Proton Spin Decomposition From Basis Light-Front Quantization

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With

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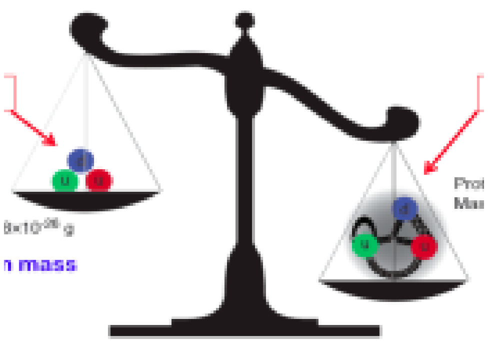
Outline

- **B**asis **L**ight-**F**ront **Q**uantization (**BLFQ**)
  - Light-front Quantization
  - QCD Light-front Hamiltonian
  - BLFQ Procedure
- Application to Proton
  - Form Factors (FFs)
  - Parton Distribution Functions (PDFs)
  - Generalized Parton Distribution Functions (GPDs)
- Conclusion and Outlook

Major Questions in Nuclear Physics

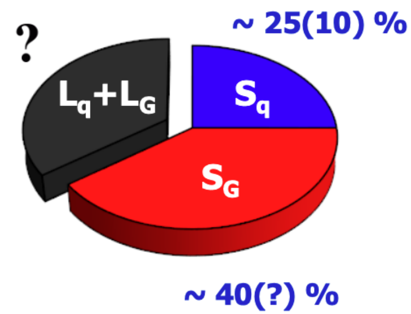
Origin of mass



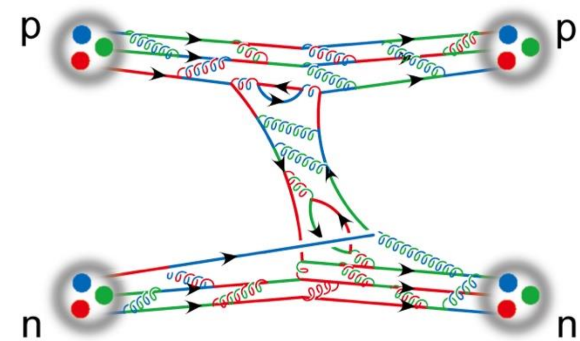
Spin puzzle

Ortibal angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$



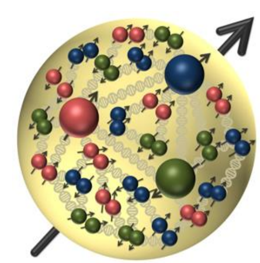
Nuclear force



We need to know the structure of nucleon beyond 1D and how does it emerge from QCD ?

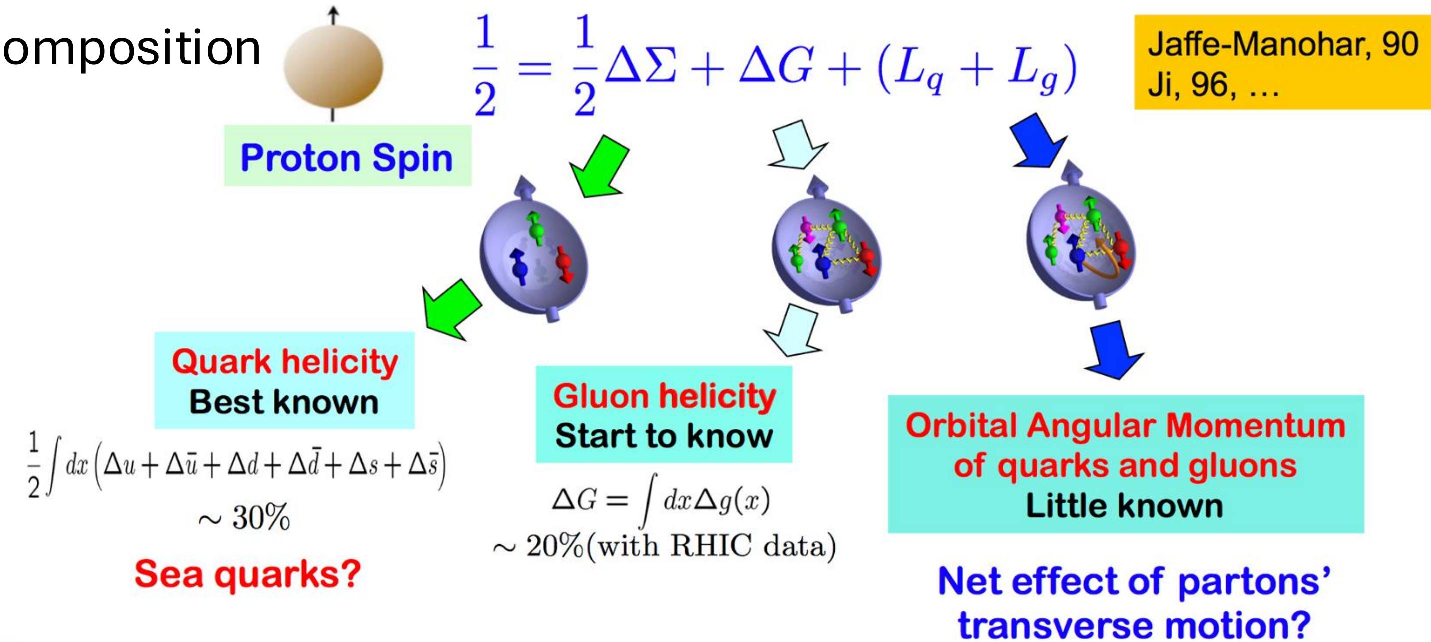
$$\mathcal{L}_{QCD} = \left(\bar{\psi}_q (i\not{D} - m_q)\psi_q \right) - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha}$$

?

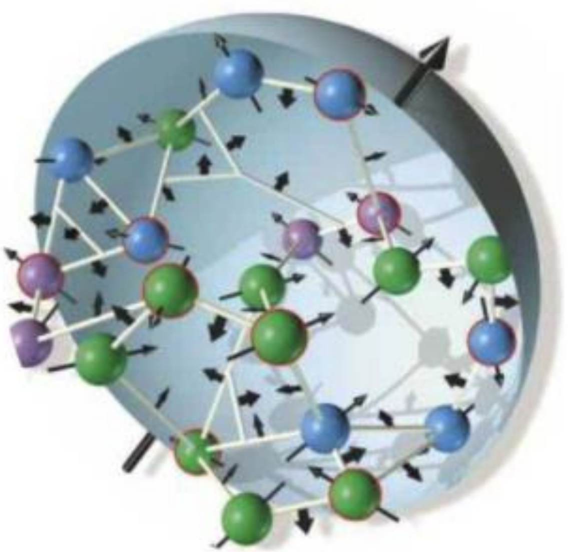


Spin Decomposition

- Jaffe-Manohar Decomposition



1. In the quark model $\Delta\Sigma = 1$
2. The helicity contribution can be measured by polarized DIS



- Ji decomposition:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_{Ji}^{q} + J_g$$

Nonperturbative Approach

- Stationary Schrödinger equation universally describes bound-state structure

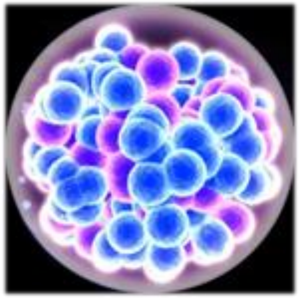
$$H|\psi\rangle = E|\psi\rangle$$

- Eigenstates $|\psi\rangle$ encode full information of the system

Nonrelativistic

atom

Nonrelativistic



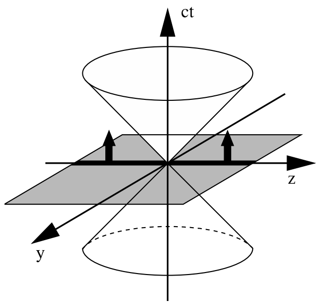
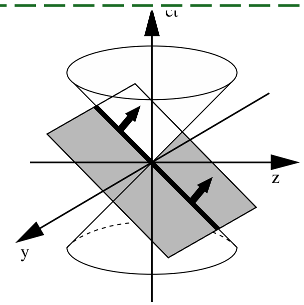
nucleus

Relativistic

nucleon

- Major challenges from relativity: frame dependence, particle number not conserving…

Light-front Quantization

Equal time quantization	Light-front quantization
t º x^0	t º $x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3$, $P^+ = P^0 + P^3$, $P^\perp = P^{1,2}$
$i\frac{\P}{\P t}\|j(t)\rangle = H\|j(t)\rangle$	$i\frac{\P}{\P x^+}\|j(x^+)\rangle = \frac{1}{2}P^-\|j(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\wedge^2}{P^+}$

[Dirac, 1949]

Main advantage:

- Simple vacuum
- Frame-independent light-front wave functions
- Minkowski spacetime
- No square roots in dispersion relation

$$\Phi^{[\gamma^+]}(x, Q^2) = \int \frac{dz^-}{8\pi} e^{\frac{ixP^+z^-}{2}} \langle P, \Lambda | \bar{\psi}(x) \gamma^+ \psi(0) | P, \Lambda \rangle \Big|_{x^+ = x^\perp = 0}$$

Basis Light-Front Quantization

- **Hamiltonian eigenvalue equation:** [Vary, et.al, 2010]

$$P^-|\beta\rangle = P_\beta^-|\beta\rangle$$

- ○ $\boldsymbol{P^-}$: Light-Front Hamiltonian
- ○ $|\boldsymbol{\beta}\rangle$: Eigenstates
- ○ $\boldsymbol{P_\beta^-}$: Eigenvalues for eigenstates

- **Basis setup:**

Fock sector expansion: $|\beta_{\text{nucleon}}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$

Single particle basis in $|qqq\rangle$: $|\alpha\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$

2-dimension harmonic oscillator	Discretized longitudinal momentum	Helicity and color
$\sum_i (2n_i + \lvert m_i \rvert + 1) \leq N_{\text{max}}$	$\sum_i k_i^+ = K_{\text{max}}$	$\Lambda = \sum_i (\lambda_i + m_i)$

- **Advantages:**
  1. Rotational symmetry in transverse plane
  2. Exact factorization between center-of-mass motion and intrinsic motion
  3. Harmonic oscillator basis supplies correct infrared behavior for hadrons

