Collision corrections to fermion spin polarization in a hot electron plasma

Shuo Fang (方 硕)

University of Science and Technology of China (USTC)

Based on: 2204.11519; 2408.09877

The 12th Circum-Pan-Pacific Symposium on High Energy Spin Physics

November 8-12, 2024 at Hefei, China

Outline

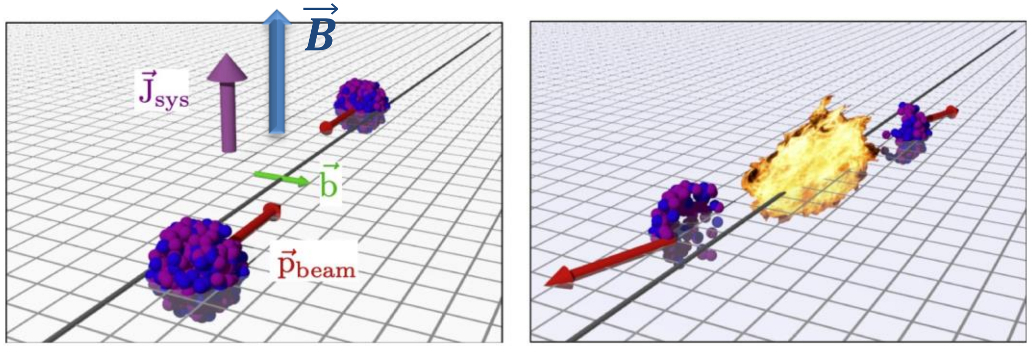
- Spin phenomena in heavy ion collisions
- Quantum kinetic theory and spin transport
- Collision corrections by solving (spin) Boltzmann equations
- Summary

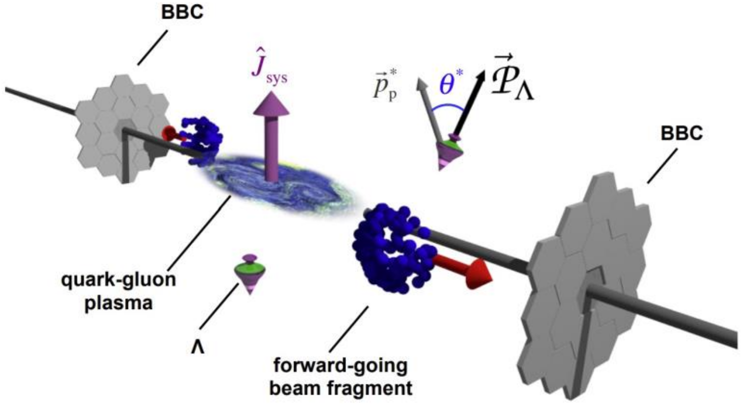
Outline

- **Spin phenomena in heavy ion collisions**
- Quantum kinetic theory and spin transport
- Collision corrections by solving (spin) Boltzmann equations
- Summary

