Effect of Coriolis Force on Shear Viscosity : A Non-Relativistic Description

Cho Win Aung,
*Collaborators:*Ashutosh Dwibedi, Jayanta Dey and Sabyasachi Ghosh

Physics Department
IIT Bhilai

Outline of the Research

- Introduction
- Formalism:
  - Mathematical building steps
  - Comparison of finite magnetic field case and finite rotation case
  - Expressions for shear viscosity components
- Results
- Summary

Introduction

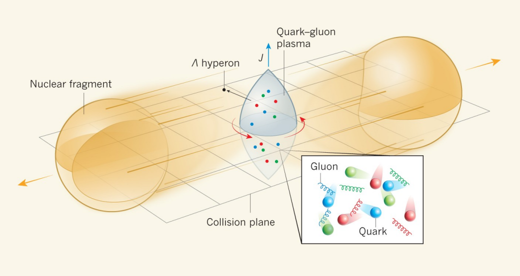

Figure: Rotating QGP medium between two heavy ions collision

- In HIC experiments, like RHIC and LHC, a huge magnetic field as well as huge vorticity.
- Magnetic field and vorticity can act as an external field, creating anisotropy in the medium.
- The QGP formed in these collisions, behave like a nearly perfect fluid system with lowest possible η/s.
- The angular momentum stuck in the medium tries to rotate the system and is termed as the local vorticity of the quark fluid.

Mathematical building steps

- In classical mechanics, the operator equation for any arbitary vector[0],

$$\left(\frac{d}{dt}\right)_s \equiv \left(\frac{d}{dt}\right)_r + \vec{\Omega}\times \quad , \tag{1}$$

- Operator on position vector $\vec{r}$

$$\vec{v}_s = \vec{v}_r + \vec{\Omega}\times\vec{r}\,, \quad \text{and} \quad \vec{a}_s = \vec{a}_r + 2(\vec{\Omega}\times\vec{v}_r) + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})\,. \tag{2}$$

where $\vec{v}_s$ and $\vec{v}_r$ with velocity in space-fixed and rotating frames.

- Arranging these terms as Newton's equation in a rotating frame, the second term in Eq. (3) is the Coriolis force.

$$m\vec{a}_s - 2m(\vec{\Omega}\times\vec{v}_r) - m\vec{\Omega}\times(\vec{\Omega}-\vec{r}) = \vec{F} \tag{3}$$

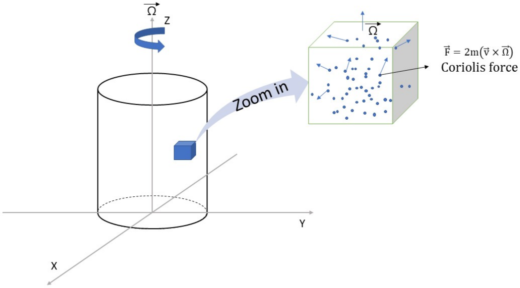

Figure: Rotating cylinder with moving particles in fluid element, which face the Coriolis force

- The particles inside any fluid element have a random part of the velocity $\vec{v}$.
- Fluid has average or effective angular velocity $\vec{\Omega}$.
- All the particles feel the Coriolis force $2m(\vec{v} \times \vec{\Omega})$.

Comparison of the finite magnetic field and finite rotation case

Finite Magnetic Field

- Due to magnetic field
- Lorentz Force : $\vec{F} = q\vec{v} \times B$
- Macroscopic expressions of energy-momentum tensors can be built.
- Basic tensors : $\vec{u}$, δ_{ij} and $b_i(B_i \equiv Bb_i)$
- $\tau_B = \frac{m}{qB}$

Finite Rotation

- Due to vorticity
- Coriolis Force : $\vec{F} = 2m\vec{v} \times \Omega$
- Macroscopic expressions of energy-momentum tensors can be built.
- Basic tensors : $\vec{u}$, δ_{ij} and $\omega_i(\Omega_i \equiv \Omega\omega_i)$
- $\tau_\Omega = \frac{1}{2\Omega}$

Expressions of shear viscosity components in finite rotation

Viscous stress tensor in macroscopic and microscopic descriptions:

$$\pi_{ij} = \eta_n C^n_{ijkl} U^{kl} = \eta_n C^n_{ij} \, .$$

$$\pi_{ij} = g \int \frac{d^3\vec{p}}{(2\pi)^3} m v_i v_j \delta f \, ,$$

where, $U_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)$. Five independent tensor components [0].

