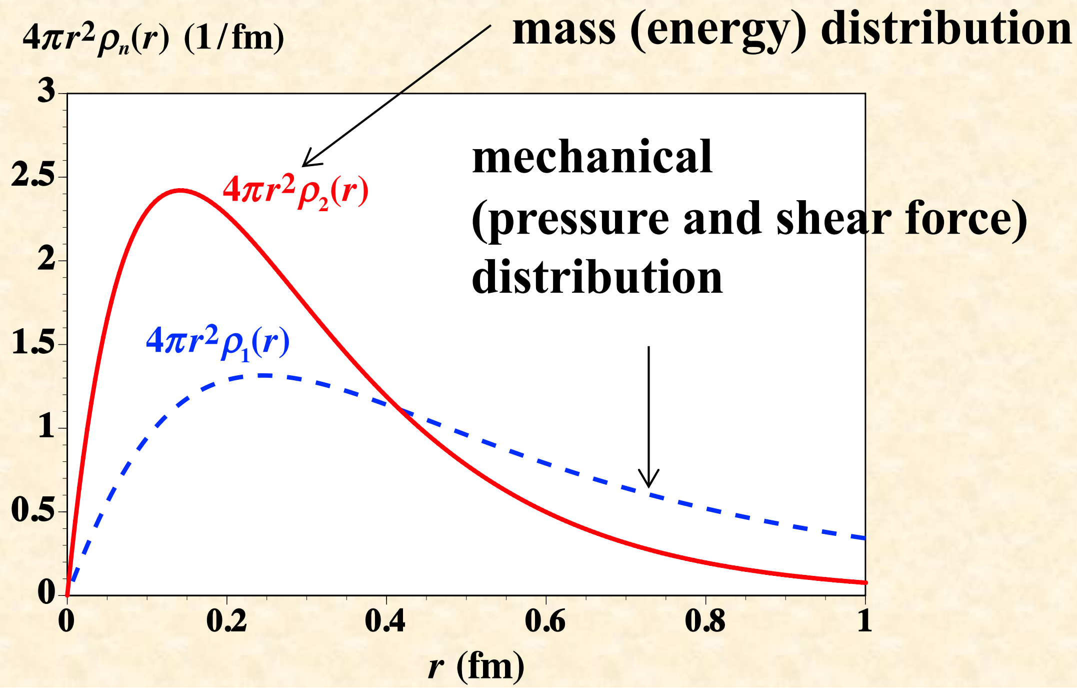
SK, Q.-T. Song, O. Teryaev
PRD 97 (2018) 014020.

Related theoretical studies:

- A. Freeseand I. C. Cloet, PRC 100 (2019) 015201;
- P. E. Shanahan and W. Detmold, PRD 99 (2019) 014511;
- C. D. Roberts *et al.*, Prog. Part. Nucl. Phys.120 (2021) 103883;
- J.-L. Zhang et al. PLB 815 (2021) 136158;
- June-Young Kim and Hyun-Chul Kim, PRD 104 (2021) 074019;
- Ho-Yeon Won *et al.*, PRD 106 (2022) 114009.

Proton mass radius:
R. Wang, W. Kou, Y.-P. Xie, X. Chen, PRD 103 (2021) L091501.





Nucleon pressure

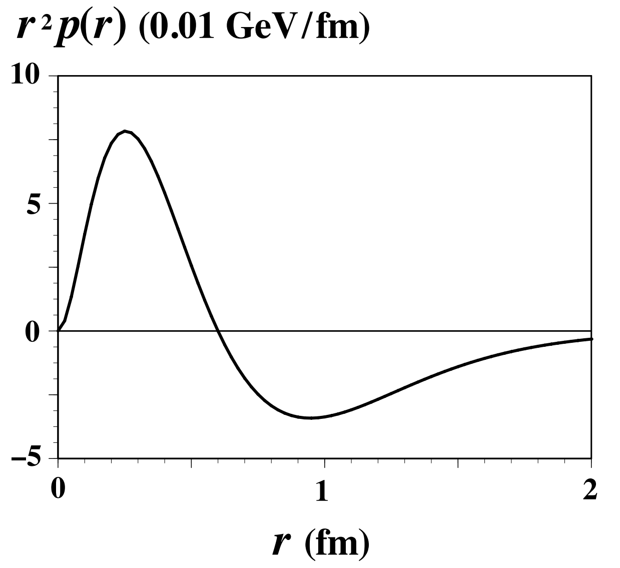
$$\langle N(p')|T_q^{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[A\gamma^{(\mu}\bar{P}^{\nu)} + B\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}Mg^{\mu\nu}\right]u(p)$$

Recent progress

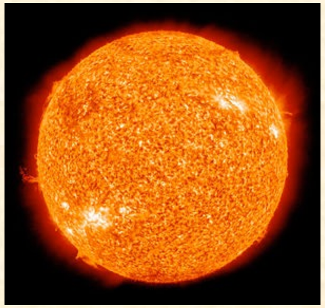
V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557 (2018) 396;

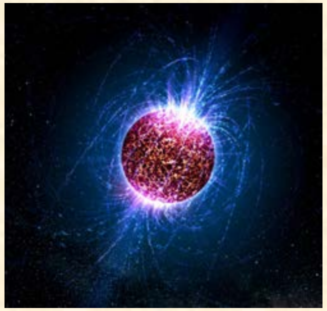
M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 1830025;

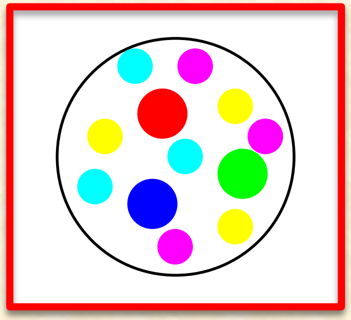
C. Lorce, H. Moutarde, and A. P. Tranwinski, Eur. Phys. J. C 79 (2019) 89.



Highest pressure in nature 1 Pa (Pascal) = 1 N/m^2







Earth atmosphere: 10^5 Pa = 1000 hPa

Center of earth: 10^{11} Pa = 100GPa

Center of Sun: 10^{16} Pa = 10 PPa

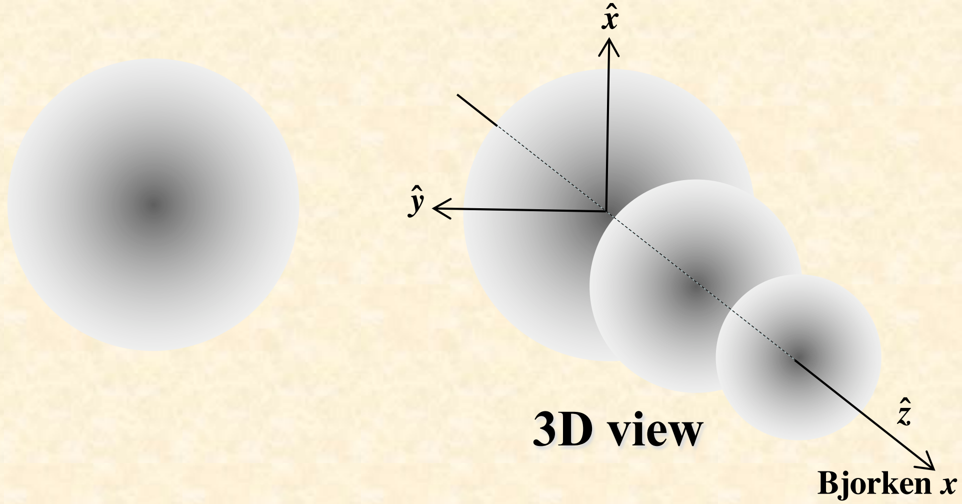
Neutron star: 10^{34} Pa

Hadron: 10^{35} Pa

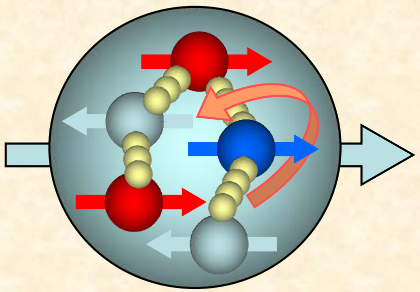
Proton (hadrons) puzzle studies by hadron tomography

Hadron tomography

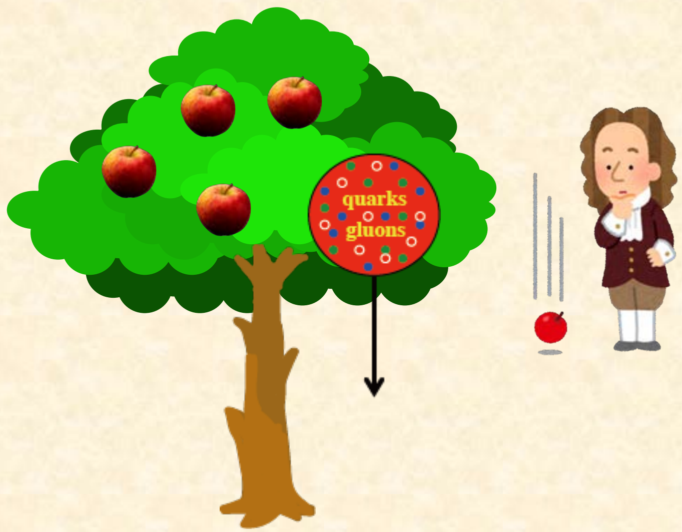
Proton radius puzzle



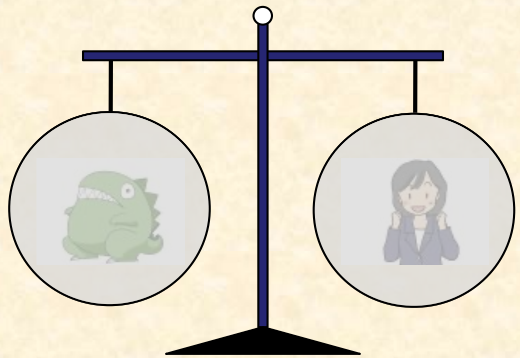
Origin of nucleon spin



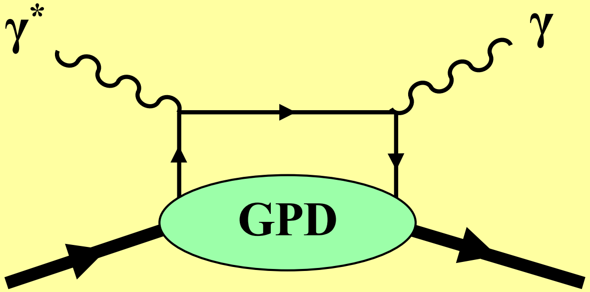
Source of gravity (mass)

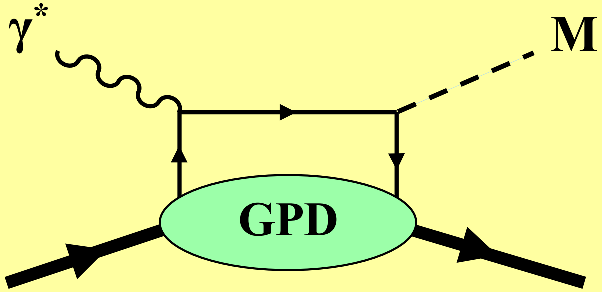


Exotic hadrons



Charged-lepton scattering on *t*-channel (spacelike) GPDs



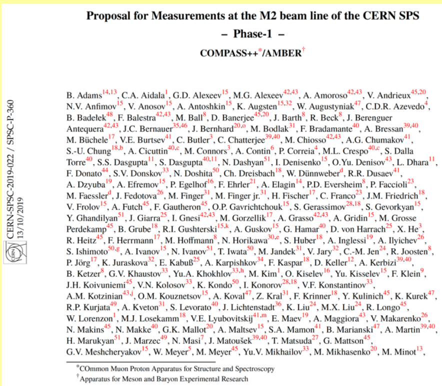


DVCS (Deeply Virtual Compton Scattering)

DVMP (Deeply Virtual Meson Production)

Jefferson Lab

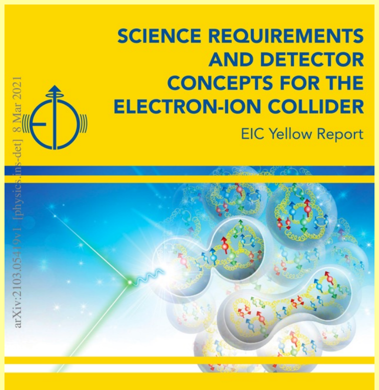
CERN-AMBER


Proposal for Measurements at the M2 beam line of the CERN SPS
– Phase-1 –
COMPASS++*/AMBER†

CERN-SPSC-2019-022 / SPSC-P-360
13/10/2019

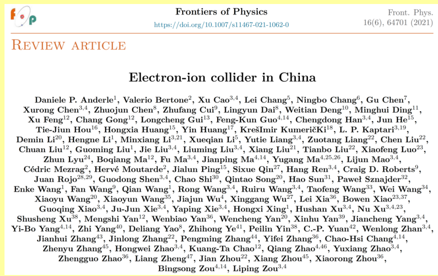
*COmmon Muon Proton Apparatus for Structure and Spectroscopy
†Apparatus for Meson and Baryon Experimental Research

EIC-US


SCIENCE REQUIREMENTS AND DETECTOR CONCEPTS FOR THE ELECTRON-ION COLLIDER
EIC Yellow Report

arXiv:2103.05419v1 [physics.ins-det] 8 Mar 2021

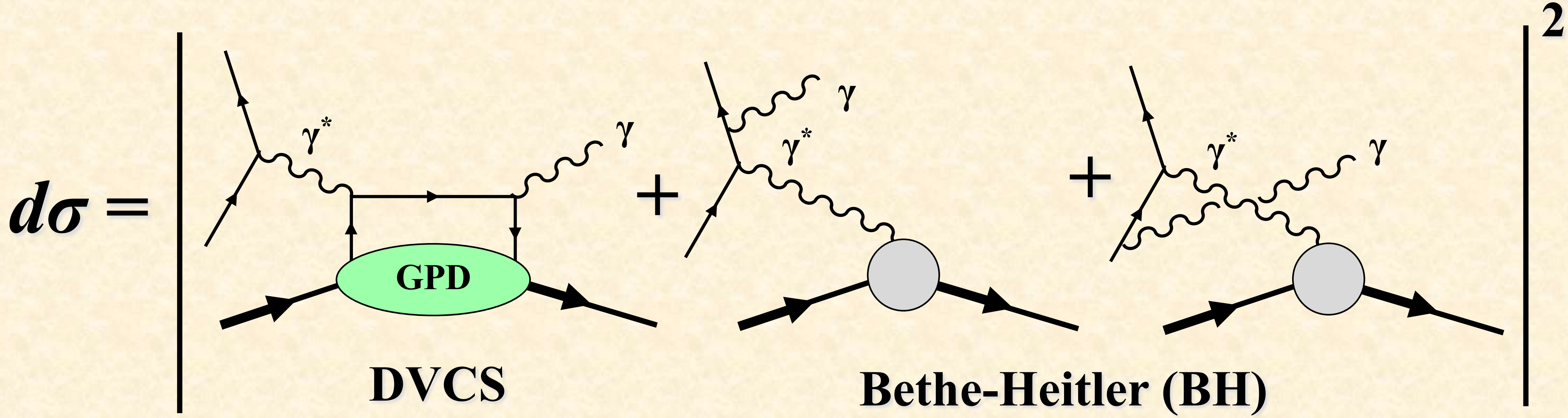
EicC


Frontiers of Physics
https://doi.org/10.1007/s11467-021-1062-0
Front. Phys. 16(6), 64701 (2021)

REVIEW ARTICLE

Electron-ion collider in China

Deeply Virtual Compton Scattering (DVCS)



$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\,dx\,dt\,d\phi} \propto \left|T_{DVCS} + T_{BH}\right|^2$$

HERMES, JLab,
COMPASS/AMBER, EIC, EicC, ⋯

e.g. Polarized beam: $d\sigma(e^{\uparrow}) - d\sigma(e^{\downarrow}) \propto T_{BH} * \mathrm{Im}\left(T_{DVCS}\right)$

$$\mathrm{Re}\,\mathcal{H}_q = e_q^2 \mathcal{P}\int_0^1 dx \left[H^q(x,\xi,t) - H^q(-x,\xi,t)\right]\left(\frac{1}{\xi - x} + \frac{1}{\xi + x}\right)$$

$$\mathrm{Im}\,\mathcal{H}_q = \pi e_q^2 \left[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)\right]$$

- Polarized beam, unpolarized target: $\mathrm{Im}\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\}$
- Unpolarized beam, longigudinally-polarized target: $\mathrm{Im}\{\mathcal{H}, \tilde{\mathcal{H}}\}$
- Polarized beam, longigudinally-polarized target: $\mathrm{Re}\{\mathcal{H}, \tilde{\mathcal{H}}\}$
- Unpolarized beam, transversely-polarized target: $\mathrm{Im}\{\mathcal{H}, \mathcal{E}\}$

Recent measurement at JLab

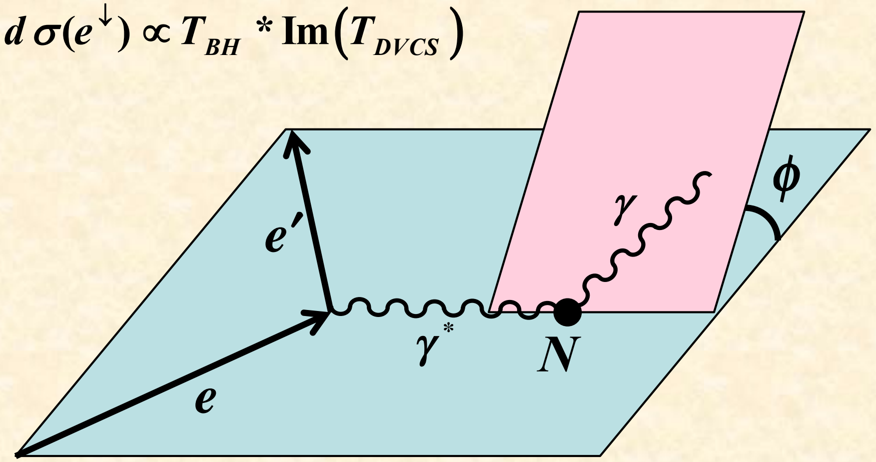
F. Georges et al., PRL 128 (2022) 252002.