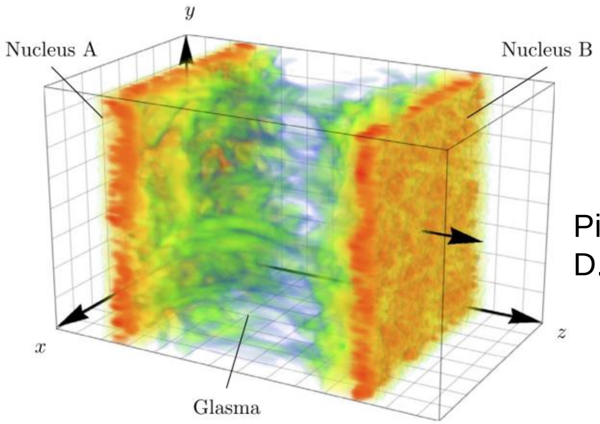
Spin phenomena in heavy ion collisions

Strong vorticity via *spin orbital coupling*

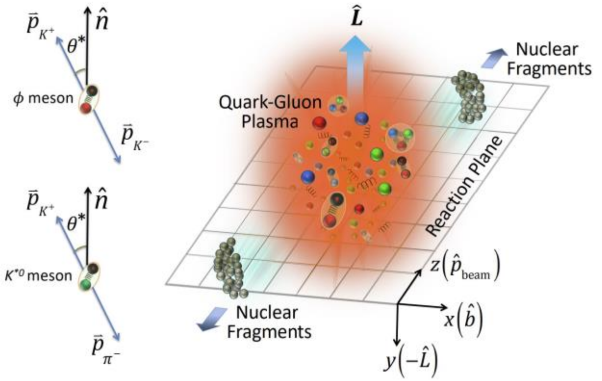




Strong color field and their *correlations*

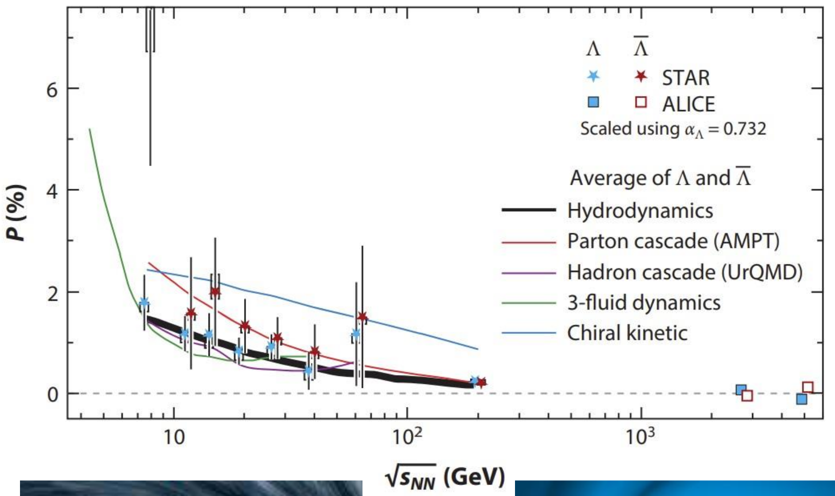


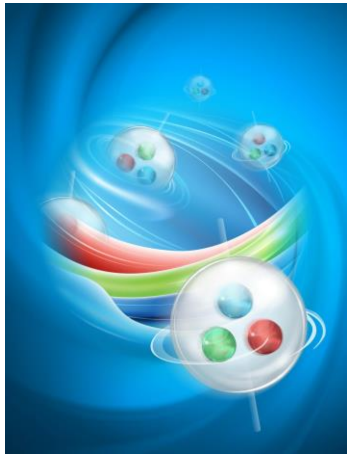
Picture from A. Ipp and D. Muller, PLB(2017)

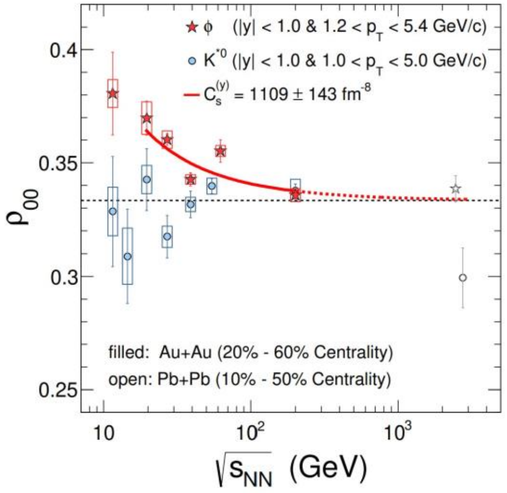


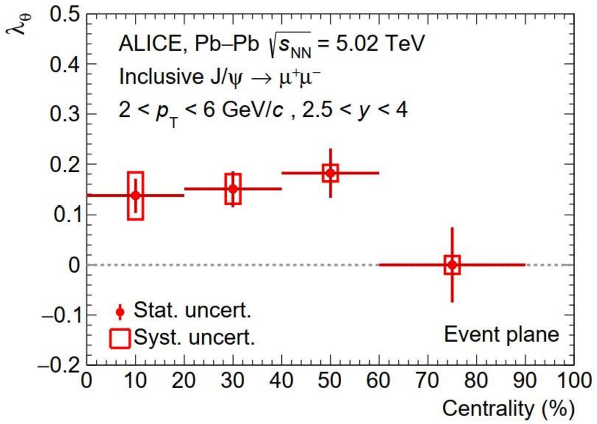
Z. T. Liang, X. N. Wang, PRL(2005); PLB(2005)
[STAR Collaboration], Nature 548, 62(2017); Nature 614,244(2023)
F. Becattini, M. A. Lisa, ARNPS(2020); F. Becattini, et al. IJMPE(2024)

Spin phenomena (a): Global polarization and alignment









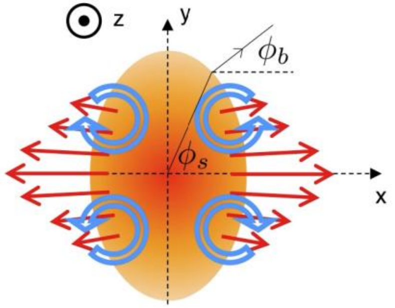
Spin polarization: Measuring the **spin-orbital coupling**;

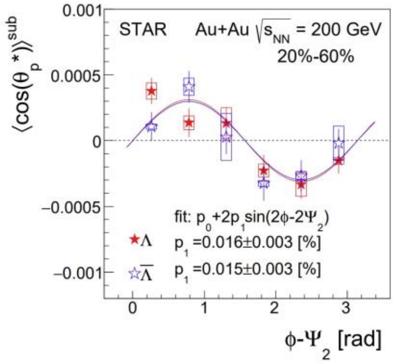
Spin alignment: Measuring the **correlations** of strong force /glasma field **fluctuations**.

[STAR Collaboration], Nature (2023); [ALICE Collaboration], PRL (2023);
Sheng, Wang, Wang, PRD(2020)
Sheng, Oliva, Liang, Wang, Wang, PRL (2023); PRD(2024);
Kumar, Muller, Yang, 2212.13354; 2304,04181;
Muller, Yang, PRD(2022);

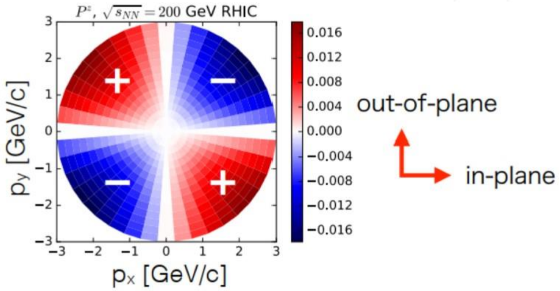
[STAR Collaboration], Nature 548, 62(2017);
F. Becattini, M. A. Lisa, ARNPS (2020)

Spin phenomena (b): Local polarization





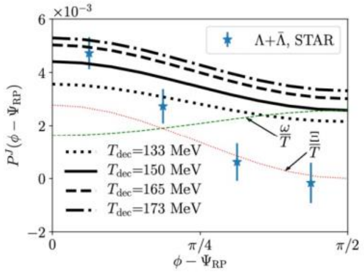
"sign puzzle"