$$\begin{aligned}
C^0_{ij} &= (3\omega_i\omega_j - \delta_{ij})(\omega_k\omega_l U_{kl} - \frac{1}{3}\vec{\nabla}\cdot\vec{u}) \\
C^1_{ij} &= 2U_{ij} + \delta_{ij}U_{kl}\omega_k\omega_l - 2U_{ik}\omega_j\omega_k - 2U_{jk}\omega_k\omega_i \\
&+ (\omega_i\omega_j - \delta_{ij})\vec{\nabla}\cdot\vec{u} + \omega_i\omega_j\omega_k\omega_l U_{kl} \\
C^2_{ij} &= 2(U_{ik}\omega_j\omega_k + U_{jk}\omega_i\omega_k - 2U_{kl}\omega_i\omega_j\omega_k\omega_l) \\
C^3_{ij} &= U_{ik}\omega_{jk} + U_{jk}\omega_{ik} - U_{kl}\omega_{ik}\omega_j\omega_l - U_{kl}\omega_{jk}\omega_i\omega_l \\
C^4_{ij} &= 2(U_{kl}\omega_{ik}\omega_j\omega_l + U_{kl}\omega_{jk}\omega_i\omega_l) \, .
\end{aligned} \quad (4)$$

$$\delta f = \sum_{n=0}^{4} C_n C_{kl}^n v_k v_l \tag{5}$$

The BTE in relaxation time approximation (RTA) at finite rotation

$$\frac{m}{T} v_i v_j U_{ij} f_0(1 - f_0) + 2(\vec{v} \times \vec{\Omega}) \cdot \frac{\partial \delta f}{\partial \vec{v}} = -\frac{\delta f}{\tau_c} \,. \tag{6}$$

$$\implies \frac{m}{T} v_i v_j U_{ij} f_0(1 - f_0) + \frac{1}{\tau_\Omega} \omega_{ij} v_j \frac{\partial \delta f}{\partial v_i} = -\frac{\delta f}{\tau_c} \,, \tag{7}$$

where, $\tau_\Omega = \frac{1}{2\Omega}.\tau_\Omega$ will play same role as the cyclotron time period $\tau_B = m/qB$ plays on the transport coefficient expressions at finite magnetic field.

- Solving the BTE to get δf, finally we get the four coefficients,

$$C_1 = -\frac{m}{2T} f_0(1-f_0)\frac{\tau_c}{1+4(\tau_c/\tau_\Omega)^2} \quad , C_2 = -\frac{m}{2T} f_0(1-f_0)\frac{\tau_c}{1+(\tau_c/\tau_\Omega)^2} \, , \tag{8}$$

$$C_3 = -\frac{m}{T} f_0(1-f_0)\frac{\tau_c(\tau_c/\tau_\Omega)}{1+4(\tau_c/\tau_\Omega)^2} \quad , C_4 = -\frac{m}{2T} f_0(1-f_0)\frac{\tau_c(\tau_c/\tau_\Omega)}{1+4(\tau_c/\tau_\Omega)^2} \, . \tag{9}$$

- Substituting the value of C_n in δf to get π_{ij} and comparing the macroscopic,

$$\eta_n = -\frac{2\, g\, m^4}{15}\int \frac{d^3 v}{(2\pi)^3} v^4 C_n \, . \tag{10}$$

- The viscosity in the absence of rotation,

$$\eta_0 = g\left(\frac{m}{2\pi}\right)^{3/2} \tau_c T^{5/2} f_{5/2}(A).$$

- General microscopic shear viscosity components due to rotation applicable for RHIC/LHC quark matter ($\mu = 0$) and compact star ($T = 0$)

$$\eta_1 = g\left(\frac{m}{2\pi}\right)^{3/2} \frac{\tau_c}{1+4\left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{5/2} f_{5/2}(A) \tag{11}$$

$$\eta_2 = g\left(\frac{m}{2\pi}\right)^{3/2} \frac{\tau_c}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{5/2} f_{5/2}(A) \tag{12}$$

$$\eta_3 = g\left(\frac{m}{2\pi}\right)^{3/2} \frac{\tau_c\left(\frac{2\tau_c}{\tau_\Omega}\right)}{1+4\left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{5/2} f_{5/2}(A) \tag{13}$$

$$\eta_4 = g\left(\frac{m}{2\pi}\right)^{3/2} \frac{\tau_c\left(\frac{\tau_c}{\tau_\Omega}\right)}{1+\left(\frac{\tau_c}{\tau_\Omega}\right)^2} T^{5/2} f_{5/2}(A)\,. \tag{14}$$

Results and Discussion

- Imagining the quark-hadron phase transition $T - \mu$ diagram, two extreme domains - (1) the early universe scenario of net quark/baryon-free domain (i.e., at $\mu = 0$), which can be produced in LHC and RHIC experiments, and (2) the compact star scenario of degenerate electron or neutron or quark matter (i.e., at $T = 0$), expected in white dwarfs and neutron stars.
- Although, we have limitations for using non-relativistic matter, which can provide some overestimation with respect to the actual relativistic matter expected in RHIC/LHC experiments and compact stars.
- Our future goal is to reach that actual scenario by developing the framework in step by step.