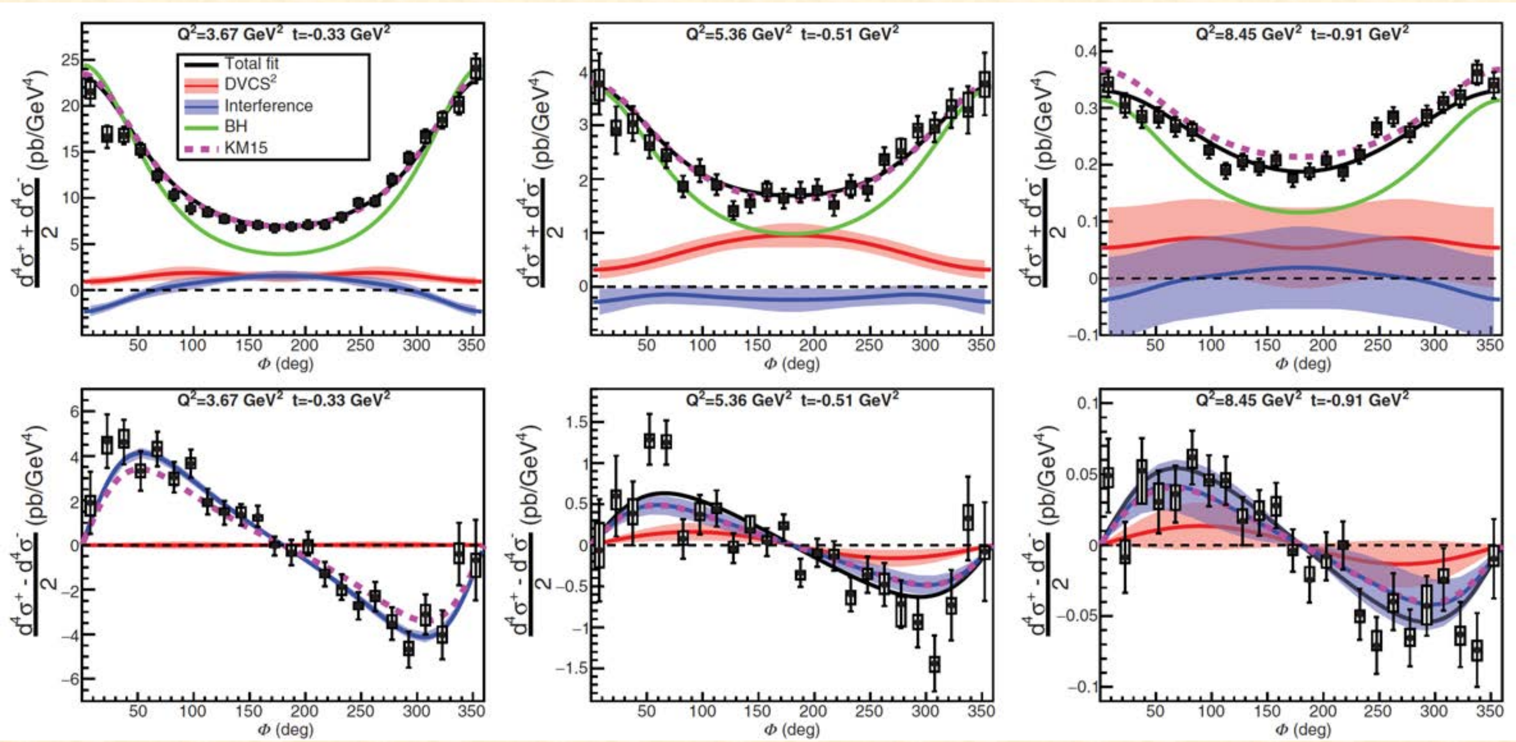
$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\, dx\, dt\, d\phi} \propto \left|T_{DVCS} + T_{BH}\right|^2,\ \text{e.g. Polarized beam: } d\sigma(e^{\uparrow}) - d\sigma(e^{\downarrow}) \propto T_{BH} * \mathrm{Im}\left(T_{DVCS}\right)$$

$$\mathrm{Re}\,\mathcal{H}_q = e_q^2 \mathcal{P}\int_0^1 dx \left[H^q(x,\xi,t) - H^q(-x,\xi,t)\right]\left(\frac{1}{\xi - x} + \frac{1}{\xi + x}\right)$$

$$\mathrm{Im}\,\mathcal{H}_q = \pi e_q^2 \left[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)\right]$$

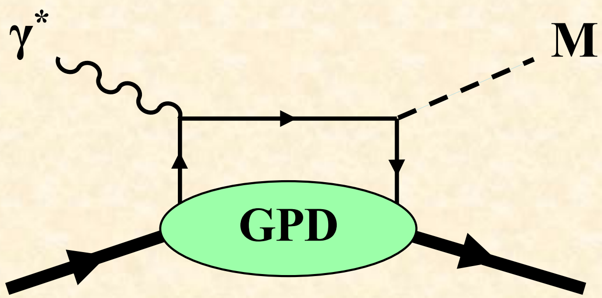
- Polarized beam, unpolarized target: $\mathrm{Im}\left\{\mathcal{H}, \tilde{\mathcal{H}}, \mathcal{E}\right\}$





ϕ dependent cross sections

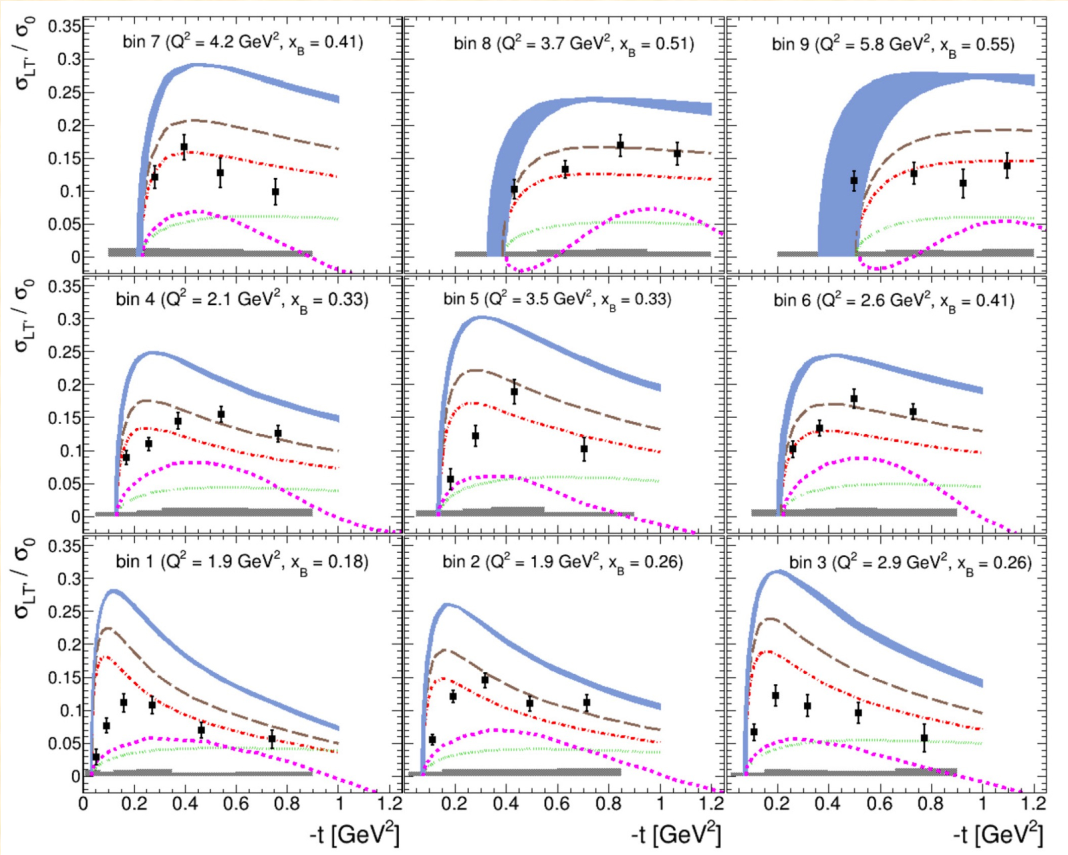
Deeply Virtual Meson Production (DVMP)



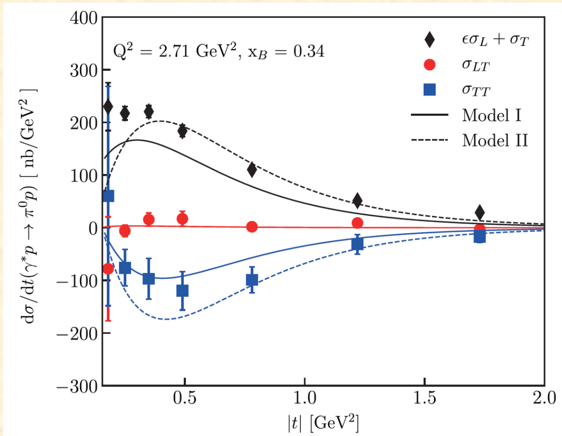
$$\frac{d\sigma(eN \to e'N'\gamma)}{dQ^2\,dx\,dt\,d\phi} \propto \sigma_T + \varepsilon\sigma_L + \varepsilon\sigma_{TT}\cos(2\phi)$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT}\cos(\phi) + P_b\sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'}\sin(\phi)$$

	Meson	Flavor
$\mathcal{H}_T, \mathcal{E}_T$	π^+	$\Delta u - \Delta d$
	π^0	$2\Delta u + \Delta d$
	η	$2\Delta u - \Delta d + 2\Delta s$
$\mathcal{H}, \mathcal{E}$	ρ^+	$u - d$
	ρ^0	$2u + d$
	ω	$2u - d$
	ϕ	g

K. Joo, EIC Asia workshop (2024).



S. Diehl *et al.*, PLB 839 (2023) 137761.

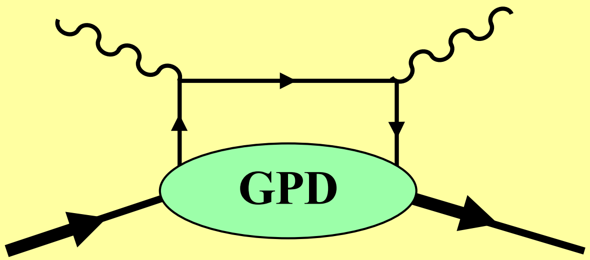


S. V. Goloskokov, Ya-Ping Xie, Xurong Chen, Chin. Phys. C 46 (2022) 123101.

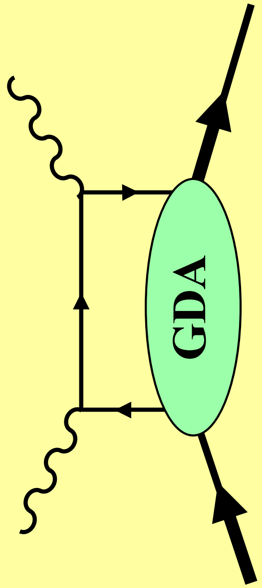
e^+e^- facilities on *s*-channel (timelike) GPDs

Generalized Distribution Amplitudes (GDAs = timelike GPDs)

***t*-channel (spacelike) GPDs** | **GDAs = *s*-channel (timelike) GPDs**



***s*-*t* crossing**



Extraction of GDAs and gravitational form factors from KEKB data.

SK, Q.-T. Song, O. Teryaev, Phys. Rev. D 97 (2018) 014020.

GPD $H_q^h(x,\xi,t)$ and GDA(= timelike GPD) $\Phi_q^{hh}(z,\zeta,W^2)$

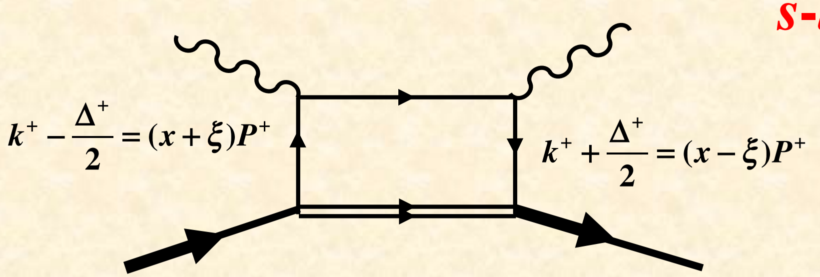
GPD: $H_q(x,\xi,t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | h(p) \rangle \big|_{y^+=0,\vec{y}_\perp=0}$, $\quad P^+ = \frac{(p+p')^+}{2}$

GDA: $\Phi_q(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p)\bar{h}(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | 0 \rangle \big|_{y^+=0,\vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2)\gamma^+\gamma_5\psi(y/2) | 0 \rangle \big|_{y^+=0,\vec{y}_\perp=0}$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

***s-t* crossing**



$p^+ = \left(P - \frac{\Delta}{2}\right)^+$ $\quad p'^+ = \left(P + \frac{\Delta}{2}\right)^+$

$P = \frac{p+p'}{2}$, $\Delta = p' - p$

JLab / COMPASS

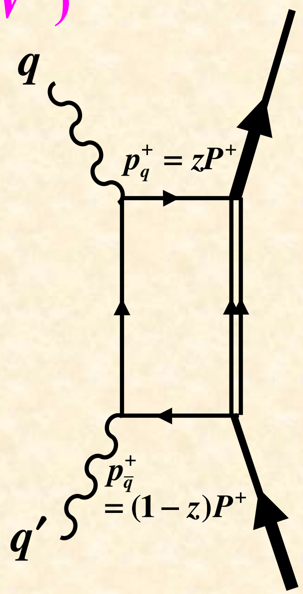
$z \Leftrightarrow \frac{1 - x/\xi}{2}$

$\zeta \Leftrightarrow \frac{1 - 1/\xi}{2}$

$W^2 \Leftrightarrow t$

KEKB

$p^+ = \zeta P^+$



$p'^+ = (1-\zeta)P^+$

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

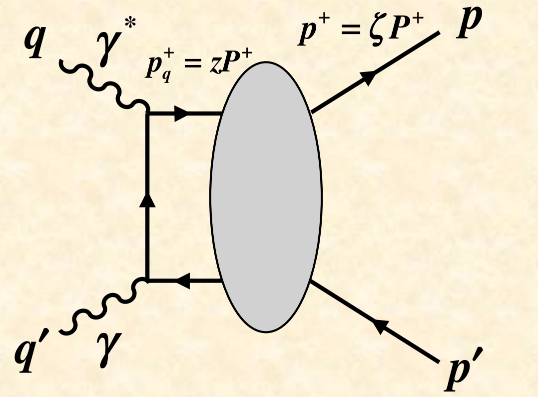
Skewdness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1+\beta\cos\theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Cross section for $\gamma^*\gamma \to \pi^0\pi^0$



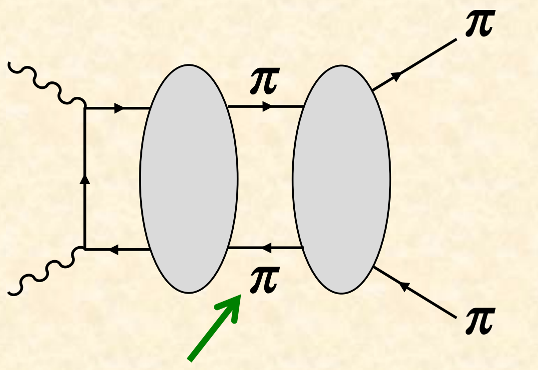
$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\,\overline{\sum_{\lambda,\lambda'}}|\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i\int d^4\xi\, e^{-i\xi\cdot q}\left\langle \pi(p)\pi(p')\middle| T J_{em}^\mu(\xi) J_{em}^\nu(0) \middle| 0\right\rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2}\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = -\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')g_T^{\mu\nu}\sum_q \frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

GDA (timelike GPD): $\Phi_q^{\pi\pi}(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-}\langle \pi(p)\pi(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|0\rangle\big|_{y^+=0,\vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}|A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$



Including intermediate resonance contributions

- Continuum: GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha(1-z)^\alpha(2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{\left[1+(s-4m_\pi^2)/\Lambda^2\right]^{n-1}}, \quad n=2 \text{ according to constituent counting rule}$$

- Resonances: There exist resonance contributions to the cross section.

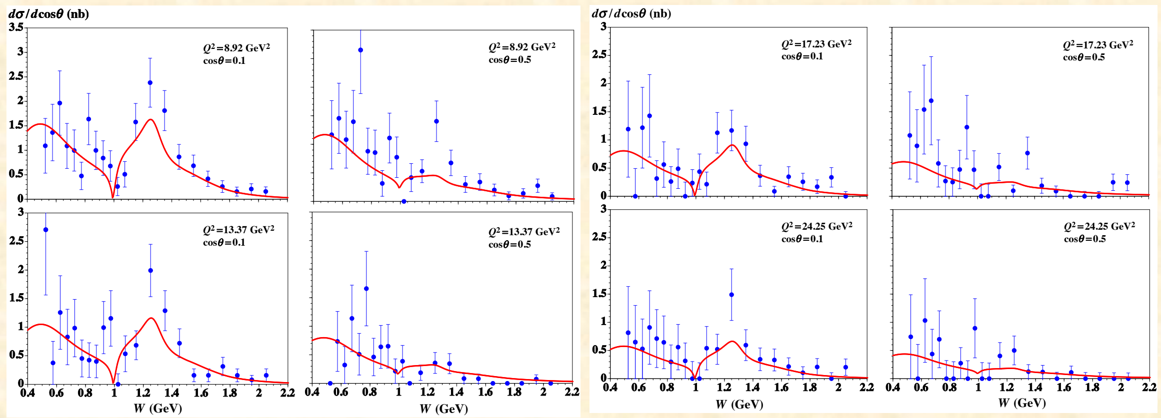
$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18N_f z^\alpha(1-z)^\alpha(2z-1)\left[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)\right]$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$\tilde{B}_{10}(W)$ = resonance $[f_0(500), f_0(980)]$ + continuum

$\tilde{B}_{12}(W)$ = resonance $[f_2(1270)]$ + continuum

$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$ $\qquad$ $Q^2 = 17.23,\ 24.25\ \text{GeV}^2$



Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

Gravitational form factors and radii for pion

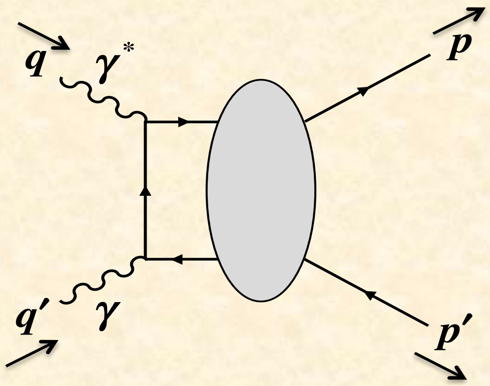
$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\langle \pi^0(p)\pi^0(p')|T_q^{++}(0)|0\rangle$$

$$\langle \pi^0(p)\pi^0(p')|T_q^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\right]$$

$$P = \frac{p+p'}{2}, \qquad \Delta = p' - p$$

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$: gravitational form factos for pion



See also Hyeon-Dong Son, Hyun-Chul Kim, PRD90 (2014) 111901.

Analysiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section

⇒ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$

⇒ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$

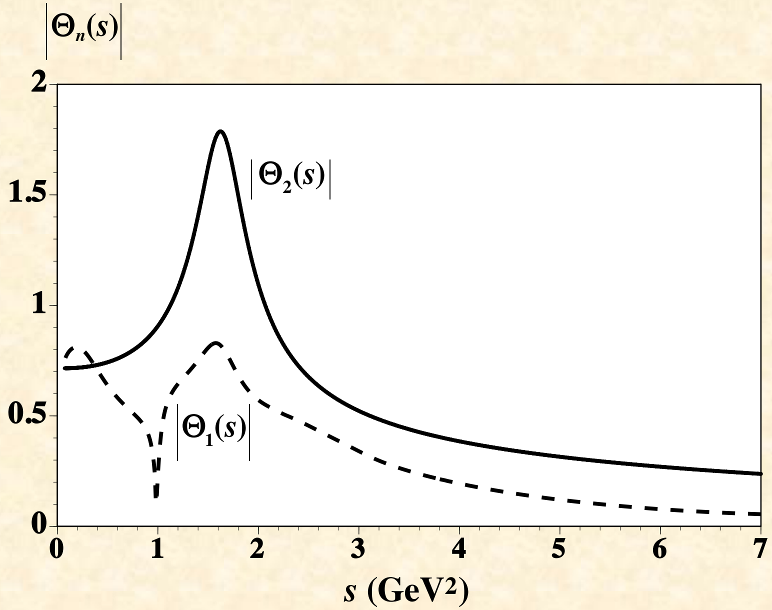
⇒ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$

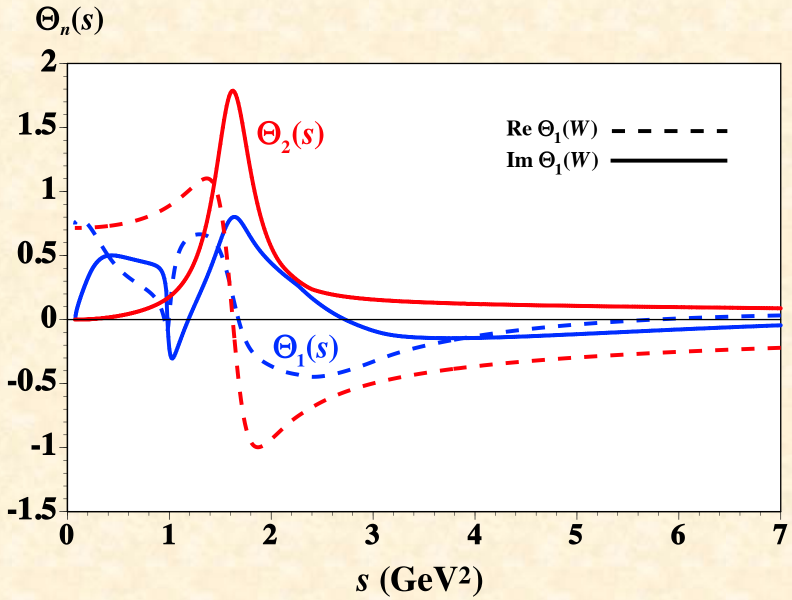
⇒ Gravitational radii of pion

Gravitational form factors:

Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.

Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009; Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008; K. Tanaka, JHEP 01 (2019) 120.