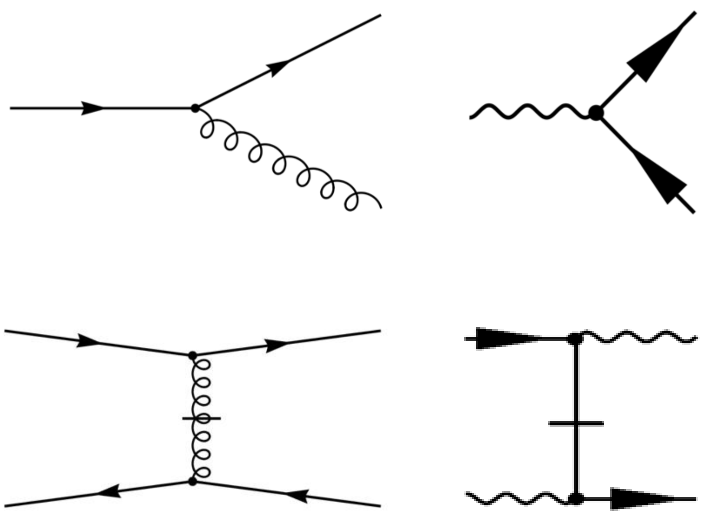
Light-front Hamiltonian

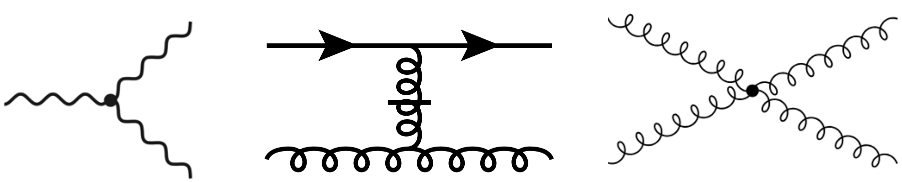
- QCD light-front Hamiltonian can be derived from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} \Longrightarrow P^{-}_{QCD} = H_K + H_I \qquad A^{+} = 0$$

$$H_K = \tfrac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^{+}\frac{(\mathrm{i}\partial^{\perp})^2 + m^2}{\mathrm{i}\partial^{+}}\psi - \tfrac{1}{2}\int \mathrm{d}^3x\, A^{i}_{a}(\mathrm{i}\partial^{\perp})^2 A^{i}{}_{a}$$

$$\begin{aligned}
H_I =& + g\int \mathrm{d}^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\psi \\
&+ \tfrac{1}{2}g^2\int \mathrm{d}^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\frac{\gamma^{+}}{\mathrm{i}\partial^{+}}\gamma_{\nu}A^{\nu}\psi \\
&- \mathrm{i}g^2\int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^{+}T^{c}\psi\frac{1}{(\mathrm{i}\partial^{+})^2}\left(\mathrm{i}\partial^{+}A^{\mu}_{a}A_{\mu b}\right) \\
&+ \tfrac{1}{2}g^2\int \mathrm{d}^3x\, \bar{\psi}\gamma^{+}T^{a}\psi\frac{1}{(\mathrm{i}\partial^{+})^2}\bar{\psi}\gamma^{+}T^{a}\psi \\
&+ \mathrm{i}g\int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^{\mu}A^{\nu a}A^{b}_{\mu}A^{c}_{\nu} \\
&- \tfrac{1}{2}g^2\int \mathrm{d}^3x\, f^{abc}f^{ade}\mathrm{i}\partial^{+}A^{\mu}_{b}A_{\mu c}\frac{1}{(\mathrm{i}\partial^{+})^2}\left(\mathrm{i}\partial^{+}A^{\nu}_{d}A_{\nu e}\right) \\
&+ \tfrac{1}{4}g^2\int \mathrm{d}^3x\, f^{abc}f^{ade}A^{\mu}_{b}A^{\nu}_{c}A_{\mu d}A_{\nu e}.
\end{aligned}$$





ψ: quark field operator
A^{a}_{μ}: gluon field operator

Progress toward First Principles

$|\mathrm{N}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,u\bar{u}\rangle + |qqq\,d\bar{d}\rangle + |qqq\,s\bar{s}\rangle + \cdots$

- **Wave Functions:**

  [PRD,102,016008] (2019) [PRD,108 9, 094002] (2023) [arXiv:2408.11298] (2024)

- **GPDs:**

  [PRD,104,094036] (2021) [PLB,847,138305] (2023)

  [PRD,105,094018] (2022) [PRD,110.056027] (2024)

  [PRD,109,014015] (2024) [arXiv:2408.09988] (2024)

  [PLB,855,138809] (2024)

- **TMDs:**

  [PLB,833,137360] (2022) [PLB,855 138831] (2024)

  [PRD,108,036009] (2023)

- **Higher-twist Distribution (GPD,TMD,DPD):**

  [PRD,109,034031] (2024) [PLB,855 138829] (2024)

  [arXiv:2410.11574] (2024)

- **Gravitational Form Factors:**

  [PRD,110,056027] (2024)

Full BLFQ

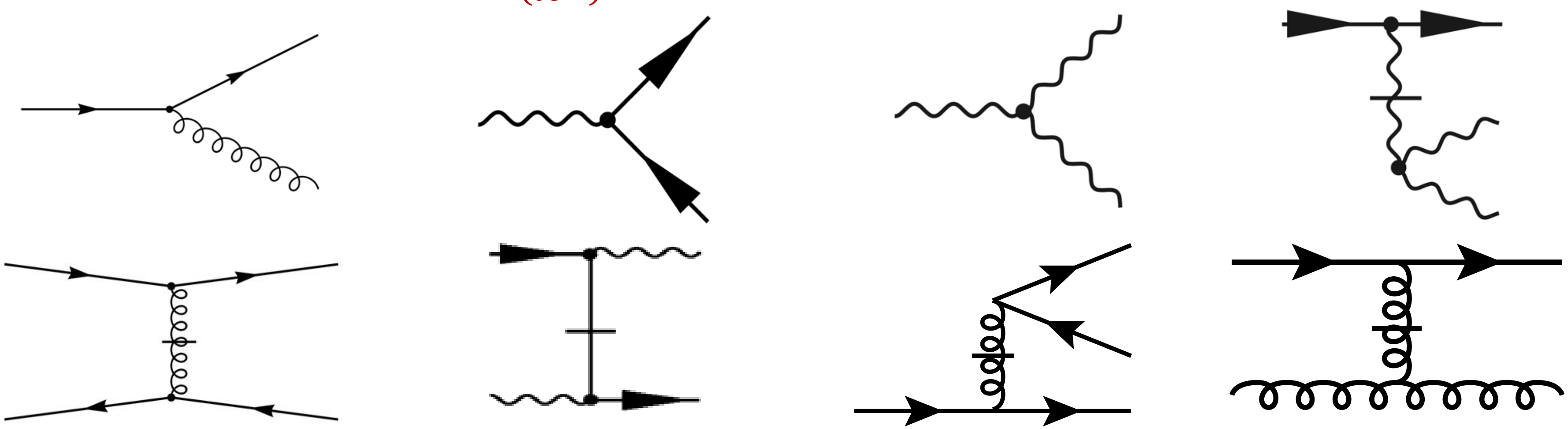
$$|N\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqgg\rangle$$

$$P^- = H_{K.E.} + H_{Interact}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_{Interact} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+ + \frac{g^2 C_F}{2}\,\bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+} A_\nu \gamma^\nu \psi$$

$$-g^2 C_F \bar{\psi}\gamma^+\psi \frac{1}{(i\partial^+)^2}\, i\partial^+ A_\mu^a A_b^\mu + igf^{abc} i\partial^\mu A_a^\nu A_\mu^b A_\nu^c$$



Fock Sector Decomposition

$$|P_{baryon}\rangle \rightarrow |qqq\rangle + |qqqg\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqgg\rangle$$

$|qqq\ q\bar{q}\rangle \sim 3$ color singlet state

1 singlet ⊗ singlet

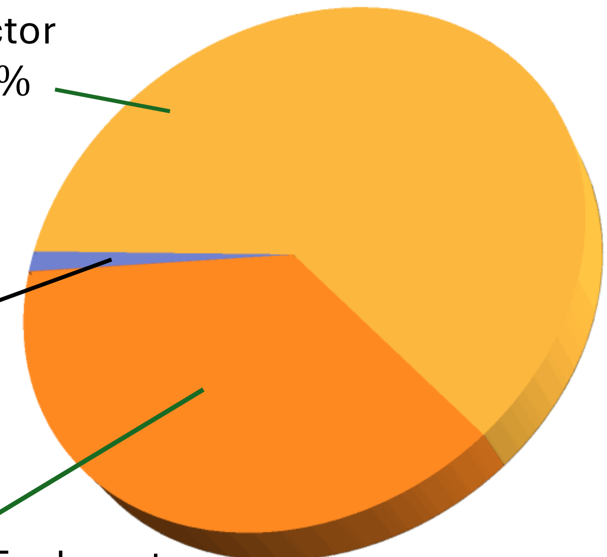
2 octet ⊗ octet

$|qqq\ gg\rangle \sim 6$ color singlet state

1 singlet ⊗ singlet

4 octet ⊗ octet

1 decuplet ⊗ octet ⊗ octet

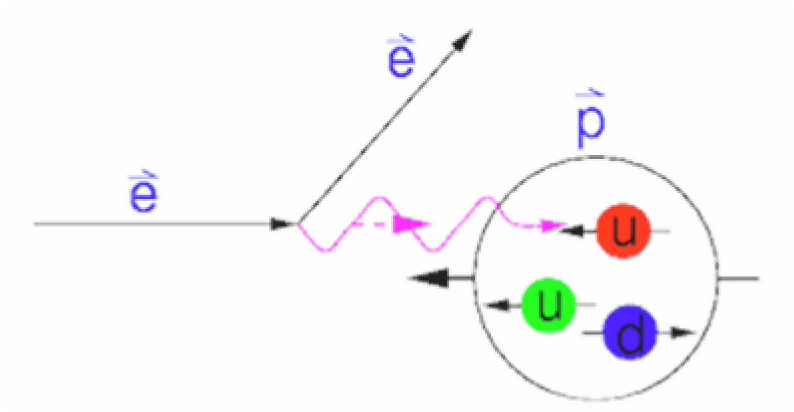


m_u	m_d	m_s	m_f	g	b	b_{inst}
0.5 GeV	0.45 GeV	0.6 GeV	3.0 GeV	2.1	0.6 GeV	3.0 GeV

Truncation parameter: $N_{\max} = 7$ and $K_{\max} = 10$

Electromagnetic Form Factors

- **Elastic scattering of proton**



[R. Hofstadter 1961]

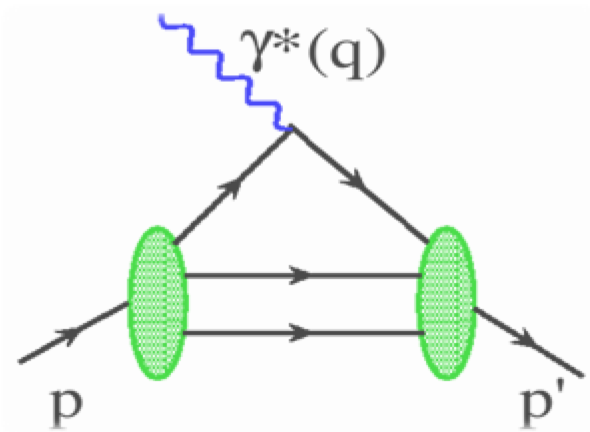
$$e(p) + h(P) \rightarrow e(p') + h(P')$$

- Elastic electron scattering established the extended nature of the proton (proton radius).

