

Hyperon decays of spin-entangled baryon-antibaryon pairs

Dr. Varvara Batozskaya

*Institute of High Energy Physics, Beijing, China
National Centre for Nuclear Research, Warsaw, Poland*

Seminar
University of Science and Technology of China
Hefei, China
17 November 2023



Narodowe Centrum Badań Jądrowych
National Centre for Nuclear Research
ŚWIERK



中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

Introduction

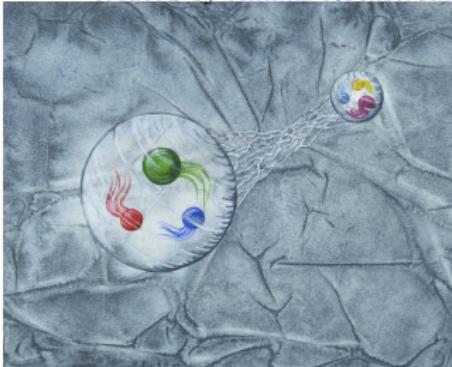
- More than 50 years of the knowledge about CP violation (CPV)
 - Confirmed only in meson decays

- SM CPV is not sufficient to explain observed matter-antimatter asymmetry
- Baryogenesis requires C and CP violation in the processes

[[PismaZh.Eksp.Teor.Fiz.5\(1967\)32](#)]

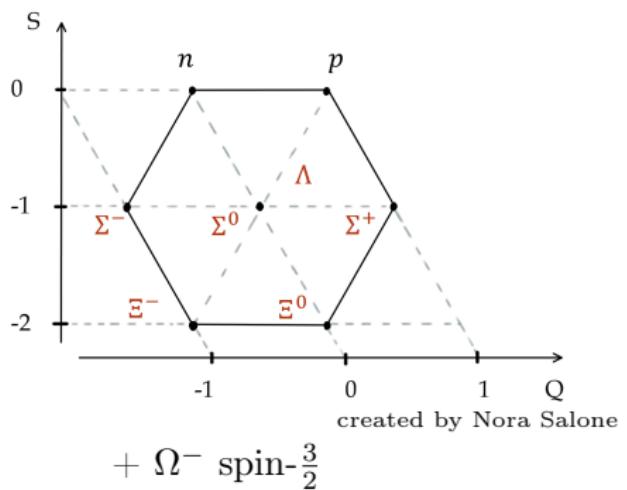
- Systematical mapping with different hadronic systems and complementary methods are needed for understanding CPV in flavour sector

created by Annika Rockström



Ground-state strange baryons

- Spin- $\frac{1}{2}$ baryon octet
- Weak $\Delta S = 1$ transitions



Hyperon	Mass [GeV/c ²]	Decay (\mathcal{B}) [%]
$\Lambda(uds)$	1.116	$p\pi^-(64.1)$ $n\pi^0(35.9)$ $pe^-\bar{\nu}_e(0.083)$
$\Sigma^-(dds)$	1.197	$n\pi^-(99.8)$ $ne^-\bar{\nu}_e(0.102)$
$\Sigma^+(uus)$	1.189	$p\pi^0(51.6)$ $n\pi^+(48.3)$ $\Lambda e^+\nu_e(0.002)$
$\Xi^0(uss)$	1.315	$\Lambda\pi^0(99.5)$ $\Sigma^+e^-\bar{\nu}_e(0.025)$
$\Xi^-(dss)$	1.322	$\Lambda\pi^-(99.9)$ $\Lambda e^-\bar{\nu}_e(0.056)$
$\Omega^-(sss)$	1.672	$\Lambda K^-(67.8)$ $\Xi^0\pi^-(23.6)$ $\Xi^-\pi^0(8.6)$ $\Xi^0e^-\bar{\nu}_e(0.56)$



Study of hyperon decays of spin-1/2 baryon

- Presentation is based on recent paper: [PRD108(2023)016011]
- **Motivation (theory):**
 - Development of formalism for SL baryon decays that allow to study the spin correlations and polarization
 - similar way as developed for hadronic hyperon decays [PRD99(2019)056008]
 - have not been done before
 - Test of CP symmetry in SL baryon decays



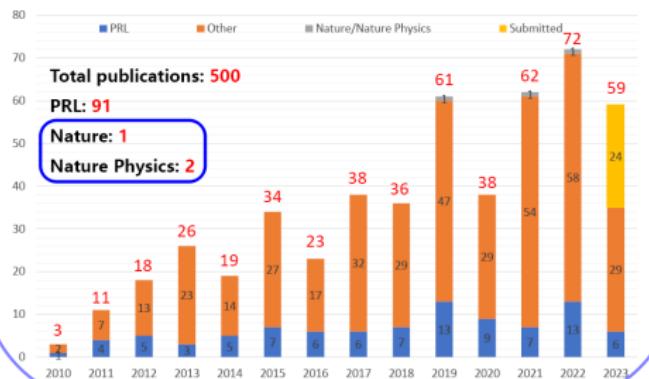
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 - have not been done before
 - Test of CP symmetry in SL baryon decays
- **Motivation (experiment):**
 - Analysis of process $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (B_1 \rightarrow SL)(\bar{B}_1 \rightarrow H) + \text{c.c.}$
 - extraction of decay parameters using provided modular method
 - some of them has been measured > 30 y.a. (hyperon sector)
 - Measurement of V_{ij} matrix elements in SL baryon decays

Process	Final state	Reference
$J/\psi \rightarrow \Lambda \bar{\Lambda}$	$(p\pi^-)(\bar{p}\pi^+)$	[NaturePhys15(2019)631] [PRL129(2022)131801]
$\psi(2S) \rightarrow \Sigma^- \bar{\Sigma}^+$	$(n\pi^-)(\bar{n}\pi^+)$	[JHEP12(2022)016]
$J/\psi \rightarrow \Sigma^+ \bar{\Sigma}^-$	$(p\pi^0)(\bar{p}\pi^0)$ $(n\pi^+)(\bar{n}\pi^-)$ $(p\gamma)(\bar{p}\pi^0)$	[PRL125(2020)052004] [PRL131(2023)191802] [PRL130(2023)211901]
$\psi(2S) \rightarrow \Sigma^+ \bar{\Sigma}^-$	$(p\pi^0)(\bar{p}\pi^0)$	[PRL125(2020)052004]
$\psi(2S) \rightarrow \Xi^0 \bar{\Xi}^0$	$(\Lambda\pi^0)(\bar{\Lambda}\pi^0)$	[PRD108(2023)L011101]
$J/\psi \rightarrow \Xi^- \bar{\Xi}^+$ $\psi(2S) \rightarrow \Xi^- \bar{\Xi}^+$	$(\Lambda\pi^-)(\bar{\Lambda}\pi^+)$	[Nature 606(2022)64] [PRD106(2022)L091101]
$\psi(2S) \rightarrow \Omega^- \bar{\Omega}^+$	$(\Lambda K^-)(\bar{\Lambda}K^+)$	[PRL126(2021)092002]