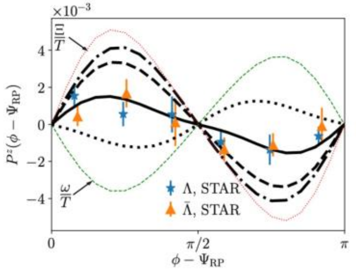


Experimental data: *[STAR Collaboration], PRL(2019)*

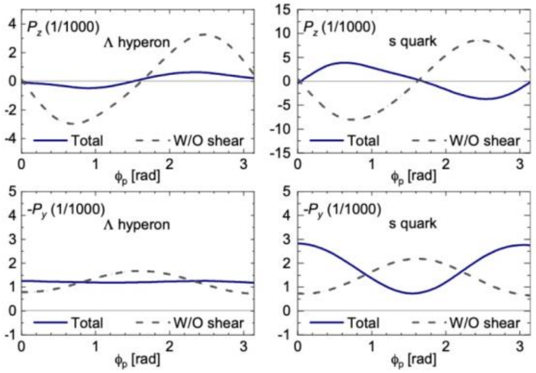
Phenomenological model:
Becattini , Karpenko, PRL(2018); Xia, Li, Tang, Wang, PRC(2018)

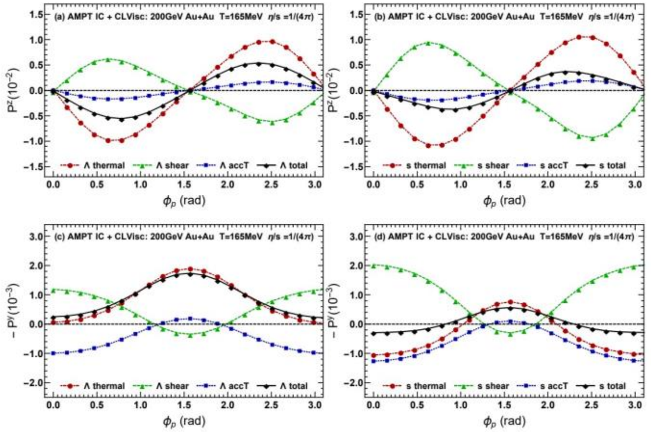
- A **shear-induced** polarization? Parameter dependence?





Local polarization:
Spin **spectrum.**



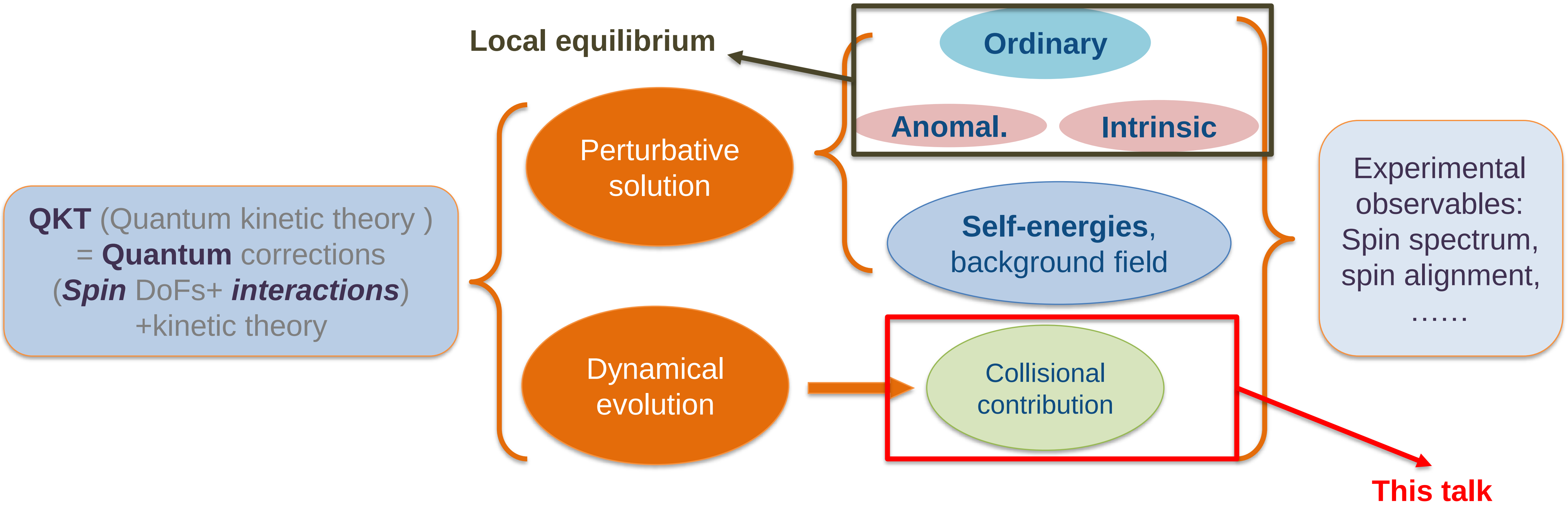


✓ **"Spin Hall effect"** in HIC

Fu, Liu, Pang et al, PRL. 127 (2021); Becattini ,et al, PRL. 127 (2021)
Yi , Pu, and Yang, PRC. 104 (2021)

Resolving “Sign puzzle” from quantum kinetic theory

Sign puzzle in local polarization urges us to go beyond the local equilibrium regime.



