Deep Inelastic Scattering

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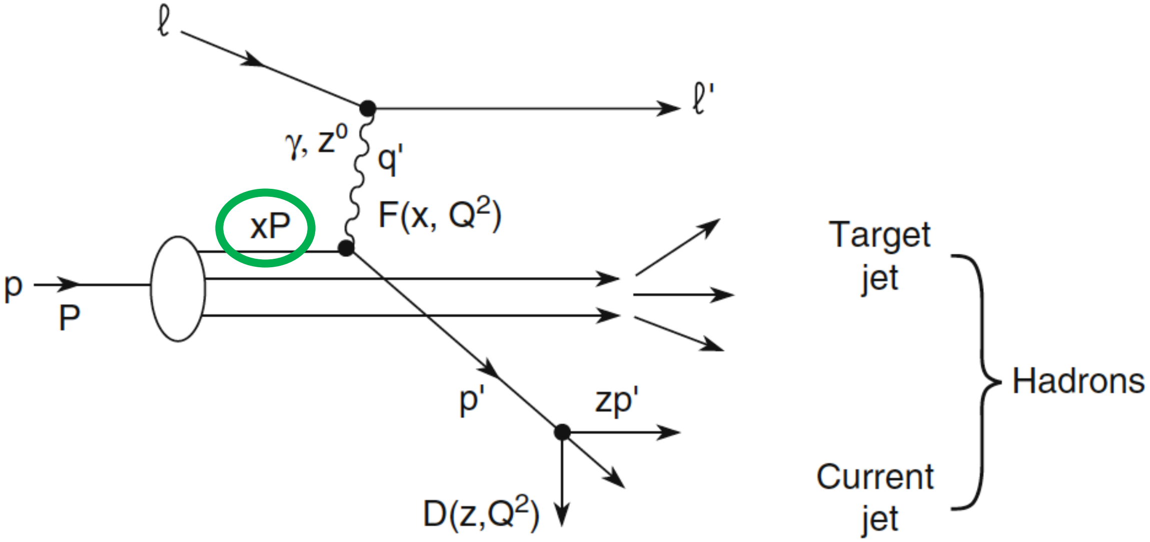
Why Deep Inelastic Scattering?

This course is titled "Collider Physics"

- Colliders are today the most powerful instrument to study the innermost structure of matter
- Proton-proton colliders are the accelerators that can reach the highest energies, for reasons that will be clear when discussing about accelerators
- Proton are very complex objects, with a complex internal structure
- The interpretation of scattering experiments needs to be based on the understanding of the proton structure
- The scattering lepton-nucleon allows us to study the structure of the proton

Deep Inelastic Scattering
→ *DIS*

Basis of QCD, the theory of hadronic interactions



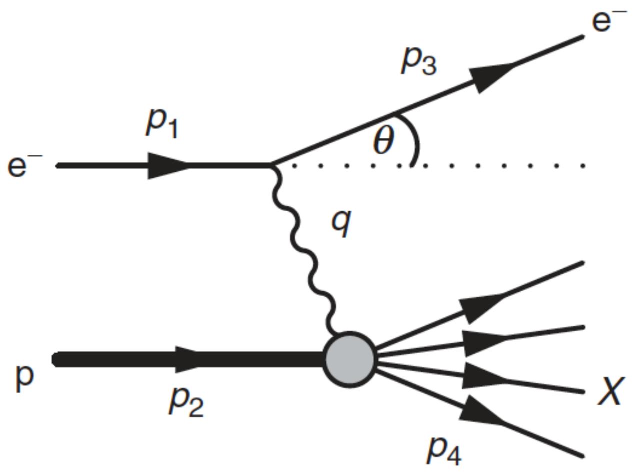
Many generation of scattering experiments.

- Initially experiments used leptons (mostly electrons) produced in accelerators and sent on a *target*
- The last generation was the HERA *collider* at Desy, Germany

30 GeV electrons against 900 GeV protons

Deep Inelastic Scattering (DIS)

- DIS =
  - *QED* interaction of a virtual photon with the constituent quarks inside the proton (electrons & muons);
  - *Weak* interactions induced by a ν can also give information on the structure of hadrons;



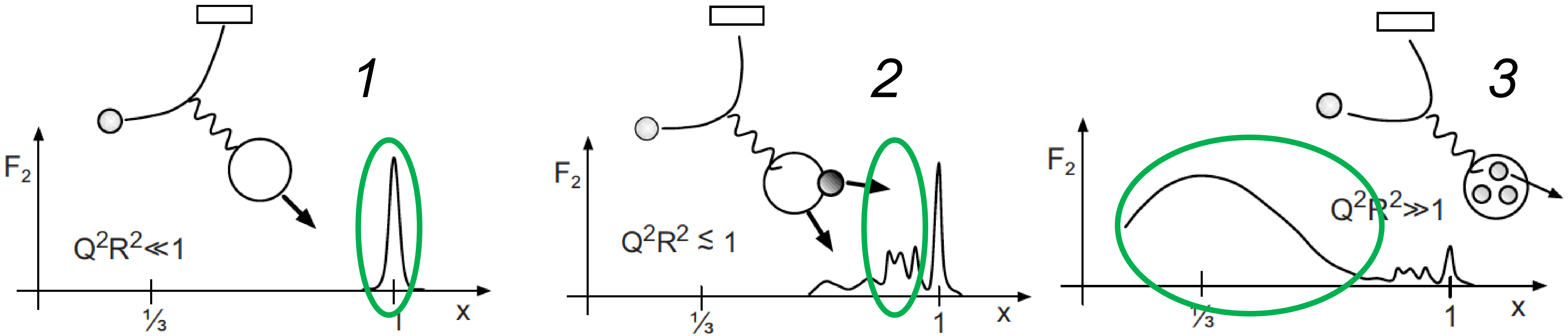
- A lot of experimental data, the measured structure functions describe the momentum distributions of the quarks.

- The proton is found to be a complex dynamical system made of quarks, gluons and antiquarks.

DIS measurements interpreted using x. This variable is generally known as “Bjorken scaling variable” and gives an indication of the inelasticity of the process.

- $x = 1 \rightarrow elastic\ scattering;$
- $x < 1 \rightarrow\ inelastic\ scattering\ on\ costituents\ of\ the\ proton$

Start to understand '*x*'



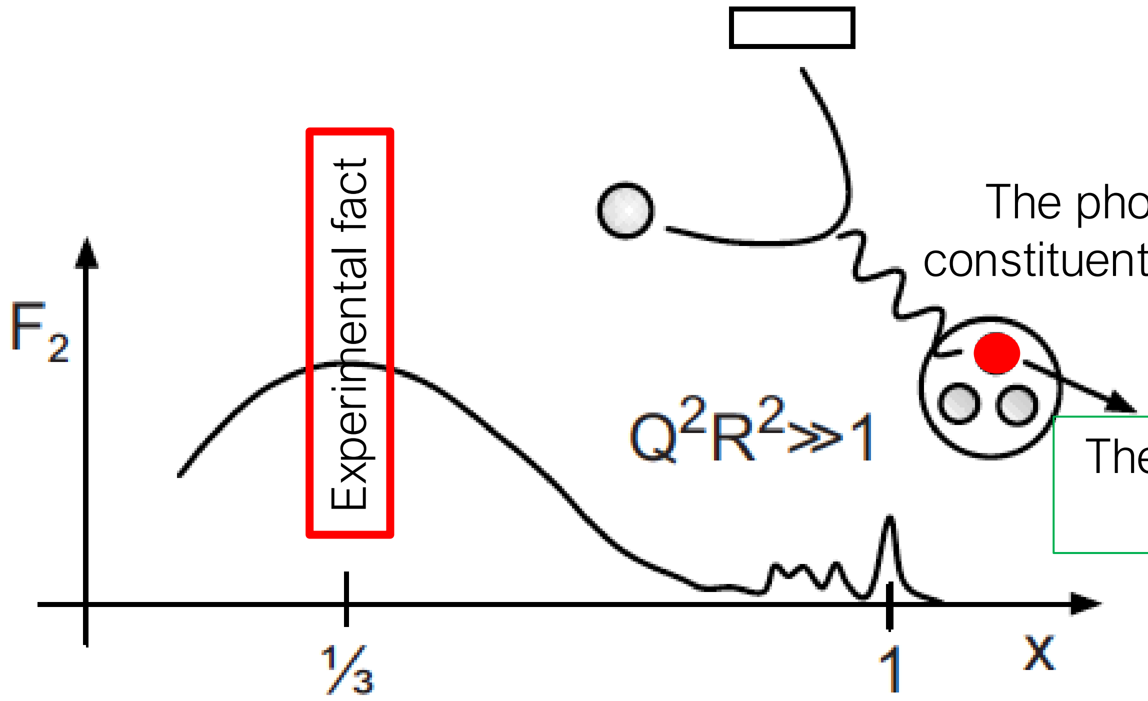
What do we see with increasing Q^2?