Spacelike gravitational form factors and radii for pion

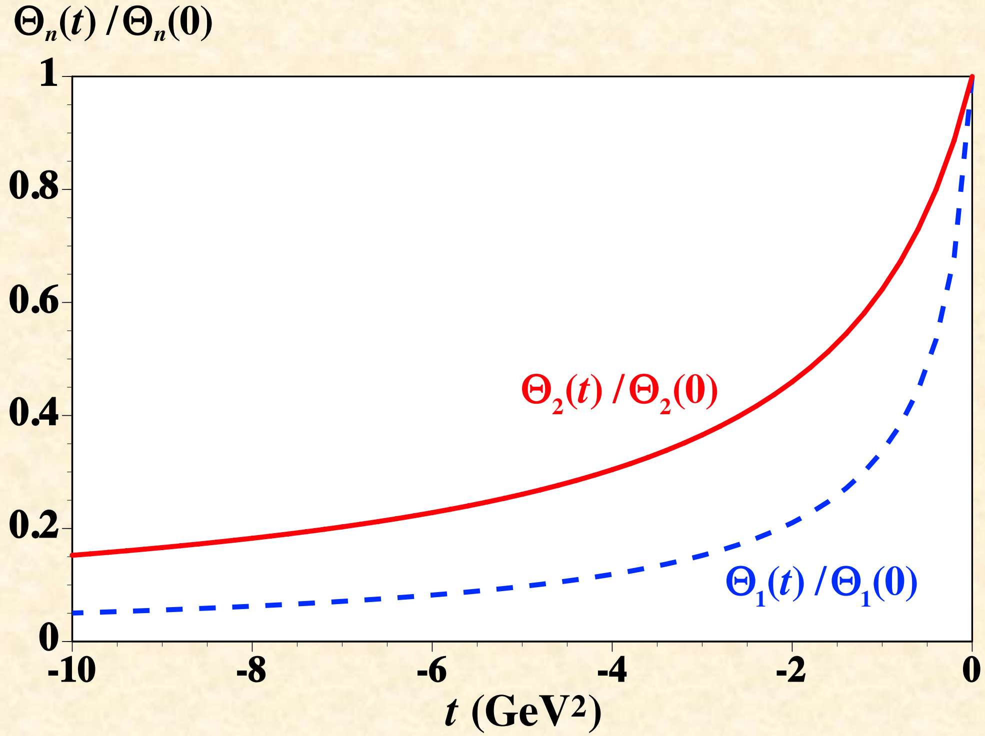
$$F(s) = \Theta_1(s),\ \Theta_1(s), \quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\operatorname{Im} F(s)}{\pi(s-t-i\varepsilon)}, \quad \rho(r) = \frac{1}{(2\pi)^3}\int d^3q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\, e^{-\sqrt{s}r} \operatorname{Im} F(s)$$

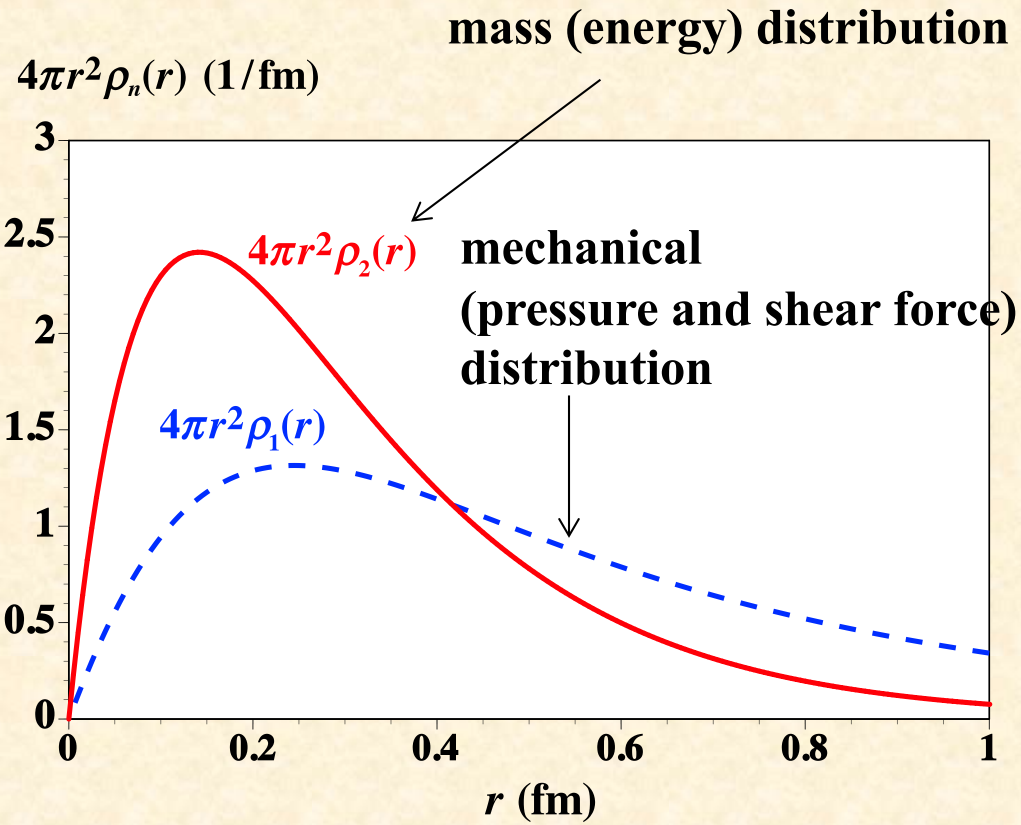
This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2\rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2\rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

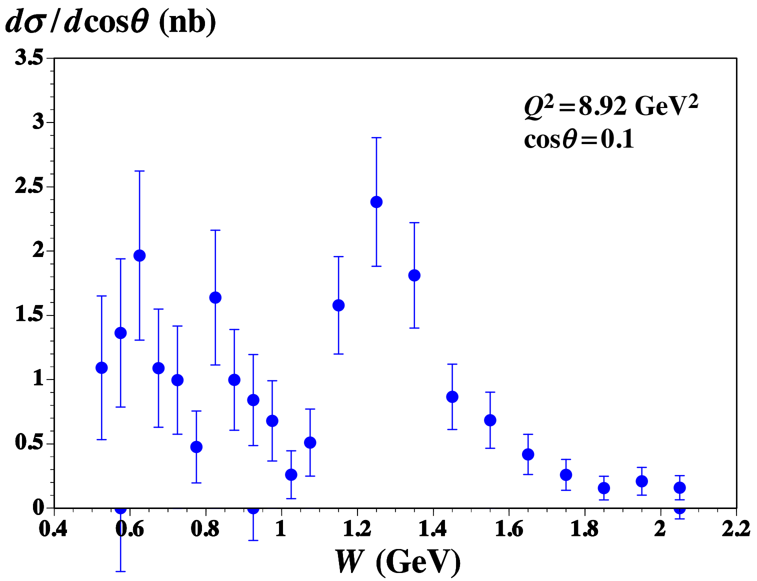
← First finding on gravitational radius from actual experimental measurements

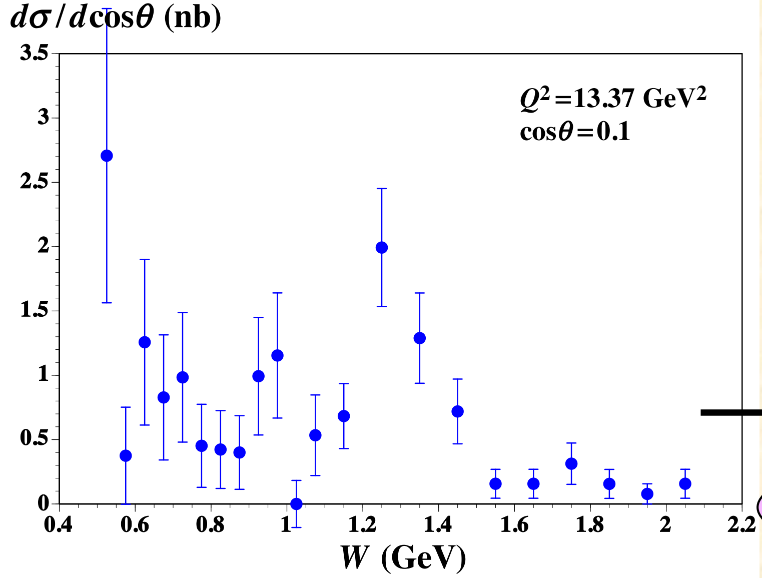
$$\Leftrightarrow \sqrt{\langle r^2\rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$



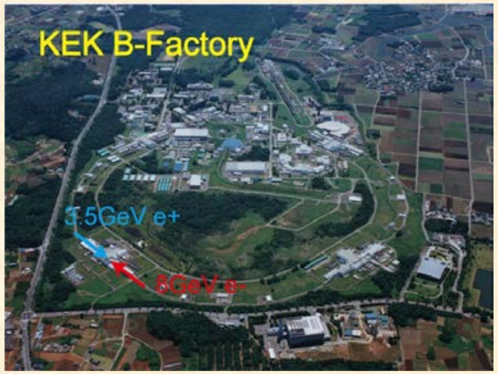


Super KEKB, ILC, FCC-ee, BES, Tau-Charm, ⋯





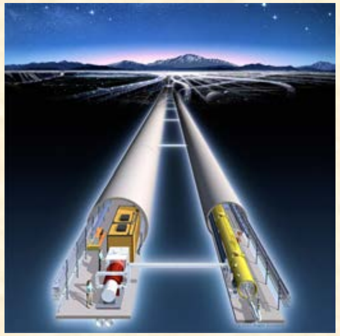
The errors are dominated by statistical errors, and they will be significantly reduced by super-KEKB.



From KEKB to ILC, FCC-ee, ⋯

- **Very Large Q^2**
- **Large W^2**

for extracting GDAs



ILC, FCC-ee, ⋯

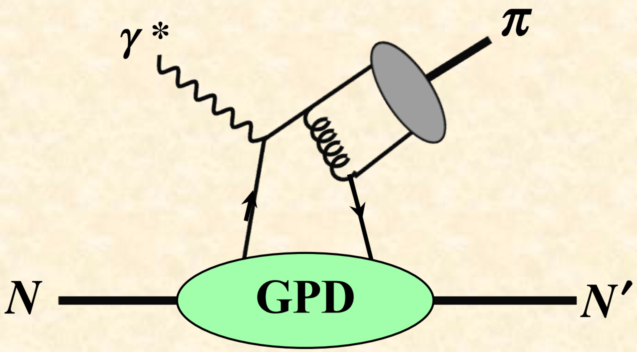
Possible studies on GPDs at hadron accelerator facilities

Chang’s talk

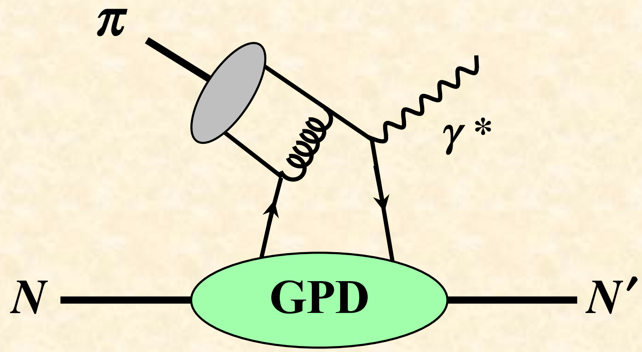
SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;
T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,
PRD 93 (2016) 114034.
J-PARC LoI 2019-07, J.-K. Ahn *et al*. (2019).
J-PARC proposal under preparation (2024),
Please get in touch with W.-C. Chang, N. Tomida
if you are interested in this project.

GPD projects at JLab /EIC and J-PARC

JLab / EIC

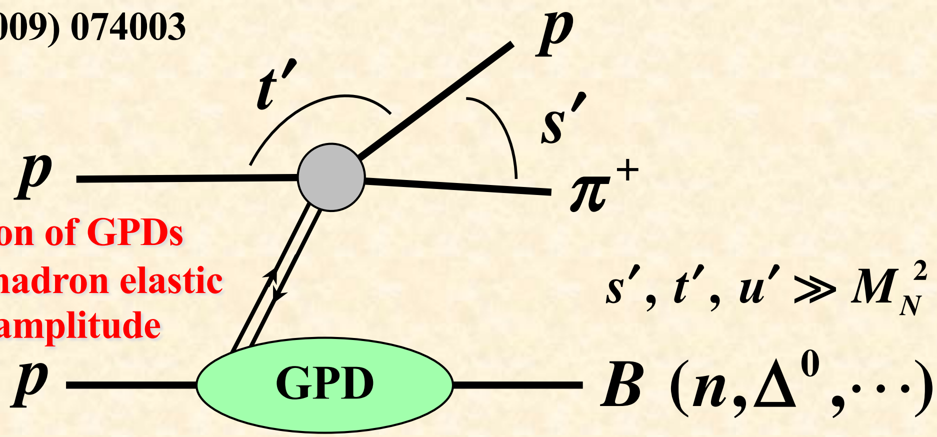


J-PARC



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



Investigation of GPDs
with 2→3 hadron elastic
scattering amplitude

$s',\ t',\ u' \gg M_N^2$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$\pi + N \to \gamma + \gamma + N'$

$h + M_B \to h' + \gamma + M_D$

$h + M_B \to h' + M_C + M_D$

Physics of J-PARC high-momentum beamline

2019 — 2021 — 2023 — 2025

Approved (E16, E50)

E16: Hadron masses in nuclear medium

. . .

E50: Charmed baryons

GPDs with E16 (proposal?)

e.g. $p+p \rightarrow p+\pi^{+}+n$

GPDs with E50 (LoI)

e.g. $\pi^{-}+p \rightarrow \mu^{+}\mu^{-}+n$

There is a possibility for high-energy hadron physics, including nucleon structure, ...

Proposal

Electron pair spectrometer at the J-PARC 50-GeV PS to explore the chiral symmetry in QCD

April 28, 2006
June 07, 2006 rev.1

S. Yokkaichi[1], H. En'yo, M. Naruki, R. Muto, T. Tabaru
RIKEN
K. Ozawa, H. Hamagaki
Center for Nuclear Study, Graduate School of Science, University of Tokyo
K. Shigaki
Graduate School of Science, Hiroshima University
S. Sawada, M. Sekimoto
High Energy Accelerator Research Organization (KEK)
F. Sakuma, K. Aoki
Department of Physics, Kyoto University

KEK/J-PARC-PAC 2012-19

Charmed Baryon Spectroscopy via the (π, D^{*-}) reaction

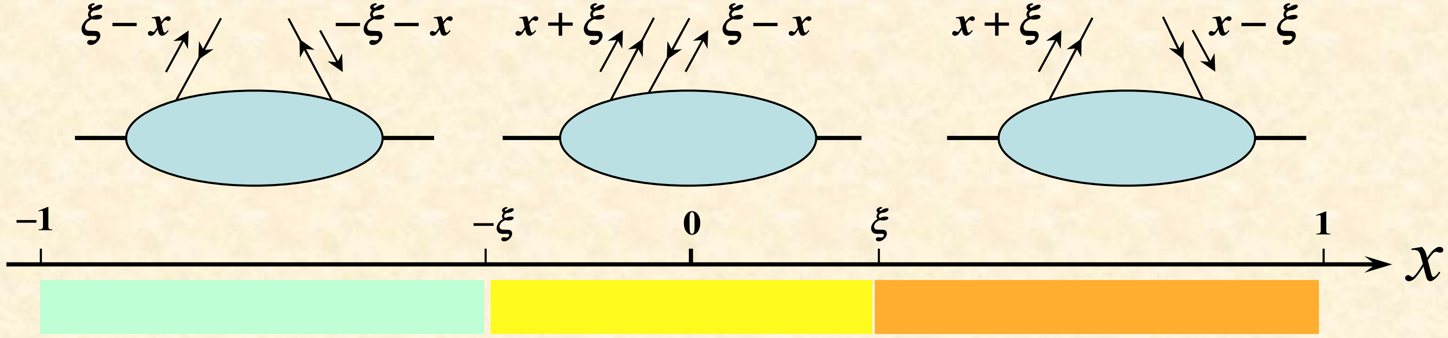
Y. Morino, T. Nakano,* H. Noumi†,* K. Shirotori, Y. Sugaya, and T. Yamaga
Research Center for Nuclear Physics (RCNP), Osaka University, 10-1, Mihogaoka, Ibaraki, Osaka, 567-0047, Japan
K. Ozawa‡
Institute of Particle and Nuclear Studies(IPNS), High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan
T. Ishikawa
Research Center for Electron Photon Science, Tohoku University, 1-2-1, Mikamine, Taihaku-ku, Sendai, Miyagi 982-0826, Japan
Y. Miyachi
Physics Department, Yamagata University, 1-4-12, Kojirakawa-machi, Yamagata 990-8560, Japan
K. Tanida
Department of Physics and Astronomy, Seoul National University, Seoul 151-747, Korea

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

GPDs in different x regions and GPDs at hadron facilities



$-1 < x < \xi \quad (x + \xi < 0,\ x - \xi < 0)$

$\xi < x < 1 \quad (x + \xi > 0,\ x - \xi > 0)$

$-\xi < x < \xi \quad (x + \xi > 0,\ x - \xi < 0)$

Quark distribution

Emission of quark with momentum fraction $x+\xi$
Absorption of quark with momentum fraction $x-\xi$

$q\bar{q}$(meson)-like distribution amplitude

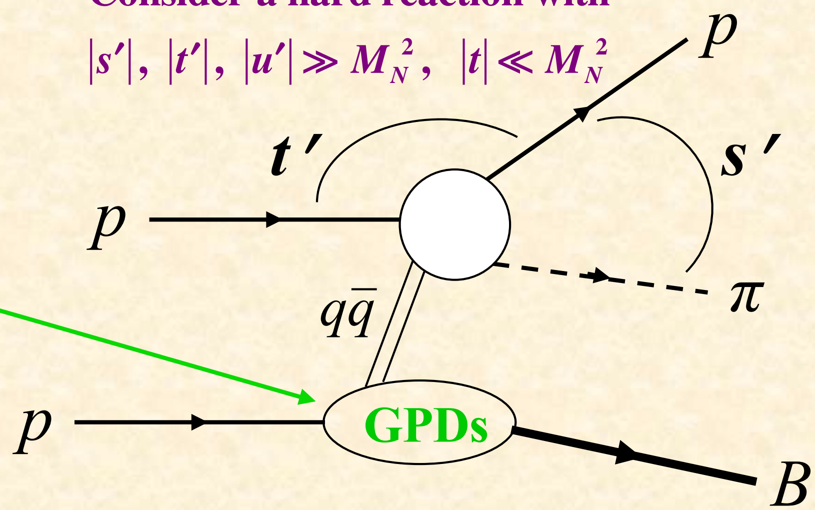
Emission of quark with momentum fraction $x+\xi$
Emission of antiquark with momentum fraction $\xi-x$

Antiquark distribution

Emission of antiquark with momentum fraction $\xi-x$
Absorption of antiquark with momentum fraction $-\xi-x$

Consider a hard reaction with

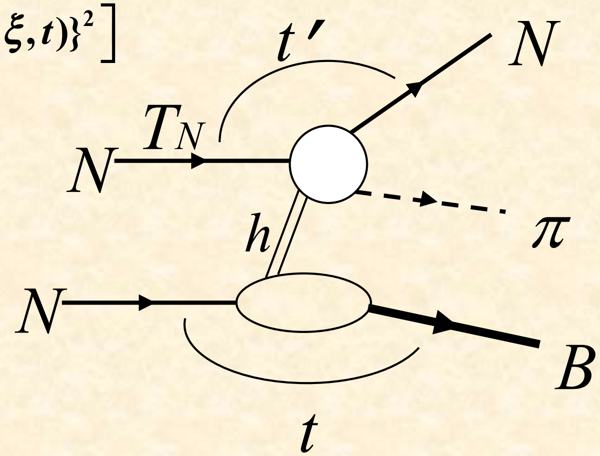
$|s'|,\ |t'|,\ |u'| \gg M_N^2,\quad |t| \ll M_N^2$



Efremov-Radyushkin
-Brodsky-Lepage (ERBL) region

Cross section estimate (ξ dependence)

$$\frac{d\sigma_{NN\to N\pi B}}{d\xi\, dt\, dt'} \propto \frac{d\sigma_{MN\to\pi N}}{dt'}\Big[8(1-\xi^2)\{H(x,\xi,t)\}^2 + 16\xi^2 H(x,\xi,t)E(x,\xi,t) - \frac{t}{m_N^2}(1+\xi)^2\{E(x,\xi,t)\}^2 + 8(1-\xi^2)\{\tilde{H}(x,\xi,t)\}^2 + 18\xi^2 \tilde{H}(x,\xi,t)\tilde{E}(x,\xi,t) - \frac{2t\xi^2}{m_N^2}\{\tilde{E}(x,\xi,t)\}^2\Big]$$

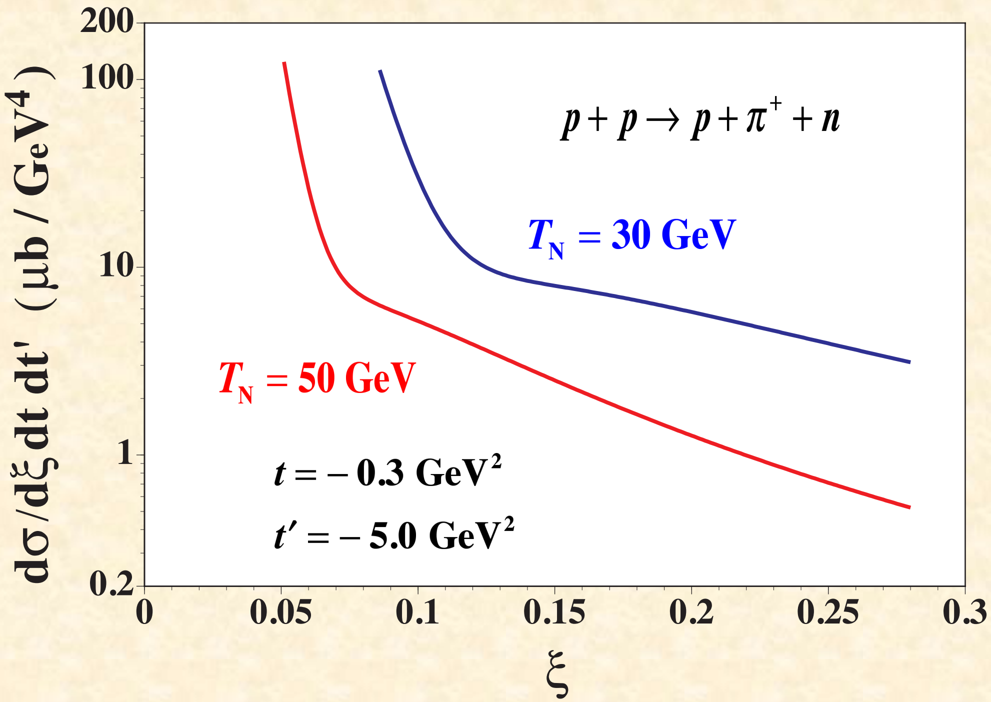


Skewdness parameter: $\xi = \dfrac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

$\dfrac{d\sigma}{d\xi dt dt'}\left(\dfrac{\mu b}{\text{GeV}^2}\right)$ as a function of ξ

at fixed $T_N = 30\ (50)$ GeV,

$t = -0.3\ \text{GeV}^2,\quad t' = -5\ \text{GeV}^2.$



At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π- and ρ-like intermediate states.