Non-leptonic decays of spin-1/2 baryon

BESIII publications (May 9, 2023)



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$\psi(2S) \rightarrow \Omega^- \bar{\Omega}^+$	$(\Lambda K^-)(\bar{\Lambda}K^+)$	[PRL126(2021)092002]

Production process

[PRD99(2019)056008]

- Two spin-1/2 particle state:

$$\rho_{1/2, \overline{1}/\overline{2}} = \frac{1}{4} \sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}} \sigma_\mu^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_1}$$

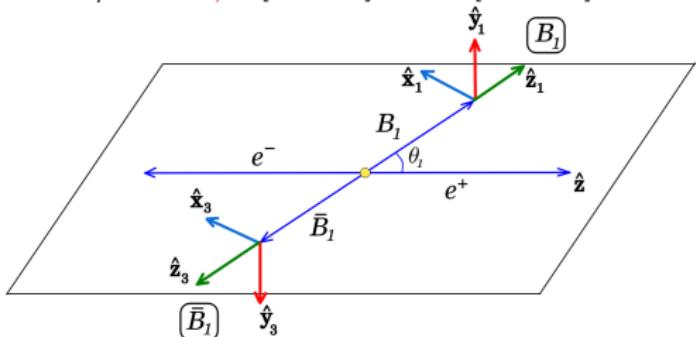
$$C_{\mu\bar{\nu}} \propto \begin{pmatrix} 1 + \alpha_\psi \cos^2 \theta & 0 & \beta_\psi \sin \theta \cos \theta & 0 \\ 0 & \sin^2 \theta & 0 & \gamma_\psi \sin \theta \cos \theta \\ -\beta_\psi^2 \sin \theta \cos \theta & 0 & \alpha_\psi \sin^2 \theta & 0 \\ 0 & -\gamma_\psi \sin \theta \cos \theta & 0 & -\alpha_\psi - \cos^2 \theta \end{pmatrix}$$

B₁ transverse polarization

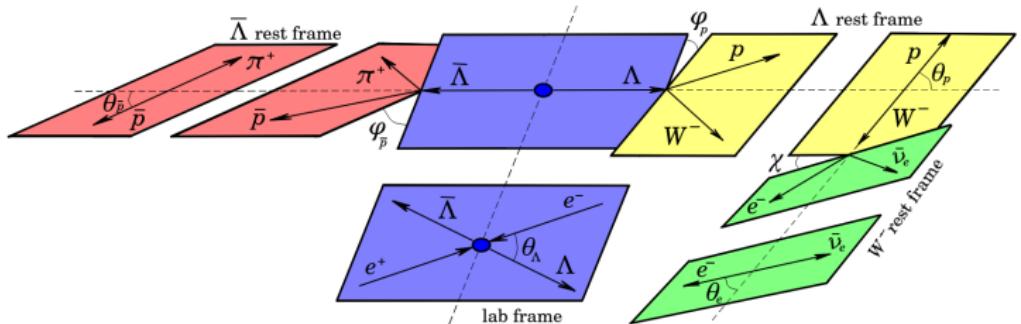
B̄₁ transverse polarization

spin correlations

- $\beta_\psi = \sqrt{1 - \alpha_\psi^2} \sin(\Delta\Phi)$ and $\gamma_\psi = \sqrt{1 - \alpha_\psi^2} \cos(\Delta\Phi)$
- Main parameters of $C_{\mu\bar{\nu}}$: θ ; $\alpha_\psi \in [-1, +1]$, $\Delta\Phi \in [-\pi, +\pi]$



Semileptonic/Hadronic Baryon decays



- $\bar{B}_1 \rightarrow \bar{B}_3 + \pi^+$
 - $a_{\mu\nu}$ for $\{\frac{1}{2} \rightarrow \frac{1}{2} + 0\}$
 - $\sigma_{\mu}^{\bar{B}_1} \rightarrow \sum_{\nu=0}^3 a_{\mu\nu} \sigma_{\nu}^{\bar{B}_3}$
 - Helicity amplitudes:
 $B_{\frac{1}{2}}, B_{-\frac{1}{2}}$
 - Main parameters:
 $\Omega_3 = \{\bar{\varphi}_3, \bar{\theta}_3, 0\}$
 $\bar{\alpha}_D, \bar{\phi}_D$
- $B_1 \rightarrow B_2 + W_{\text{off-shell}}^- (\rightarrow l^- \bar{\nu}_l)$
 $\mathcal{B}_{\mu\nu}$ for $\{\frac{1}{2} \rightarrow \frac{1}{2} + \{0, \pm 1, t\}\}$
- $\sigma_{\mu}^{B_1} \rightarrow \frac{3(q^2 - m_l^2)}{4\pi^3} \sum_{\nu=0}^3 \mathcal{B}_{\mu\nu} \sigma_{\nu}^{B_2}$
- $H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}, H_{\frac{1}{2}t}, H_{-\frac{1}{2}t}$
- $\Omega_2 = \{\varphi_2, \theta_2, 0\}, \Omega_l = \{\varphi_l, \theta_l, 0\}$
 $q^2 \in (m_l^2, (M_1 - M_2)^2), g_{\text{av}}^D(q^2), g_w^D(q^2)$

where $g_{\text{av}}^D(q^2) = F_1^A(q^2)/F_1^V(0), g_w^D(q^2) = F_2^V(q^2)/F_1^V(0)$



Hadronic baryon decay

- Momenta and masses: $B_1(p_1, M_1) \rightarrow B_2(p_2, M_2) + M(p_3, m_3)$
- Decay can be described by transition matrix [PRL55(1985)162]:

$$\langle B_2(p_2) | \mathcal{M} | B_1(p_1) \rangle = \bar{u}(p_2) [A_S + A_P \vec{\sigma} \cdot \hat{n}] u(p_1)$$



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- Asymmetry parameters [PRD99(2019)056008]:

$$\alpha = -2 \frac{\Re(A_S * A_P)}{|A_S|^2 + |A_P|^2} = \frac{|B_{\frac{1}{2}}|^2 - |B_{-\frac{1}{2}}|^2}{|B_{\frac{1}{2}}|^2 + |B_{-\frac{1}{2}}|^2},$$

$$\beta = -2 \frac{\Im(A_S * A_P)}{|A_S|^2 + |A_P|^2} = 2 \frac{\Im(B_{\frac{1}{2}} B_{-\frac{1}{2}}^*)}{|B_{\frac{1}{2}}|^2 + |B_{-\frac{1}{2}}|^2},$$

$$\gamma = \frac{|A_S|^2 - |A_P|^2}{|A_S|^2 + |A_P|^2} = 2 \frac{\Re(B_{\frac{1}{2}} B_{-\frac{1}{2}}^*)}{|B_{\frac{1}{2}}|^2 + |B_{-\frac{1}{2}}|^2},$$

where $\beta = \sqrt{1 - \alpha^2} \sin \phi$ and $\gamma = \sqrt{1 - \alpha^2} \cos \phi$