See above 3 different cases

Q2 ↑ wave length of the probe particle ↓

1. The Q^2 of the reaction is ~low, the **nucleon** is seen by the exchanged photon as **a unique object**. We have elastic scattering
2. The Q^2 of the reaction is not as ~low as in 1, not enough to probe the inner structure but enough to **excite the nucleon**
3. The Q^2 of the reaction is ~large enough to see the internal structure of the proton and the photon *scatters elastically* on one of the **internal constituents of the nucleon**

More Understanding of 'x'



The constituent of the nucleon is scattered elastically

The constituent of the quark cannot be observed → it 'transforms' into several hadrons ("it hadronizes") (will see this later)

$x \sim$ fraction of proton momentum carried by the struck component

The peak at ~1/3 can be understood as the "most probable" x value corresponding to the *elastic scattering of the photon and one of the nucleon constituents.*

If we assume that the 'x' budget is equally shared by '**n**' nucleon constituents then

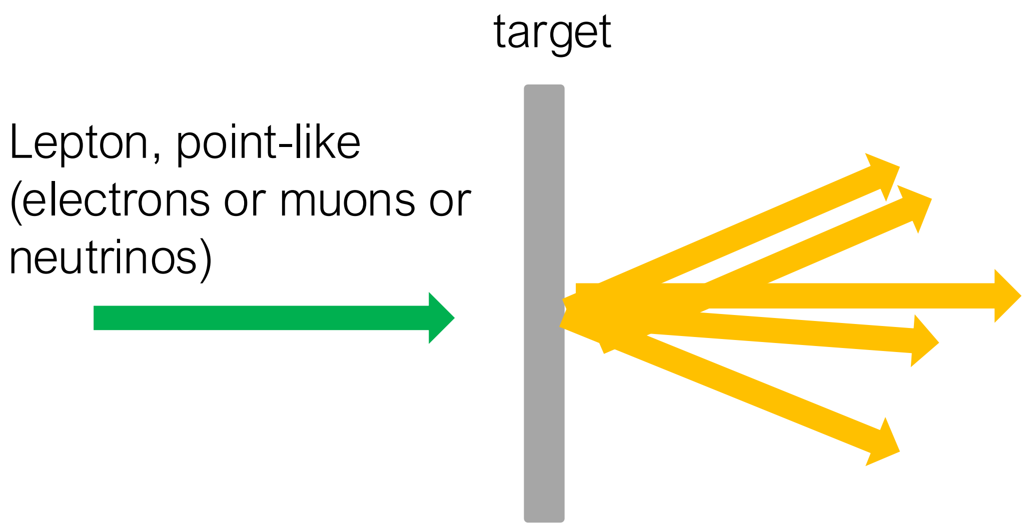
Elaborate more in the following!

$$x = \frac{1}{n}\frac{Q^2}{2Pq} = \frac{1}{n}\frac{Q^2}{2M\nu}$$

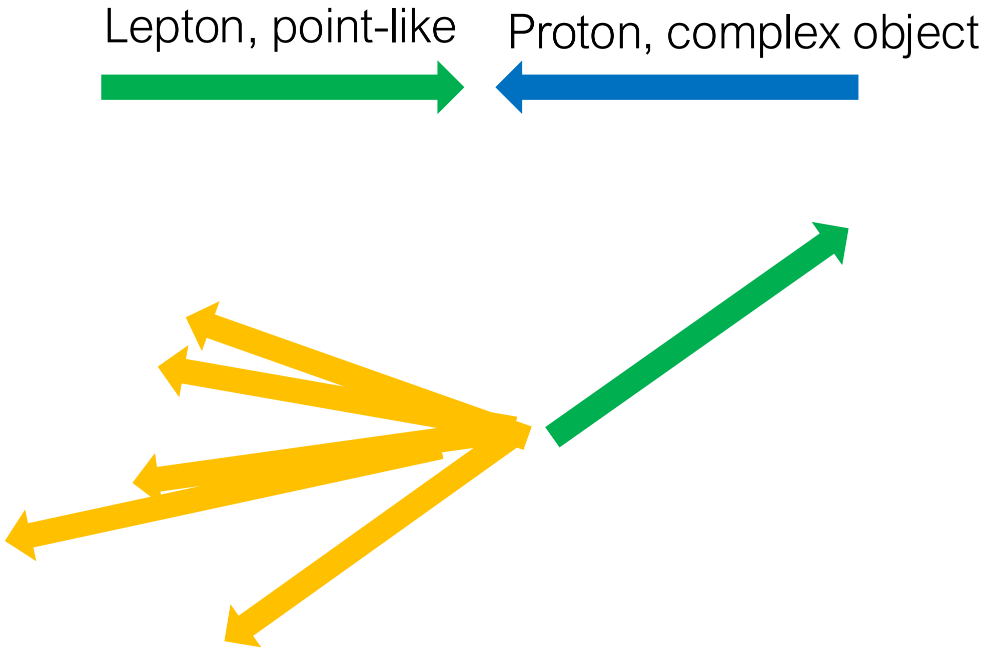
This term is equal to 1 in case of elastic scattering

$\frac{1}{3} = \frac{1}{n}$ → there are 3 components in the nucleon

Laboratory & Center of Mass System



Laboratory System



Center of Mass System

Beam + target ~ appears as collider in cms
Beam + Beam MUCH better option

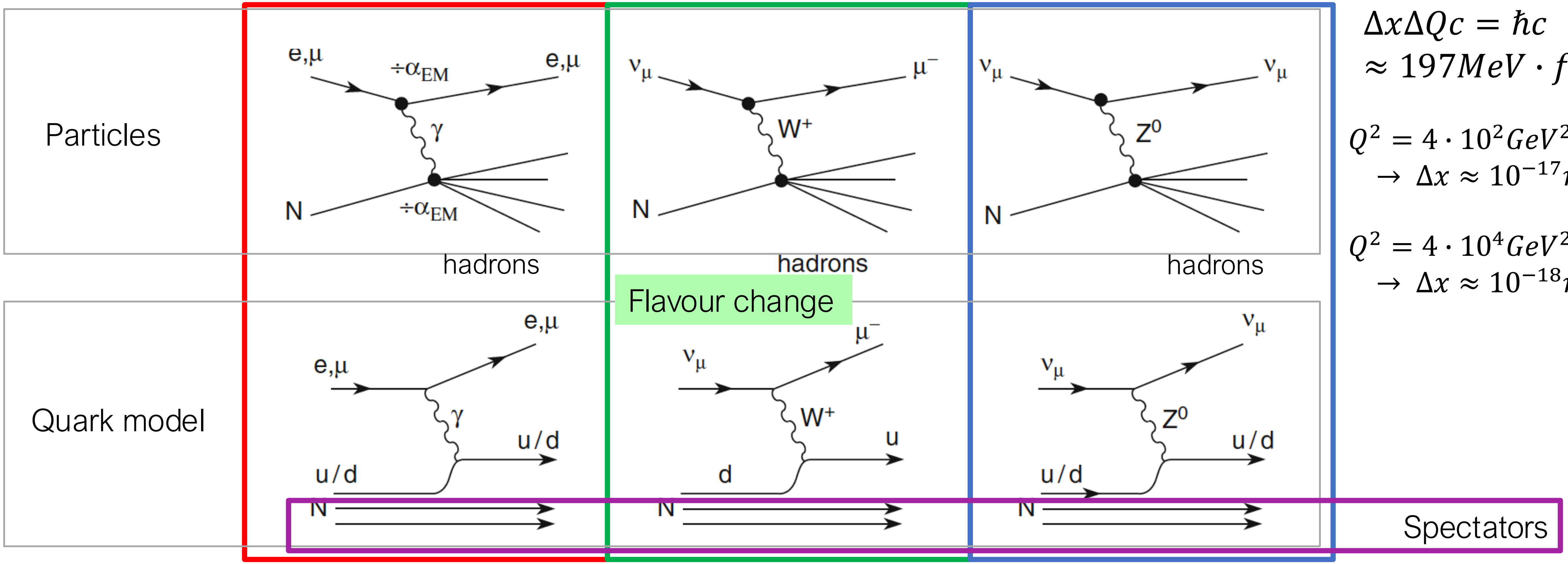
Inelastic Lepton-Nucleus Scattering

$ep: \quad e^{\pm} + p \to e^{\pm} + X^{+}$

$\mu p: \quad \mu^{\pm} + p \to \mu^{\pm} + X^{+}$

$\nu_\mu p(CC): \nu_\mu + p \to \mu^- + X^{++}, \ \overline{\nu}_\mu + p \to \mu^+ + X^0$