For the details, please look at
SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003.

Exclusive Drell-Yan $\pi^- + p \to \mu^+\mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{\tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$

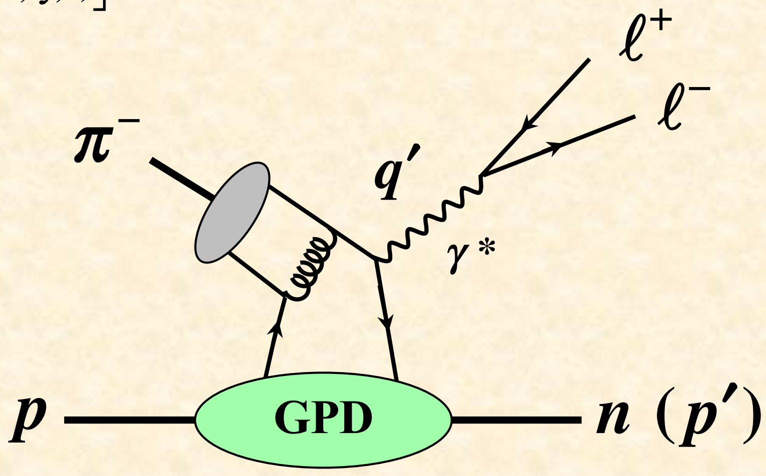
$$Q'^2 = q'^2,\ \ t=(p-p')^2,\ \ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s-m_\pi^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p(p')|\bar{q}(-z/2)\gamma^+\gamma_5 q(z/2)|p(p)\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_p^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_p^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

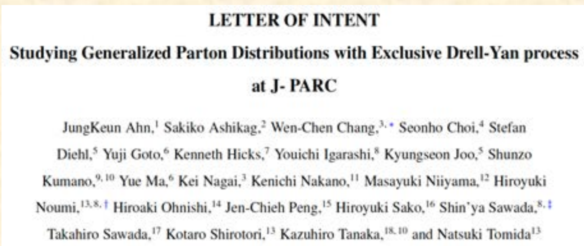
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle n(p')|\bar{q}_d(-z/2)\gamma^+\gamma_5 q_u(z/2)|p(p)\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

$$\tilde{H}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde{H}^d(x',\xi,t) - \tilde{H}^u(x',\xi,t)\right]$$

$$\tilde{E}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon}\right]\left[\tilde{E}^d(x',\xi,t) - \tilde{E}^u(x',\xi,t)\right]$$



T. Sawada, W.-C. Chang, SK, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.


LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,[1] Sakiko Ashikag,[2] Wen-Chen Chang,[3,*] Seonho Choi,[4] Stefan Diehl,[5] Yuji Goto,[6] Kenneth Hicks,[7] Youichi Igarashi,[8] Kyungseon Joo,[5] Shunzo Kumano,[9,10] Yue Ma,[6] Kei Nagai,[3] Kenichi Nakano,[11] Masayuki Niiyama,[12] Hiroyuki Noumi,[13,8,†] Hiroaki Ohnishi,[14] Jen-Chieh Peng,[15] Hiroyuki Sako,[16] Shin'ya Sawada,[8,‡] Takahiro Sawada,[17] Kotaro Shirotori,[13] Kazuhiro Tanaka,[18,10] and Natsuki Tomida[13]

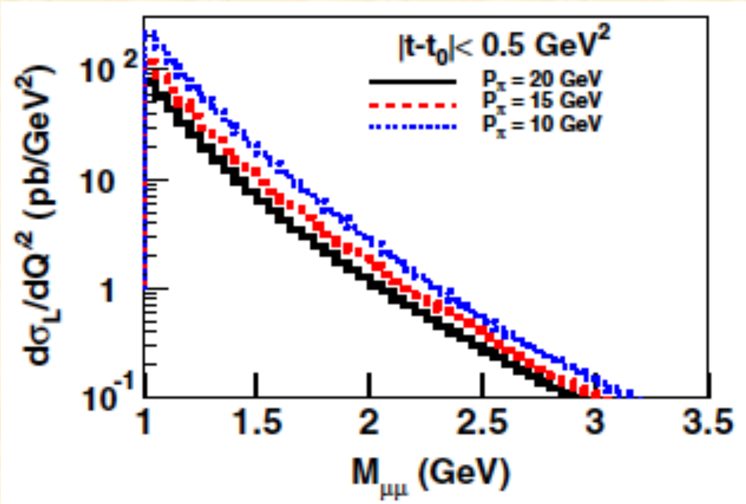
LoI for a J-PARC experiment

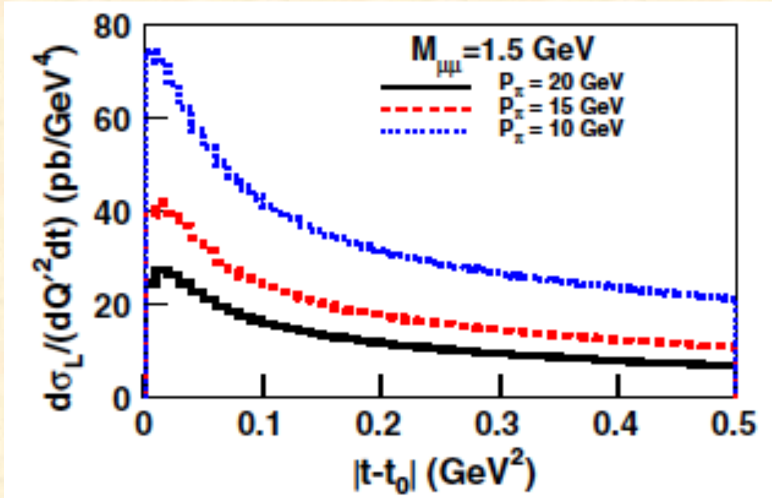
$$\pi^-(\bar{u}d) + p(uud) \to n(udd) + \gamma^*(\to \ell^+\ell^-)$$

Expected Drell-Yan events at J-PARC

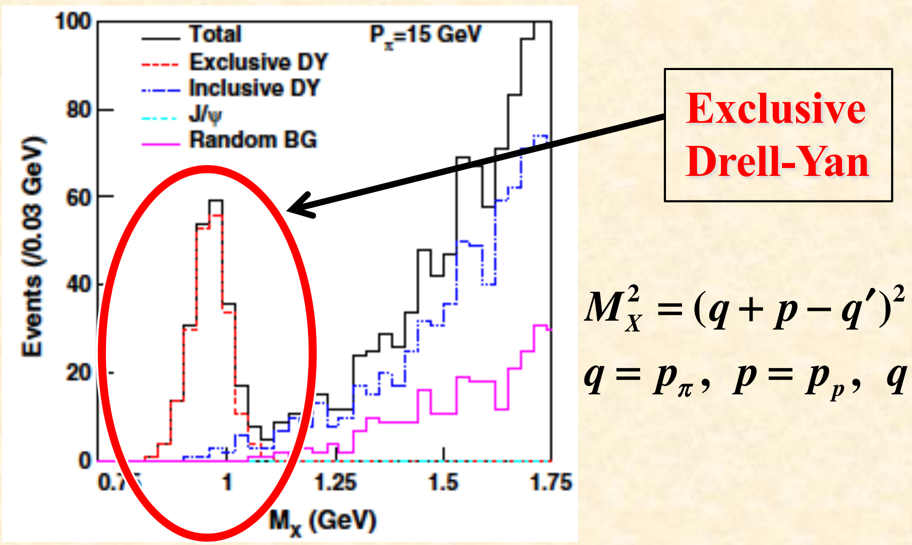
$$Q'^2 = q'^2,\ \ t = (p - p')^2,\ \tau = \frac{Q'^2}{2p \cdot q_\pi} \simeq \frac{Q'^2}{s - m_N^2}$$

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{\tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$



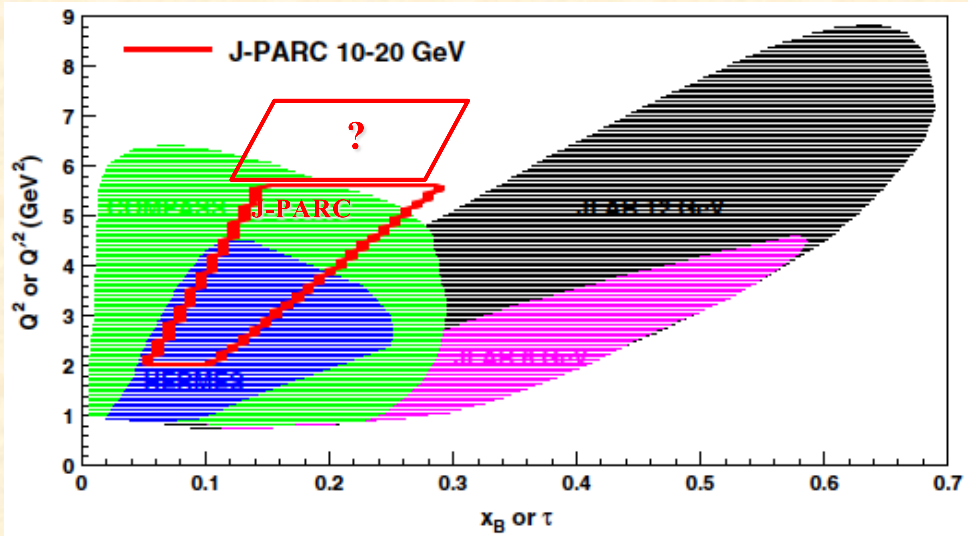


Missing mass

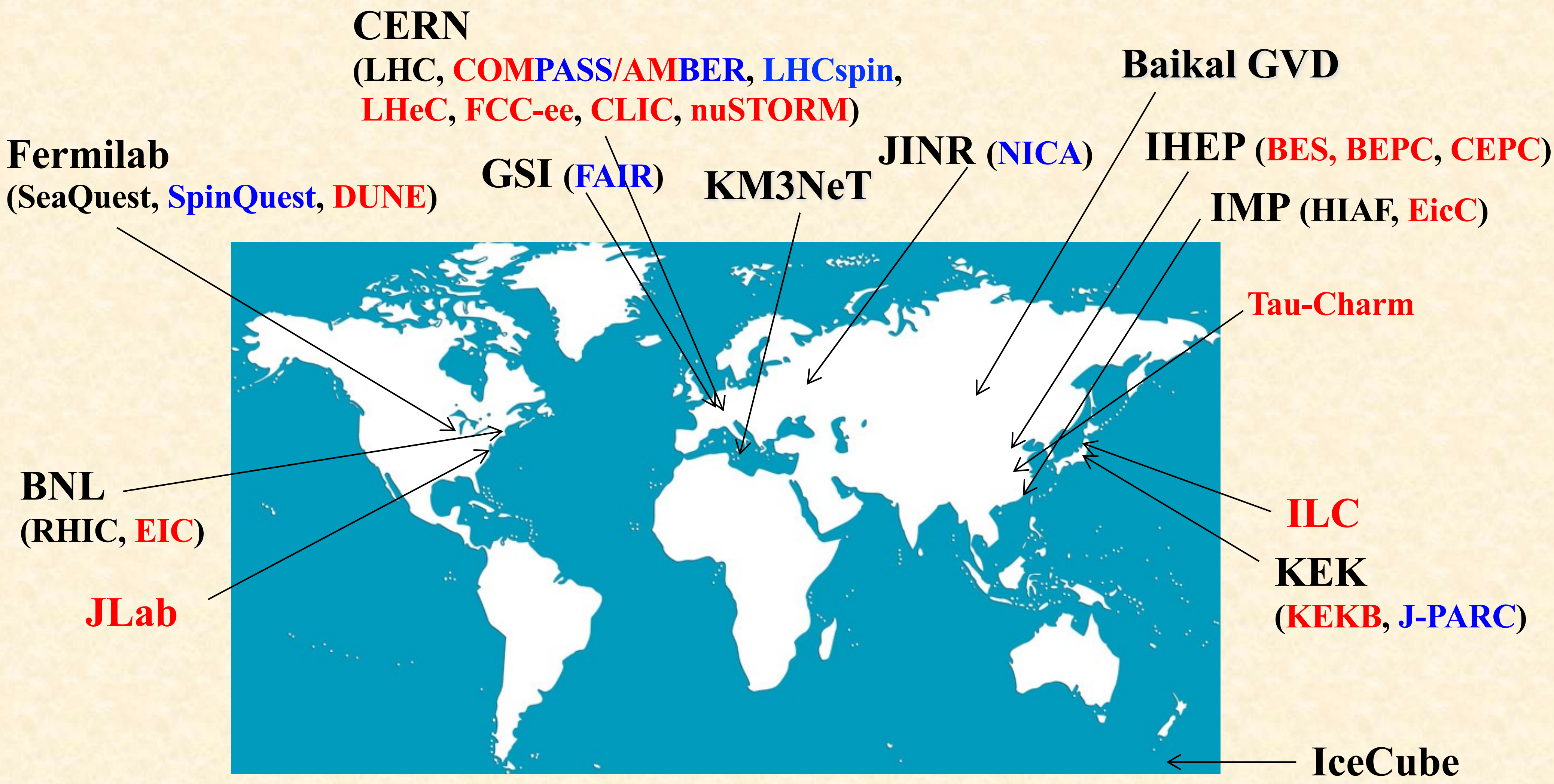


$$M_X^2 = (q + p - q')^2$$

$$q = p_\pi,\ \ p = p_p,\ \ q = p_{\mu^+\mu^-}$$



High-energy hadron physics experiments: hadron facilities



Facilities on hadron structure functions on GPDs including future possibilities.
Hadron accelerator facilities. Lepton accelerator facilities.

Possible GPD studies at neutrino facilities

X. Chen, SK, R. Kunitomo, S. Wu, Y.-P. Xie,

Euro. Phys. J. A 60 (2024) 208, 1-18

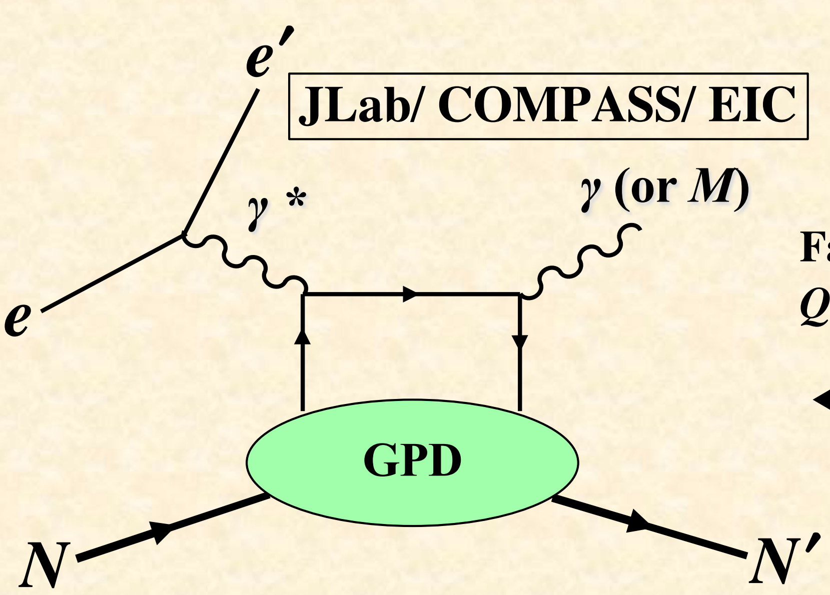
See also

SK, EPJ Web Conf. 208 (2019) 07003.

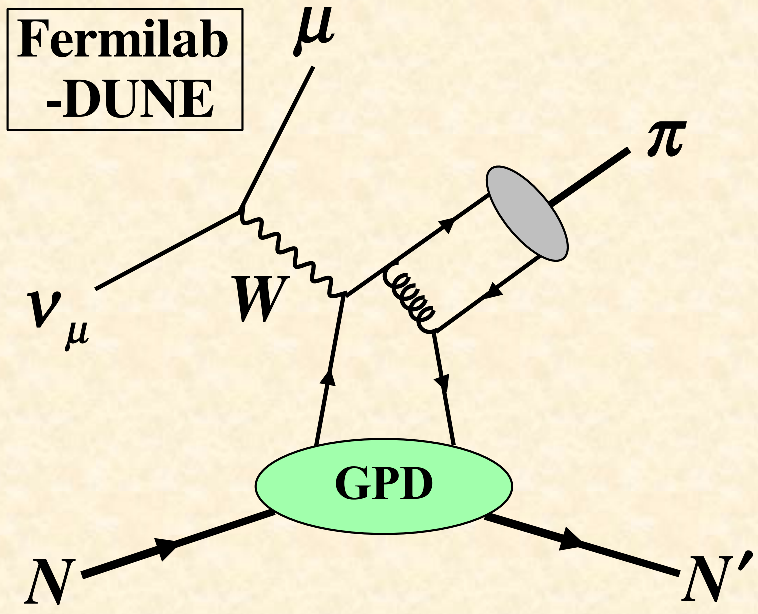
EIC yellow report, R. Abdul Khalek *et al*., arXiv:2103.05419,
Sec. 7.5.2, Neutrino physics by SK and R. Petti.

SK and R. Petti, PoS (NuFact2021) 092.

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)

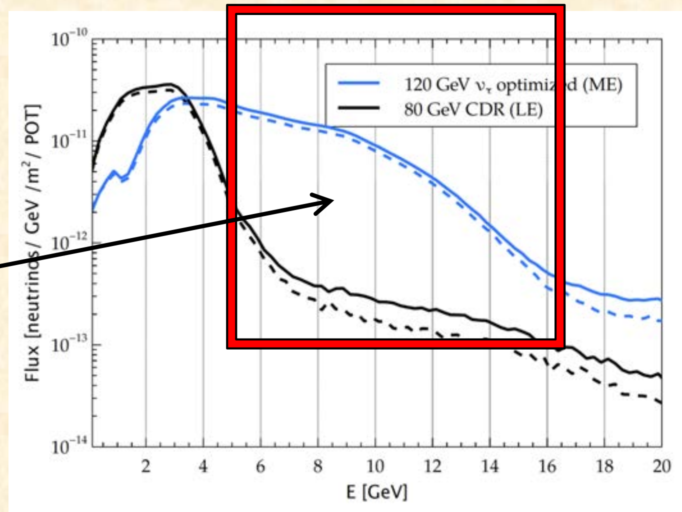


Factorization condition:
$Q^2 \gg |t|, \Lambda^2_{QCD}$



Deep Underground Neutrino Experiment (DUNE)
at Long-Baseline Neutrino Facility (LBNF)

High-energy part of the LBNF ν beam can be used for the GPD studies.