Hidaka, Pu, Wang, Yang, PPNP(2022)

Outline

- Spin phenomena in heavy ion collisions
- **Quantum kinetic theory and spin transport**
- Collision corrections by solving (spin) Boltzmann equations
- Summary

QKT and spin Boltzmann equation (a)

- We define the gauge invariant 2-point Wigner function for fermions,

$$S^{<}_{\alpha\beta}(x,p) = -\int d^4y e^{-ip\cdot y}\langle :\bar{\psi}_\beta(x+\frac{y}{2})e^{\frac{y}{2}\cdot\overleftarrow{D}(x)}\otimes e^{-\frac{y}{2}\cdot\overrightarrow{D}(x)}\psi_\alpha(x-\frac{y}{2}) :\rangle$$

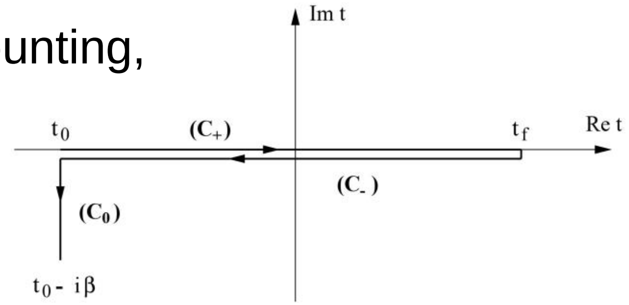
- For massless fermions, in the Clifford components

$$S = \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu,$$

- Based on the **quantum nature** of **spin polarization**, we adopt the power counting,

$$\mathcal{V}^\mu \sim \mathcal{O}(\hbar^0), \mathcal{A}^\mu \sim \mathcal{O}(\hbar^1),$$

Phase-space **particle current** Phase-space **spin current**



- Along the Schwinger-Keldysh contour, one derives the Kadanoff-Baym equations up to O(ℏ),

$$p_\mu\gamma^\mu S^< + \frac{i\hbar}{2}\gamma^\mu\partial_{X,\mu}S^< = \frac{i\hbar}{2}(\Sigma^< S^> - \Sigma^> S^<),$$

$$p_\mu S^<\gamma^\mu - \frac{i\hbar}{2}\partial_{X,\mu}S^<\gamma^\mu = -\frac{i\hbar}{2}(S^>\Sigma^< - S^<\Sigma^>),$$

Σ: Self-energies

Blaizot, Iancu, Phys. Rept.(2002);
Hidaka, Pu, Yang, PRD(2017);

QKT and spin Boltzmann equation (b)

- Under quasi-particle approximations, we can perturbatively solve the Wigner functions.

- The perturbative solutions are

Distribution functions of vector and axial charge

$$S^{\mu\nu}_{(n)} = \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma}{2n \cdot p},$$

$$\mathcal{V}^{\lessgtr,\mu}(X,p) = 2\pi p^\mu \delta(p^2) f^{\lessgtr}_{\rm V}(X,p),$$

$$\mathcal{A}^{\lessgtr,\mu}(X,p) = 2\pi p^\mu \delta(p^2) f^{\lessgtr}_{\rm A}(X,p) + 2\pi\hbar\delta(p^2) S^{\mu\alpha}_{(n)} \left(\partial^X_\alpha f^{\lessgtr}_{\rm V}(X,p) - C_{{\rm V},\alpha}[f^{\lessgtr}_{\rm V}]\right)$$

- The kinetic equations are as follows:

Boltzmann eq. : $\partial_{X,\mu}\mathcal{V}^{<,\mu} = \Sigma^{<}_{\rm V}\cdot\mathcal{V}^{>} - \Sigma^{>}_{\rm V}\cdot\mathcal{V}^{<} \equiv 2\pi\delta(p^2)\mathcal{C}_{\rm V},$

Spin Boltzmann eq. : $\partial_{X,\mu}\mathcal{A}^{<,\mu} = \Sigma^{<}_{\rm V}\cdot\mathcal{A}^{>} - \Sigma^{>}_{\rm V}\cdot\mathcal{A}^{<} - \Sigma^{<}_{\rm A}\cdot\mathcal{V}^{>} + \Sigma^{>}_{\rm A}\cdot\mathcal{V}^{<} \equiv 2\pi\delta(p^2)\mathcal{C}_{\rm A}.$

- It has been proven the following distributions make vector and axial collision kernels zero:

$$f^{<}_{\rm V,leq}(x,p) = \frac{1}{e^{\beta u\cdot p - \alpha}+1},$$

$$f^{<}_{\rm A,leq}(x,p) = -\frac{\hbar}{2} f^{<}_{\rm V,leq}(x,p) f^{>}_{\rm V,leq}(x,p)\omega^s_{\mu\nu} S^{\mu\nu}_{(u)}(x,p),$$

$$\omega^s_{\mu\nu} = \Omega_{\mu\nu},$$

Spin chemical potential
= Thermal vorticity

Y. Hidaka, S. Pu, D.-L. Yang, PRD(2018);
D.-L. Yang, K. Hattori, Y. Hidaka, JHEP(2020);
SF, S. Pu, D.-L. Yang, PRD(2022)

Off-eq. corrections to spin polarization

- Spin polarization spectrum is directly related to axial vector WF $\mathcal{A}^{<,\mu}$,

Becattini, et al. Annals Phys.(2013)
Fang, Pang, Wang, Wang, PRC(2016)

$$\mathcal{P}^{\mu}(t;\mathbf{p})=\hbar\frac{\int_{\Sigma}\mathrm{d}\Sigma\cdot p\int\frac{\mathrm{d}p_0}{2\pi}\boxed{\mathcal{A}^{<,\mu}(x,p)}}{2m\int_{\Sigma}\int\frac{\mathrm{d}p_0}{2\pi}\mathrm{d}\Sigma\cdot\mathcal{V}^{<}(x,p)}.\qquad \mathcal{A}^{<,\mu}=\mathcal{A}^{<,\mu}_{\mathrm{leq}}+\boxed{\mathcal{A}^{<,\mu}_{\mathrm{collision}}}+\ldots$$