- Possible CPV tests:

$$A_{\text{CP}} = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} \quad \text{and} \quad \Phi_{\text{CP}} = \frac{\phi + \bar{\phi}}{2}$$



Semileptonic baryon decay

- Momenta and masses: $B_1(p_1, M_1) \rightarrow B_2(p_2, M_2) + l^-(p_l, m_l) + \bar{\nu}_l(p_{\bar{\nu}_l}, 0)$
- FF for the weak current-induced baryonic $1/2^+ \rightarrow 1/2^+$ transitions [EPJ C59 (2009) 27]:

$$\langle B_2(p_2) | J_\mu^{V+A} | B_1(p_1) \rangle = \bar{u}(p_2) \left[\gamma_\mu (F_1^V(q^2) - F_1^A(q^2)\gamma_5) - \frac{i\sigma_{\mu\nu}q^\nu}{M_1} (F_2^V(q^2) - F_2^A(q^2)\gamma_5) \right. \\ \left. + \frac{q^\mu}{M_1} (F_3^V(q^2) - F_3^A(q^2)\gamma_5) \right] u(p_1)$$

where $q_\mu = (p_1 - p_2)_\mu$

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- For $B_1 \rightarrow B_2 e^- \bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \rightarrow 0 \implies F_3^{V,A}(q^2) \rightarrow 0$
- $H_{\lambda_2 \Delta_W} = (H_{\lambda_2 \Delta_W}^V + H_{\lambda_2 \Delta_W}^A)$ with ($\lambda_2 = \pm 1/2; \Delta_W = t, 0, \pm 1$): $H_{\lambda_2 \Delta_W}^{V,A} \equiv H_{\lambda_2 \Delta_W}^{V,A}(F_{1,2}^{V,A}(q^2))$

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vector helicity amplitudes

$$H_{\frac{1}{2}1}^V = \sqrt{2Q_-} \left(-F_1^V - \frac{M_+}{M_1} F_2^V \right),$$

$$H_{\frac{1}{2}0}^V = \sqrt{\frac{Q_-}{q^2}} \left(M_+ F_1^V + \frac{q^2}{M_1} F_2^V \right),$$

$$H_{\frac{1}{2}t}^V = \sqrt{\frac{Q_+}{q^2}} \left(M_- F_1^V + \frac{q^2}{M_1} F_3^V \right),$$

axial-vector helicity amplitudes

$$H_{\frac{1}{2}1}^A = \sqrt{2Q_+} \left(F_1^A - \frac{M_-}{M_1} F_2^A \right),$$

$$H_{\frac{1}{2}0}^A = \sqrt{\frac{Q_+}{q^2}} \left(-M_- F_1^A + \frac{q^2}{M_1} F_2^A \right),$$

$$H_{\frac{1}{2}t}^A = \sqrt{\frac{Q_-}{q^2}} \left(-M_+ F_1^A + \frac{q^2}{M_1} F_3^A \right)$$

where $Q_\pm = (M_1 \pm M_2)^2 - q^2 \equiv M_\pm^2 - q^2$, $H_{-\lambda_2, -\Delta_W}^{V,A} = \pm H_{\lambda_2, \Delta_W}^{V,A}$



Semileptonic Baryon decays (1)

- Initial baryon B_1 with spin-density matrix $\rho_1^{\kappa\kappa'}$ transforms to final baryon B_2 with spin-density matrix $\rho_2^{\lambda_2\lambda'_2}$

$$\rho_2^{\lambda_2\lambda'_2} = T^{\kappa\kappa', \lambda_2\lambda'_2} \rho_1^{\kappa\kappa'}$$

- Transition tensor:

$$T^{\kappa\kappa', \lambda_2\lambda'_2} = \frac{1}{4\pi} \sum_{\underline{\lambda}_W, \underline{\lambda}'_W} T_{\underline{\lambda}_W, \underline{\lambda}'_W}^{\kappa\kappa', \lambda_2\lambda'_2}(q^2, \Omega_2) L_{\underline{\lambda}_W, \underline{\lambda}'_W}(q^2, \Omega_l)$$

- Hadronic tensor**

$$T_{\underline{\lambda}_W, \underline{\lambda}'_W}^{\kappa\kappa', \lambda_2\lambda'_2}(q^2, \Omega_2) = H_{\lambda_2 \underline{\lambda}_W} H_{\lambda'_2 \underline{\lambda}'_W}^* D_{\kappa, \lambda_2 - \lambda_W}^{1/2*}(\Omega_2) D_{\kappa', \lambda'_2 - \lambda'_W}^{1/2}(\Omega_2)$$

- Lepton tensor** with $\varepsilon = m_l^2/(2q^2)$

$$L_{\underline{\lambda}_W, \underline{\lambda}'_W}(q^2, \Omega_l) = \frac{8(q^2 - m_l^2)}{4\pi} \left[\ell_{\underline{\lambda}_W, \underline{\lambda}'_W}^{\text{nf}}(\Omega_l) + \varepsilon \ell_{\underline{\lambda}_W, \underline{\lambda}'_W}^{\text{f}}(\Omega_l) \right]$$

- nonflip($\underline{\lambda}_W = \mp 1$) : $|h_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}^l|^2 = 8\delta(\lambda_l + \lambda_\nu)(q^2 - m_l^2)$
- flip($\underline{\lambda}_W = 0, t$) : $|h_{\lambda_l=\pm\frac{1}{2}, \lambda_\nu=\pm\frac{1}{2}}^l|^2 = 8\delta(\lambda_l - \lambda_\nu)\varepsilon(q^2 - m_l^2)$



Semileptonic Baryon decays (2)

$$\sigma_\mu^{B_1} \longrightarrow \frac{3(q^2 - m_l^2)}{4\pi^3} \sum_{\nu=0}^3 \mathcal{B}_{\mu\nu} \sigma_\nu^{B_2}$$

- $\mathcal{B}_{\mu\nu}$ can be obtained by inserting Pauli matrices for mother and daughter baryons in $T^{\kappa\kappa',\lambda_2\lambda'_2}$ tensor expression

$$\mathcal{B}_{\mu\nu} = \frac{2\pi^3}{3(q^2 - m_l^2)} \sum_{\lambda_2, \lambda'_2 = -1/2}^{1/2} \sum_{\kappa, \kappa' = -1/2}^{1/2} T^{\kappa\kappa', \lambda_2\lambda'_2} \sigma_\mu^{\kappa, \kappa'} \sigma_\nu^{\lambda'_2, \lambda_2}$$