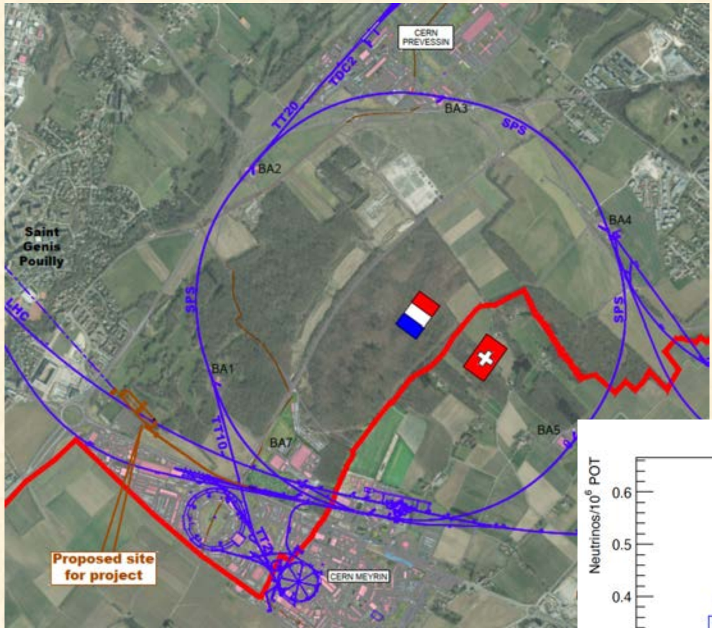


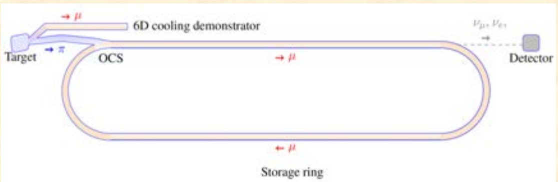
J. Rout *et al.*, PRD 102 (2020) 116018

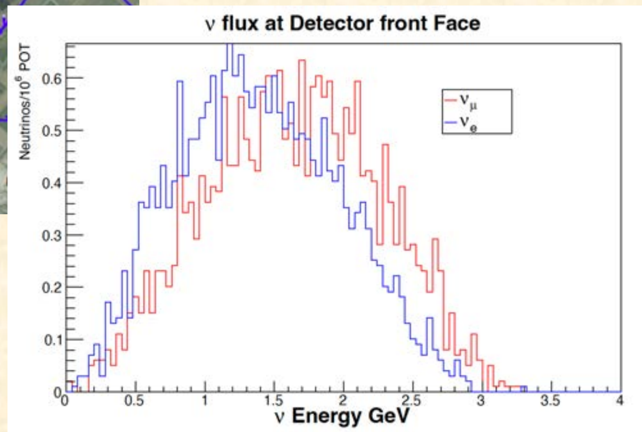
nuSTORM (Neutrinos from Stored Muons)

CERN

Feasibility Study, C. C. Ahdida *et al*., (2020);
L. A. Ruso *et al*., arXiv:2203.07545.

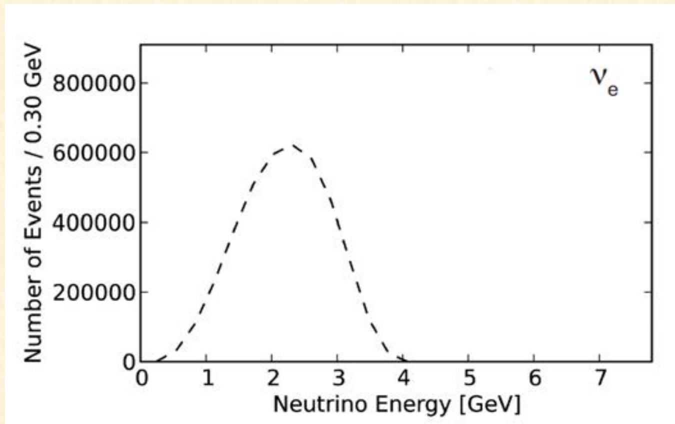






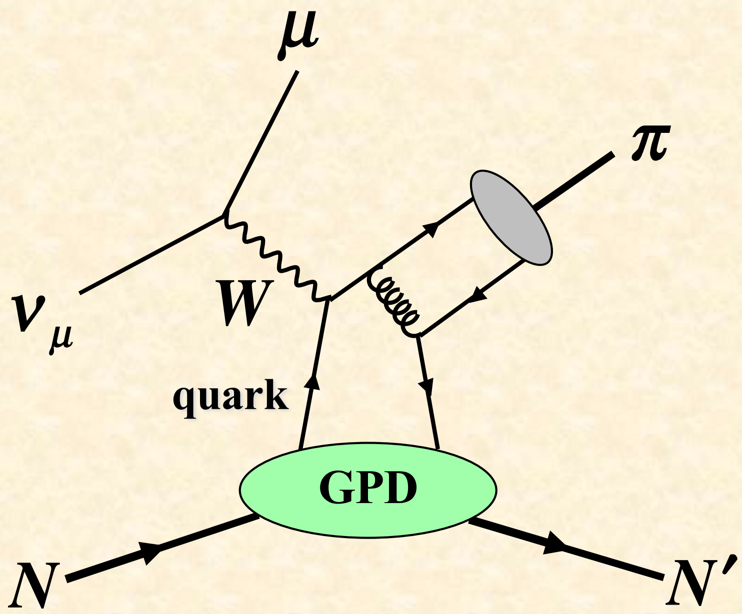
Fermilab

Letter of Intent, arXiv:1206.0294,
P. Kyberd *et al*. (2012);
Proposal, D. Adey *et al*., arXiv:1308.6822.
No recent update.



At this stage, the considered beam energy is not high enough for structure-function studies; however, high-energy option could be possible. (personal communications: Xianguo Lu)
→ SK's talk at the nuSTORM-collaboration meeting on July, 15, 2024
They could be interested in the higher-energy possibility.

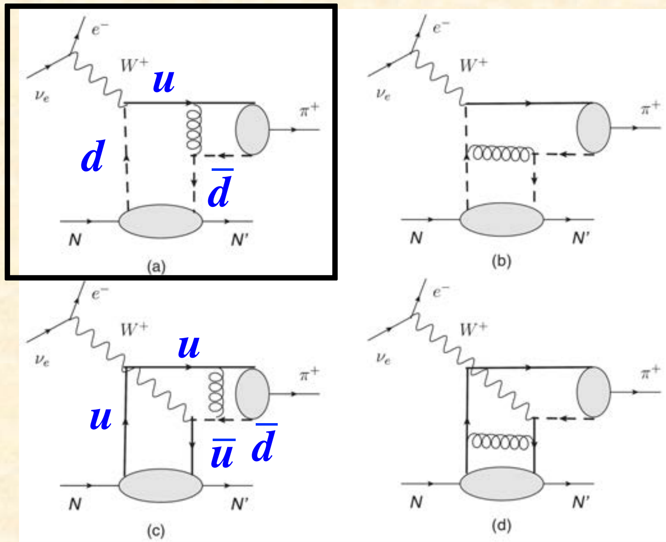
Recent work on pion production in neutrino reaction for GPD studies



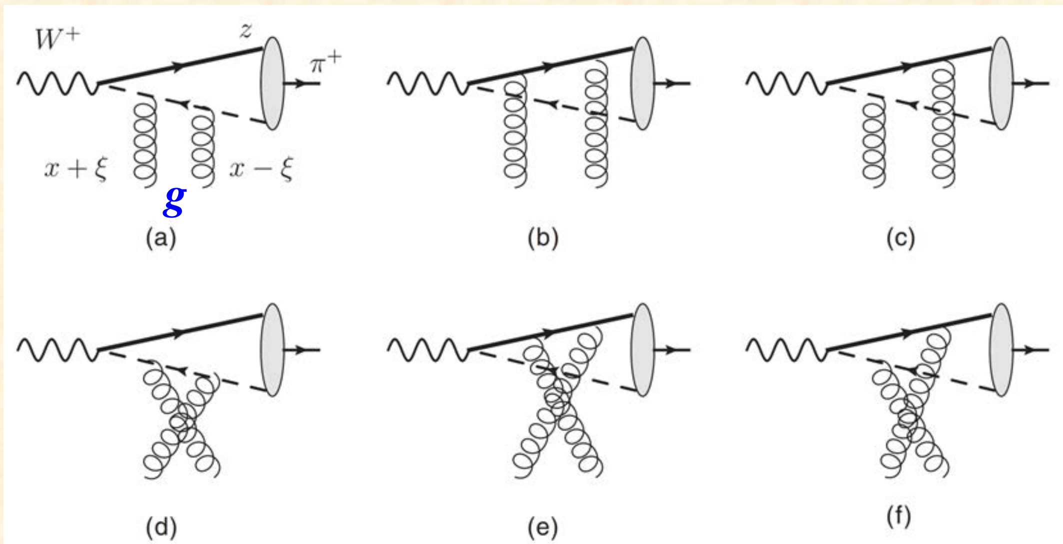
B. Pire, L. Szymanowski, and J. Wagner, Phys. Rev. D 95, 114029 (2017).

There are several processes to contribute to the pion-production cross section, including the gluon GPD terms.

Quark GPDs



Gluon GPDs



Cross section formalism

B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).

Cross section

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} = \Gamma\, \varepsilon\, \sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32\,(2\pi)^4 (s-m_N^2)^2\, y\, (1-\varepsilon)\sqrt{1+4x^2 m_N^2/Q^2}}$$

$$\sigma_L = \varepsilon_L^{*\mu} W_{\mu\nu} \varepsilon_L^\nu = \frac{1}{Q^2}\Big[(1-\xi^2)\Big\{\left|C_q \mathcal{H}_q + C_g \mathcal{H}_g\right|^2 + \left|C_q \tilde{\mathcal{H}}_q\right|^2\Big\} + \frac{\xi^4}{1-\xi^2}\Big\{\left|C_q \mathcal{E}_q + C_g \mathcal{E}_g\right|^2 + \left|C_q \tilde{\mathcal{E}}_q\right|^2\Big\}$$

$$-2\xi^2\, \mathrm{Re}\Big\{(C_q \mathcal{H}_q + C_g \mathcal{H}_g)(C_q \mathcal{E}_q + C_g \mathcal{E}_g)^*\Big\} - 2\xi^2\, \mathrm{Re}\Big\{C_q \tilde{\mathcal{H}}_q (C_q \tilde{\mathcal{E}}_q)^*\Big\}\Big]$$

Quark contributions

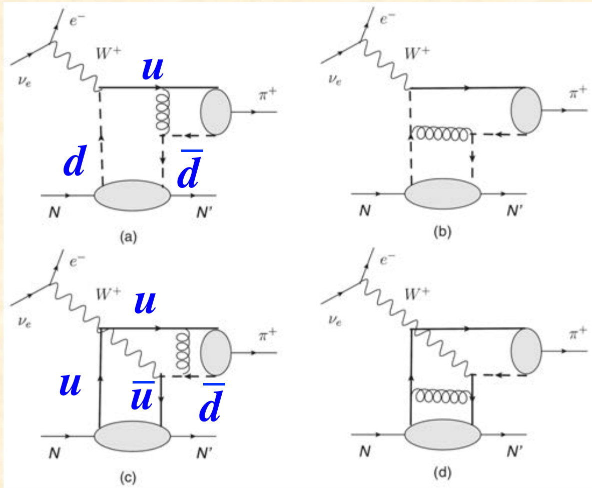
$$T_q = -i\frac{C_q}{2Q} N(p')\left[\mathcal{H}_q \hat{n} + \mathcal{E}_q \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} - \tilde{\mathcal{H}}_q \hat{n}\gamma_5 - \tilde{\mathcal{E}}_q \frac{\gamma_5 n\cdot\Delta}{2m_N}\right] N(p)$$

$$\mathcal{F}_q = 2f_\pi \int \frac{dz\, \phi_\pi(z)}{1-z} \int dx\, \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

$$= \text{(pion distribution amplitude)} \cdot \text{(quark GPD)}$$

$$F_q(x,\xi,t) \equiv F_d(x,\xi,t) - F_u(-x,\xi,t)$$

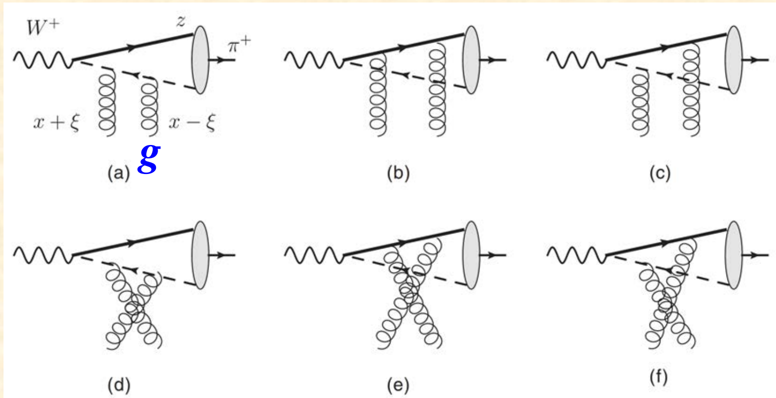
$$F = H,\, E,\, \tilde{H},\, \tilde{E}$$



Gluon contributions

$$T_g = -i\frac{C_g}{2Q} N(p')\left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N}\right] N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi} \int \frac{dz\, \phi_\pi(z)}{z(1-z)} \int dx\, \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$



GK (Goloskokov-Kroll) - 2013 parametrization

P. Kroll, H. Moutarde, F. Sabatie,
Eur. Pjys. J. C 73 (2013) 2278.

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

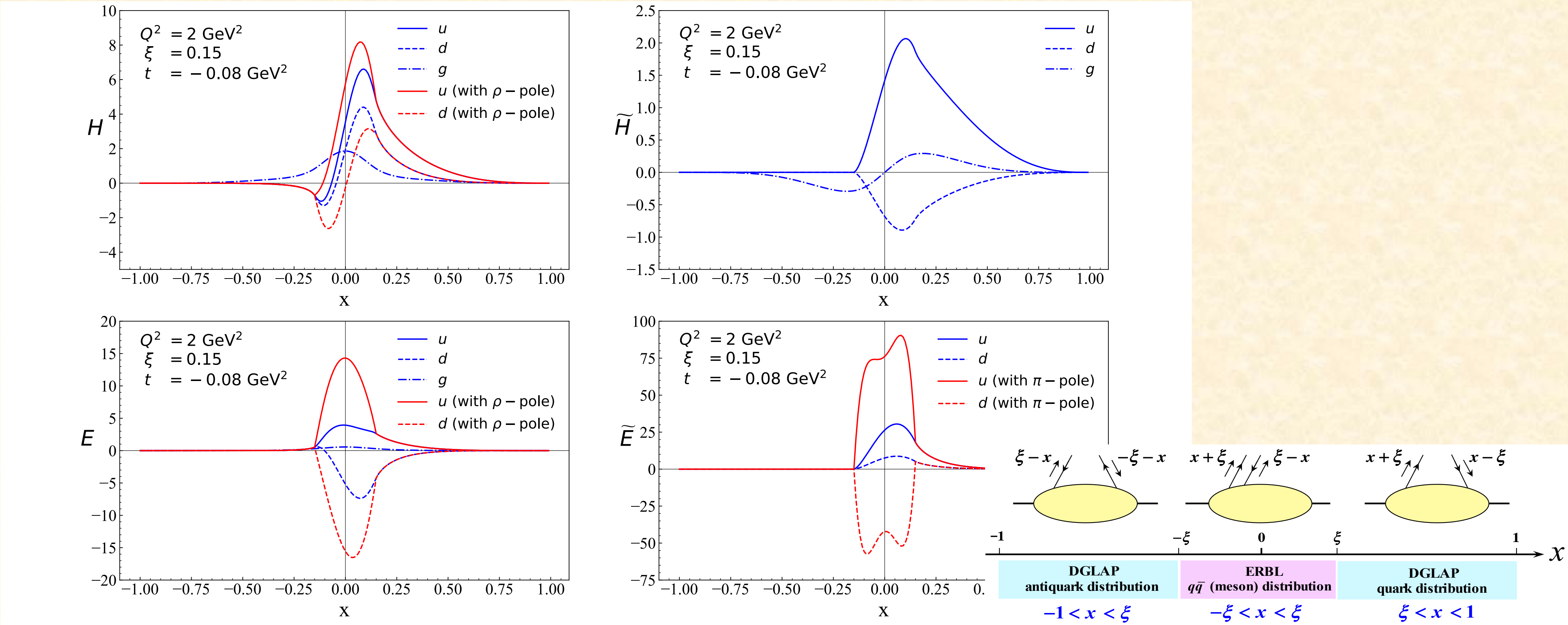
$$F_i(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x) f_i(\beta,\alpha,t) + D_i(x',t)\Theta(\xi^2-x^2)$$

$$f_i(\beta,\alpha,t) = F_i(\beta,\xi=0,t=0)e^{tp_{h_i}(\beta)} \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)} \frac{\left[(1-|\beta|)^2-\alpha^2\right]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\Theta(\xi^2-x^2) = \begin{cases} 1 & \xi^2 > x^2 \\ 0 & \xi^2 < x^2 \end{cases}, \quad p_{h_i}(\beta) = -\alpha'_{h_i}\ln\beta + b_{h_i}$$

$$F_i(\beta,\xi=0,t=0) = \beta^{-\delta_i}(1-\beta)^{2n_i+1}\sum_{j=0}^{3} c_{f_j}\beta^{j/2},$$

parameters determined by global analysis



Cross sections

π^+ production: $\nu p \to \ell^- \pi^+ p$

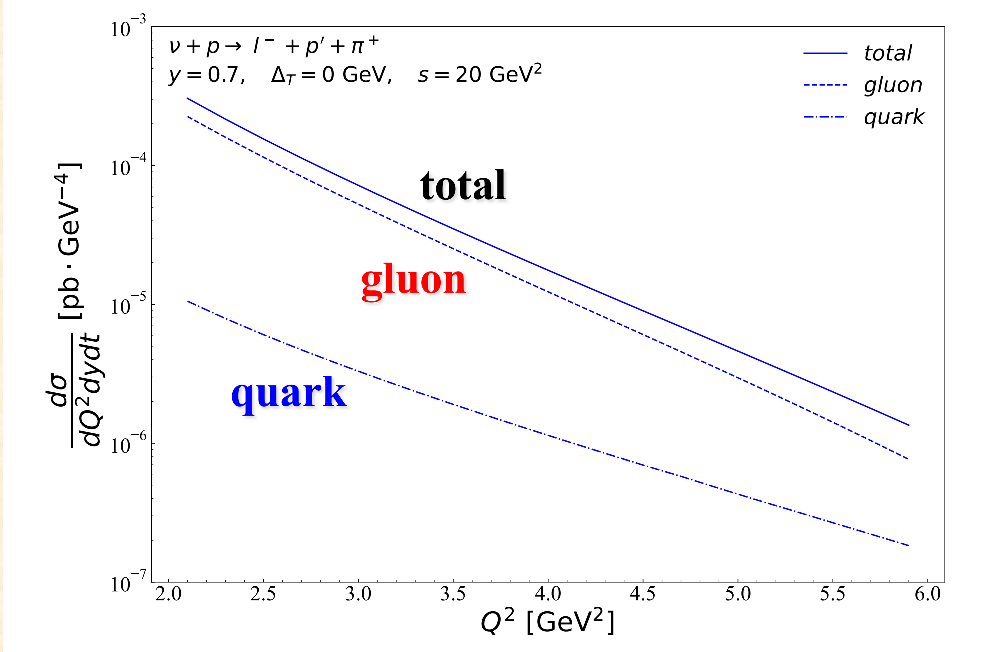
$$\mathcal{F}_q = 2 f_\pi \int \frac{dz\, \phi_\pi(z)}{1-z} \int dx \frac{F_q(x,\xi,t)}{x-\xi+i\varepsilon}$$

$$\mathcal{F}_g = \frac{8 f_\pi}{\xi} \int \frac{dz\, \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x,\xi,t)}{x-\xi+i\varepsilon}$$