- Fourier transformation of these form factors provide spatial distributions (charge and magnetization distributions).

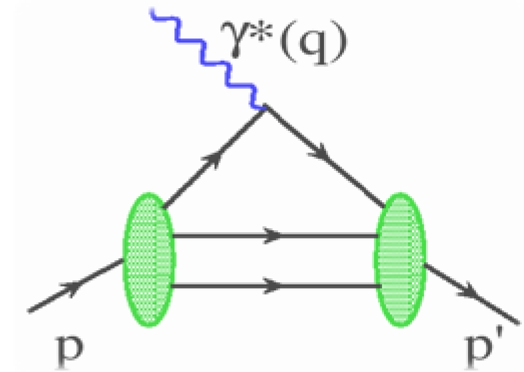
$$\langle N(p') | J^{\mu}(0) | N(p) \rangle =$$
$$\bar{u}(p') \left[\gamma^{\mu} \underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N} q_{\nu} \underbrace{F_2(q^2)} \right] u(p)$$

Dirac Form Factor   Pauli Form Factor

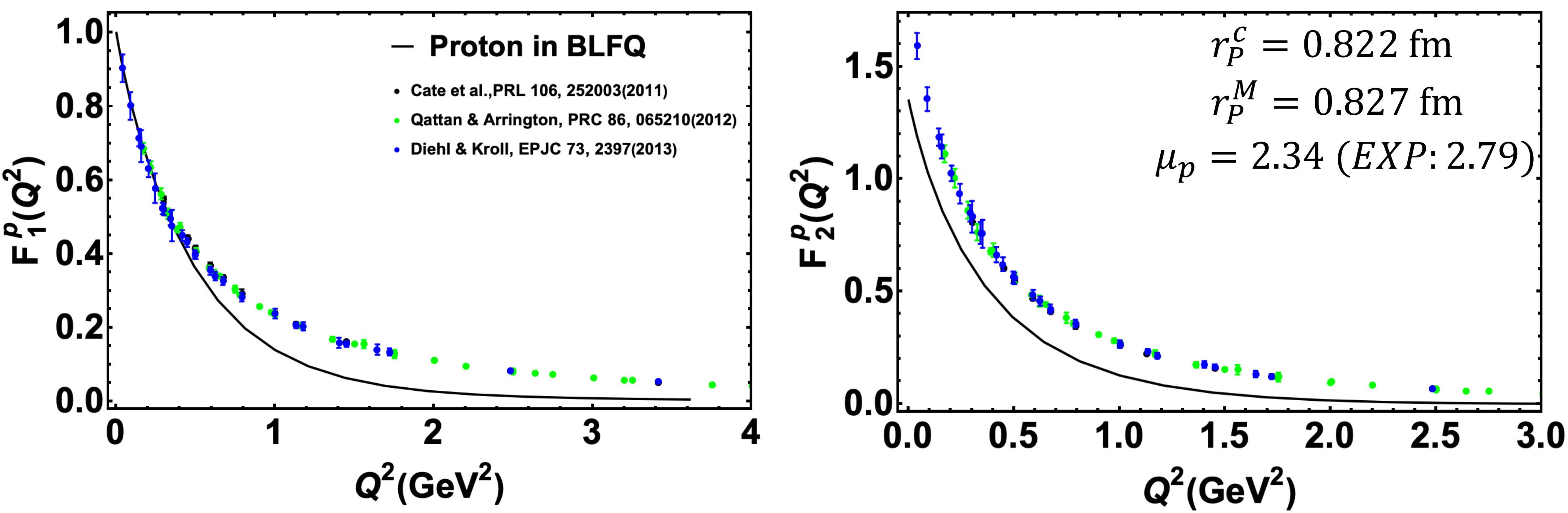


Nucleon Form Factors

$$\langle N(p')|J^{\mu}(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^{\mu}\underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N}q_{\nu}\underbrace{F_2(q^2)}\right]u(p)$$



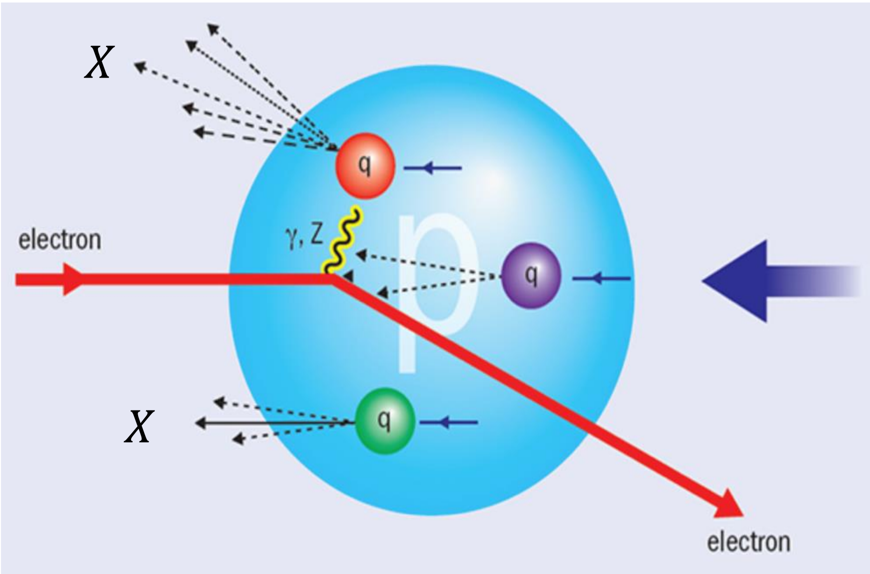
Preliminary results



- BLFQ results qualitatively agree with the experimental data for Dirac and Pauli FFs

Parton Distribution Functions (PDF)

➢ **Deep Inelastic Scattering (SLAC 1968)**



$$e(p) + h(P) = e'(p') + X(P')$$

✧ **Localized probe:**

$$Q^2 = -(p-p')^2 \gg 1\ \mathrm{fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1\ \mathrm{fm}$$

Discovery of spin ½ quarks and partonic structure

➢ **Parton distribution functions** (PDFs) are extracted from **DIS** processes.

$$\Phi^{[\gamma^+]}(x, Q^2) = \int \frac{dz^-}{8\pi}\, e^{\frac{ixP^+z^-}{2}} \langle P,\Lambda|\bar{\psi}(x)\gamma^+\psi(0)|P,\Lambda\rangle\Big|_{x^+=x^\perp=0}$$

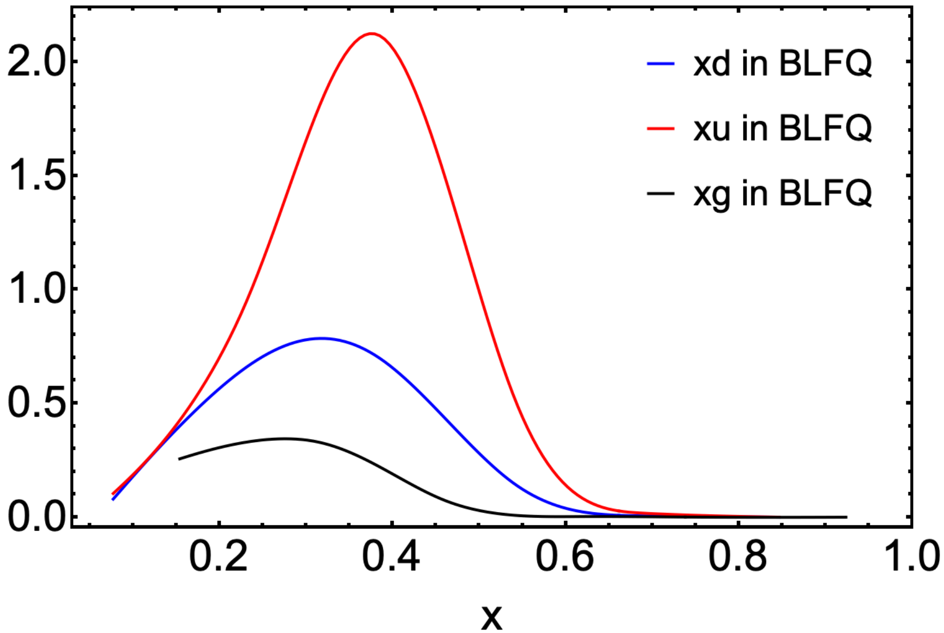
➢ PDFs encode the distribution of longitudinal momentum and polarization carried by the constituents

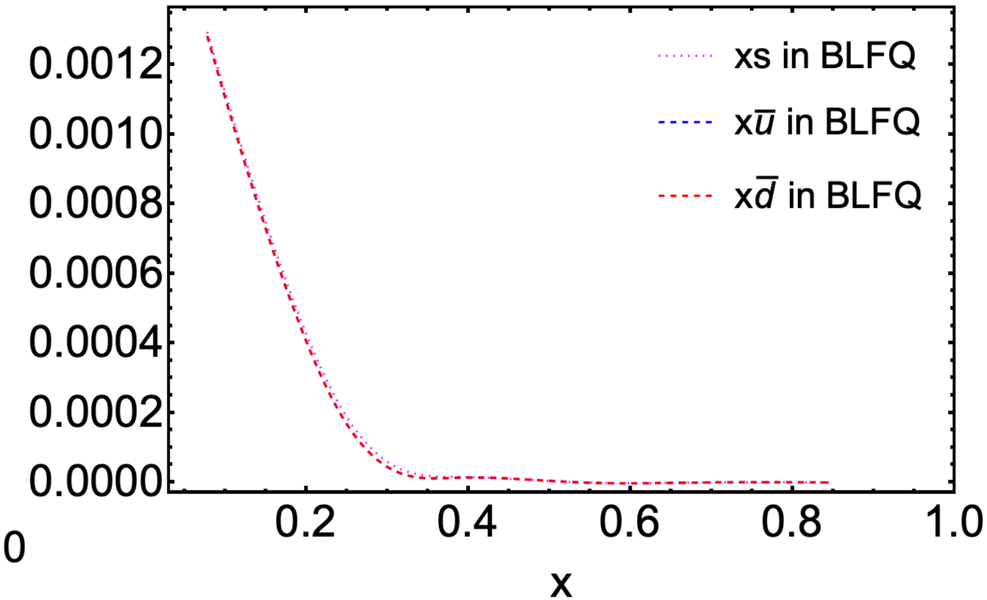
Unpolarized Parton Distribution Function