↓

$$\mathcal{B}_{\mu\nu} = \sum_{\kappa=0}^3 \mathcal{R}_{\mu\kappa}(\Omega_2) b_{\kappa\nu}(q^2, \Omega_l)$$

- $\mathcal{R}_{\mu\kappa}$ - 4×4 rotation matrix
- $b_{\kappa\nu}$ coefficients correspond to $B_1 \rightarrow B_2$ transition where axes orientation of the r.s. are aligned $\Omega_2 = \{0, 0, 0\}$

$$b_{\kappa\nu} = \frac{\pi}{6(q^2 - m_l^2)} \sum_{\Delta_W, \Delta'_W} \sum_{\lambda_2, \lambda'_2} H_{\lambda_2 \Delta_W} H_{\lambda'_2 \Delta'_W}^* \sigma_\kappa^{\lambda_2 - \Delta_W, \lambda'_2 - \Delta'_W} \sigma_\nu^{\lambda'_2, \lambda_2} L_{\Delta_W, \Delta'_W}(q^2, \Omega_l)$$

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- Replace $b_{\kappa\nu}$ by other decay matrices
→ can describe $B_1 \rightarrow B_2 + \pi$, $B_1 \rightarrow B_2 + \gamma$ and others



Rotation matrix $\mathcal{R}_{\mu\nu}$

- 4D rotation matrix $\mathcal{R}_{\mu\nu}(\Omega)$ with $\Omega \equiv \{\varphi, \theta, \chi\}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta \cos \chi \cos \varphi - \sin \chi \sin \varphi & -\cos \theta \sin \chi \cos \varphi - \cos \chi \sin \varphi & \sin \theta \cos \varphi \\ 0 & \cos \theta \cos \chi \sin \varphi + \sin \chi \cos \varphi & \cos \chi \cos \varphi - \cos \theta \sin \chi \sin \varphi & \sin \theta \sin \varphi \\ 0 & -\sin \theta \cos \chi & \sin \theta \sin \chi & \cos \theta \end{pmatrix}$$

- BESIII: $\mathcal{R}_{\mu\nu}(\Omega_2) = \mathcal{R}_{\mu\nu}(\varphi_2, \theta_2, 0)$



Decay matrices $b_{\kappa\nu}^i$

$$B_1 \rightarrow B_2 + \pi$$

$$b_{\kappa\nu}^D \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_D \\ 0 & \gamma_D & -\beta_D & 0 \\ 0 & \beta_D & \gamma_D & 0 \\ \alpha_D & 0 & 0 & 1 \end{pmatrix}$$

$$B_1 \rightarrow B_2 + \gamma$$

$$b_{\kappa\nu}^\gamma \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha_\gamma & 0 & 0 & -1 \end{pmatrix}$$



Decay matrices $b_{\kappa\nu}^i$

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$$B_1 \rightarrow B_2 + \gamma$$

$$b_{\kappa\nu}^\gamma \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha_\gamma & 0 & 0 & -1 \end{pmatrix}$$

$$B_1 \rightarrow B_2 + W_{\text{off-shell}}^- (\rightarrow l^- \bar{\nu}_l): b_{\kappa\nu}^{\text{SLW}} = b_{\kappa\nu}^{\text{nf}} + \varepsilon b_{\kappa\nu}^{\text{f}}$$

$$b_{\kappa\nu}^{\text{nf}} = \begin{pmatrix} b_{00}^{\text{nf}} & -\Re(\mathcal{I}_{01}^{\text{nf}}) & \Im(\mathcal{I}_{10}^{\text{nf}}) & b_{03}^{\text{nf}} \\ -\Re(\mathcal{I}_{10}^{\text{nf}}) & \Re(\mathcal{E}_{00}^{\text{nf}} + \mathcal{E}_{11}^{\text{nf}}) & -\Im(\mathcal{E}_{00}^{\text{nf}} + \mathcal{E}_{11}^{\text{nf}}) & \Re(\mathcal{I}_{13}^{\text{nf}}) \\ \Im(\mathcal{I}_{10}^{\text{nf}}) & \Im(\mathcal{E}_{00}^{\text{nf}} - \mathcal{E}_{11}^{\text{nf}}) & \Re(\mathcal{E}_{00}^{\text{nf}} + \mathcal{E}_{11}^{\text{nf}}) & -\Im(\mathcal{I}_{13}^{\text{nf}}) \\ b_{30}^{\text{nf}} & -\Re(\mathcal{I}_{31}^{\text{nf}}) & \Im(\mathcal{I}_{31}^{\text{nf}}) & b_{33}^{\text{nf}} \end{pmatrix}$$

$$b_{\kappa\nu}^{\text{f}} = \begin{pmatrix} b_{00}^{\text{f}} & -\Re(\mathcal{I}_{01}^{\text{f}}) & \Im(\mathcal{I}_{10}^{\text{f}}) & b_{03}^{\text{f}} \\ -\Re(\mathcal{I}_{10}^{\text{f}}) & \Re(\mathcal{E}_{00}^{\text{f}} - \mathcal{E}_{11}^{\text{f}}) & -\Im(\mathcal{E}_{00}^{\text{f}} - \mathcal{E}_{11}^{\text{f}}) & \Re(\mathcal{I}_{13}^{\text{f}}) \\ \Im(\mathcal{I}_{10}^{\text{f}}) & \Im(\mathcal{E}_{00}^{\text{f}} + \mathcal{E}_{11}^{\text{f}}) & \Re(\mathcal{E}_{00}^{\text{f}} + \mathcal{E}_{11}^{\text{f}}) & -\Im(\mathcal{I}_{13}^{\text{f}}) \\ b_{30}^{\text{f}} & -\Re(\mathcal{I}_{31}^{\text{f}}) & \Im(\mathcal{I}_{31}^{\text{f}}) & b_{33}^{\text{f}} \end{pmatrix}$$