$\nu_\mu p(NC): \ \nu_\mu + p \to \nu_\mu + X^{+}, \ \overline{\nu}_\mu + p \to \overline{\nu}_\mu + X^{+}.$



$\Delta x \Delta Q c = \hbar c$
$\approx 197 MeV \cdot fm$

$Q^2 = 4 \cdot 10^2 GeV^2/c^2$
$\to \ \Delta x \approx 10^{-17} m$

$Q^2 = 4 \cdot 10^4 GeV^2/c^2$
$\to \ \Delta x \approx 10^{-18} m$

N = p / n

Electromagnetic | Weak Charged Current | Weak Neutral Current

Electron – Proton scattering: History

Studying the nucleon's constituents the wave length of the probe particle λ has to be small compared to the nucleon's radius, R

$$\lambda \ll R \;\; \rightarrow \;\; Q^2 \gg \hbar^2/R^2$$

Large Q^2 values are needed → high energies are required.

- The first generation ~1960 @ **SLAC** 25 GeV electrons on a target
- The second generation ~ 1980 @ **CERN** using beams of *muons* of up to 300 GeV (*).

Beams on target

- The last generation ~1990 → 2007 @ **DESY** Collider **HERA:** 30 GeV electrons against 900 GeV protons (see next slides).

Collider

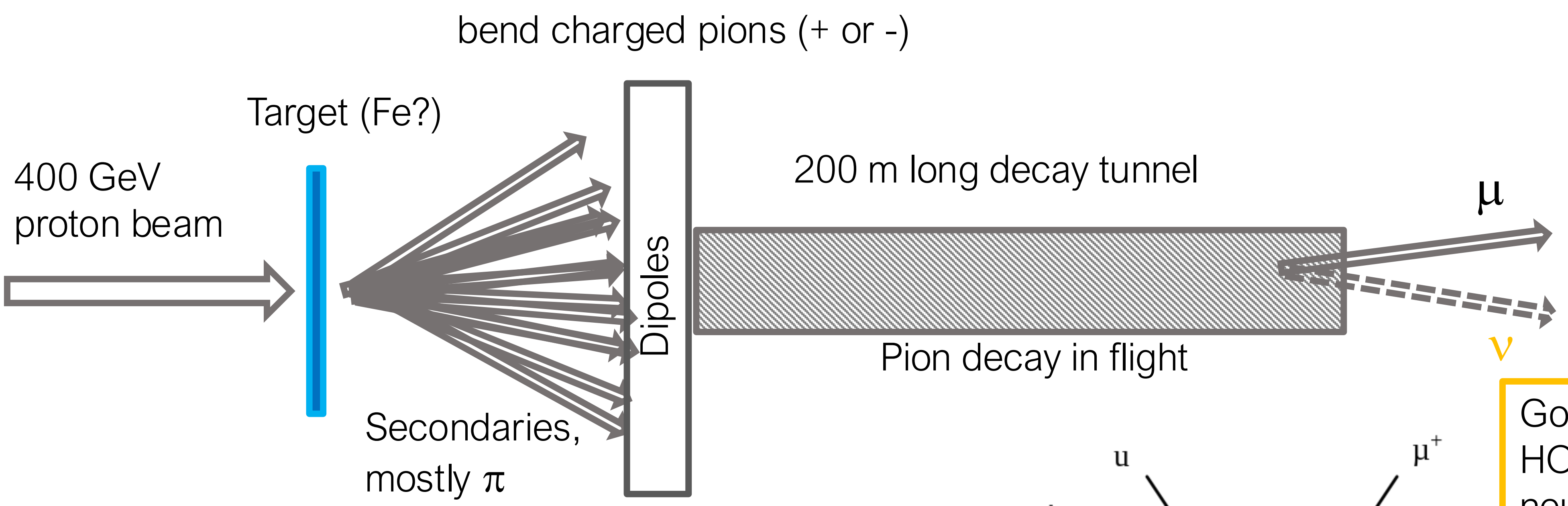
- ➢ In the SLAC experiments, the basic properties of the quark and gluon structure of the hadrons were established.
- ➢ The second and the third generations of experiments are at the basis of the

Quantum Chromodynamics,

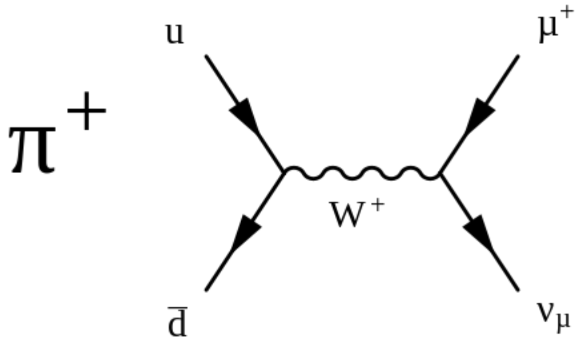
the theory of the strong interaction.

(*) Protons of 400 GeV on a target produced pions which were kept confined in a 200 meters tunnel. During the flight part of the pions decayed into muons which were collected into a beam with energies up to 300 GeV.

Producing Muon Beams



Goes ~undetected
HOWEVER may give a
neutrino beam



$\pi^0 \rightarrow \gamma + \gamma$

p + Fe → X (~1/3 π^+ + 1/3 π^- + 1/3 π^0) + ~ p (1/2 energy)