- Parton distribution functions with five Fock sectors
  - Qualitative behavior agree with experimental results
  - Endpoint behavior improves with $|qqqgg\rangle$ Fock sector included
  - Five-particle sector contributions are small due to Fock sector truncation (no $|qqq\ q\bar{q}\ g\rangle$ $|qqq\ ggg\rangle$),

Preliminary results

All results at the initial scale





Unpolarized PDFs

➢ Parton distribution functions with five Fock sectors

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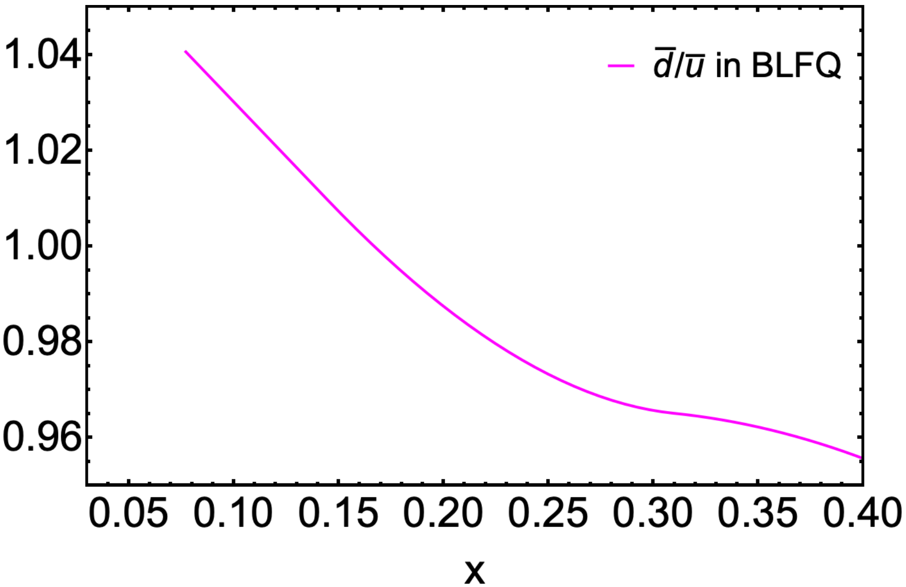
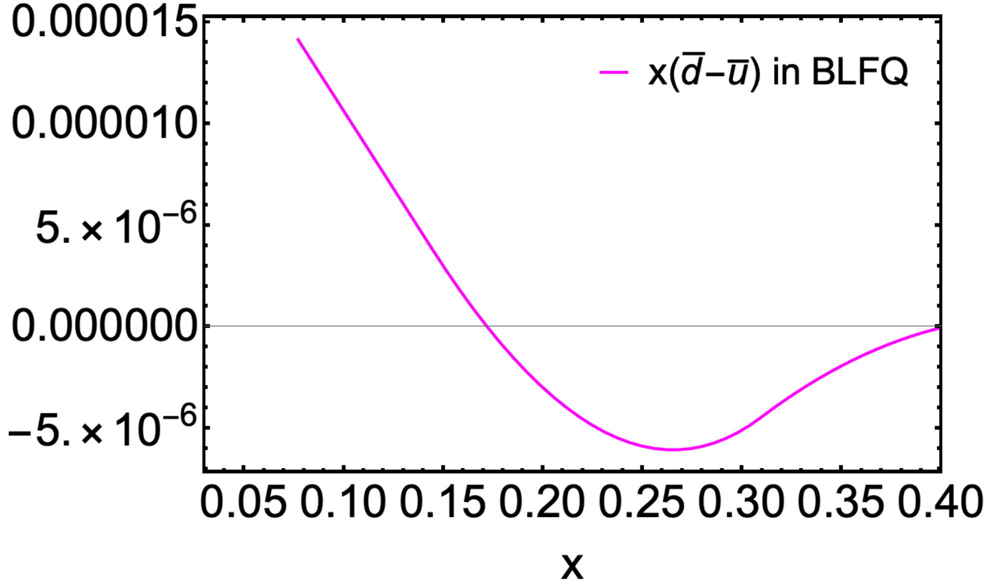
Preliminary results

All results at the initial scale





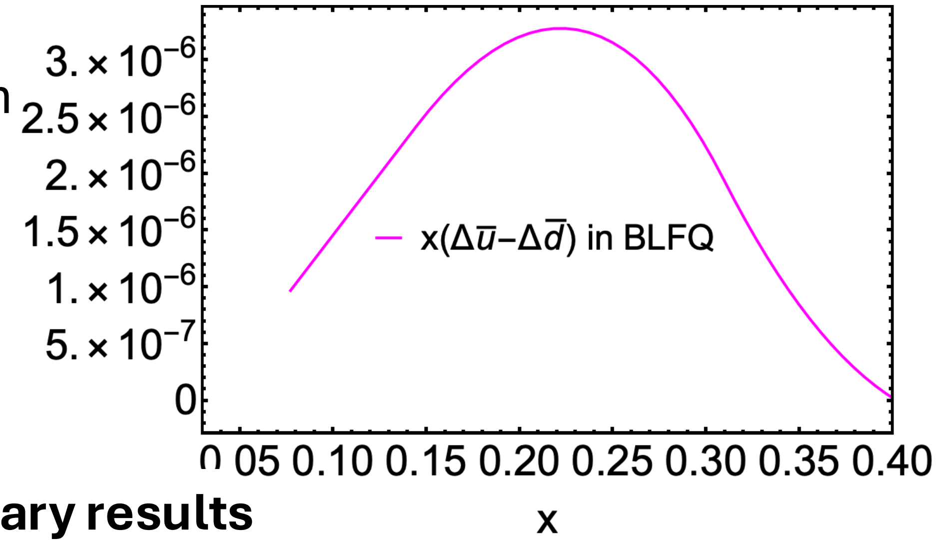
Helicity PDFs

- Helicity PDFs with five particle parton distribution

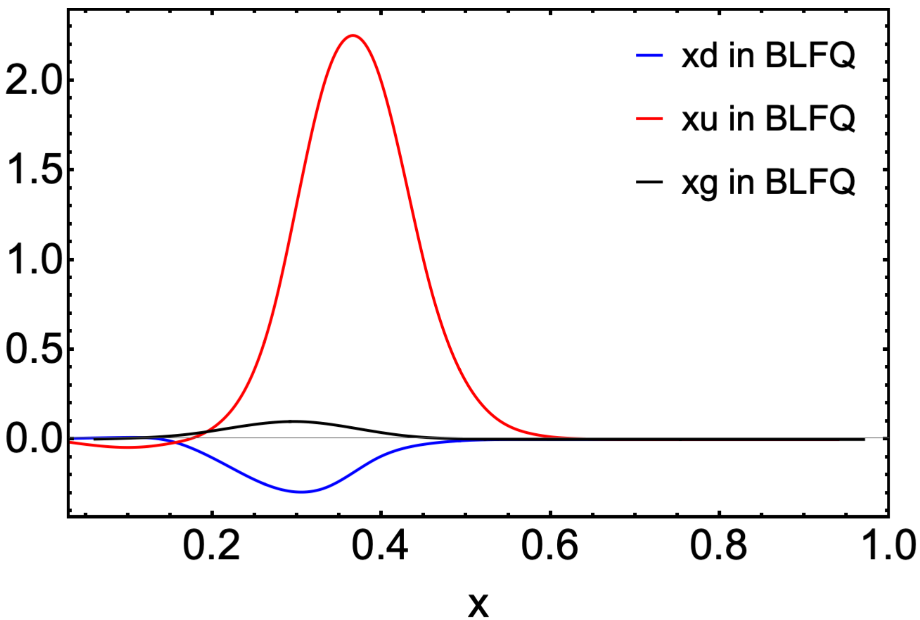
Sea asymmetry qualitatively agrees with JAM results

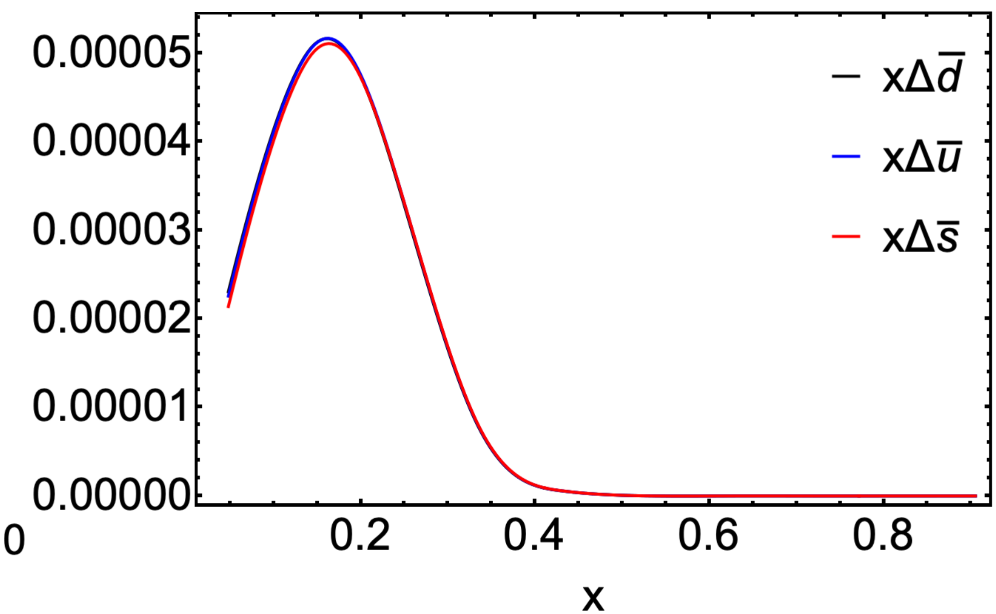
$\Delta\Sigma_u = 0.84 \qquad \Delta\Sigma = 0.653$