Our focus

- We expect to investigate the near-equilibrium spin polarization depending on f_V:
  - **Scenario (I)**: Vector charge (near) out of equilibrium:

$$\delta\mathcal{A}^{<,\mu}_{(\mathrm{I})}(X,p)=2\pi p^{\mu}\delta(p^2)\delta f^{<}_{\mathrm{A}}(X,p)-2\pi\hbar\delta(p^2)S^{\mu\alpha}_{(u)}C_{\mathrm{V},\alpha}[\delta f^{<}_{\mathrm{V}}]+\mathcal{O}(\partial^2).$$

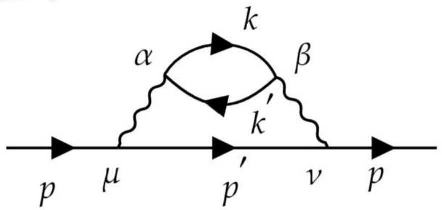
  - **Scenario (II)**: Vector charge in equilibrium:

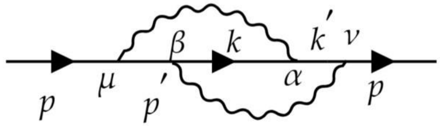
$$\delta\mathcal{A}^{<,\mu}_{(\mathrm{II})}(X,p)=2\pi p^{\mu}\delta(p^2)\delta f^{<}_{\mathrm{A}}(X,p).$$

Solving Boltzmann eq.

Solving **Spin** Boltzmann eq.

- It is convenient to consider the elastic scattering process:





SF, S. Pu, D.-L. Yang, PRD(2022)
SF, S. Pu, 2408.09877

Outline

- Spin phenomena in heavy ion collisions
- Quantum kinetic theory and spin transport
- **Collision corrections by solving (spin) Boltzmann equations**
- Summary

Novel matching condition

- Matching condition mapping distribution moments to hydrodynamical quantities, defining thermodynamical variables: *G.S. Denicol, D. Rischke, Lecture Notes in Phys.(2022)*

$$\varepsilon = \varepsilon_0(\alpha_0, \beta_0) = \langle E_{\mathbf{p}}^2 \rangle_0, \quad n = n_0(\alpha_0, \beta_0) = \langle E_{\mathbf{p}} \rangle_0, \quad \Longrightarrow \quad \langle E_{\mathbf{p}} \rangle_\delta = \langle E_{\mathbf{p}}^2 \rangle_\delta = 0.$$

- In QKT, since our local equilibrium eliminates spin chemical potential, we need to add another **novel matching condition** for f_A:

$$n_{\mathrm{A}}(x) = n_{\mathrm{A},0}(\alpha_0, \beta_0), \quad \Longrightarrow \quad \int_p 2\pi\delta(p^2)(f_{\mathrm{A}}^< - f_{\mathrm{A,leq}}^<)E_{\mathbf{p}} = 0.$$

which can also be derived from properties of SBE collision kernel:

$$\mathcal{J}_5^\mu(x) = \int_p 2\pi\delta(p^2)\mathcal{A}^{<,\mu}(x,p).$$

$$\partial_\mu \int_p 2\pi\delta(p^2) G_p \mathcal{A}^\mu(x,p) = \int_p 2\pi\delta(p^2) G_p \mathcal{C}_{\mathrm{A}}.$$

Only possible choice: $C_A = 1$ $\Leftrightarrow$ $\partial_\mu \mathcal{J}_5^\mu(x) = 0,$

- Within the validity domain of **gradient expansion**, it simplifies the collision correction in Scenario (I):

$$\mathcal{O}(\partial) \ll \mathcal{O}(Te^4) \ll \mathcal{O}(T)$$

i.e. $Kn = \frac{\lambda}{L} \sim (Te^4)^{-1}\partial \ll 1$

$$\delta\mathcal{A}_{(\mathrm{I})}^{<,\mu}(X,p) = -2\pi\hbar\delta(p^2)S^{(u),\mu\alpha}C_{\mathrm{V},\alpha}[\delta f_{\mathrm{V}}^<] + \mathcal{O}(\partial^2).$$

*which **only** depends on the **off-eq. correction** of **vector distribution***

It also demands at least $\delta f_A \sim O(\partial^2)$.

SF, S. Pu, 2408.09877

Solving SBE for Scenario (II) (a)

- Using systematical gradient expansion,

G.S. Denicol, D. Rischke, Lecture Notes in Phys.(2022)

Axial dist.

$$f_{\rm A} = f_{\rm A,leq} + \delta f_{\rm A} = {\rm Kn} f_{\rm A}^{(1)} + {\rm Kn}^2 f_{\rm A}^{(2)} + \mathcal{O}({\rm Kn}^3),$$

Axial collision kernel

$$\begin{aligned}\mathcal{C}_{\rm A}[\delta f_{\rm A}, f_{\rm V,leq}] &= {\rm Kn}\,\mathcal{C}_{\rm A}^{(1)}[\delta f_{\rm A}, f_{\rm V,leq}] + {\rm Kn}^2\mathcal{C}_{\rm A}^{(2)}[\delta f_{\rm A}, f_{\rm V,leq}] + \mathcal{O}({\rm Kn}^3)\\ &= \int_{p',k,k'} W_{pk\to p'k'}(2\pi)^4\delta(k'^2)\delta(k^2)\delta(p'^2)\delta(p^2) f^{<}_{\rm V,leq}(k') f^{<}_{\rm V,leq}(p') f^{>}_{\rm V,leq}(k) f^{>}_{\rm V,leq}(p)\\ &\times\left[\frac{\delta f^{<}_{\rm A}(p')}{f^{<}_{\rm V,leq}(p') f^{>}_{\rm V,leq}(p')} - \frac{\delta f^{<}_{\rm A}(p)}{f^{<}_{\rm V,leq}(p) f^{>}_{\rm V,leq}(p)}\right].\end{aligned}$$

Very special for spin collision kernel!