$\pi^- \rightarrow \mu^- + \bar{\nu}$

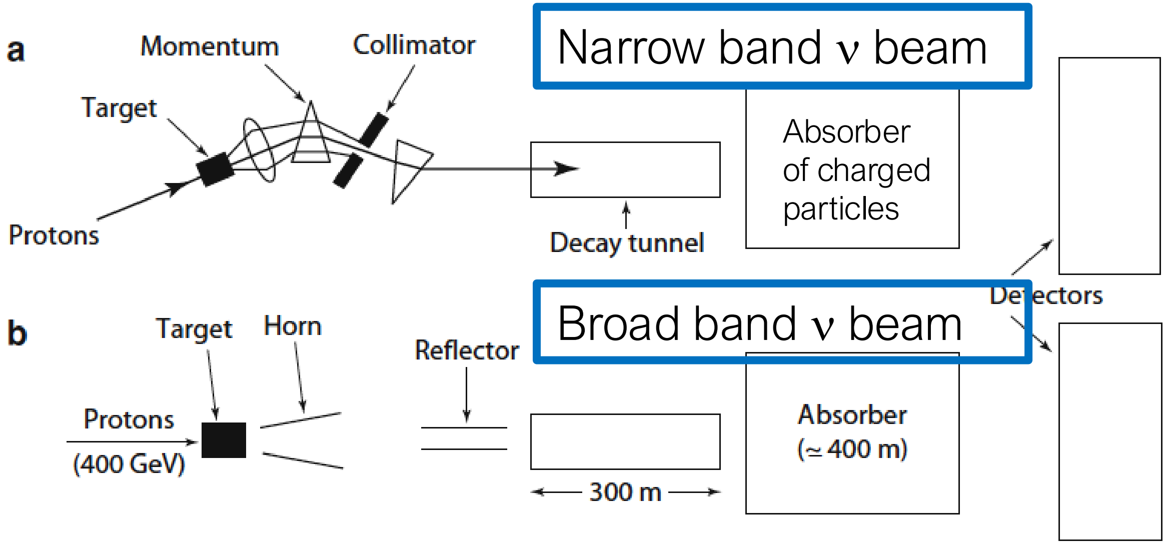
$\pi^+ \rightarrow \mu^+ + \nu$

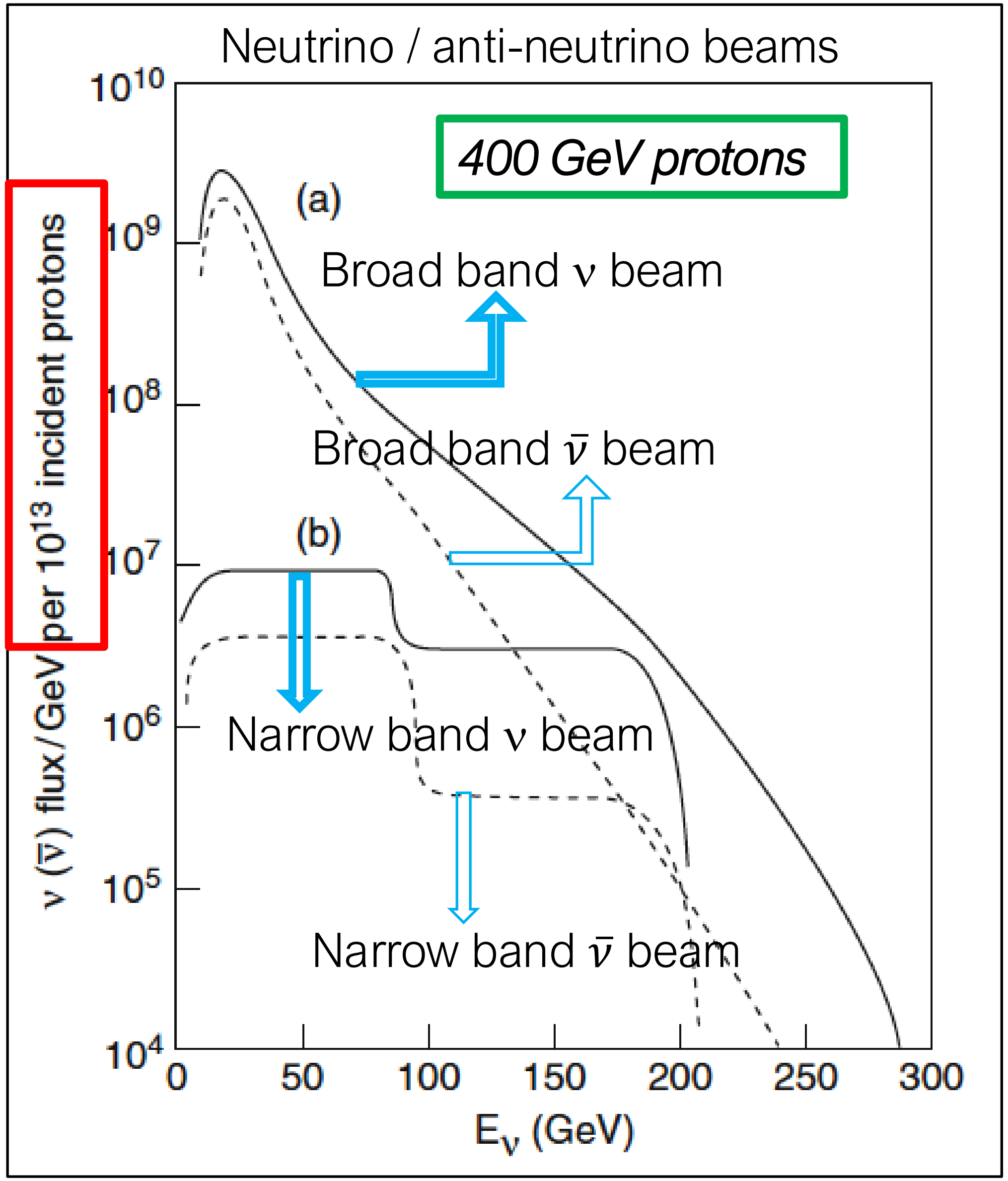
π^+ *gives* ν *and* μ^+

π^- *gives* $\bar{\nu}$ *and* μ^-

Producing Neutrino Beams

$$\pi^- \rightarrow \mu^- + \bar{\nu}$$
$$\pi^+ \rightarrow \mu^+ + \nu$$





Narrow band ν beam: ~ selected in momentum ~ low intensity
Broad band ν beam: ~ not selected in momentum ~ high intensity

Experiments:

The mean free path in iron of 10GeV neutrinos is $\lambda \approx 2.6 \cdot 10^9 Km$ (~ 20 cm for hadrons!). This means that only a very small fraction $3 \cdot 10^{-13}$ of 10 GeV neutrinos interact in a meter of iron. With a flux of 10^{12} neutrinos (for 10^{13} accelerated protons incident on the target), there are only 0.3 interactions in one meter of iron.

→ very long and massive detectors

Hera, Hadron-Electron Ring in Desy-DE

Circular $e + p$ accelerator @ Desy, Hamburg-DE.

- 15 to 30 m underground and circumference of 6.3 km. Leptons and protons → two independent rings

- At HERA, 27.5 GeV ***electrons (or positrons***) collided with 920 GeV protons, cms energy of 318 GeV (*).
- electrons or positrons: 450 MeV, 7.5 GeV, 14 GeV, 27.5 GeV.
- Protons: 50 MeV, 7 GeV, 40 GeV, 920 GeV.

- 4 interaction regions, 4 experiments H1, ZEUS, HERMES and Hera-B.
- About 40 minutes to fill the machine

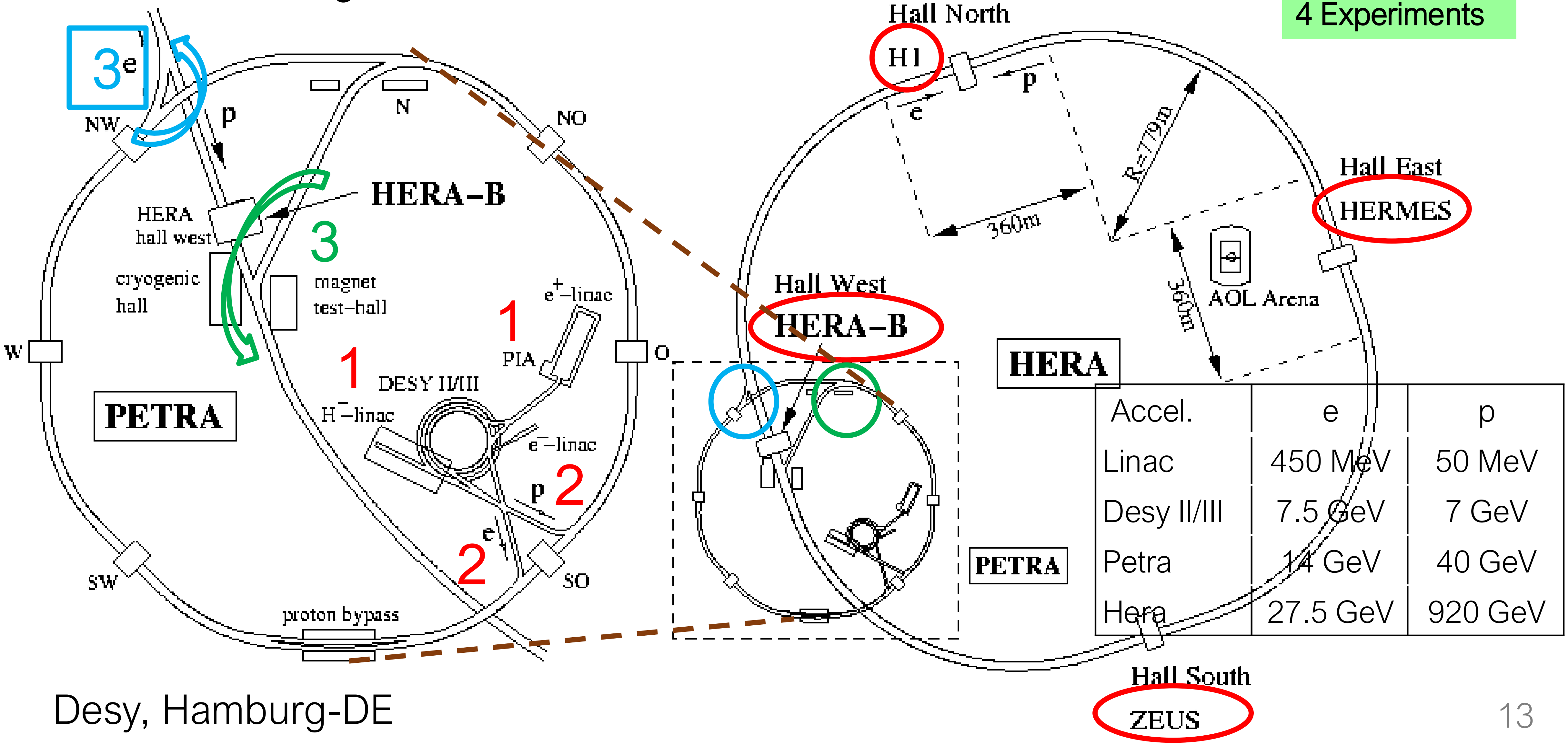
- Operated between 1992 and 2007.