$\Delta\Sigma_d = -0.19 \qquad \Delta G = 0.094$



Preliminary results



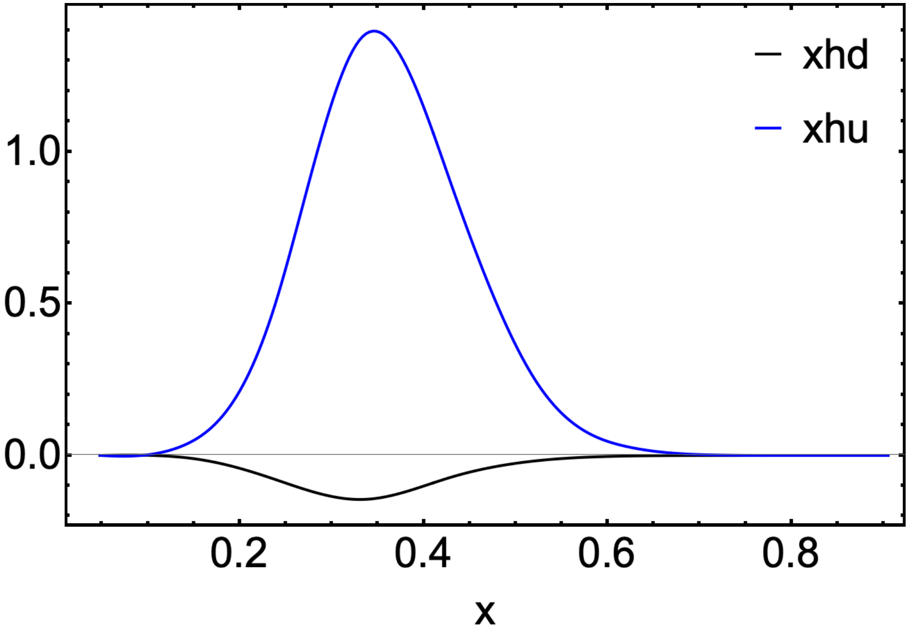


Transversity PDFs

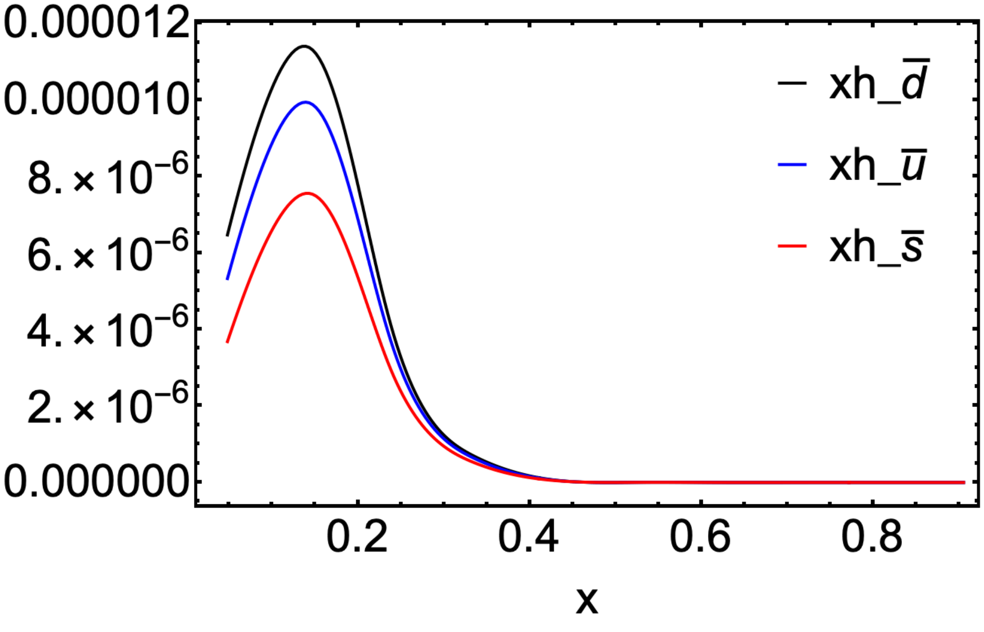
- u has opposite sign of d
- Qualitatively consistent with the experimental data
- Asymmetry between $\bar{u}$, $\bar{d}$, and $\bar{s}$

Tensor Charge: $\delta u = 0.91$, $\delta d = -0.10$ At initial scale

Preliminary results

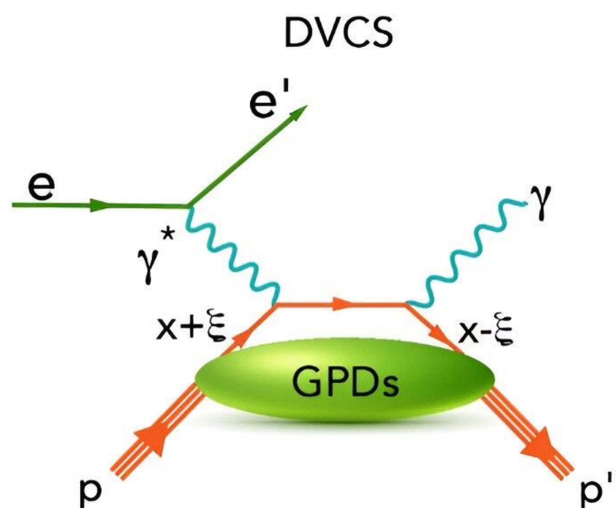


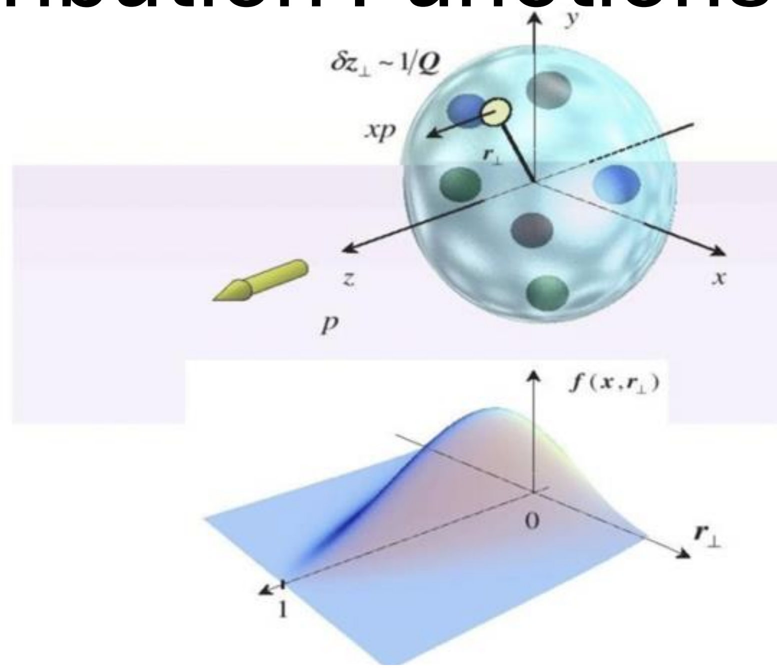
Preliminary results



Generalized Parton Distribution Functions

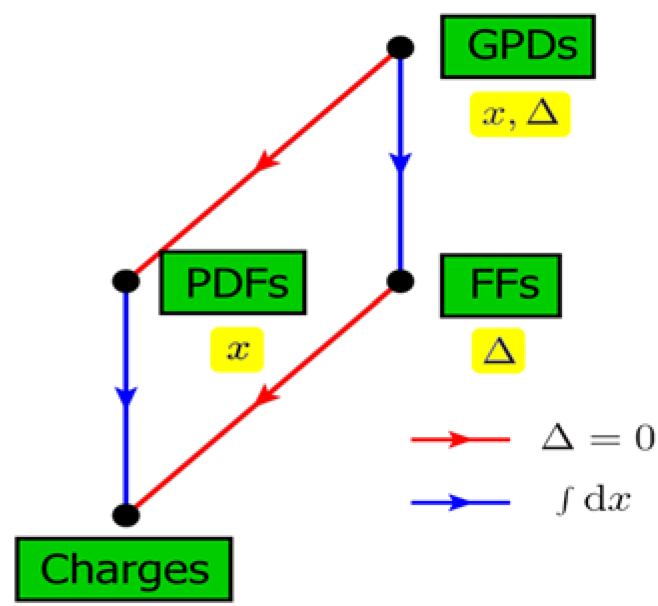
- Deeply Virtual Compton Scattering (DVCS)



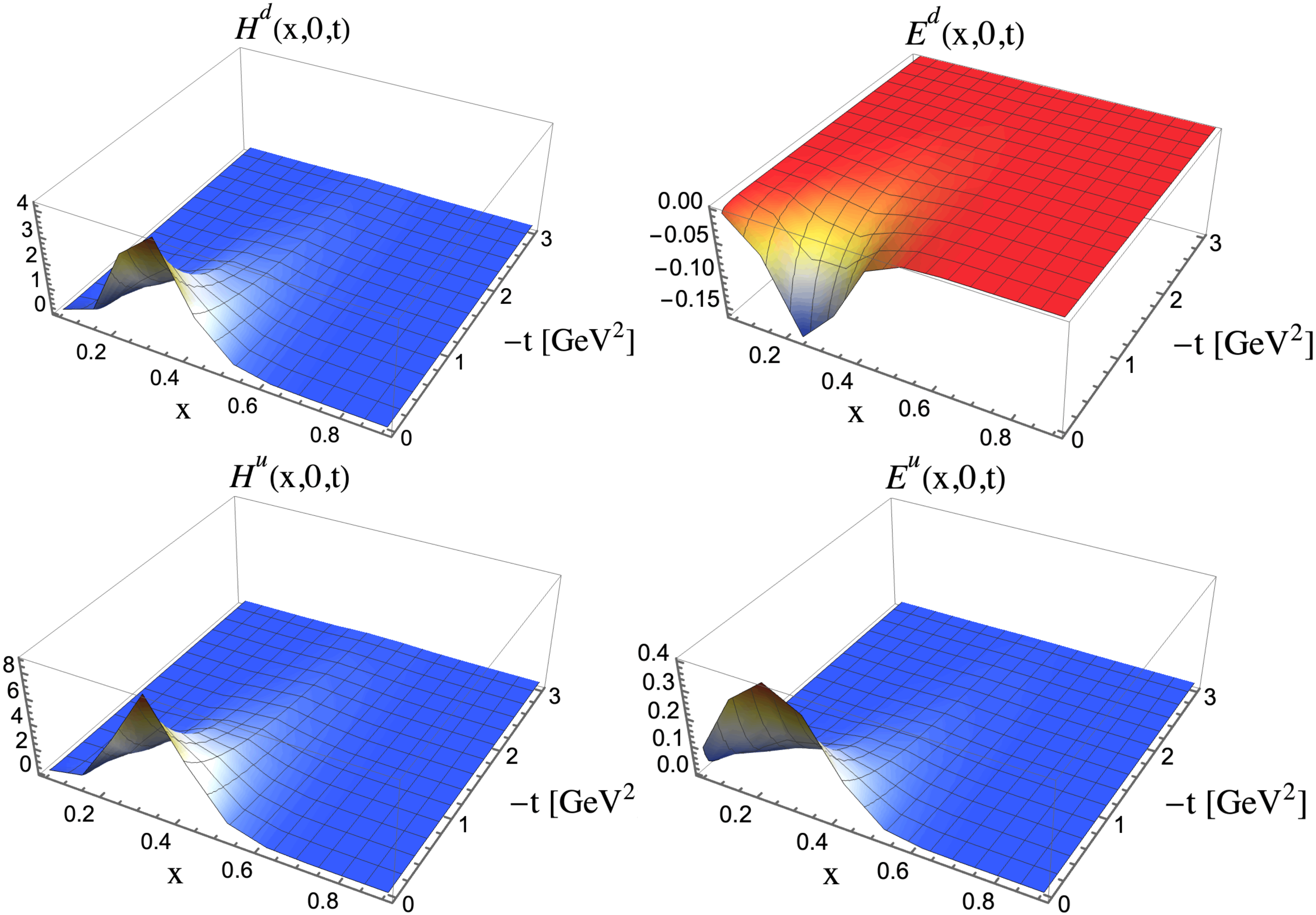


- Encode the information about three-dimensional spatial structure the spin and orbital angular momentum
- With increasing momentum transfer (t), peaks of distributions shift to larger x