Polarization \vec{P} of baryon B_2

- Represent first row of $b_{0\kappa}$ matrix

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \frac{1}{b_{00}^{\text{nf}} + \varepsilon b_{00}^{\text{f}}} \begin{bmatrix} -\cos \varphi_l & \sin \varphi_l & 0 \\ \sin \varphi_l & \cos \varphi_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Re(\mathcal{I}_{01}) \\ \Im(\mathcal{I}_{01}) \\ b_{03}^{\text{nf}} + \varepsilon b_{03}^{\text{f}} \end{bmatrix}$$

where

$$b_{00/03}^{\text{nf}} = \frac{1}{4}(1 \mp \cos \theta_l)^2 |H_{\frac{1}{2}1}|^2 + \frac{1}{4}(1 \pm \cos \theta_l)^2 |H_{-\frac{1}{2}-1}|^2 + \frac{1}{2} \sin^2 \theta_l (|H_{-\frac{1}{2}0}|^2 - |H_{\frac{1}{2}0}|^2),$$

$$\begin{aligned} b_{00/03}^{\text{f}} = & |H_{\frac{1}{2}t}|^2 - |H_{-\frac{1}{2}t}|^2 + \frac{1}{2} \sin^2 \theta_l (|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2) - \cos^2 \theta_l (|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2) \\ & - \cos \theta_l \Re(H_{\frac{1}{2}0}^* H_{\frac{1}{2}t} + H_{-\frac{1}{2}0}^* H_{-\frac{1}{2}t}), \end{aligned}$$

$$\mathcal{I}_{01}^{\text{nf}} = \pm \frac{1}{2\sqrt{2}} \sin \theta_l \left[(1 \pm \cos \theta_l) H_{-\frac{1}{2}-1}^* H_{\frac{1}{2}0} + (1 \mp \cos \theta_l) H_{-\frac{1}{2}0}^* H_{\frac{1}{2}1} \right],$$

$$\mathcal{I}_{01}^{\text{f}} = \frac{1}{\sqrt{2}} \sin \theta_l \left[(H_{-\frac{1}{2}-1}^* H_{\frac{1}{2}t} - H_{-\frac{1}{2}t}^* H_{\frac{1}{2}1}) + \cos \theta_l (H_{-\frac{1}{2}0}^* H_{\frac{1}{2}1} - H_{-\frac{1}{2}-1}^* H_{\frac{1}{2}0}) \right].$$



Joint angular distribution (1)

- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (\textcolor{red}{B}_1 \rightarrow B_3 \pi^-)(\bar{B}_1 \rightarrow \bar{B}_3 \pi^+)$

$$\text{Tr} \rho_{B_3 \bar{B}_3} \propto \sum_{\mu, \bar{\nu}=0}^3 C_{\mu \bar{\nu}} \textcolor{red}{a}_{\mu 0}^{B_1 B_3} a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3}$$

- $C_{\mu \bar{\nu}} \equiv C_{\mu \bar{\nu}}(\theta_1; \alpha_\psi, \Delta \Phi)$
- $\textcolor{red}{a}_{\mu 0}^{B_1 B_3} \equiv a_{\mu 0}(\theta_3, \varphi_3; \alpha_{B_1})$
- $a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3} \equiv a_{\bar{\nu} 0}(\bar{\theta}_3, \bar{\varphi}_3; \bar{\alpha}_{B_1})$



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-
- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (\textcolor{red}{B}_1 \rightarrow B_2 W_{\text{off-shell}}^-(\rightarrow l^- \bar{\nu}_l))(\bar{B}_1 \rightarrow \bar{B}_3 \pi^+)$

$$\text{Tr} \rho_{B_2 \bar{B}_3} \propto \sum_{\mu, \bar{\nu}=0}^3 C_{\mu \bar{\nu}} \mathcal{B}_{\mu 0}^{B_1 B_2} a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3} \equiv \sum_{\mu, \bar{\nu}=0}^3 C_{\mu \bar{\nu}} \sum_{\kappa=0}^3 \mathcal{R}_{\mu \kappa}(\Omega_2) b_{\kappa 0}^{B_1 B_2}(q^2, \Omega_l) a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3}$$

- $\mathcal{B}_{\mu 0}^{B_1 B_2} \equiv \mathcal{R}_{\mu \kappa}(\theta_2, \varphi_2) b_{\kappa 0}(\theta_l, \phi_l, q^2; g_{\text{av}}^{B_1}, g_w^{B_1})$



Joint angular distribution (2)

- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (B_1 \rightarrow B_3\pi^-)(\bar{B}_1 \rightarrow \bar{B}_3\pi^+)$

$$d\Gamma \propto \mathcal{W}(\xi; \alpha_\psi, \Delta\Phi, \alpha_{B_1}, \bar{\alpha}_{B_1}) =$$

$1 + \alpha_\psi \cos^2 \theta_1$	Cross section	Spin correlations
$+ \alpha_{B_1} \bar{\alpha}_{B_1} (\sin^2 \theta_1 (st_3 cp_3 \bar{s}t_3 \bar{c}p_3 + \alpha_\psi st_3 sp_3 \bar{s}t_3 \bar{s}p_3) - (\cos^2 \theta_1 + \alpha_\psi) ct_3 \bar{c}t_3)$		
$+ \alpha_{B_1} \bar{\alpha}_{B_1} \sqrt{1 - \alpha_\psi^2} \cos(\Delta\Phi) \sin \theta_1 \cos \theta_1 (st_3 cp_3 \bar{c}t_3 - ct_3 \bar{s}t_3 \bar{c}p_3)$		
$+ \sqrt{1 - \alpha_\psi^2} \sin(\Delta\Phi) \sin \theta_1 \cos \theta_1 (-\alpha_{B_1} st_3 sp_3 + \bar{\alpha}_{B_1} \bar{s}t_3 \bar{s}p_3)$		Polarization



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- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (B_1 \rightarrow B_2 W_{\text{off-shell}}^-(\rightarrow l^-\bar{\nu}_l))(\bar{B}_1 \rightarrow \bar{B}_3\pi^+)$

$$d\Gamma \propto \mathcal{W}(\xi'; \alpha_\psi, \Delta\Phi, g_{av}^{B_1}, g_w^{B_1}, \bar{\alpha}_{B_1}) =$$

$b_{00}(1 + \alpha_\psi \cos^2 \theta_1)$	Cross section	Spin correlations
$+ b_{30} \bar{\alpha}_{B_1} (\sin^2 \theta_1 (st_2 cp_2 \bar{s}t_3 \bar{c}p_3 + \alpha_\psi st_2 sp_2 \bar{s}t_3 \bar{s}p_3) - (\cos^2 \theta_1 + \alpha_\psi) ct_2 \bar{c}t_3)$		
$+ b_{10} \bar{\alpha}_{B_1} (-\sin^2 \theta_1 (ct_2 cp_2 \bar{s}t_3 \bar{c}p_3 + \alpha_\psi ct_2 sp_2 \bar{s}t_3 \bar{s}p_3) - (\cos^2 \theta_1 + \alpha_\psi) st_2 \bar{c}t_3)$		
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$+ b_{10} \bar{\alpha}_{B_1} \sqrt{1 - \alpha_\psi^2} \cos(\Delta\Phi) \sin \theta_1 \cos \theta_1 (-ct_2 cp_2 \bar{c}t_3 + st_2 \bar{s}t_3 \bar{c}p_3)$		
$+ \sqrt{1 - \alpha_\psi^2} \sin(\Delta\Phi) \sin \theta_1 \cos \theta_1 (-b_{30} st_2 sp_2 + \bar{\alpha}_{B_1} \bar{s}t_3 \bar{s}p_3 + b_{10} ct_2 sp_2)$		Polarization