$$(*)\ E_{cm}(or\ cms) = \sqrt{m_p^2 + m_e^2 + 2E_pE_e(1-\beta_1\beta_2\cos(\theta))} \approx \sqrt{2E_pE_e \cdot 2}$$

HERA Accelerator Complex

Three stage acceleration: Linac and *Petra and then Hera*

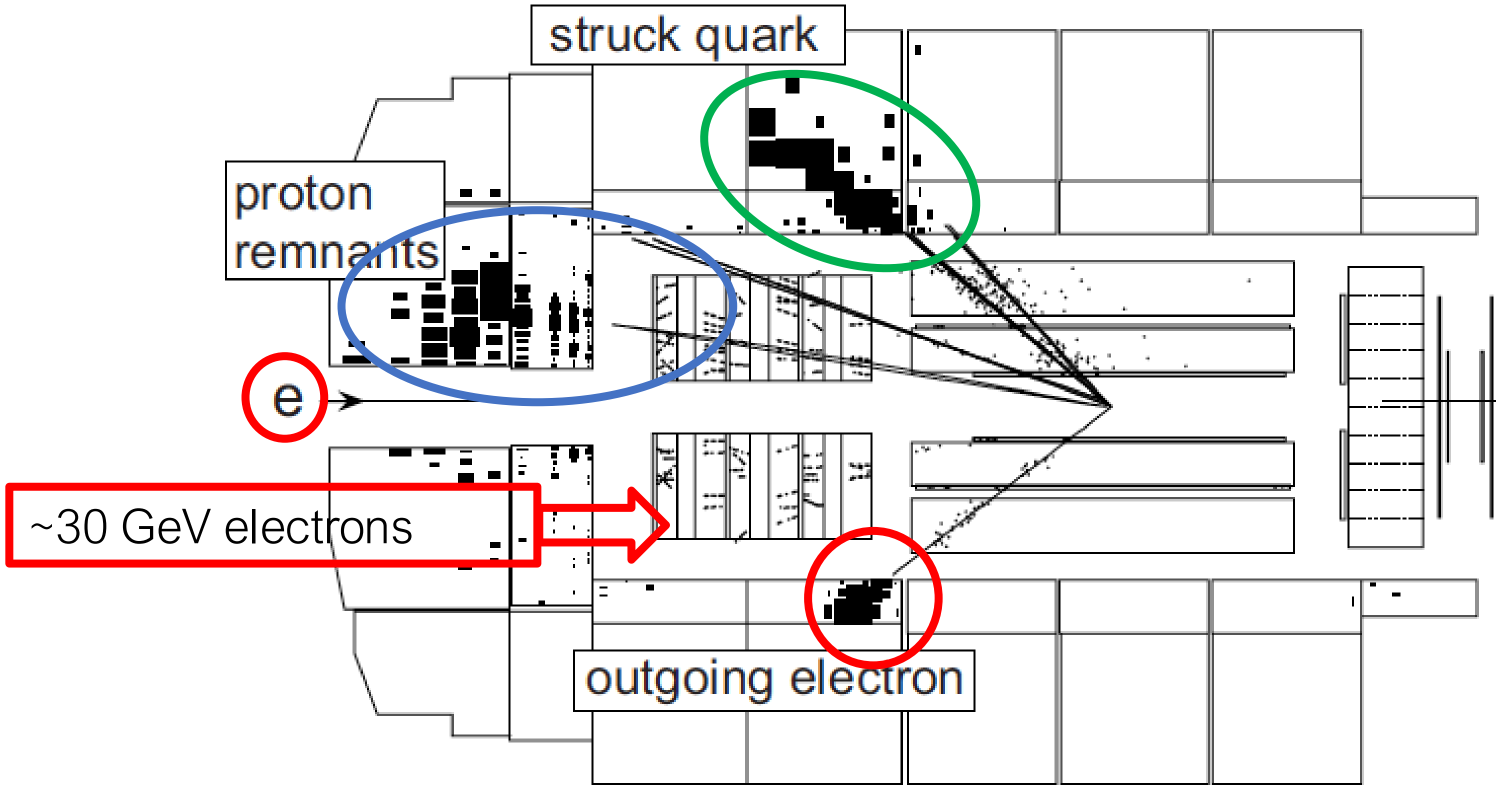
4 Experiments



Accel.	e	p
Linac	450 MeV	50 MeV
Desy II/III	7.5 GeV	7 GeV
Petra	14 GeV	40 GeV
Hera	27.5 GeV	920 GeV

Desy, Hamburg-DE

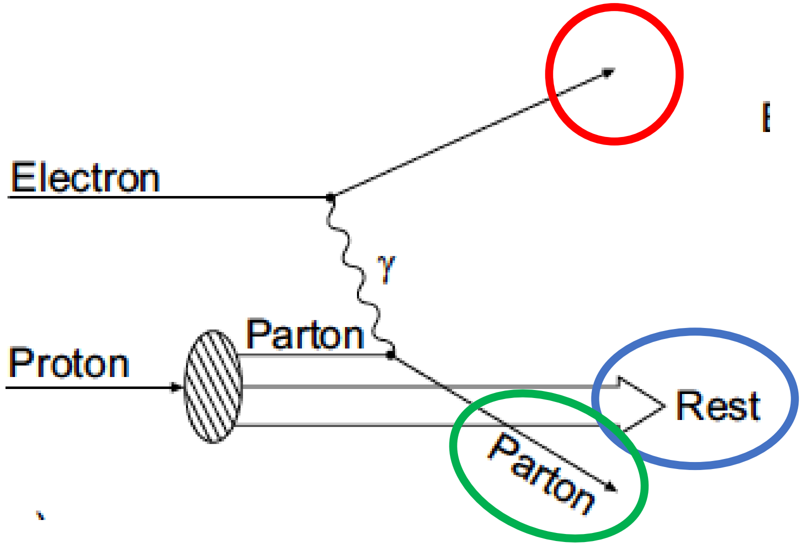
Display of one DIS event in Hera



To calculate the momentum transfer Q^2 and the **energy loss** ν=E'-E, the **energy** and the **scattering angle** of the electron have to be determined in the experiment.

Very asymmetric event topology!

Up to 920 GeV protons

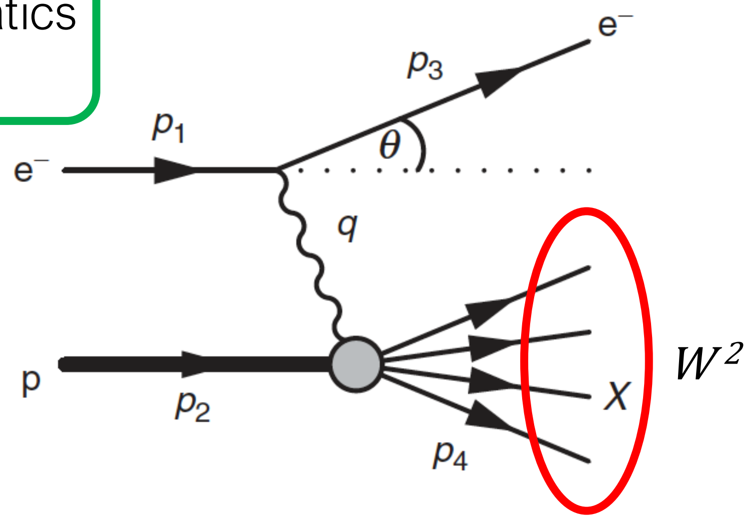


The direction of all charged particles is measured in the inner tracking detector. The energy of the scattered electron is measured in the electromagnetic calorimeter, that of the hadrons in the hadron calorimeter.