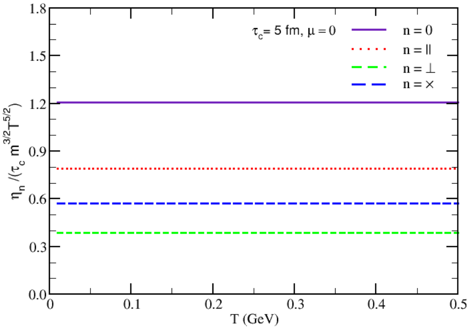

Figure: Normalized shear viscosity components against temperature

-

$$\tau_\perp = \frac{\tau_c}{1 + 4(\tau_c/\tau_\Omega)^2} \quad (15)$$

$$\tau_\parallel = \frac{\tau_c}{1 + (\tau_c/\tau_\Omega)^2} \quad (16)$$

$$\tau_\times = \frac{\tau_c(\tau_c/\tau_\Omega)}{1 + (\tau_c/\tau_\Omega)^2} \quad (17)$$

as $\eta_{\parallel,\perp,\times} \propto \tau_{\parallel,\perp,\times}$, while $\eta_0 \propto \tau_c$ only.

- The non-zero ratio τ_c/τ_Ω is the deciding factor for the ranking among $\eta_\parallel$, $\eta_\perp$, $\eta_\times$.

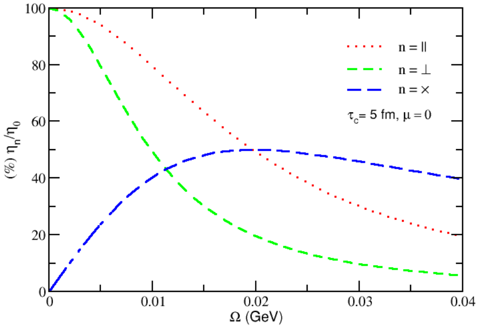

Figure: Relative percentage of shear viscosity components vs angular velocity.

- The relative $\eta_{\perp,||}$ decreases with Ω in the whole range, whereas $\eta_\times$ initially increases and then decreases with Ω.
- $\eta_{\perp,||}$ are more dominant than $\eta_\times$ in lower Ω, in higher Ω, $\eta_\times$ is more dominant.
- Both $\eta_{\perp,||}$ will merge to η_0 i.e, $\eta_{\perp,||}(\Omega \to 0) = \eta_0$.
- We can conclude that the finite (global) vorticity can create anisotropy in shear viscosity components, like in magnetic field picture.

Summary

- We have explored the equivalent role of magnetic field and rotation on shear viscosity via Lorentz force and Coriolis force.
- The shear viscosity expressions for quark matter and compact star scenario in non-relativistic case
- τ_c/τ_Ω plays the important role in deciding the values of η_n components.
- Finite global vorticity can cause anisotropic shear viscosity components.
- In the absence of rotation, we will have an isotropic shear viscosity.

Thank you!

References

- H. Goldstein, Classical mechanics(Pearson Education India, 2011)
- X.-G. Huang, A. Sedrakian, and D. H. Rischke, Annals of Physics 326, 3075 (2011)

Backup Slides

- For the case of constant angular velocity, the Euler force vanishes, but the other two forces, i.e., Coriolis and Centrifugal, remain non-zero.
- When the particle is stationary in the moving system, the centrifugal force is the only added term in the effective force.
- However, when the particle moves, the middle term known as the Coriolis effect* comes into play.
- Only the effect of Coriolis force will be considered to keep our expressions simple to understand.
- In a realistic system, both forces should be considered, but one may ignore the centrifugal force for the particular domain where particle (average) velocity is quite larger than the fluid element's angular velocity (more explicitly $v >> \Omega r/2$).

Backup Slides

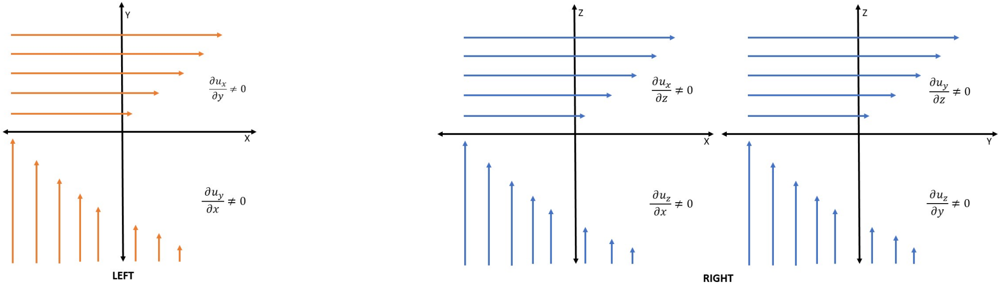

Figure: Velocity gradients along the axis of XY, ZX and ZY plane