Spin Boltzmann eq.

$$\mathcal{C}_{\rm A}^{(1)}[\delta f_{\rm A}, f_{\rm V,leq}] = 0,$$

$$\hbar\frac{\hat{\partial}_\mu S^{\mu\alpha}_{(u)}(p)}{E_{\mathbf p}} u_\alpha\left(\hat{D} f^{<}_{\rm V}(p)\right)^{(1)} + \hbar\frac{\hat{\partial}_\mu S^{\mu\alpha}_{(u)}(p)}{E_{\mathbf p}}\hat{\nabla}_\alpha f_{\rm V}^{<,(0)}(p)$$
$$+L\left(\hat{D} f^{<}_{\rm A}(p)\right)^{(2)} + L\frac{p_\perp^\mu}{E_{\mathbf p}}\hat{\nabla}_\mu f_{\rm A}^{<,(1)} = \frac{\lambda^2}{E_{\mathbf p}}\mathcal{C}_{\rm A}^{(2)}[\delta f_{\rm A}, f_{\rm V,leq}],$$

Gradient expansion:
Differential-integral eq. (spin BE)
→ Integral eq.
→ Linear eq. array ✓ **Solvable!**

Coefficient functions to be determined from axial collision kernel

From the l.h.s. of spin Boltzmann eq., it is sufficient to assume,

$$f_{\rm A}^{<,(2)}(p) = -\frac{\hbar}{2} f^{<}_{\rm V,leq}(p) f^{>}_{\rm V,leq}(p)\left\{\varphi^{\rm s,1}_{\rm A,\mathbf p}\frac{1}{\beta_0}\hat{\omega}^\alpha\hat{\nabla}_\alpha\beta_0 + \ldots + \varphi^{\rm v,1}_{\rm A,\mathbf p}\beta_0\hat{\omega}^\mu\hat{\sigma}^\alpha_\mu p_{\langle\alpha\rangle} + \ldots\right\}$$

So that the SBE reduces to,

$$\begin{aligned}{\rm l.h.s.} &= -\frac{\hbar}{2} f^{<}_{\rm V,leq}(p) f^{>}_{\rm V,leq}(p)\left[A_{\mathbf p} + B^\alpha_{\mathbf p} p_{\langle\alpha\rangle} + C^{\alpha\rho}_{\mathbf p} p_{\langle\alpha}p_{\rho\rangle} + D^{\mu\alpha\lambda}_{\mathbf p} p_{\langle\mu}p_\alpha p_{\lambda\rangle}\right],\\ {\rm r.h.s.} &= \int_{p',k,k'} W_{pk\to p'k'}(2\pi)^4\delta(k'^2)\delta(k^2)\delta(p'^2)\delta(p^2) f^{<}_{\rm V,leq}(k') f^{<}_{\rm V,leq}(p') f^{>}_{\rm V,leq}(k) f^{>}_{\rm V,leq}(p)\\ &\times\left[\left(\varphi^{\rm s,1}_{\rm A,\mathbf p} - \varphi^{\rm s,1}_{\rm A,\mathbf p'}\right)\frac{1}{\beta_0}\hat{\omega}^\alpha\hat{\nabla}_\alpha\beta_0 + \left(\varphi^{\rm s,2}_{\rm A,\mathbf p} - \varphi^{\rm s,2}_{\rm A,\mathbf p'}\right)\hat{\nabla}^\alpha\hat{\omega}_\alpha + \ldots\right]\end{aligned}$$

$$\varphi^i_{\rm A,\mathbf p} = \sum_{n=0}^{N^i_{\rm A}} \varepsilon^i_{{\rm A},n}\xi^{(n)}_{\mathbf p}(E_{\mathbf p}).$$

Solve it by choosing basis function.

SF, S. Pu, 2408.09877

Solving SBE for Scenario (II) (b)

- The Spin BE now becomes the following **linear eq. arrays**:

$$\sum_{n=0}^{N_A^{s,i}} \mathcal{A}^{s}_{A,rn}\varepsilon^{s,i}_{A,n} = \alpha^{s,i}_{A,r}\ , \qquad \sum_{n=0}^{N_A^{v,j}} \mathcal{A}^{v}_{A,rn}\varepsilon^{v,j}_{A,n} = \alpha^{v,j}_{A,r},$$

$$\sum_{n=0}^{N_A^{ts,i}} \mathcal{A}^{ts}_{A,rn}\varepsilon^{ts,k}_{A,n} = \alpha^{ts,k}_{A,r}\ , \qquad \sum_{n=0}^{N_A^{tt}} \mathcal{A}^{tt}_{A,rn}\varepsilon^{tt}_{A,n} = \alpha^{tt}_{A,r},$$

Collision integral e.g.

$$\mathcal{A}^{s}_{A,rn} = \lambda^2 \int_{p,p',k,k'} W_{pk\to p'k'}(2\pi)^4\delta(p^2)\delta(k'^2)\delta(k^2)\delta(p'^2) \times f^{<}_{V,leq}(k')f^{<}_{V,leq}(p')f^{>}_{V,leq}(k)f^{>}_{V,leq}(p)E_{\mathbf{p}}^{r-1}(E^n_{\mathbf{p}} - E^n_{\mathbf{p}'}),$$