DIS Variables

$$p_1 = (E_1, 0, 0, E_1), \quad p_3 = (E_3, E_3 \sin\theta, 0, E_3\cos\theta) \text{ and } q = (E_1 - E_3, \mathbf{p}_1 - \mathbf{p}_3),$$
$$p_2 = (m_p, 0, 0, 0)$$

- Elastic Scattering e against point-like proton: 1 variable only determines kinematics
- Inelastic Scattering electron against *complex* proton: 2 variables are needed.



Possible choices:

- Q^2 negative of the 4-momentum of the virtual photon $\boldsymbol{Q^2 = -q^2}$;

$$Q^2 = -(p_1 - p_3)^2 = -2m_e^2 + 2p_1 \cdot p_3 = -2m_e^2 + 2E_1E_3 - 2\mathrm{p}_1\mathrm{p}_3\cos\theta.$$

At high Q^2 the electron mass can be neglected and $p \sim E$

$$Q^2 \approx 2E_1E_3(1 - \cos\theta) = 4E_1E_3\sin^2\frac{\theta}{2},$$

- Bjorken $\boldsymbol{x} = \dfrac{\boldsymbol{Q^2}}{\boldsymbol{2p_2 \cdot q}}$

Get physical meaning by computing the invariant mass of the hadronic system W^2

$$W^2 \equiv p_4^2 = (q + p_2)^2 = q^2 + 2p_2 \cdot q + p_2^2$$
$$\Rightarrow \quad W^2 + Q^2 - m_p^2 = 2p_2 \cdot q, \quad \rightarrow \quad x = \frac{Q^2}{Q^2 + W^2 - m_p^2}$$

Elasticity of the interaction

If $\boldsymbol{x = 1} \rightarrow W^2 = m_p^2 \rightarrow$ elastic scattering

$$0 \le x \le 1.$$

You always have one baryon (p lightest) $\quad W^2 \equiv p_4^2 \ge m_p^2$

dimensionless

DIS Variables

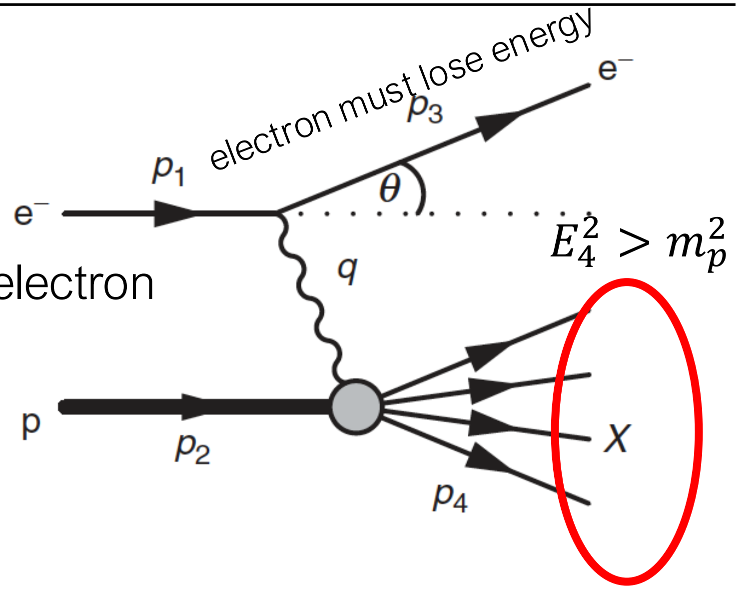
The inelasticity y

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}.$$

dimensionless

In the frame where the proton is at rest

$p_1 = (E_1, 0, 0, E_1)$ incoming electron
$p_2 = (m_p, 0, 0, 0)$ proton at rest
$p_3 = (E_3, E_3 \cdot \sin(\theta), 0, E_3 \cdot \cos(\theta))$ outgoing electron
$q = (E_1 - E_3, \boldsymbol{p_1} - \boldsymbol{p_3})$



$$y = \frac{m_p(E_1 - E_3)}{m_p E_1} = 1 - \frac{E_3}{E_1}$$

Relative energy lost by the e (%)

$$0 \leq y \leq 1.$$

The electron energy loss ν

$$\nu \equiv \frac{p_2 \cdot q}{m_p},$$

In the system where the proton is at rest ν is the energy lost by the incoming electron

$$\nu = E_1 - E_3,$$

DIS at ~Low Q^2

$p_1 = (E_1, 0,0, E_1)$ incoming electron
$p_2 = (m_p, 0,0,0)$ proton at rest
$p_3 = (E_3, E_3 \cdot \sin(\theta), 0, E_3 \cdot \cos(\theta))$
$q = (E_1 - E_3, \boldsymbol{p_1} - \boldsymbol{p_3})$

At ~low Q^2 both elastic and inelastic scattering can happen

Fixed target experiment at DESY (Germany)

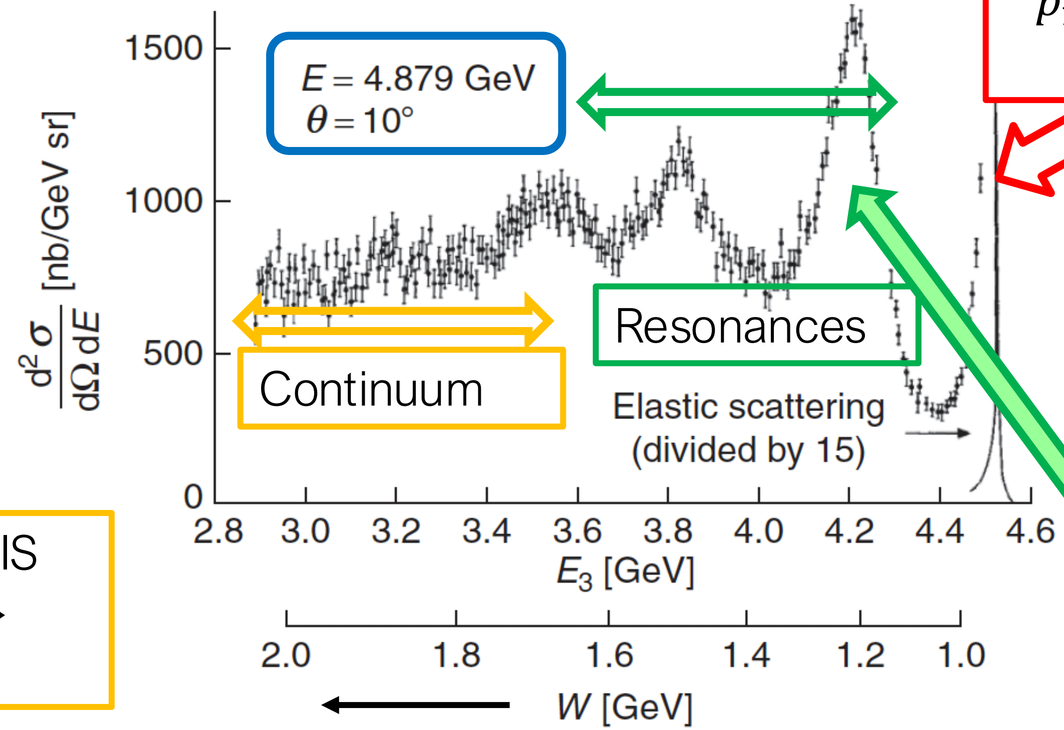
2 variables needed to describe kinematics $\rightarrow \frac{d^2\sigma}{d\Omega dE} \rightarrow E, \Omega\ (\rightarrow \theta)$: Final electron energy & electron scattering angle

$$W^2 = (p_2 + q)^2 = p_2^2 + 2p_2 \cdot q + q^2 = m_p^2 + 2p_2 \cdot (p_1 - p_3) + (p_1 - p_3)^2$$
$$\approx \left[m_p^2 + 2m_p E_1\right] - 2\left[m_p + E_1(1 - \cos\theta)\right] E_3.$$

resonances = excited bound states of the proton (quarks composition uud), which subsequently decay strongly. Width Γ, lifetime τ

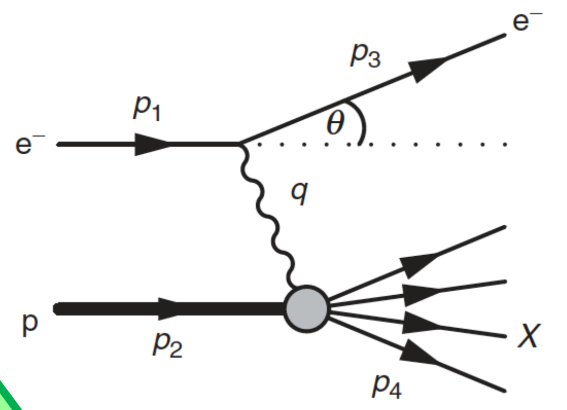
$$\Gamma = \frac{1}{\tau}$$

The continuum: start of DIS → the proton is broken → multi-particle final states.