Form factors

- Neglecting possible CP-odd weak phase, $\text{FF}(l^-, \bar{\nu}_l) = \text{sign} \text{ FF}(l^+, \nu_l)$
- In limit of exact SU(3) symmetry, F_2^A and $F_3^V \rightarrow 0$



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- FF parametrization for **hyperons** [PLB478(2000)417][EPJC81(2021)226]:

$$F_i^{V,A}(q^2) = \frac{F_i^{V,A}(0)}{1 - \frac{q^2}{M_{V,A}^2}} \frac{1}{1 - \alpha_{\text{BK}} \frac{q^2}{M_{V,A}^2}} \implies F_i^{V,A}(q^2) = F_i^{V,A}(0) \left[1 + r_i^{V,A} q^2 + \dots \right]$$

with $r^{V,A} = 2/m_{V,A}^2$ [AnnRevNuclPartSci34(1984)351] [AnnRevNuclPartSci53(2003)39]

- $\Delta S = 0$: $m_V = 0.84$ GeV [RivNuovoCim2(1972)241], $m_A = 1.08$ GeV [BNL-24848]
- $|\Delta S| = 1$: $m_V = m_{K^*(892)} = 0.89$ GeV, $m_A = m_{K^*(1270)} = 1.27$ GeV

Decay	$\mathcal{B}(\times 10^{-4})$	$g_{av}^D(0)$ [a]	$g_w^D(0)$ [a]	$M_1 - M_2$ [MeV]	Ref.
$\Lambda \rightarrow p e^- \bar{\nu}_e$	8.32(14)	0.718(15)	1.066	177	[1, 2]
$\Sigma^+ \rightarrow \Lambda e^+ \nu_e$ [b]	0.20(05)	0.01(10)	2.4(17)	74	[1]
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$	5.63(31)	0.25(5)	0.085	206	[2, 3]

[a] $g_{av} = F_1^A(0)/F_1^V(0)$ and $g_w = F_2^V(0)/F_1^V(0)$

[b] Since $F_1^\Sigma = 0$, g_{av} and g_w are defined as F_1^V/F_1^A and F_2^V/F_1^A , respectively

[1] PTEP2022 083C01(2022) [2] AnnRevNuclPartSci53(2003)39 [3] ZPhysC21(1983)1

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- FF parametrization for **charm baryons**:

[EPJC76(2016)628] [PRD93(2016)034008] [PRD80(2009)074011][PRC72(2005)035201]

and many others



Form factors for charm baryons (1)

- Light-front approach [[Chin.Phys.C42\(2018\)093101](#)]:

$$F_i(q^2) = F_i(0) / \left(1 \mp \frac{q^2}{m_{\text{fit}}^2} + \delta \left(\frac{q^2}{m_{\text{fit}}^2} \right)^2 \right)$$

where m_{fit} , δ fitted from numerical results

- Pole-dominance model:

SU(4)-symmetry limit [[PRD40\(1989\)2944](#)], MIT bag model [[PRD40\(1989\)2955](#)]:

$$F_i^{V,A}(q^2) = F_i^{V,A}(0) \left[1 + r_i^{V,A} q^2 \right] \quad \text{with} \quad r^{V,A} = n/m_{V,A}^2$$

- $|\Delta C| = 1, \Delta S = 0$: $m_V = m_{D^*} = 2.01$ GeV, $m_A = m_{D^{*0}} = 2.42$ GeV
- $|\Delta C| = |\Delta S| = 1$: $m_V = m_{D_s^*} = 2.11$ GeV, $m_A = m_{D_{s1}} = 2.54$ GeV

Form factors for charm baryons (2)



- Relativistic quark model based on quasi-potential approach with QCD-motivated potential:

$$F_i(q^2) = \frac{1}{1 - q^2/(M_{\text{pole}}^{F_i})^2} \sum_{n=0}^{n_{\max}} a_n^{F_i} [z(q^2)]^n$$

where $z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$ with $t_0 = (M_1 - M_2)^2$

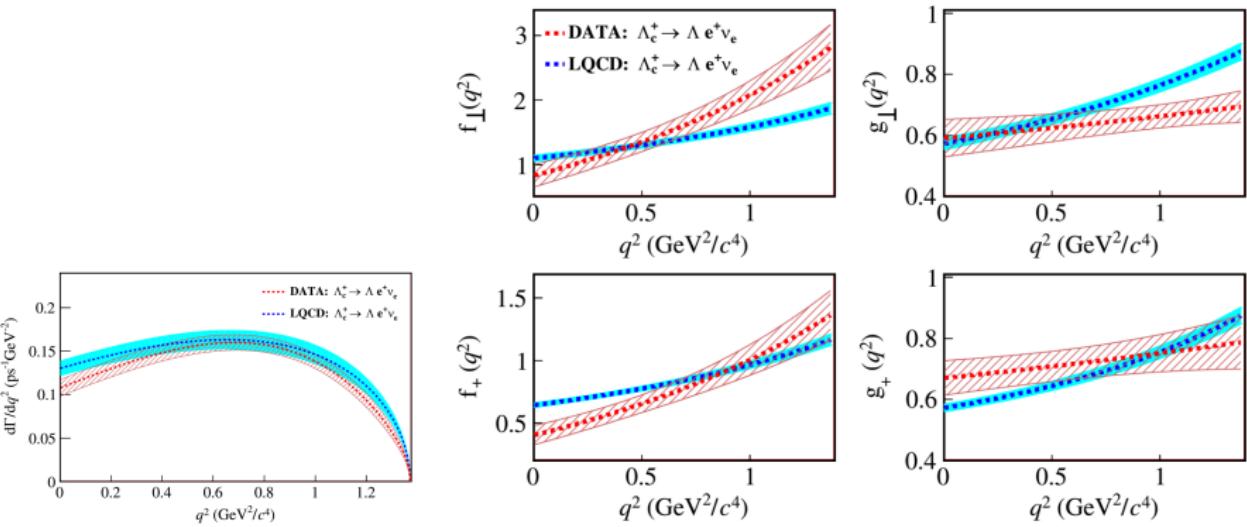
Decay	$\sqrt{t_+}$	$m(F_{1,2}^V)$ [GeV]	$m(F_3^V)$ [GeV]	$m(F_{1,2}^A)$ [GeV]	$m(F_3^A)$ [GeV]	$M_1 - M_2$ [GeV]	Ref.
$\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$	$m_D + m_K$	2.11	2.32	2.46	1.97	1.17	[1]
$\Xi_c \rightarrow \Xi l \nu_l$	$m_{D_s} + m_K$	2.11	2.54	2.54	1.97	1.15	[2]
$\Xi_c \rightarrow \Lambda l \nu_l$	$m_D + m_\pi$	2.01	2.42	2.42	1.87	1.35	[2]