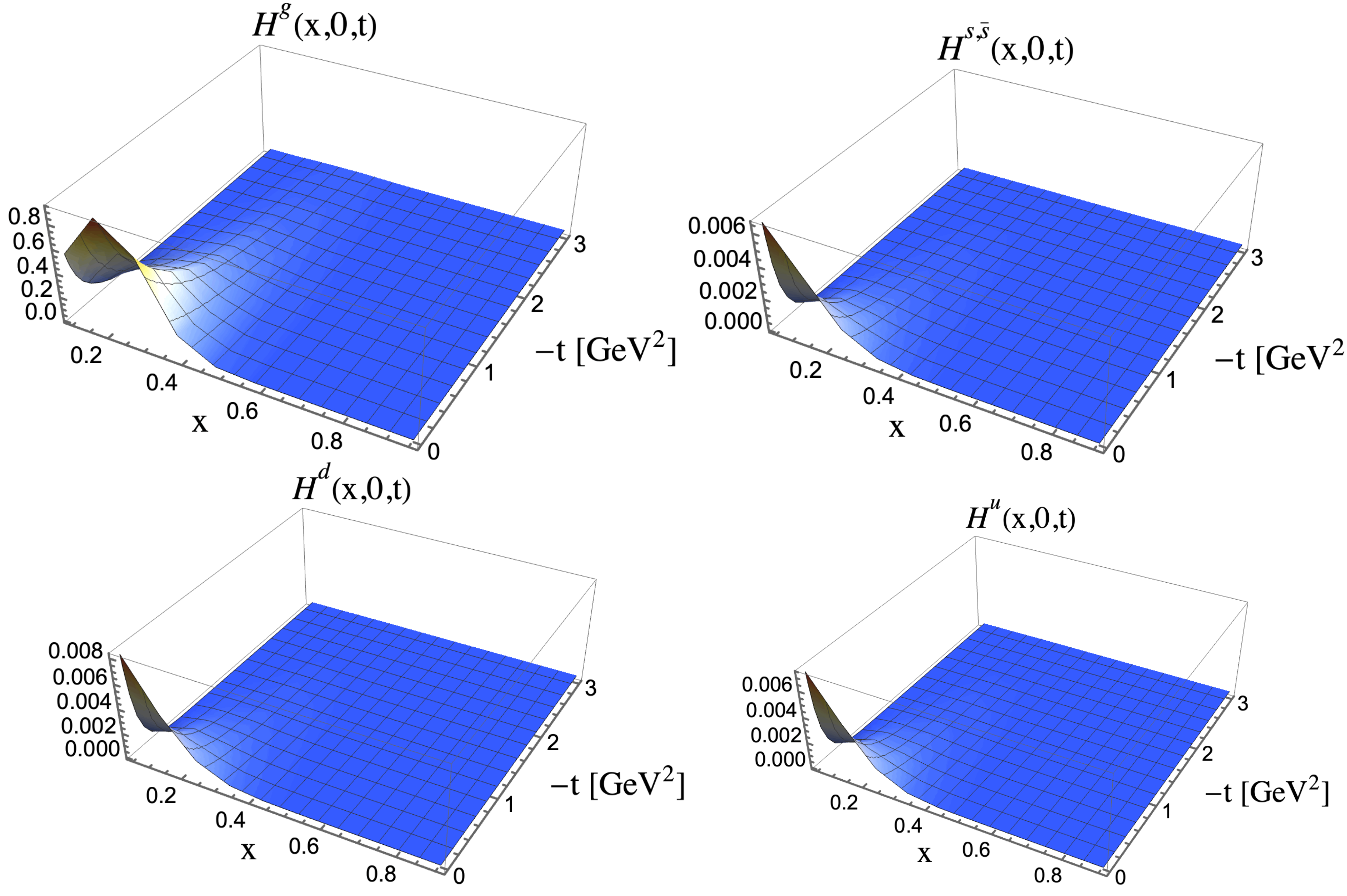
$$t = \Delta^2, x = \frac{k^+}{P^+}, \boldsymbol{\zeta = \frac{\Delta^+}{P^+} = 0} \qquad b \overset{FT}{\rightarrow} \Delta_\perp$$



Generalized Parton Distribution Functions



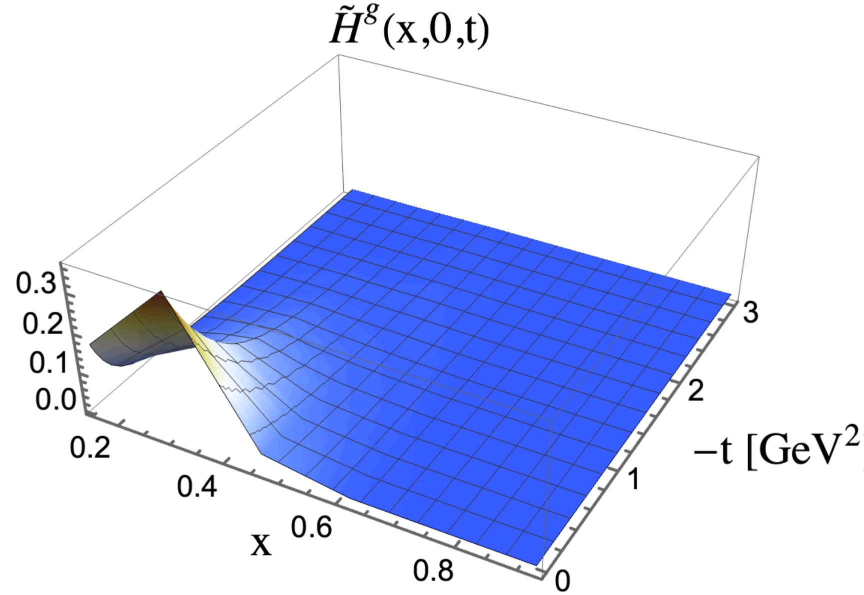
Generalized Parton Distribution Functions

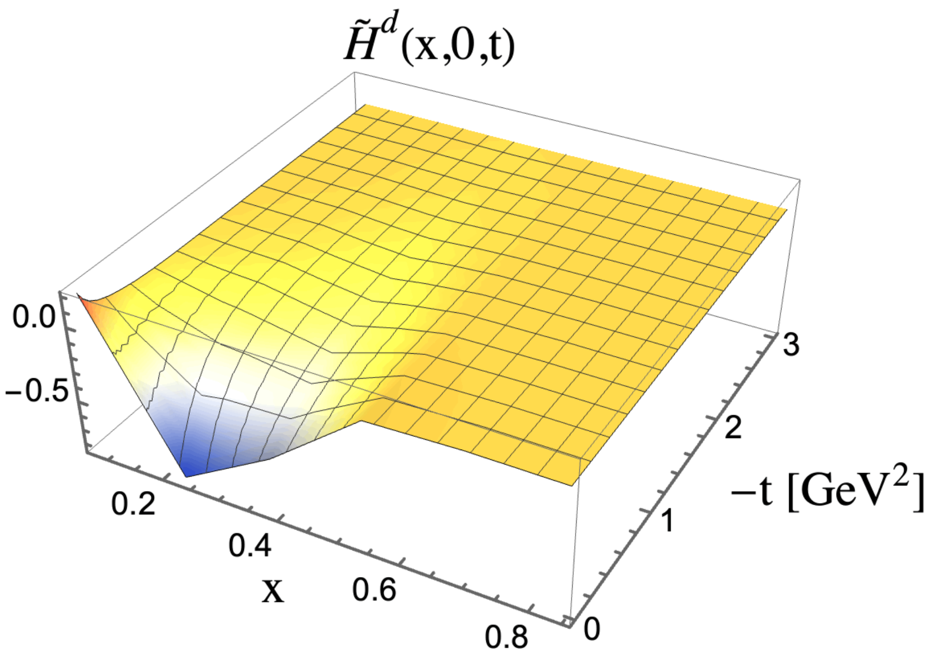


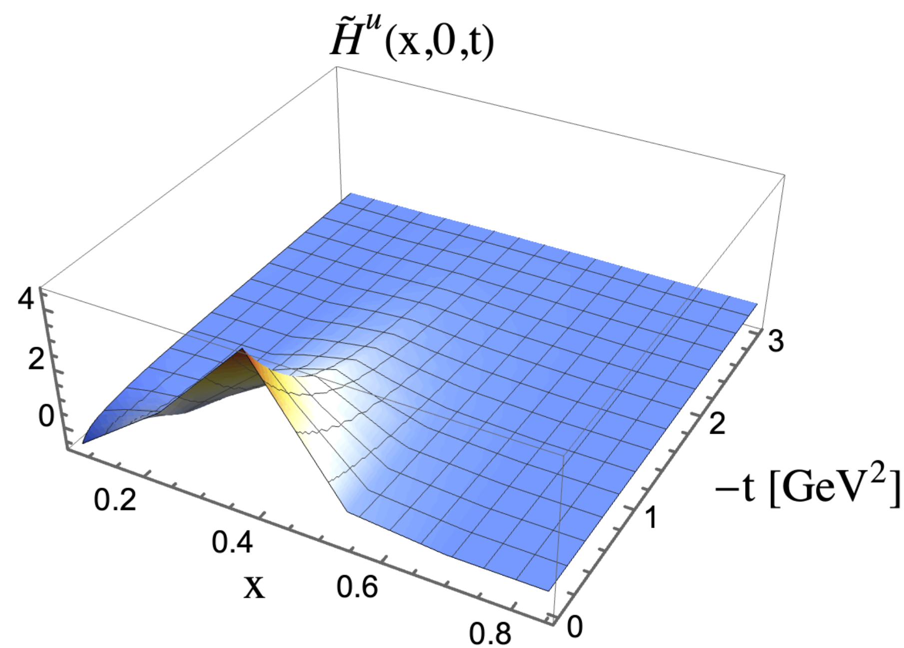
Generalized Parton Distribution Functions