Thermodynamical integral e.g.

$$\alpha^{s,1}_{A,r} = -\alpha^{s,2}_{A,r} = -\frac{1}{3}\beta\int_p 2\pi\delta(p^2)E^r_{\mathbf{p}} f^{<}_{V,leq}(p) f^{>}_{V,leq}(p),$$

Coefficient to be solved

- Within a simplest choice: (i) Polynomial basis (to simplify the collision integral); $\xi^{(r)}(E_{\mathbf{p}}) = E^r_{\mathbf{p}}$
  (ii) Minimal truncation (to simplify solving of the linear eq. array).

$$N_A^{s,i} = 1\ ,\ N_A^{v,i} = N_A^{ts,i} = N_A^{tt} = 0.$$

- Within the **Hard Thermal Loop approximation** we derive the collision integral. We finally get,

$$\delta\mathcal{A}^{<,\mu}_{(II)}(X,p) = -\hbar\pi p^\mu \delta(p^2) f^{<}_{V,leq}(p) f^{>}_{V,leq}(p)\frac{\beta^2}{e^4 \ln e^{-1}}\mathcal{F},$$

$\propto \tau_{spin} \sim \frac{T^{-1}}{e^4 \ln e^{-1}}$, proportional to **spin relaxation time**

with

$$\mathcal{F} = \left(E_{\mathbf{p}} - d_1\frac{1}{\beta_0}\right) d_2\omega^\alpha\nabla_\alpha\beta_0 + \ldots + d_4\beta_0\omega^\mu\sigma^\alpha_\mu p_{\langle\alpha\rangle} + \ldots + d_7\beta_0^2\epsilon^{\mu\nu\sigma\langle\rho}\sigma^{\alpha\rangle}_\mu u_\sigma\nabla_\nu\alpha_0 p_{\langle\alpha}p_{\rho\rangle} + \ldots + d_9\beta_0^3\omega^{\langle\mu}\sigma^{\alpha\lambda\rangle}p_{\langle\mu}p_\alpha p_{\lambda\rangle}$$

SF, S. Pu, 2408.09877

Coupling independence in Scenario (I)

- However, when utilizing **Chapman-Enskog expansion** in scenario (I):

$$\delta\mathcal{A}^{<,\mu}_{(\mathrm{I})}(X,p) = 2\pi\hbar\delta(p^2)\beta_0\left[-g_1(E_{\mathbf{p}})\frac{\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma}{2E_{\mathbf{p}}}\nabla_\nu\alpha_0 - g_2(E_{\mathbf{p}})\frac{\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma}{2E_{\mathbf{p}}}\sigma_{\nu\alpha}p^\alpha\right]$$

The corrections are **Coupling-Independent** manifestly!

- Recall that in CKT:

$$S^{<,\mu}_{\chi}(p,X) = 2\pi\theta(u\cdot p)\left[\delta(p^2)p^\mu f_\chi + \delta(p^2)\hbar\chi\frac{\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha}{2u\cdot p}(\Delta_\beta f_\chi - \mathcal{C}_\beta[f_\chi])\right]$$

The collision part is,

$$S^{<,\mu}_{\chi,\mathcal{C}}(p,X) = -2\pi\theta(u\cdot p)\delta(p^2)\hbar\chi\frac{\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha}{2u\cdot p}\mathcal{C}_\beta[f_\chi]$$

$$= \int_{\mathrm{BCD}} C_{\mathrm{ABCD}}\overline{\Delta}^\mu$$

$$\sim \int_{p',q,q'} \delta\boldsymbol{r}\left(|\mathcal{M}|^2\delta f_p\right) f_{p'}(1-f_q)(1-f_{q'})$$

$$|\mathcal{M}|^2\delta f_p \sim \tau\tau^{-1}p\cdot\partial f_0 = p\cdot\partial f_0 \propto \tau^0.$$

Interaction cancellation

$$\delta\boldsymbol{r} \approx \frac{\boldsymbol{p}}{2|\boldsymbol{p}|^3}\times\Delta\boldsymbol{p} = \boldsymbol{\Omega}\times\Delta\boldsymbol{p}$$

Physical origin: Coordinate ***side-jump*** due to ***Berry curvature***

Remark on $e\to 0$ limit: Boltzmann equation is **free-streaming** when $e\to 0$, then the **gradient expansion breaks down**!

$$\mathcal{O}(\partial) \ll \mathcal{O}(Te^4)$$ ✘

SF, S. Pu, 2408.09877
J.-Y. Chen, D.T. Son, M.A. Stephanov, PRL(2015)
Y. Hidaka, S. Pu, D.-L. Yang, PRD(2017)

Physical origin of spin polarization (I)

Spin polarization in medium is a ***spin Hall effect*** for spin transport, we probe the spin current as response to **electric fields**. It is illustrating to compare it with condensed matter community.