[1] [PRL118(2017)082001]

[2] [EPJC79(2019)695]

$\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$ FFs

- First measurement by BESIII [PRL129(2022)231803]
 - $z(q^2)$ expansion
- Comparison with LQCD calculation [PRL118(2017)082001]
 - Different kinematic behaviour for $FF(q^2)$
 - Agreement for decay rate
- $\{F_1^V, F_2^V, F_1^A, F_2^A\} \rightarrow \{f_+, f_\perp, g_+, g_\perp\}$





Summary

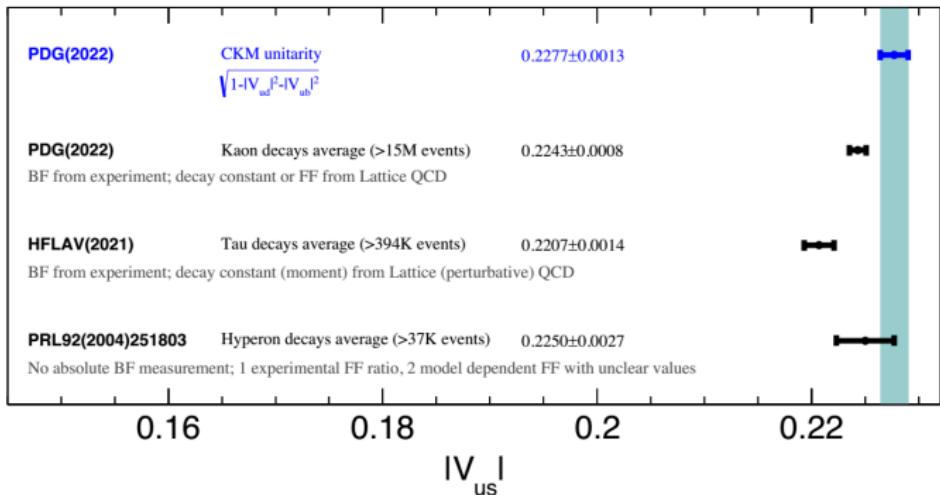
- General formalism [PRD108(2023)016011] can be applied to data analyses performing in e^+e^- collider experiments
- Modular description is very flexible:
 - Non-leptonic, semileptonic, radiative and electromagnetic decays of baryons with spin 1/2
 - One- and two-step decays
- Different FFs parametrization can be taken into account



Summary

- General formalism [PRD108(2023)016011] can be applied to data analyses performing in e^+e^- collider experiments
- Modular description is very flexible:
 - Non-leptonic, semileptonic, radiative and electromagnetic decays of baryons with spin 1/2
 - One- and two-step decays
- Different FFs parametrization can be taken into account
- Neglecting hadronic CP-violating effects, CP-symmetry tests can be performed using FFs
- Measurement of FFs and \mathcal{BR} will allow to measure CKM matrix elements V_{ij} within one data analysis

V_{us} measurement



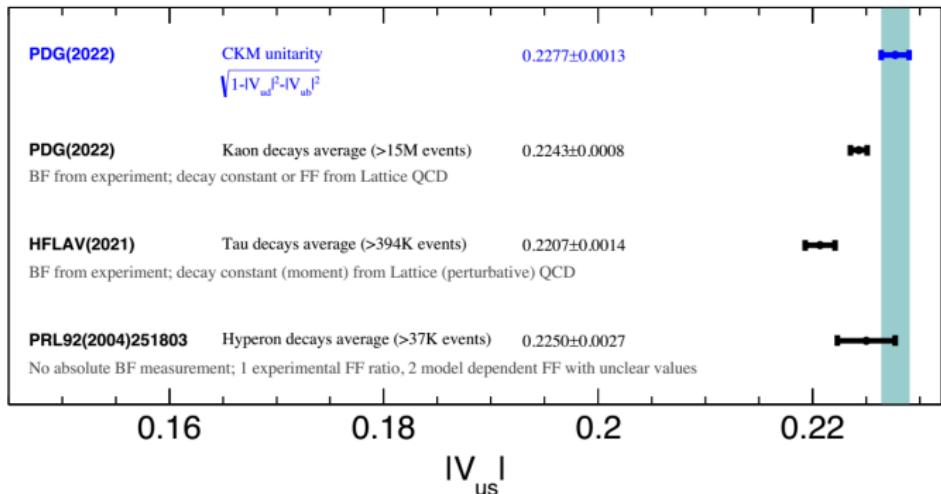
$s \rightarrow u$ transition SL:

$$\frac{d\Gamma}{dq^2} \propto G_F^2 |V_{us}|^2 V_{Ph}(q^2) (q^2 - m_l^2) b_{00}$$

Decay Process	Rate (μsec^{-1})	g_1/f_1	V_{us} [PRL92(2004)251803]
$\Lambda \rightarrow p e^- \bar{\nu}$	3.161(58)	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \rightarrow n e^- \bar{\nu}$	6.88(24)	-0.340(17)	0.2282 ± 0.0049
$\Xi^- \rightarrow \Lambda e^- \bar{\nu}$	3.44(19)	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$	0.876(71)	$1.32(+.22/- .18)$	0.209 ± 0.027
Combined			0.2250 ± 0.0027



V_{us} measurement



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Thank you for your attention! 谢谢您的关注!



Backups



"I ALWAYS BACK UP EVERYTHING."

$$B_1 \rightarrow B_2 \gamma^* \rightarrow B_2 l^+ l^-$$

- Differential decay rate

$$\frac{d\Gamma}{dq^2} \propto \frac{\alpha_{em}^2}{q^2} V_{Ph}(q^2) \left(1 - \frac{4m_l^2}{q^2}\right) b_{00}^{em}$$

- Unrotated decay matrix:

$$b_{\kappa\nu}^{em} = \frac{1}{2(q^2 - 4m_l^2)} \sum_{\lambda_\gamma, \lambda'_\gamma} \sum_{\lambda_2, \lambda'_2} H_{\lambda_2 \lambda_\gamma} H_{\lambda'_2 \lambda'_\gamma}^* \sigma_\kappa^{\lambda_2 - \lambda_\gamma, \lambda'_2 - \lambda'_\gamma} \sigma_\nu^{\lambda'_2, \lambda_2} L_{\lambda_\gamma, \lambda'_\gamma}(q^2, \Omega_l)$$