- Polarized GPDs for valence quark and gluon at zero skewness

$$t = \Delta^2, x = \frac{k^+}{P^+}, \boldsymbol{\zeta = \frac{\Delta^+}{P^+} = 0}$$



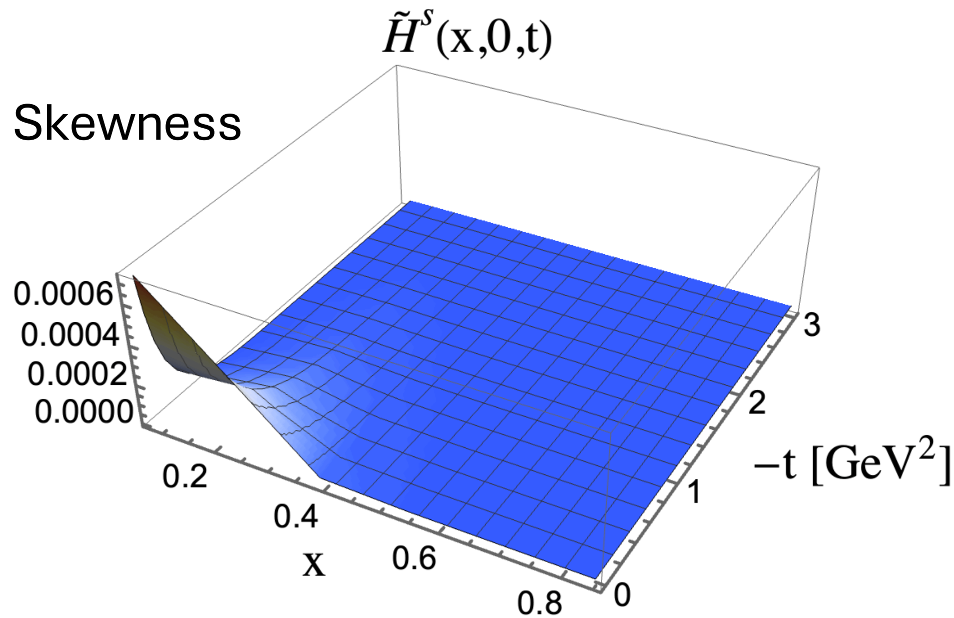


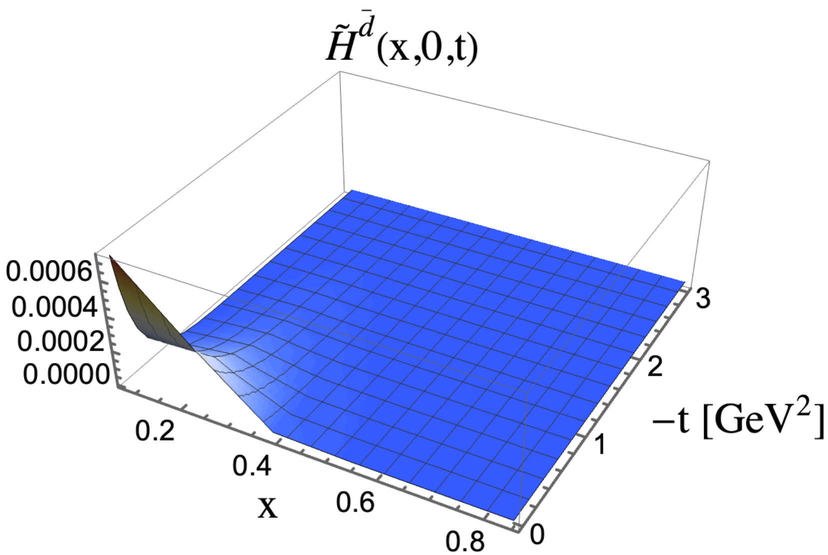


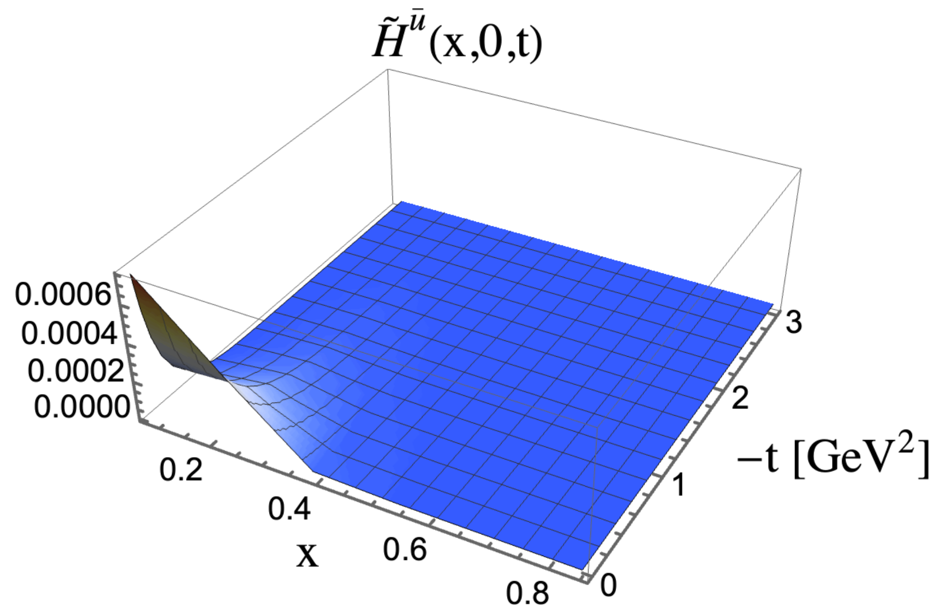
Generalized Parton Distribution Functions

➢ Polarized GPDs for sea quark at zero Skewness

$$t = \Delta^2, x = \frac{k^+}{P^+}, \boldsymbol{\zeta = \frac{\Delta^+}{P^+} = 0}$$







Spin Decomposition

- Using generalized parton distributions to calculate the angular momentum

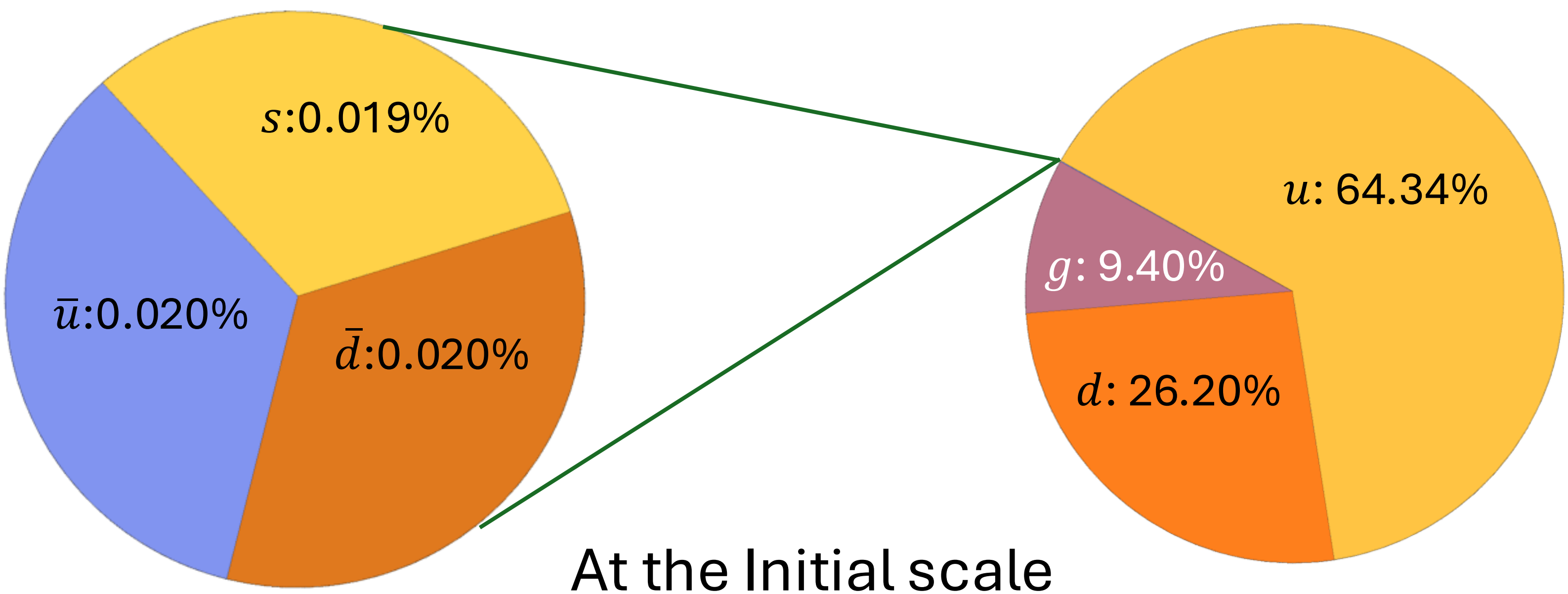
$$J_{q,g} = \int dx\, \frac{x}{2}[H_{q,g}(x,0,0) + E_{q,g}(x,0,0)]$$