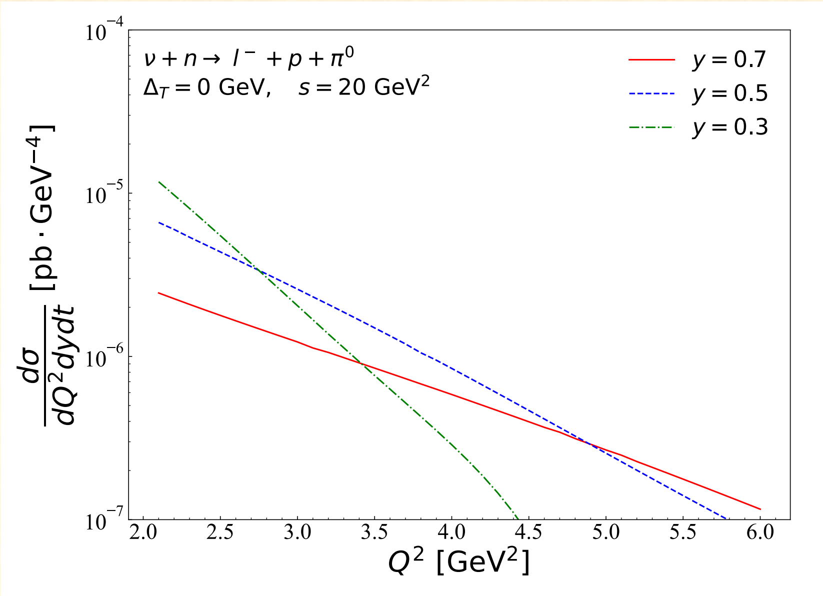
$$\frac{\mathcal{F}_q}{\mathcal{F}_g} \sim \frac{\xi}{8} = \frac{0.1 \sim 0.3}{8} = 0.01 \sim 0.04$$

$$= \text{a few } \% \ll 1$$

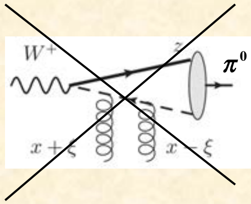
gluon $\gg$ quark



π^0 production: $\nu n \to \ell^- \pi^0 p$



no gluon



no gluon for π^0

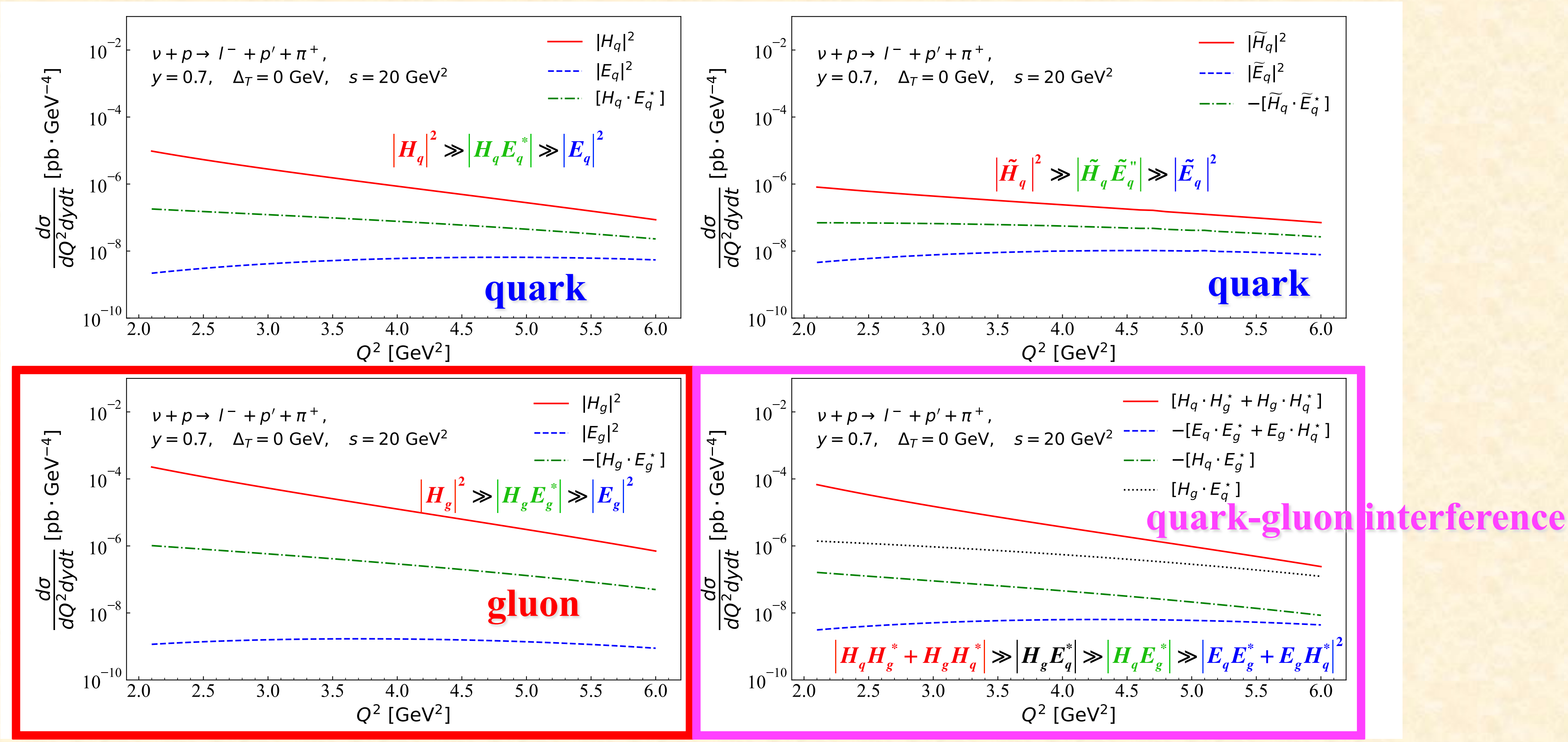
Neutrino GPD studies are complementary to the charged-lepton projects.

- **Gluon GPDs could be probed in charged-pion production.**
- **Quark GPDs could be probed in π^0 production.**
- **Flavor dependece of quark GPDs could be investigated.**

Contribution of each term to the π^+-production cross section

$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} \propto \frac{1}{Q^2}\Bigg[(1-\xi^2)\left\{\left|C_q\mathcal{H}_q + C_g\mathcal{H}_g\right|^2 + \left|C_q\tilde{\mathcal{H}}_q\right|^2\right\} + \frac{\xi^4}{1-\xi^2}\left\{\left|C_q\mathcal{E}_q + C_g\mathcal{E}_g\right|^2 + \left|C_q\tilde{\mathcal{E}}_q\right|^2\right\}$$

$$-2\xi^2\,\mathrm{Re}\left\{(C_q\mathcal{H}_q + C_g\mathcal{H}_g)(C_q\mathcal{E}_q + C_g\mathcal{E}_g)^*\right\} - 2\xi^2\,\mathrm{Re}\left\{C_q\tilde{\mathcal{H}}_q(C_q\tilde{\mathcal{E}}_q)^*\right\}\Bigg]$$

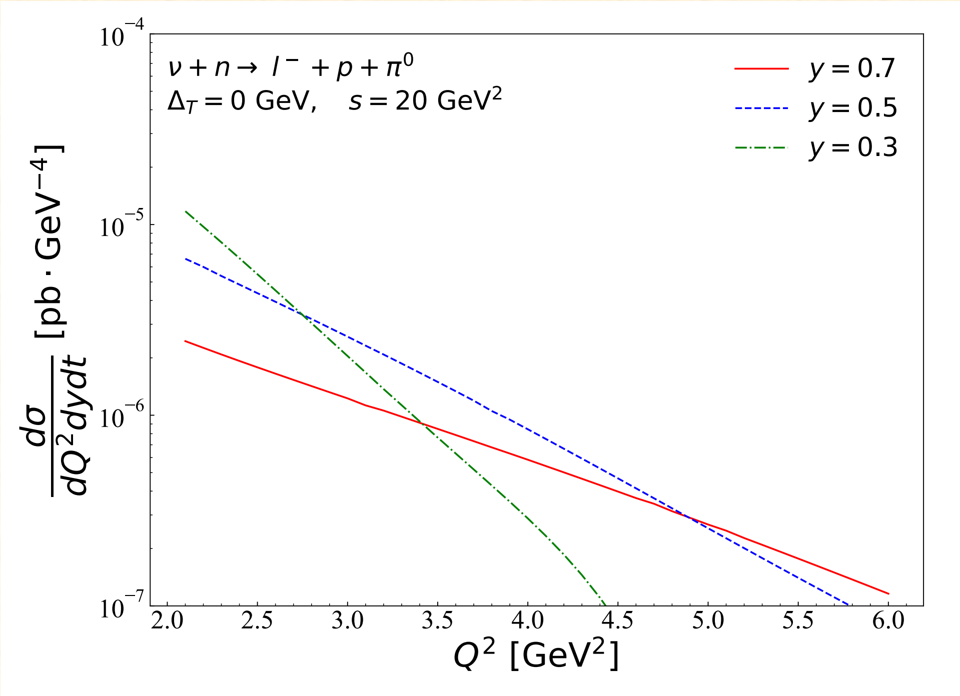


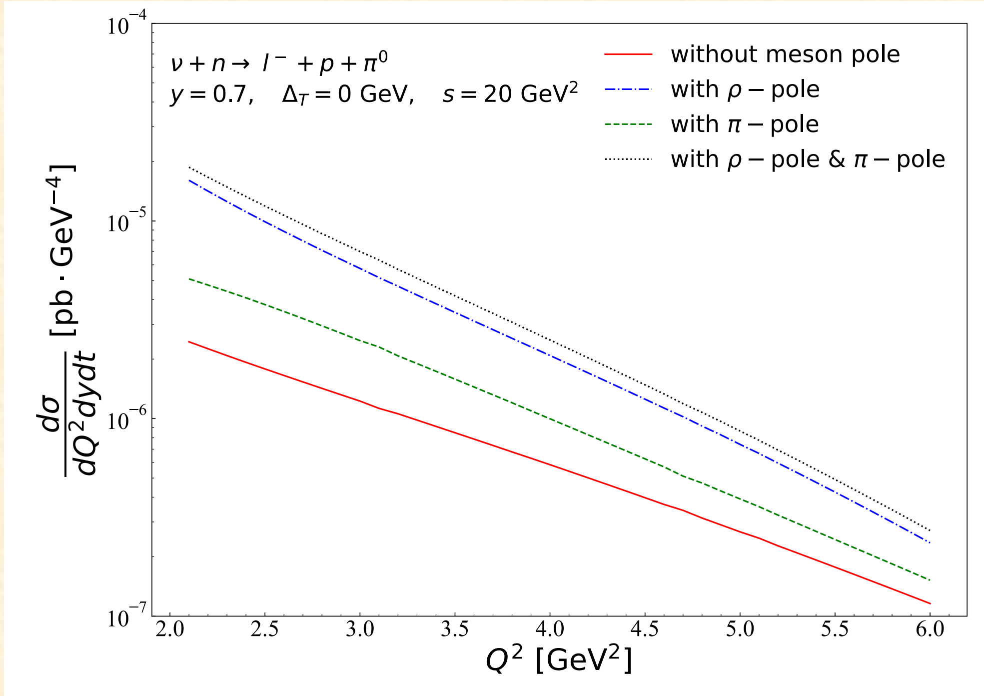
$$H_g > H_q > \tilde{H}_q > E_q,\ \tilde{E}_q,\ E_g$$

- π^+ production is senstive to gluon $\mathcal{H}_g$.
- Sizable quark-gluon interference $\mathcal{H}_q\mathcal{H}_g$.

Contribution of each term to the π^0-production cross section

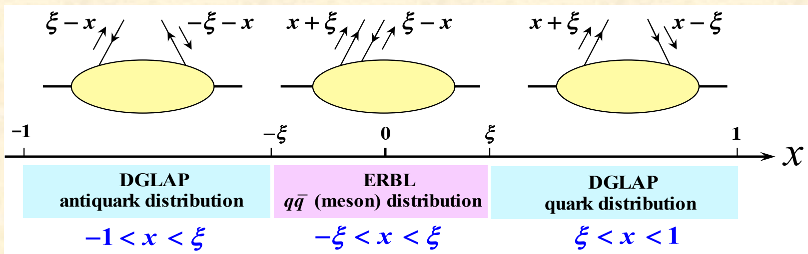
$$\frac{d\sigma(\nu_\ell N \to \ell^- N'\pi)}{dy\, dQ^2\, dt\, d\phi} \propto \frac{1}{Q^2}\Bigg[(1-\xi^2)\Big\{\big|C_q\mathcal{H}_q + \cancel{C_g\mathcal{H}_g}\big|^2 + \big|C_q\tilde{\mathcal{H}}_q\big|^2\Big\} + \frac{\xi^4}{1-\xi^2}\Big\{\big|C_q\mathcal{E}_q + \cancel{C_g\mathcal{E}_g}\big|^2 + \big|C_q\tilde{\mathcal{E}}_q\big|^2\Big\}$$
$$-2\xi^2\,\mathrm{Re}\Big\{(C_q\mathcal{H}_q + \cancel{C_g\mathcal{H}_g})(C_q\mathcal{E}_q + \cancel{C_g\mathcal{E}_g})^*\Big\} - 2\xi^2\,\mathrm{Re}\Big\{C_q\tilde{\mathcal{H}}_q(C_q\mathcal{E}_q)^*\Big\}\Bigg]$$





$\cancel{H_g} > H_q > \tilde{H}_q > E_q,\ \tilde{E}_q,\ \cancel{E_g}$

- π^0 production is senstive to quark $\mathcal{H}_q$.
- GPDs in the ERBL (Efremov-Radyushkin-Brodsky-Lepage) region could be probed.



Gravitational form factors, Prospects on neutrino GPD project

Nucleon mass: $M = \left\langle N(p) \middle| \int d^3x\, T^{00}(x) \middle| N(p) \right\rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)}q(x) + \frac{1}{4}g^{\mu\nu}F^2(x) - F^{\mu\alpha}(x)F_{\alpha}^{\ \nu}(x) = T_q^{\mu\nu}(x) + T_g^{\mu\nu}(x)$$

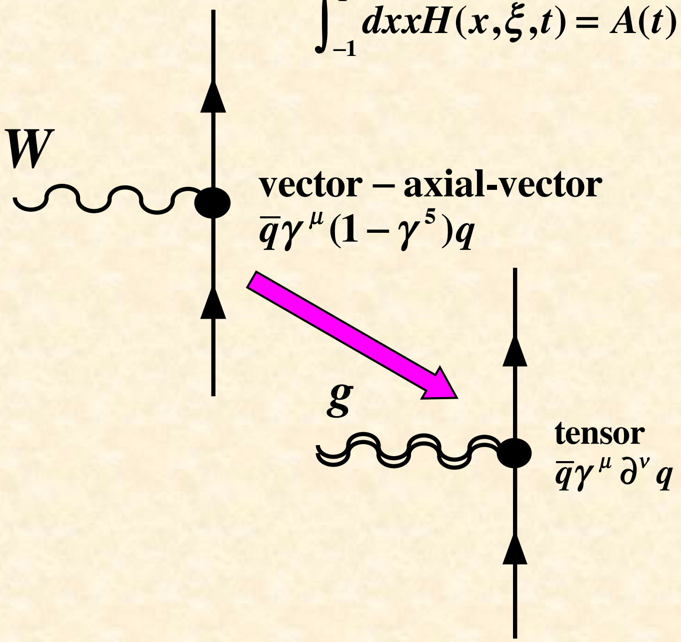
Gravitational form factors: A, B, C, D

$$\left\langle N(p') \middle| \int d^3x\, T^{\mu\nu}(x) \middle| N(p) \right\rangle = u(p')\left[A\gamma^{\{\mu}\bar{P}^{\nu\}} + B\frac{\bar{P}^{\{\nu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M} + CMg^{\nu\nu} + D\frac{\Delta^\mu\Delta^\nu - g^{\nu\nu}\Delta^2}{M} \right]u(p)$$

T^{00}: $$\left\langle N(p') \middle| \int d^3x\, T^{00}(x) \middle| N(p) \right\rangle = 2ME\left[A(t) - \frac{t}{4M^2}\{A(t) - 2B(t) + D(t)\} \right]$$

GPDs and gravitational form factors:

$$\int_{-1}^{1} dx\,xH(x,\xi,t) = A(t) + \xi^2 D(t), \quad \int_{-1}^{1} dx\,xE(x,\xi,t) = B(t) - \xi^2 D(t)$$



- Neutrino-scattering experiments (LBNF, nuSTORM) are valuable and complementary to JLab, AMBER, KEK-B, and the other facility projects in the sense that the cross sections are sensitive to quark flavor.

- This project is already in progress.

  The new detector, which was the basis of various GPD measurements, was selected by the DUNE collaboration to be part of the near detector complex (R. Petti, 2021).

  nuSTORM at CERN?

GPDs for exotic hadrons

(If transition GPDs could be studied, this exotic-hadron project becomes realistic.)

H. Kawamura and SK, Phys. Rev. D 89 (2014) 054007.

Constituent counting rule for exotic hadrons:
H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006.

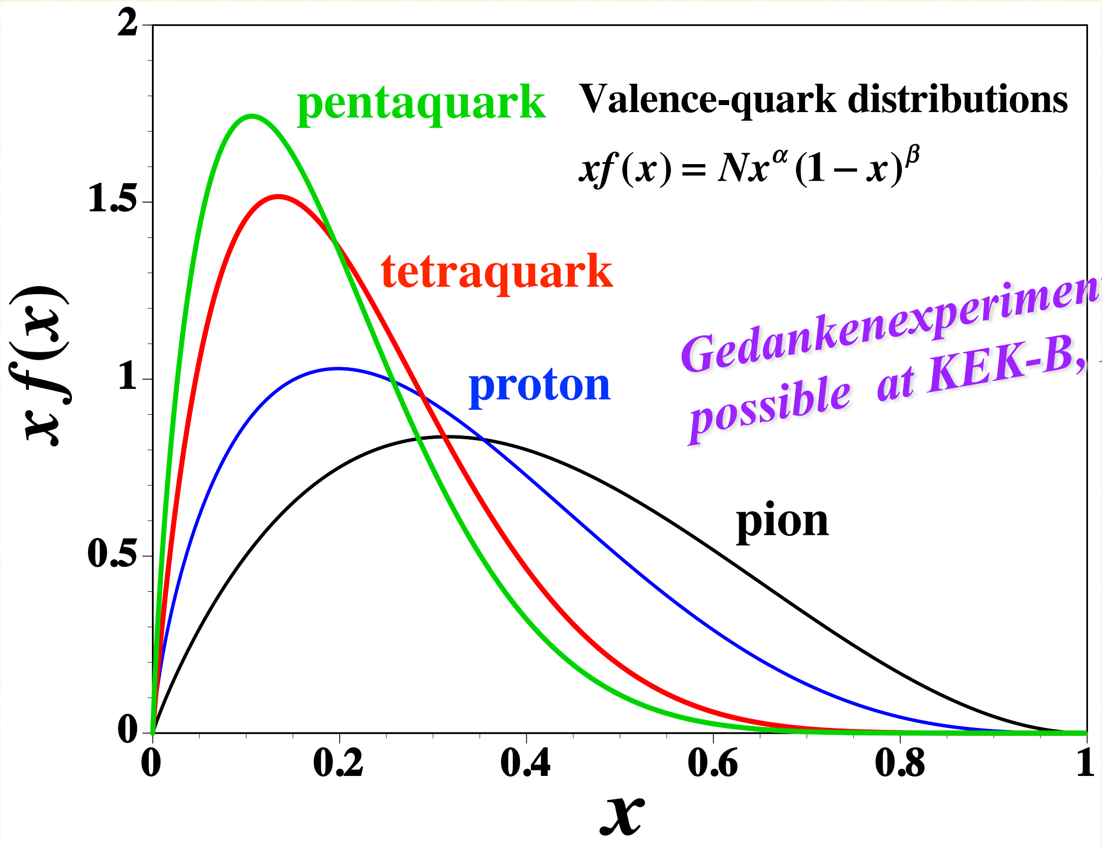
Simple function of GPDs

$H_q^h(x,t) = f(x)F(t,x)$

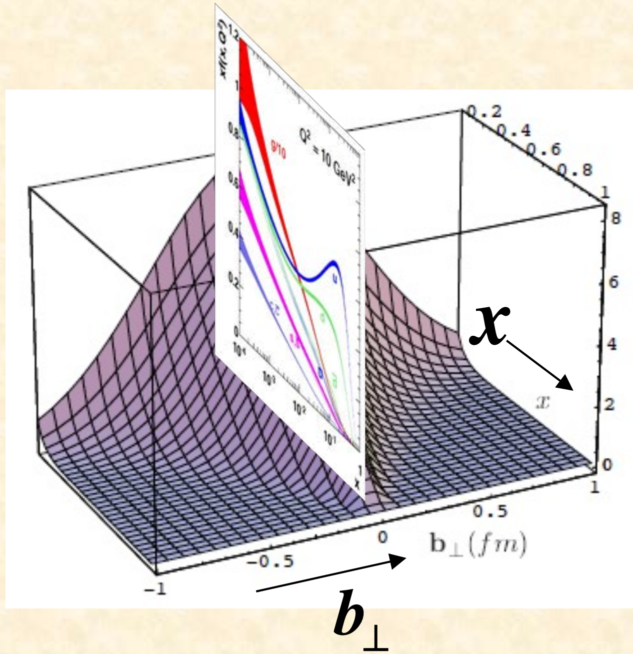
M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaeghen, PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n}(1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx\, f(x) = n$
- Constituent conting rule at $x \to 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx\, x f(x)$