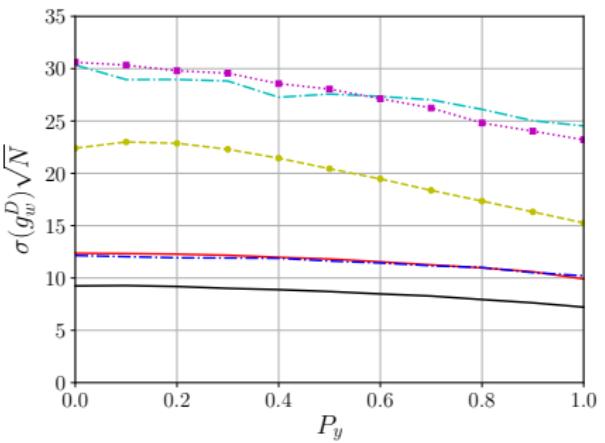
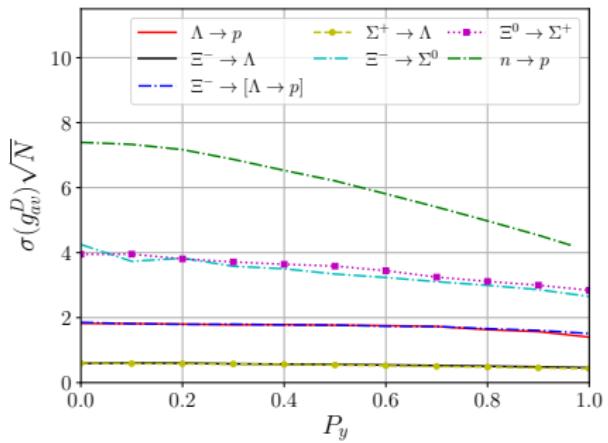
where $\lambda_\gamma = \{-1, 0, 1\}$ for γ^* decay and its elements:

$$b_{\kappa\nu}^{em} = \begin{pmatrix} b_{00}^{em} & b_{01}^{em} & b_{02}^{em} & 0 \\ b_{01}^{em} & b_{11}^{em} & b_{12}^{em} & b_{13}^{em} \\ b_{02}^{em} & b_{12}^{em} & b_{22}^{em} & b_{23}^{em} \\ 0 & -b_{13}^{em} & -b_{23}^{em} & b_{33}^{em} \end{pmatrix}$$

- Non zero FFs: $H_{\frac{1}{2}1}^V = H_{-\frac{1}{2}-1}^V$ and $H_{\frac{1}{2}0}^V = H_{-\frac{1}{2}0}^V$
- Full definition of $b_{\kappa\nu}$ elements is provided in [PRD108(2023)016011]



g_{av} and g_w sensitivity



Joint angular distribution (1)

- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow B_1 \rightarrow B_2 W_{\text{off-shell}}^- (\rightarrow l^- \bar{\nu}_l)$

$$\text{Tr} \rho_{B_2} \propto \sum_{\mu=0}^3 C_{\mu 0} \mathcal{B}_{\mu 0}^{B_1 B_2} = \sum_{\mu=0}^3 C_{\mu 0} \sum_{\kappa=0}^3 \mathcal{R}_{\mu \kappa}(\Omega_2) b_{\kappa 0}^{B_1 B_2}(q^2, \Omega_l)$$

- $C_{\mu 0} \equiv (1, P_x, P_y, P_z)$
- $\mathcal{B}_{\mu 0}^{B_1 B_2} \equiv \mathcal{R}_{\mu \kappa}(\theta_2, \phi_2) b_{\kappa 0}(\theta_l, \phi_l, q^2; g_{\text{av}}^{B_1}, g_w^{B_1})$
- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (B_1 \rightarrow B_2 W_{\text{off-shell}}^- (\rightarrow l^- \bar{\nu}_l)) (\bar{B}_1 \rightarrow \bar{B}_3 \pi^+)$

$$\text{Tr} \rho_{B_2 \bar{B}_3} \propto \sum_{\mu, \bar{\nu}=0}^3 C_{\mu \bar{\nu}} \mathcal{B}_{\mu 0}^{B_1 B_2} a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3}$$

- $C_{\mu \bar{\nu}} \equiv C_{\mu \bar{\nu}}(\theta_1; \alpha_\psi, \Delta \Phi)$
- $\mathcal{B}_{\mu 0}^{B_1 B_2} \equiv \mathcal{R}_{\mu \kappa}(\theta_2, \phi_2) b_{\kappa 0}(\theta_l, \phi_l, q^2; g_{\text{av}}^{B_1}, g_w^{B_1})$
- $a_{\bar{\nu} 0}^{\bar{B}_1 \bar{B}_3} \equiv a_{\bar{\nu} 0}(\bar{\theta}_3, \bar{\phi}_3; \bar{\alpha}_{B_1})$

Joint angular distribution (2)

- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (\Xi^- \rightarrow \Lambda(\rightarrow p e^- \bar{\nu}_e) \pi^-)(\bar{\Xi}^+ \rightarrow \bar{\Lambda}(\rightarrow \bar{p} \pi^+) \pi^+)$

$$\text{Tr}\rho_{p\bar{p}} \propto \sum_{\mu, \bar{\nu}=0}^3 C_{\mu\bar{\nu}}^{\Xi\bar{\Xi}} \sum_{\mu'=0}^3 a_{\mu\mu'}^{\Xi\Lambda} \mathcal{B}_{\mu'0}^{\Lambda p} \sum_{\bar{\nu}'=0}^3 a_{\bar{\nu}\bar{\nu}'}^{\Xi\bar{\Lambda}} a_{\bar{\nu}'0}^{\bar{\Lambda}\bar{p}}$$

- $\mathcal{B}_{\mu'0}^{\Lambda p} \equiv \mathcal{R}_{\mu'\kappa}(\theta_p, \phi_p) b_{\kappa 0}(\theta_e, \phi_e, q^2; g_{\text{av}}^\Lambda, g_{\text{w}}^\Lambda)$
- $e^+e^- \rightarrow J/\psi, \psi(2S) \rightarrow (\Xi^- \rightarrow \Lambda(\rightarrow p \pi^-) e^- \bar{\nu}_e)(\bar{\Xi}^+ \rightarrow \bar{\Lambda}(\rightarrow \bar{p} \pi^+) \pi^+)$

$$\text{Tr}\rho_{p\bar{p}} \propto \sum_{\mu, \bar{\nu}=0}^3 C_{\mu\bar{\nu}}^{\Xi\bar{\Xi}} \sum_{\mu'=0}^3 \mathcal{B}_{\mu\mu'}^{\Xi\Lambda} a_{\mu'0}^{\Lambda p} \sum_{\bar{\nu}'=0}^3 a_{\bar{\nu}\bar{\nu}'}^{\Xi\bar{\Lambda}} a_{\bar{\nu}'0}^{\bar{\Lambda}\bar{p}}$$

- $\mathcal{B}_{\mu\mu'}^{\Xi\Lambda} \equiv \mathcal{R}_{\mu\kappa}(\theta_\Lambda, \phi_\Lambda) b_{\kappa\mu'}(\theta_e, \phi_e, q^2; g_{\text{av}}^\Xi, g_{\text{w}}^\Xi)$