$$\frac{1}{2} = J_u + J_d + J_g + J_{sea}$$

- Small sea quark contributions at initial scale compared to valence and gluon

Sea quark Contribution: 0.059%

Valence and gluon contribution



At the Initial scale

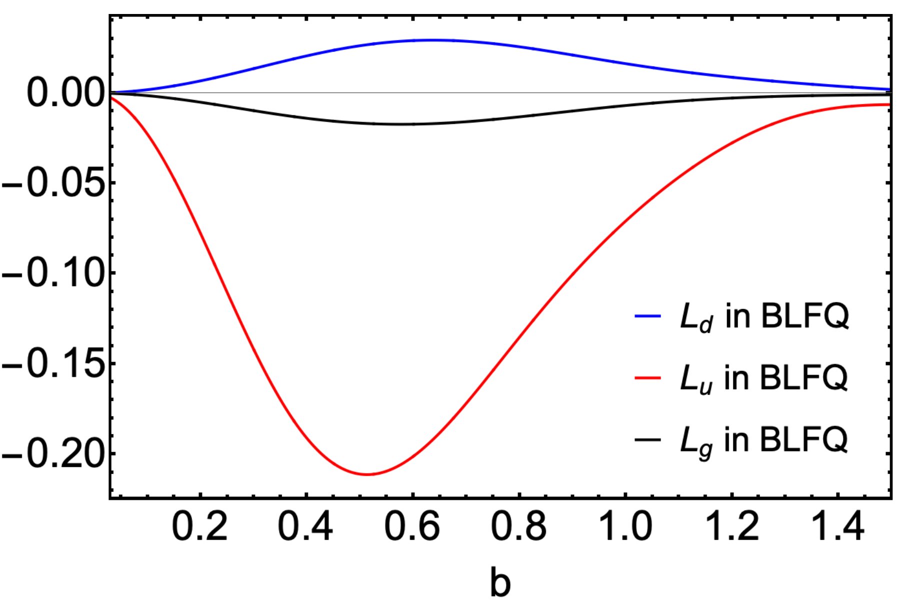
Spin Decomposition

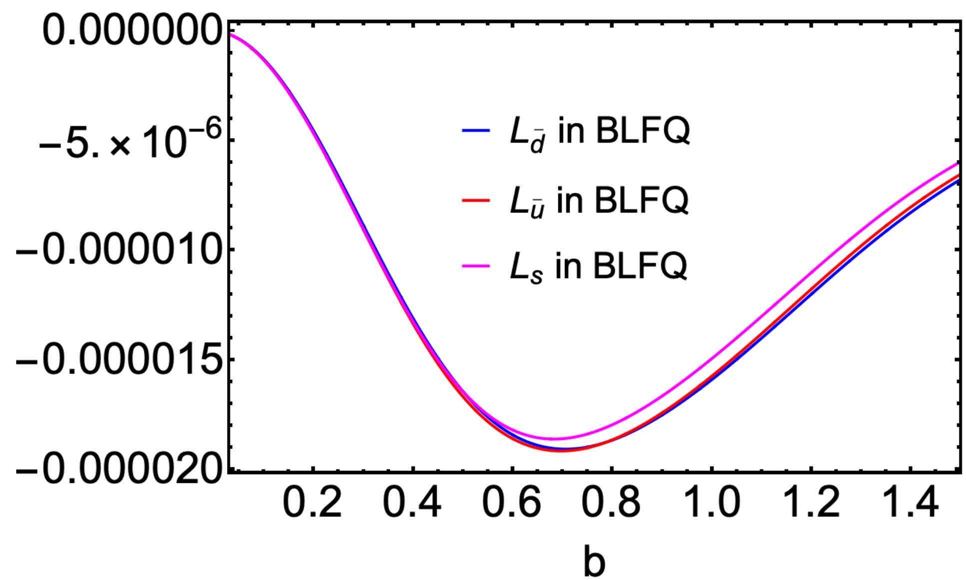
➢ Orbital angular momentum distribution at light-cone gauge ($A^+ = 0$)

$$\langle J^z\rangle(b_\perp) = \langle L^z\rangle(b_\perp) + \langle S^z\rangle(b_\perp)$$

$$\langle L^z_{q,g}\rangle(b) = -\frac{s^z}{2} b \frac{d\,\tilde{L}(b)}{d\,b} \qquad \tilde{L}(b) = \frac{1}{2}\int dx\, x(H(x,b) + E(x,b)) - \widetilde{H}(x,b)$$

➢ In the light-cone gauge, the orbital angular momentum can be extracted





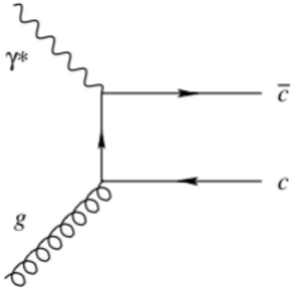
Conclusions

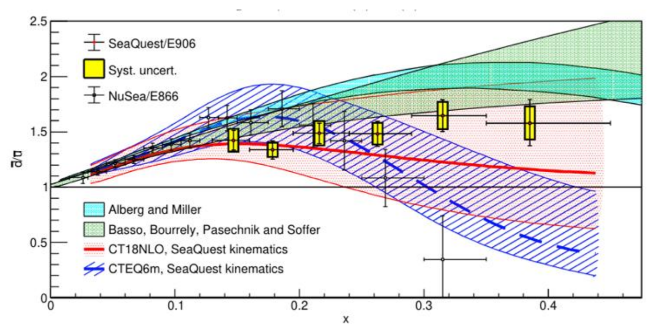
- BLFQ: a non-perturbative Hamiltonian approach based on QCD
- $|qqq\rangle + |qqqg\rangle + |qqq\ q\bar{q}\rangle + |qqq\ gg\rangle$ Fock sectors included
- Incorporates all QCD interactions other than four-gluon interactions
- Results qualitatively agree with global fitting
- Utilizes 3D structures to analyze spin decomposition
- Progressing towards a First-Principles Approach

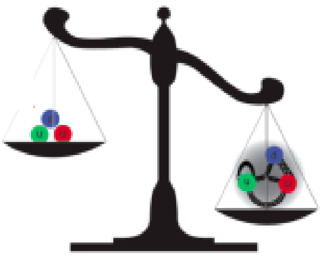
Outlook

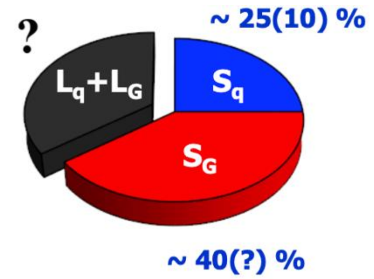
Current status

- **Full QCD interaction**
  - **Fock sector expansion**
    $|qqq\ q\bar{q}\ g\rangle$ and $|qqq\ ggg\rangle$......
    - **Intrinsic charm**
    - **Sea asymmetry**
    - **Origin of spin and mass**
- **Deutron calculation**
  $|qqq\ qqq\rangle + |qqq\ qqq\ g\rangle$
  - **EMC effect**



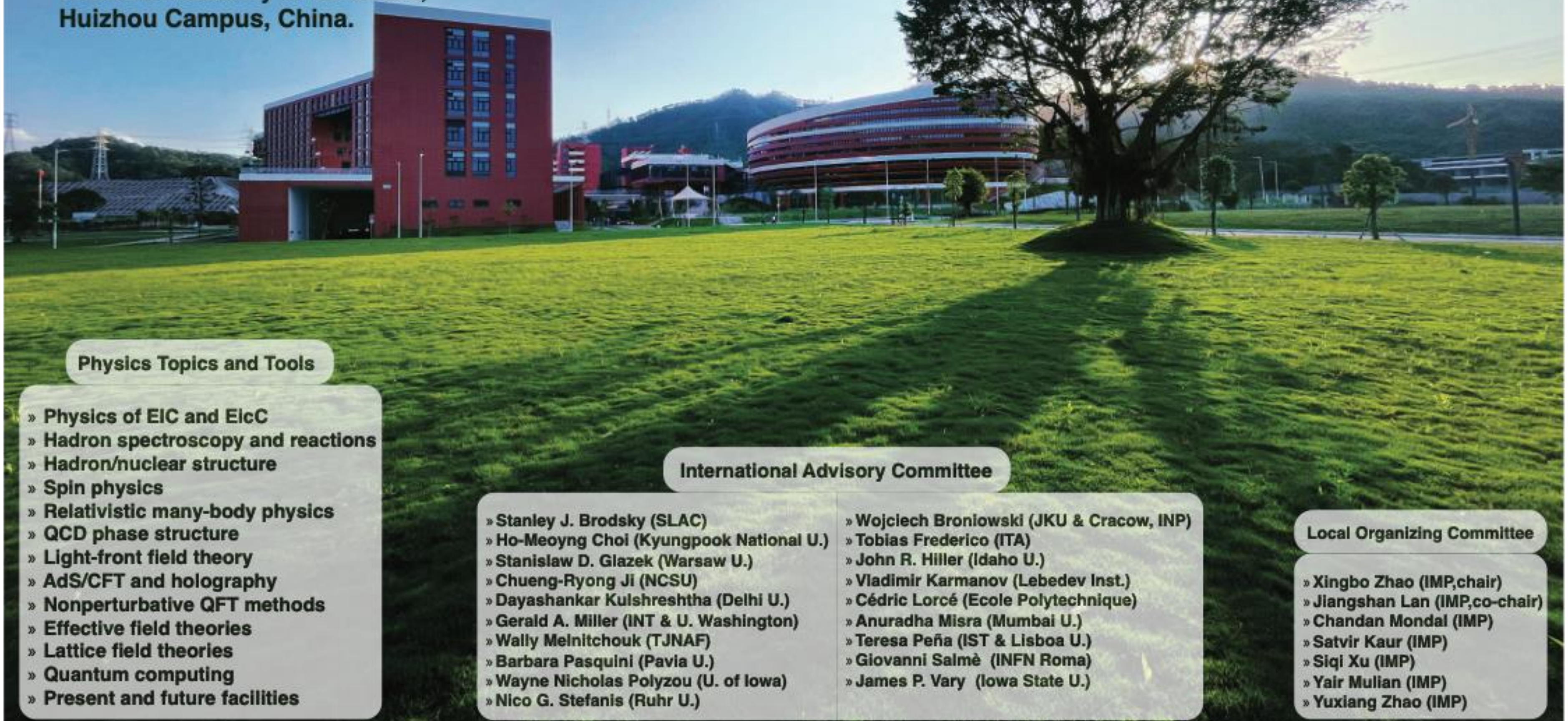






LIGHT CONE 2024

Hadron Physics in the EIC era



The Institute of Modern Physics, Chinese Academy of Sciences, Huizhou Campus, China.

November 25-29, 2024

Physics Topics and Tools

- Physics of EIC and EicC
- Hadron spectroscopy and reactions
- Hadron/nuclear structure
- Spin physics
- Relativistic many-body physics
- QCD phase structure
- Light-front field theory
- AdS/CFT and holography
- Nonperturbative QFT methods
- Effective field theories
- Lattice field theories
- Quantum computing
- Present and future facilities

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Registration and abstract submission opens : 1st April, 2024
Abstract submission deadline : 15th November, 2024
Registration closes : 15th November, 2024

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Thank you!

See you in Huizhou☺