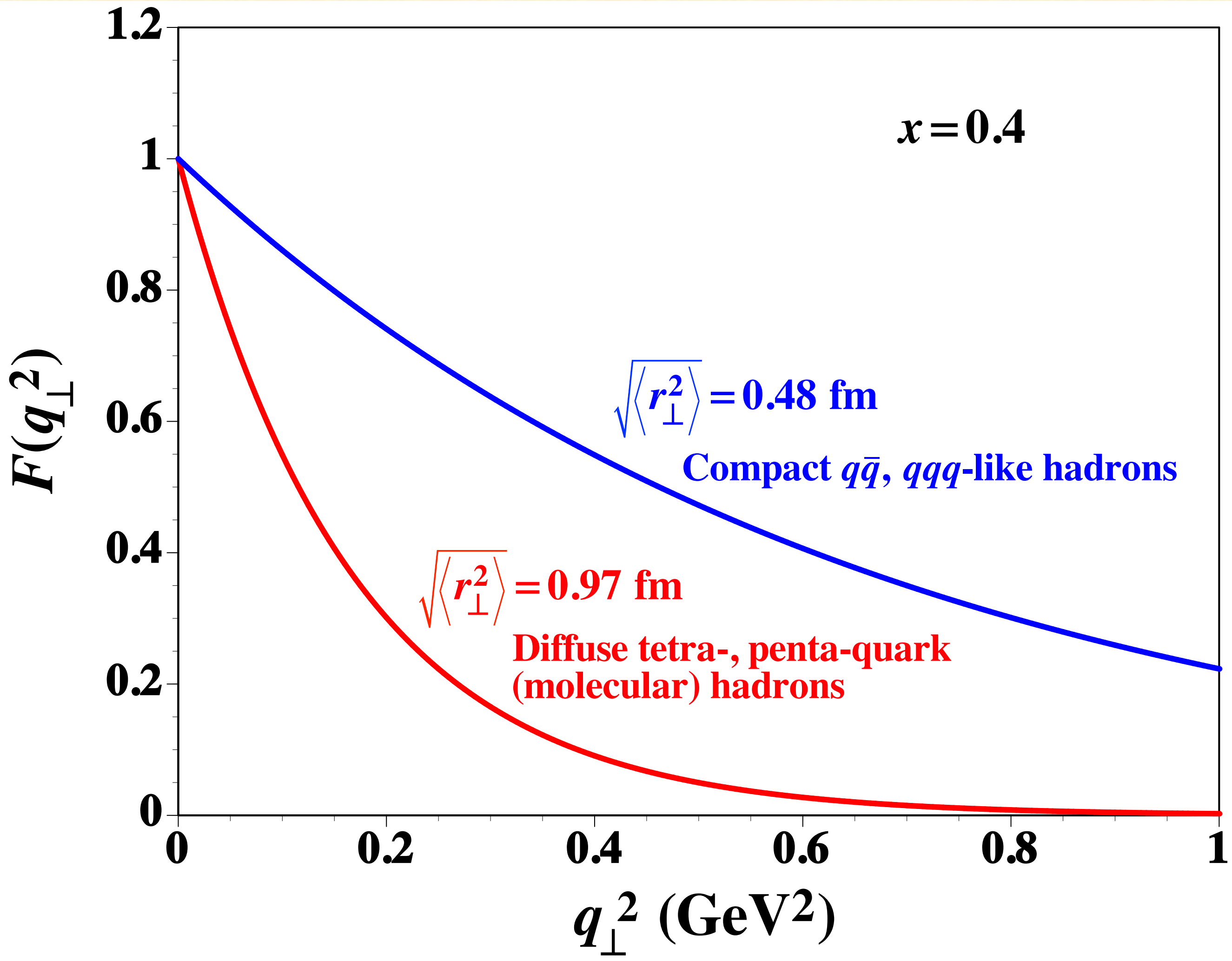
Gedankenexperiment, but possible at KEK-B, ILC, ...

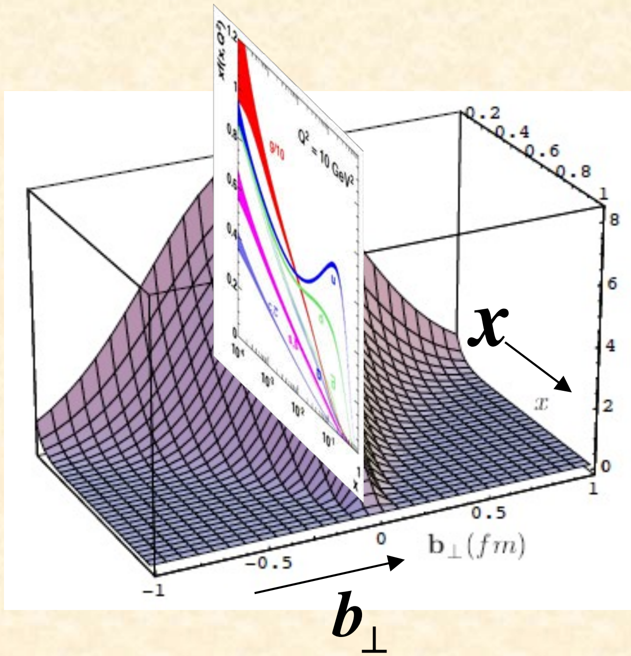


Recent study on Zc(3900),
C. Han, X. Wang, W. Kou, X. Chen,
arXiv:2407.05923 (2024).

Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$





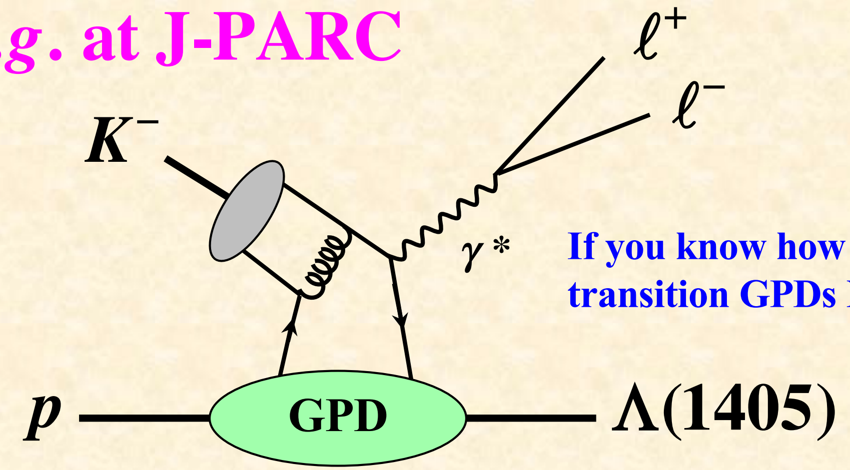
GPDs for exotic hadrons !?

Because stable targets do not exist for exotic hadrons, it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or → $s \leftrightarrow t$ crossed qunatity = GDAs at KEKB, Linear Collider

***e.g.* at J-PARC**

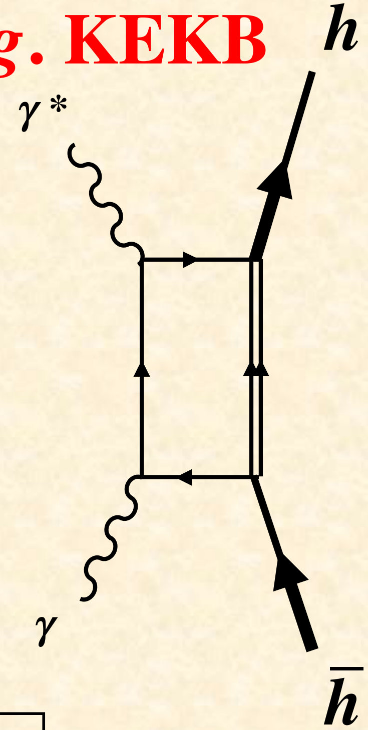


If you know how to handle this kind of transition GPDs N→Λ, please inform me.

$$K^-(\bar{u}s) + p(uud) \rightarrow \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

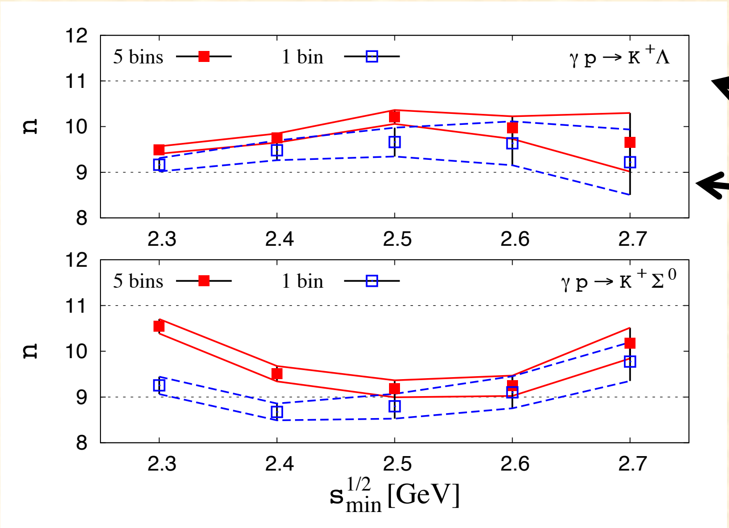
Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate

***e.g.* KEKB**



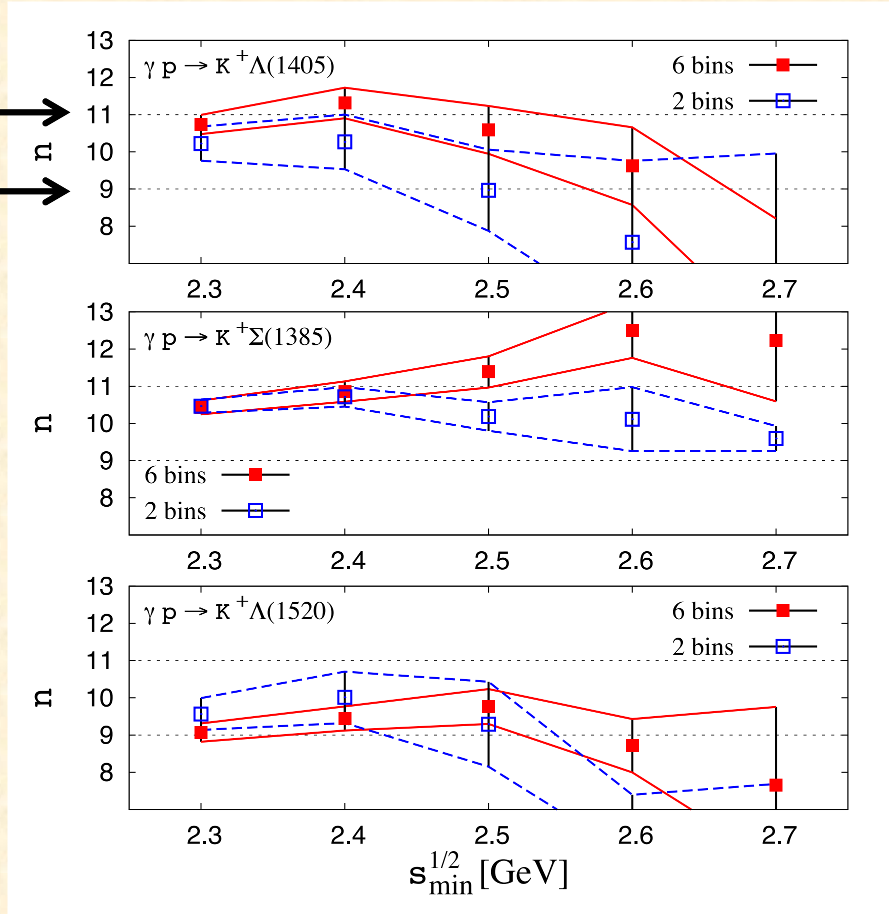
See H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

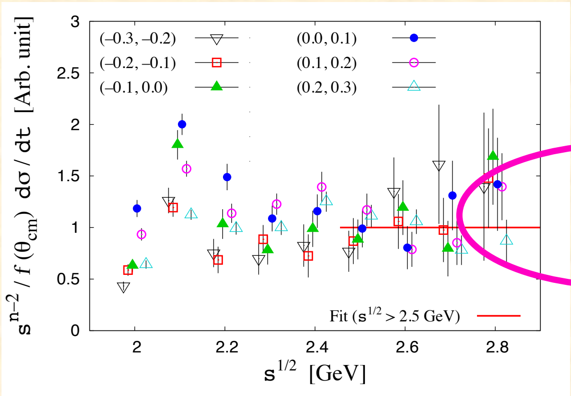
JLab hyperon productions including Λ(1405)



$n_\Lambda = 5$

$n_\Lambda = 3$





Range of 12 GeV JLab!

- Λ. Λ(1520) and Σ seem to be consistent with ordinary baryons with $n = 3$.
- Λ(1405) looks penta-quark at low energies but $n \sim 3$ at high energies???
- Σ(1385) : $n = 5$???

→ In order to clarify the nature of Λ(1405) $\left[qqq,\ \bar{K}N,\ qqqq\bar{q}\right]$, the JLab 12-GeV experiment plays an important role!

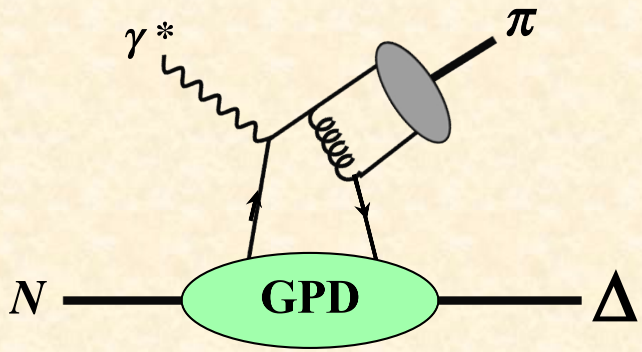
W.-C. Chang, SK, T. Sekihara, PRD 93 (2016) 034006.

Transition GPDs for exotic hadrons

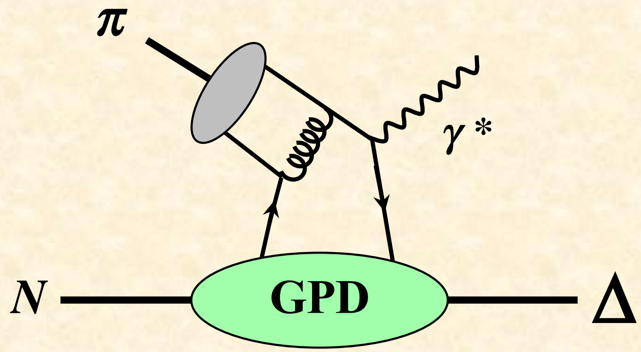
S. Diehl *et al*. (SK, 15th author),
arXiv:2405.15386, submitted for Eur. Phys. J. A

Transition GPDs from N to Δ

JLab / EIC



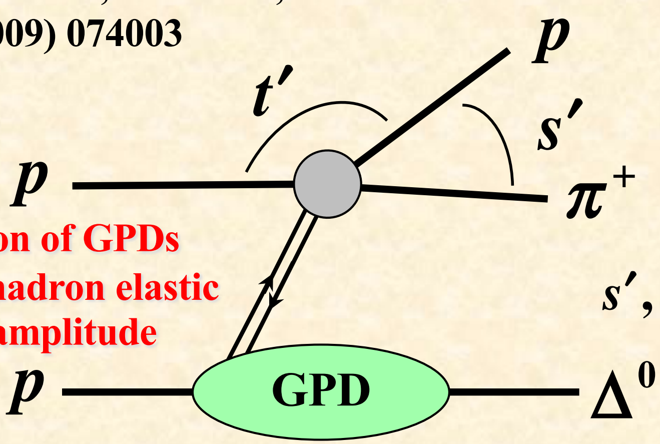
J-PARC



In future

$K^- + p \rightarrow \Lambda_{1405} + \gamma^*$?

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003



Investigation of GPDs with 2→3 hadron elastic scattering amplitude

$s',\ t',\ u' \gg M_N^2$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

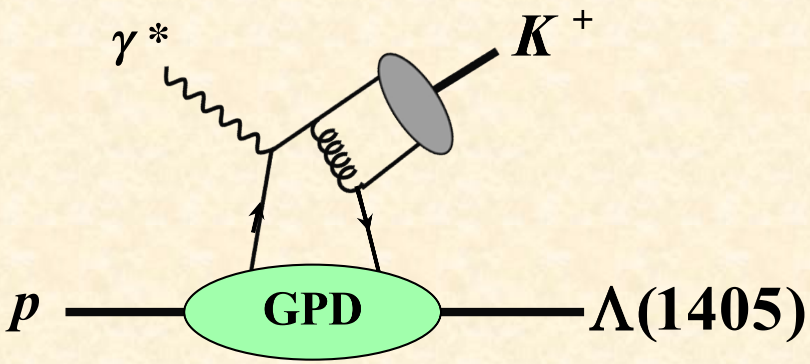
$\pi + N \rightarrow \gamma + \gamma + N'$

$h + M_B \rightarrow h' + \gamma + M_D$

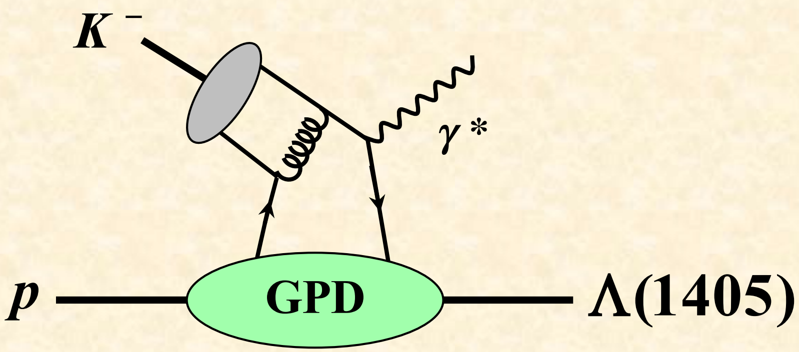
$h + M_B \rightarrow h' + M_C + M_D$

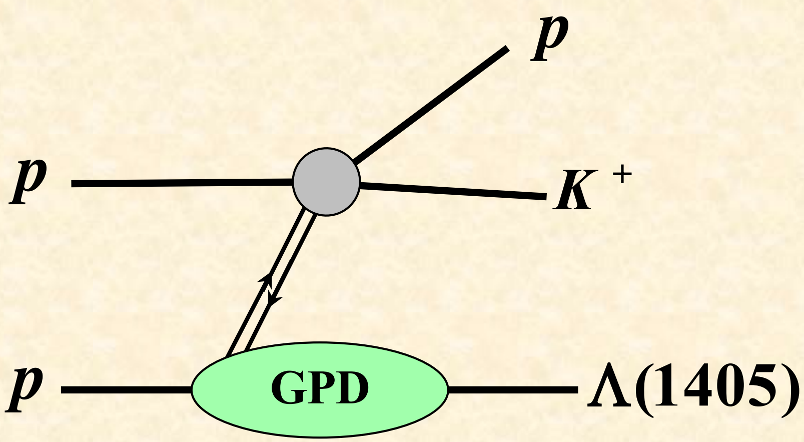
Transition GPDs for exotic hadrons

JLab / EIC



J-PARC



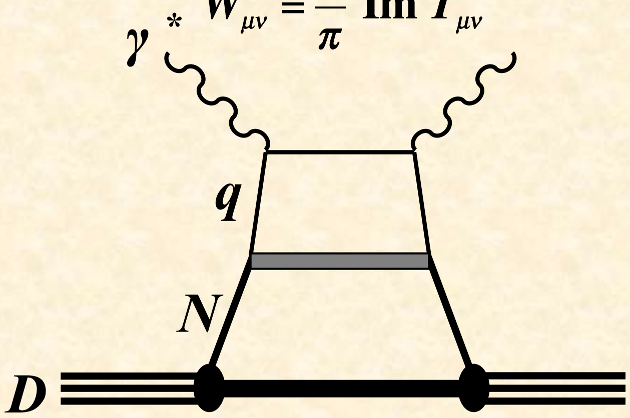


However, there is no theoretical study on the N→Λ(1405) transition GPDs at this stage.