$p_1 = 4.879, \theta = 10°, E_3 \approx 4.5\ GeV \rightarrow m_p$
$\rightarrow$ *elastic scattering*

$\Delta^+ \rightarrow p\pi^0$



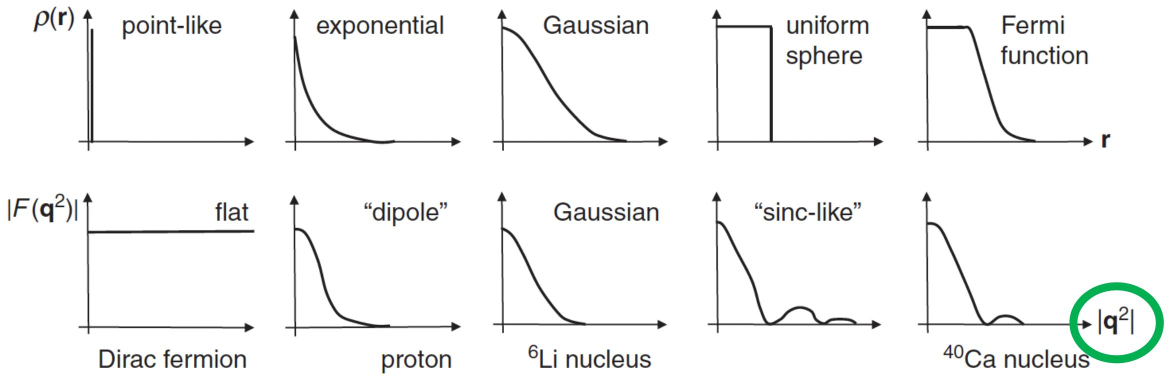
Low Q^2 DIS measurements

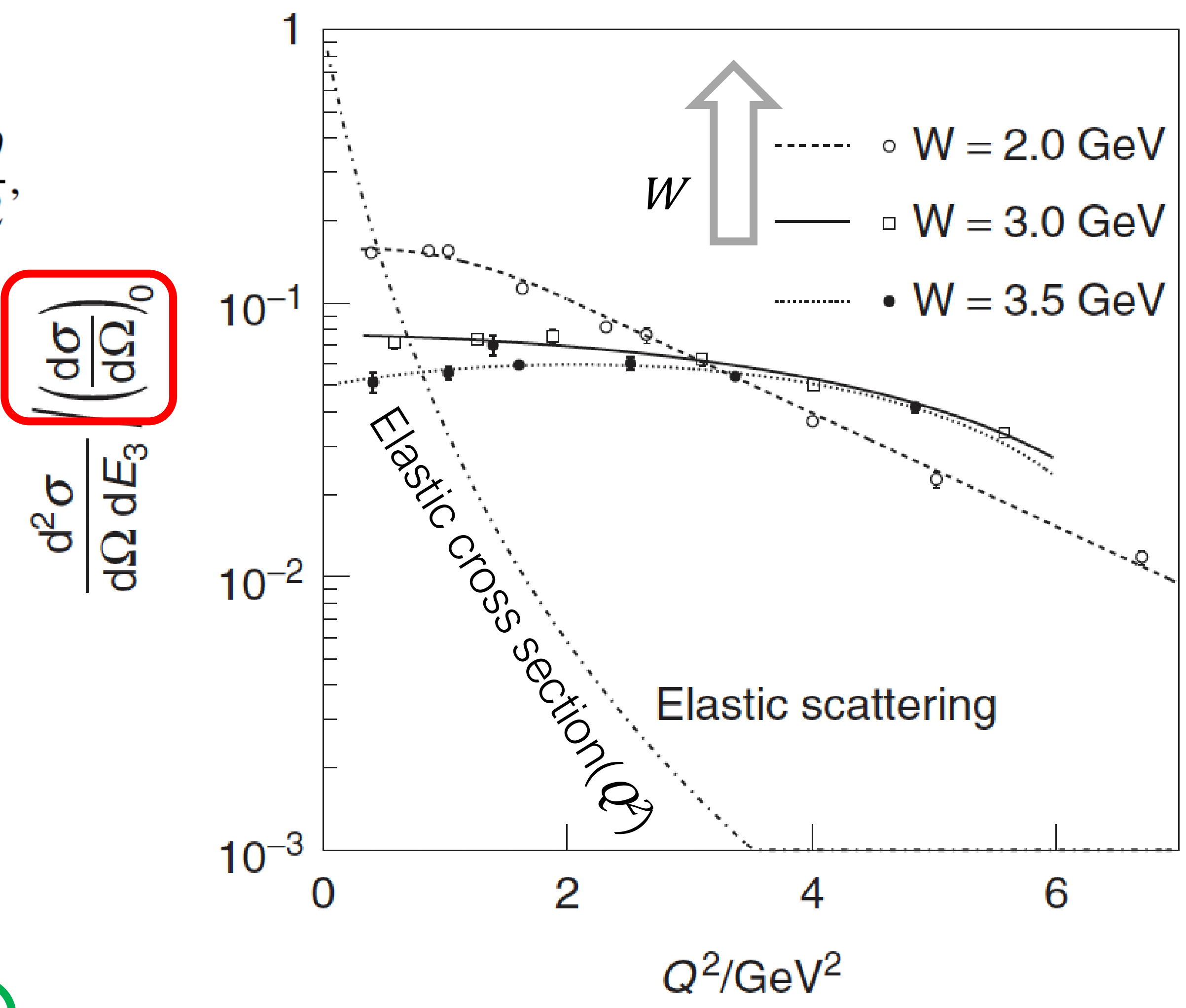
$$\frac{d\sigma}{d\Omega} = \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right)\left(\frac{d\sigma}{d\Omega}\right)_0$$

Mott cross section (point-like proton)

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)}\left(\frac{E_3}{E_1}\right)\cos^2\frac{\theta}{2},$$

- At high values of W → dependence on Q^2 much weaker than for elastic scattering;
- The higher the mass (→ more in DIS region) the smaller the dependence;
- Qualitatively explained by Form Factor $\frac{F_{Dirac}(Q^2)}{F_{Proton}(Q^2)}$





Rosenbluth Formula Rewritten (elastic *ep* scattering)

Rosenbluth formula is the most general expression for the ***elastic*** scattering cross section

$$ep \rightarrow ep$$

Puts into evidence electric and magnetic form factors

$$G_E, G_M$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

$$\tau = \frac{Q^2}{4m_p^2}$$

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}.$$

Can be rewritten by

- Using Q^2 and y (for elastic scattering x=1 and y depends only on Q^2)

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

- introducing two new functions: $f_1(Q^2)$, magnetic only, and $f_2(Q^2)$, magnetic and electric;