- Here we list the different contributions of spin polarization current, which is defined as

$$S^{\mu\rho\sigma}(x) = 4\epsilon^{\mu\nu\rho\sigma}\int_p \mathcal{A}_\nu^<(p,x) = 4\epsilon^{\mu\nu\rho\sigma}\int_p \frac{j_+^\leq(p,x) - j_-^\leq(p,x)}{2}$$

In **heavy ion physics,** the effective electric field comes from the anisotropy of the medium, and is defined in the phase space :

Effective electric field from medium anistropy

$$[p^\alpha \xi_{\mu\alpha} + p^\alpha \omega_{\mu\alpha} - \partial_\mu(\beta\mu_\chi)]\frac{\partial f_\chi^0}{\partial\varepsilon} \equiv E_\mu^{\text{eff}}\frac{1}{\varepsilon}\frac{\partial f_\chi^0}{\partial\varepsilon}$$

Such effective electric field is corrected by **self-energies**

$$E_{\text{eff}}^\mu = \varepsilon p_\alpha \xi^{\mu\alpha} \to \varepsilon p_\alpha \xi^{\mu\alpha}\left[1 + \frac{g^2T^2}{E^2}G_{\omega_1}(E_{\boldsymbol{q}}, \boldsymbol{q}) + ...\right]$$

In **condensed matter physics,** the electric field is an external classical source input in the table-based experiment:

$$J_\mu(x) = -\sigma_{\mu\nu}E^\nu(x),$$

N. Nagaosa, et al, RMP.82.1539

We probe the **phase-space spin current** in general in heavy ion physics.

SF, S. Pu, 2408.09877
SF, S. Pu, D.-L. Yang, PRD(2024)

Physical origin of spin polarization (II)

We consider the **coordinate space spin current** for illustration:

In **HIC physics** $\boldsymbol{J}_\chi = \boldsymbol{J}_\chi^{\mathrm{Hall}} + \boldsymbol{J}_\chi^{\mathrm{int}} + \boldsymbol{J}_\chi^{\mathrm{adist}} + \boldsymbol{J}_\chi^{\mathrm{sj-1}} + \boldsymbol{J}_\chi^{\mathrm{skew}},$

Hall current $\boldsymbol{J}_\chi^{\mathrm{Hall}} = \int_p 2\pi\theta(u\cdot p)\delta(p^2)|\boldsymbol{p}|\,[\boldsymbol{v}f_\chi^0]\,,$

Intrinsic from Berry curvature $\boldsymbol{J}_\chi^{\mathrm{int}} = \int_p 2\pi\theta(u\cdot p)\delta(p^2)|\boldsymbol{p}|\,[-\hbar\chi\varepsilon_{\boldsymbol{p}}\boldsymbol{\Omega}\times\boldsymbol{\nabla}f_\chi^0]\,,$

Fermion Berry curvature

Anomalous from Novel detailed balance $\boldsymbol{J}_\chi^{\mathrm{adist}} = \int_p 2\pi\theta(u\cdot p)\delta(p^2)|\boldsymbol{p}|\left[-\boldsymbol{v}\frac{\partial f_\chi^0}{\partial\varepsilon}\frac{\chi}{2}\hat{\boldsymbol{p}}\cdot\boldsymbol{\omega}\right],$

Side-jump from collisions $\boldsymbol{J}_\chi^{\mathrm{sj-1}} = \int_p 2\pi\theta(u\cdot p)\delta(p^2)|\boldsymbol{p}|\,[-\hbar\chi\varepsilon_{\boldsymbol{p}}\boldsymbol{\Omega}\times\boldsymbol{\mathcal{C}}[\delta f_\chi]]$

Skew-scattering From collisions $\boldsymbol{J}_\chi^{\mathrm{skew}} = \int_p 2\pi\theta(u\cdot p)\delta(p^2)|\boldsymbol{p}|\,[\boldsymbol{v}\delta f_\chi]\,.$

Anomalous Hall current

One to one correspondence

In **condensed matter physics**

$$\delta\boldsymbol{r} = \boldsymbol{r}-\boldsymbol{r}' = \frac{\hbar^2}{4m^2c^2}\boldsymbol{\sigma}\times\boldsymbol{k}.$$

The coordinate side-jump

$$\boldsymbol{J}^{\mathrm{int}} = -e^2\boldsymbol{E}\times\int[\mathrm{d}\boldsymbol{k}]f_{\boldsymbol{k}}\boldsymbol{\Omega},$$

$$\sigma_{xy}^{\mathrm{sj-1}} = e\sum_l\frac{g_l}{E_y}\sum_{l'}\omega_{l'l}(\delta\boldsymbol{r}_{l'l})_x,$$

$$\sigma_{xy}^{\mathrm{adist}} = e\sum_l\frac{g_l^{\mathrm{adist}}}{E_y}(v_{0l})_x,$$

$$\sigma_{xy}^{\mathrm{sk}} = e\sum_l\frac{g_l}{E_y}(v_{0l})_x.$$

g_l : the off-equilibrium corrections to distribution from solving Boltzmann equation

g_l^{adist} : modified equilibrium distribution functions in the presence of electirc field

$\omega_{ll'}$: scattering amplitude in the Born level

v_{0l} : velocity operator

J.-Y. Chen, D.T. Son, M.A. Stephanov, PRL(2015)
Y. Hidaka, S. Pu, D.-L. Yang, PRD(2017)
SF, S. Pu, 2408.09877

N. Nagaosa, et al, RMP.82.1539
N. A. Sinitsyn, et al, PRB.75.045315

Outline

- Spin phenomena in heavy ion collisions
- Quantum kinetic theory and spin transport
- Collision corrections by solving (spin) Boltzmann equations
- **Summary**

Summary and outlook

Summary

We have investigated the off-equilibrium corrections for fermion spin polarization in a hot electron plasma. Such collision sources mainly contain two parts in leading gradient:

1. Side-jump part: **independent of interactions**
2. Extrinsic part: proportional to **spin relaxation time**

Outlook

1. Reproduce the side-jump contribution from *linear response theory*;
2. Generalize to a real QCD plasma.

Welcome to Hefei and thanks for your attention！