GPDs of hadrons with spin $\geqq 1$

“Standard-model” prediction for b_1 of deuteron

$$W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu}$$



Standard model of the deuteron

$$b_1(x) = \int \frac{dy}{y}\,\delta_T f(y) F_1^N(x/y,\,Q^2), \qquad y = \frac{M p\cdot q}{M_N P\cdot q} \simeq \frac{2p^-}{P^-}$$

$$\delta_T f(y) = f^0(y) - \frac{f^+(y)+f^-(y)}{2}$$

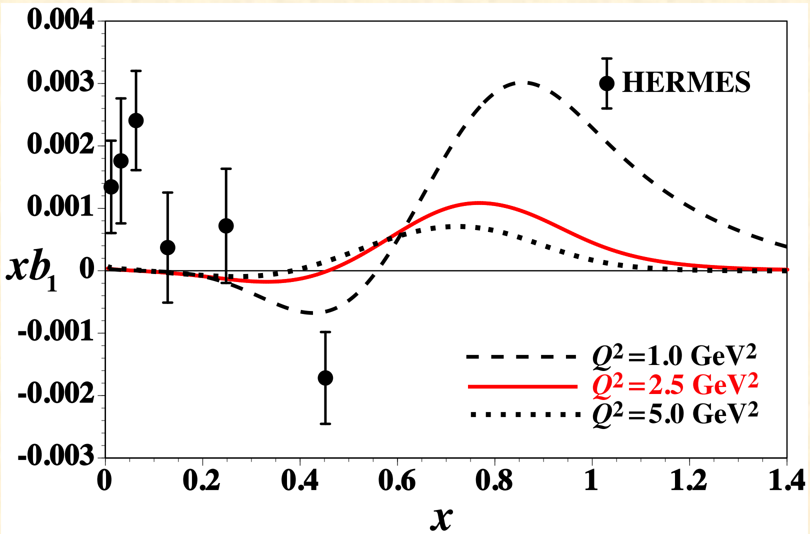
$$= \int d^3p\; y\left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p) + \frac{3}{16\pi}|\phi_2(p)|^2\right](3\cos^2\theta - 1)\,\delta\!\left(y - \frac{p\cdot q}{M_N \nu}\right)$$

S-D term　　D-D term

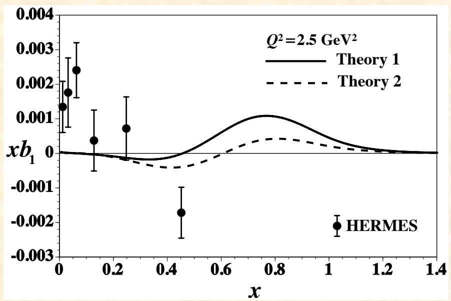
Nucleon momentum distribution:

$$f^H(y) \equiv f_\uparrow^H(y) + f_\downarrow^H(y) = \int d^3p\; y\,|\phi^H(\vec{p})|^2\,\delta\!\left(y - \frac{E - p_z}{M_N}\right)$$

D-state admixture: $\phi^H(\vec{p}) = \phi^H_{\ell=0}(\vec{p}) + \phi^H_{\ell=2}(\vec{p})$

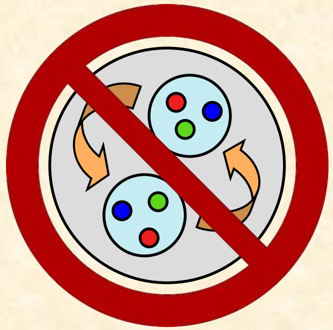


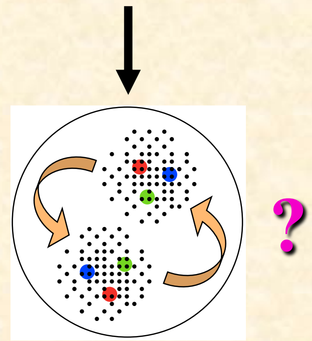
W. Cosyn, Yu-Bing Dong, SK, M. Sargsian,
Phys. Rev. D 95 (2017) 074036.



$|b_1(\text{theory})| \ll |b_1(\text{HERMES})|$
at $x < 0.5$

Standard convolution model does not work for the deuteron tensor structure!?





G. A. Miller, PRC 89 (2014) 045203,
Interesting suggestions:
hidden-color, 6-quark, · · ·

$$|6q\rangle = |NN\rangle + |\Delta\Delta\rangle + |CC\rangle + \cdots$$

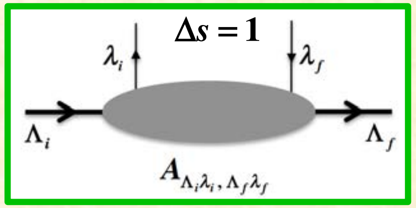
Gluon transversity $\Delta_T g$

Note on our notations:
Tensor-polarized gluon distribution: $\delta_T g$
Gluon transversity: $\Delta_T g$

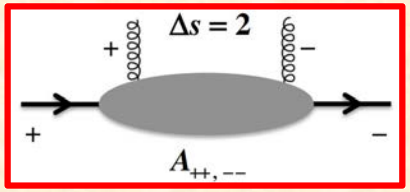
Helicity amplitude $A(\Lambda_i, \lambda_i, \Lambda_f, \lambda_f)$, conservation $\Lambda_i - \lambda_i = \Lambda_f - \lambda_f$

Longitudinally-polarized quark in nucleon: $\Delta q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ +\frac{1}{2}+\frac{1}{2}\right) - A\left(+\frac{1}{2}-\frac{1}{2},\ +\frac{1}{2}-\frac{1}{2}\right)$

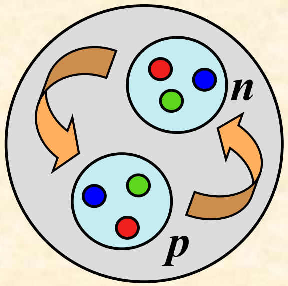
Quark transversity in nucleon: $\Delta_T q(x) \sim A\left(+\frac{1}{2}+\frac{1}{2},\ -\frac{1}{2}-\frac{1}{2}\right)$, $\lambda_i = +\frac{1}{2} \to \lambda_f = -\frac{1}{2}$ quark spin flip ($\Delta s = 1$)



Gluon transversity in deuteron: $\Delta_T g(x) \sim A(+1+1,\ -1-1)$, ~~$A\left(+\frac{1}{2}+1,\ -\frac{1}{2}-1\right)$~~ not possible for nucleon



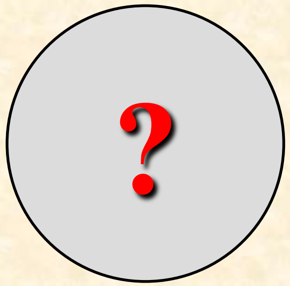
Note: Gluon transversity does not exist for spin-1/2 nucleons.



S + D waves

$b_1\ (\delta_T q,\ \delta_T g) \neq 0 \iff$ still $\Delta_T g = 0$

What would be the mechanism(s) for creating $\Delta_T g \neq 0$?



Physics beyond "the standard model" in nuclear physics?

(Physics beyond the standard model in particle physics???)

JLab PAC-38 (2011) proposal, PR12-11-110

Full approval in 2023

The Deuteron Tensor Structure Function b_1

2011

A Proposal to Jefferson Lab PAC-38.
(Update to LOI-11-003)

J.-P. Chen (co-spokesperson), P. Solvignon (co-spokesperson),
K. Allada, A. Camsonne, A. Deur, D. Gaskell,
C. Keith, S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

N. Kalantarians (co-spokesperson), O. Rondon (co-spokesperson)

PR12-13-011

The Deuteron Tensor Structure Function b_1

2023

A Proposal to Jefferson Lab PAC-40
(Update to PR12-11-110)

K. Allada, A. Camsonne, J.-P. Chen,†
A. Deur, D. Gaskell, M. Jones, C. Keith, J. Pierce,
P. Solvignon,† S. Wood, J. Zhang
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

O. Rondon Aramayo,† D. Crabb, D. B. Day,
C. Hanretty, D. Keller,† R. Lindgren, S. Liuti, B. Norum,
Zhihong Ye, X. Zheng
University of Virginia, Charlottesville, VA 22903

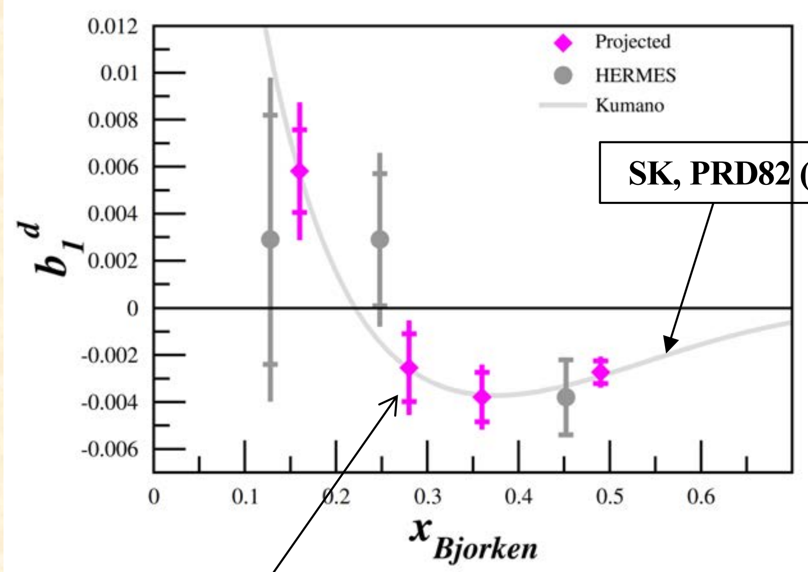
N. Kalantarians†
Hampton University, Hampton VA 23668

T. Badman, J. Calarco, J. Dawson,
S. Phillips, E. Long,† K. Slifer†‡, R. Zielinski
University of New Hampshire, Durham, NH 03861

G. Ron
Hebrew University of Jerusalem, Jerusalem

W. Bertozzi, S. Gilad, J. Huang
A. Kelleher, V. Sulkosky
Massachusetts Institute of Technology, Cambridge, MA 02139

***b_1* experiment**



SK, PRD82 (2010) 017501

Expected errors by JLab

Gluon transversity

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

M. Jones, C. Keith, J. Maxwell*, D. Meekins
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606

W. Detmold, R. Jaffe, R. Milner, P. Shanahan
Laboratory for Nuclear Science, MIT, Cambridge, MA 02139

D. Crabb, D. Day, D. Keller, O. A. Rondon
University of Virginia, Charlottesville, VA 22904

J. Pierce
Oak Ridge National Laboratory, Oak Ridge, TN 37831

Letter of Intent at Jefferson Lab (middle 2020's)

Jefferson Lab,
Electron accelerator ~12 GeV

LoI, arXiv:1803.11206

A Letter of Intent to Jefferson Lab PAC 44, June 6, 2016

Search for Exotic Gluonic States in the Nucleus

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D. Crabb, D. Day, D. Keller, O. A. Rondon

University of Virginia, Charlottesville, VA 22904

J. Pierce

Oak Ridge National Laboratory, Oak Ridge, TN 37831

For development of polarized deuteron target, see D. Keller, D. Crabb, D. Day Nucl. Inst. Meth. Phys. Res. A981 (2020) 164504.

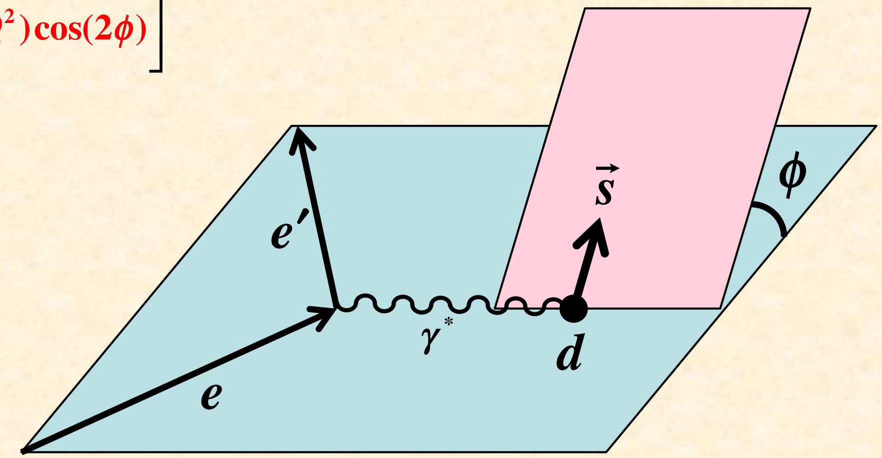
Electron scattering with polarized-deuteron target

$$\left.\frac{d\sigma}{dx\,dy\,d\phi}\right|_{Q^2 \gg M^2} = \frac{e^4 ME}{4\pi^2 Q^4}\left[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{1}{2}x(1-y)\Delta(x,Q^2)\cos(2\phi)\right]$$

$$\Delta(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q e_q^2 x^2 \int_x^1 \frac{dy}{y^3}\Delta_T g(y,Q^2)$$

By looking at the deuteron-polarization angle ϕ,
the quark transversty $\Delta_T g$ can be measured.

Lattice QCD estimates:
W. Detmold and P. E. Shanahan,
PRD 94 (2016) 014507; 95 (2017) 079902.



References on GPDs of spin-1 and 3/2 hadrons

- **GPDs of the spin-1 deuteron**

  E. L. Berger, F. Cano, M. Diehl, and B. Pire, Phys. Rev. Lett. 87 (2001) 142302, 1-4.

- **Transversity GPDs of the spin-1 deuteron**

  W. Cosyn and B. Pire, Phys. Rev. D 98 (2018) 074020.

- **GPDs of spin-3/2 hadrons**

  D. Fu, B.-D. Sun, and Y.-B. Dong, Phys. Dev. D 106 (2022) 116012.

- **Transversity GPDs of spin-3/2 hadrons**

  D. Fu, Y.-B. Dong, and S. Kumano, Phys. Dev. D 109 (2024) 096006.

I may miss your papers.

- **Model studies on ρ meson**

  B.D. Sun, Y.B. Dong, Phys. Rev. D 96 (2017) 036019; 99, (2019) 016023; 101 (2020) 096008;

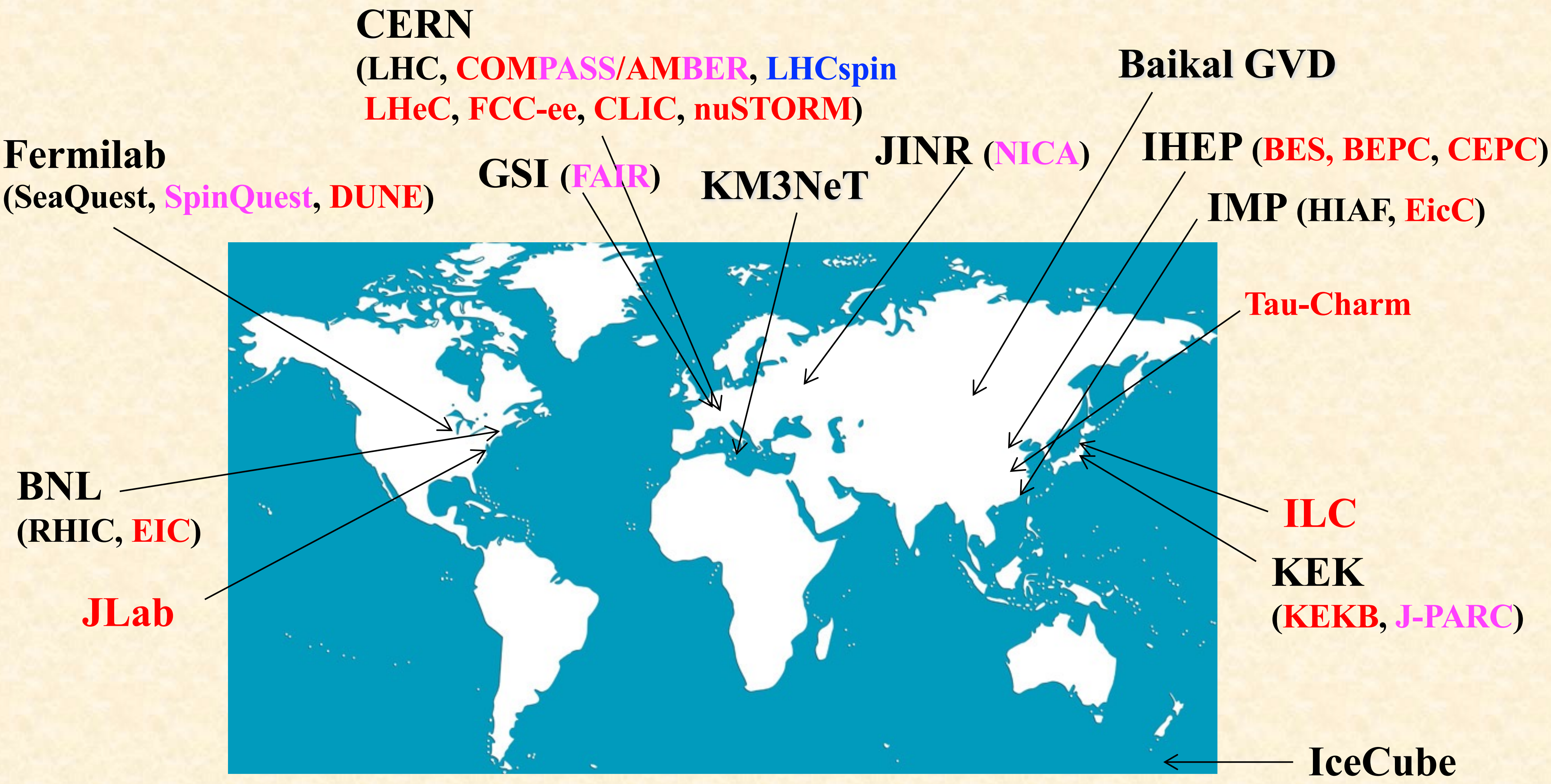
  N. Kumar, Phys. Rev. D 99 (2019) 014039.

- **PDFs, TMDs, Fragmentation functions: brief summary**

  S. Kumano, Euro. Phys. J. A 60 (2024) 205.

Future prospects on GPD projects

High-energy hadron physics experiments

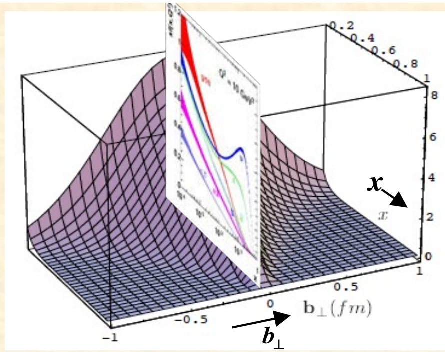


Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

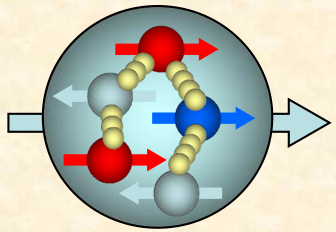
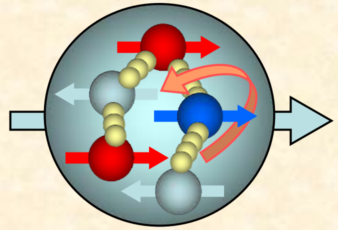
By hadron tomography

3D view of hadrons



Origin of nucleon spin

By the tomography, we determine


or

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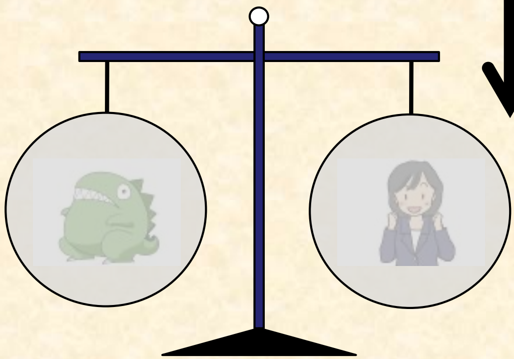
Exotic hadrons



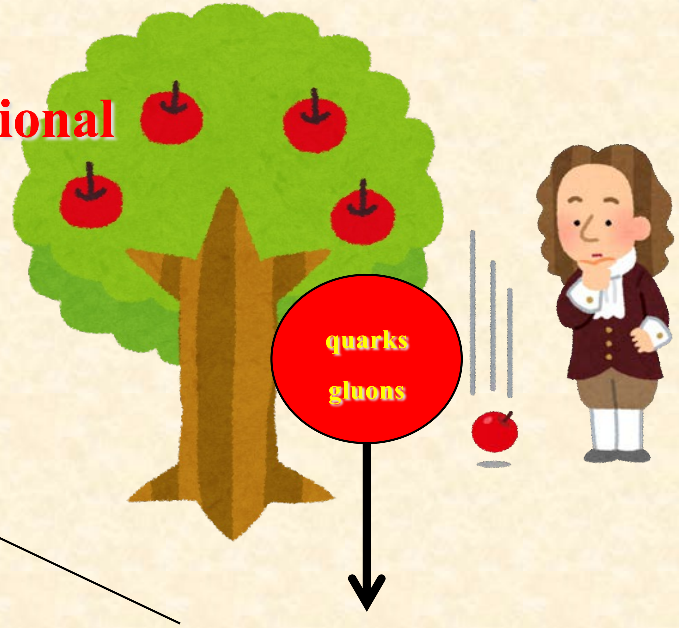
By tomography, we determine

or

.

Origin of gravitational source (mass)

By tomography,
we determine gravitational sources in terms of quarks and gluons.



Summary on GPDs

Hadron-tomography and gravitational form factors

- Puzzle to find the origin of hadron masses and pressures in terms of quark and gluon degrees of freedom
- Puzzle to find the origin of nucleon spin
- Exotic hadron candidates could be studied in the same tomography method.
- There are world-wide lepton and hadron accelerator facilities which has been used and could be used in future for our studies. In addition to the JLab/AMBER/EIC type charged-lepton scattering projects, the GPD studies are possible by neutrino and hadron beam facilities and e^+e^- colliders.

Time has come to understand the gravitational sources in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.

The End

The End