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

Elastic scattering, proton doesn't break, $x=1 \rightarrow y$ depends on Q^2 only

$$x = \frac{Q^2}{2p_2 \cdot q}$$

$$1 = \frac{Q^2}{2p_2 \cdot q} \rightarrow 2p_2 \cdot q = Q^2$$

$$y = \frac{Q^2}{2 \cdot p_2 \cdot p_1} \quad \boxed{p_2, p_1\ fixed}$$

Structure Functions: Inelastic Scattering

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

Elastic scattering: 1 variable $f(Q^2)$

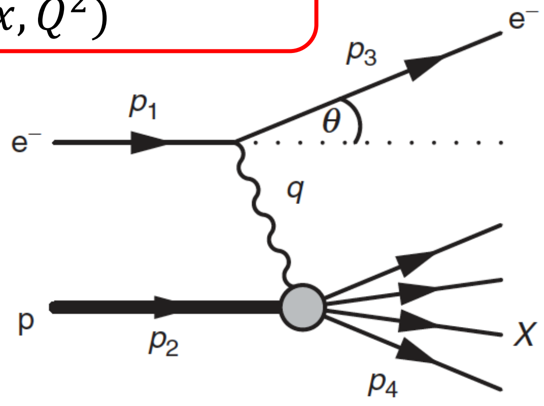
Lorenz Invariant

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$

Inelastic scattering: 2 variables $F(x, Q^2)$

$F_2(x, Q^2)$ Electric & magnetic term

$F_1(x, Q^2)$ Purely magnetic term



If $\tau = m_p^2 y^2 / Q^2 \ll 1$ $(DIS\ region)$

$$\frac{d^2\sigma}{dx\, dQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1 - y)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right].$$

In a fixed target experiment $e^- p$ the values of $Q^2, x\ and\ y$ can be determined by the measurement of the energy and direction of the scattered electron,E_3, θ

$$Q^2 = 4E_1 E_3 \sin^2\frac{\theta}{2}, \quad x = \frac{Q^2}{2m_p(E_1 - E_3)} \quad \text{and} \quad y = 1 - \frac{E_3}{E_1}$$

The double differential cross section $\frac{d^2\sigma}{dx dQ^2}$ can me constructed by counting how many events you have between

$$x \to x + \Delta x,\ Q^2 \to Q^2 + \Delta Q^2$$

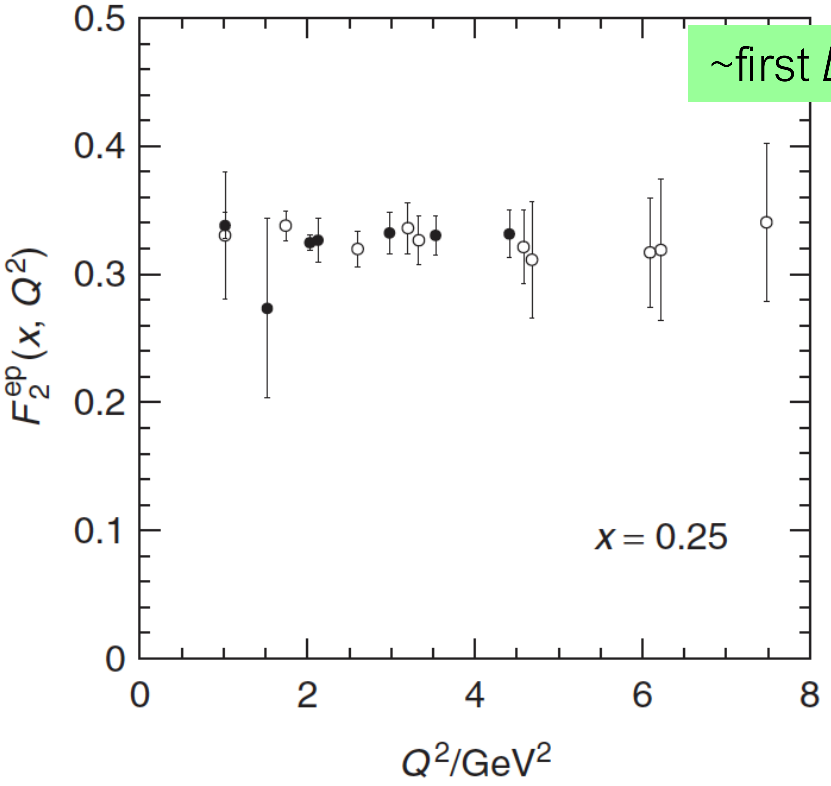
$\frac{d^2\sigma}{dx dQ^2}$ →separate the contributions from $F1(x, Q^2)$ and $F2(x, Q^2)$

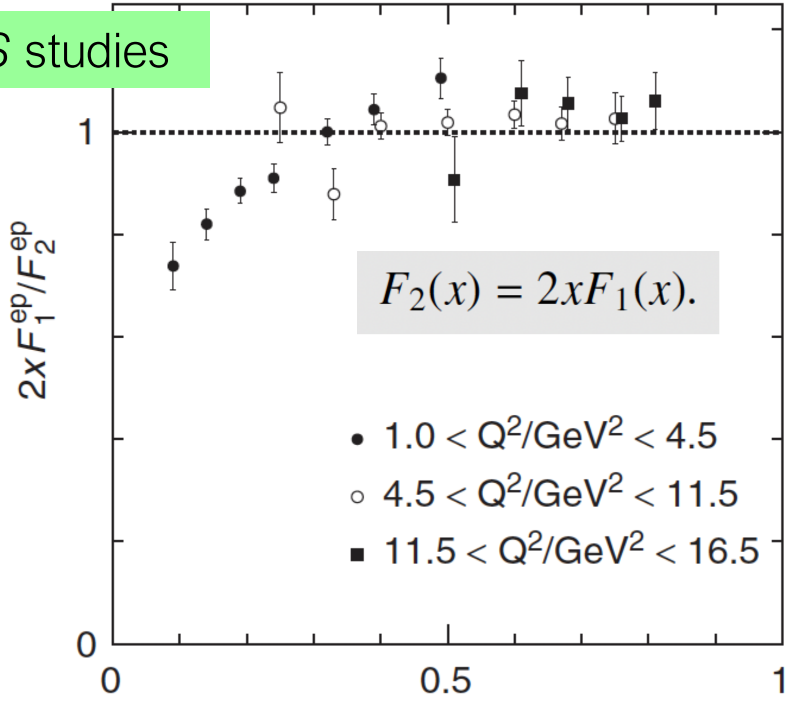
Structure Functions & Scaling Violations

F_1 and F_2 in inelastic scattering
$ep \rightarrow e + X$

- Experiment at the Stanford Linear Accelerator Centre (SLAC) in California.
- e^- between 5 GeV and 20 GeV on a hydrogen target.
- The scattering angle and energy of the electron was measured using a large movable spectrometer.

~first *DIS* studies





$F_1(x, Q_2)$ and $F_2(x, Q_2)$ are (almost) independent of Q^2

$$F_1(x, Q^2) \rightarrow F_1(x) \quad \text{and} \quad F_2(x, Q^2) \rightarrow F_2(x).$$

→ Point-like scattering object

$$F_2(x) = 2xF_1(x).$$

Implies: ep *inelastic* scattering = *elastic* scattering of eq pointlike spin 1/2 constituent of the p. Electric and magnetic contributions are defined by the Dirac equation

$eq \to eq$ elastic scattering

Compute Matrix element using the technique introduced before for computing $\mathcal{M}$ in

- $e^+e^- \to \mu^+\mu^-$
- $e^-p \to e^-p$ (Rutherford & Mott & Rosenbluth formulas)

1. Define currents

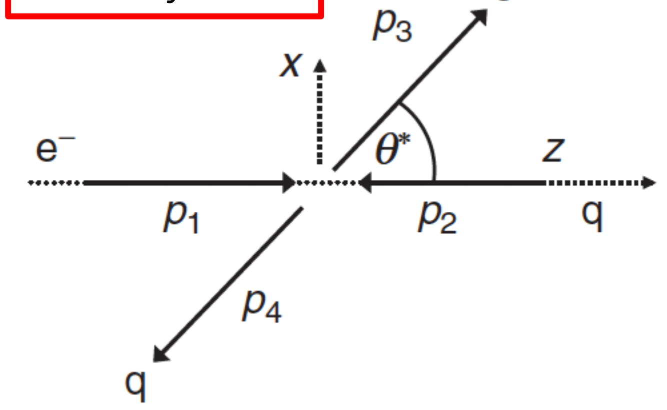
$$\overline{u}(p_4)[-iQ_q e\gamma^\nu]u(p_2)$$

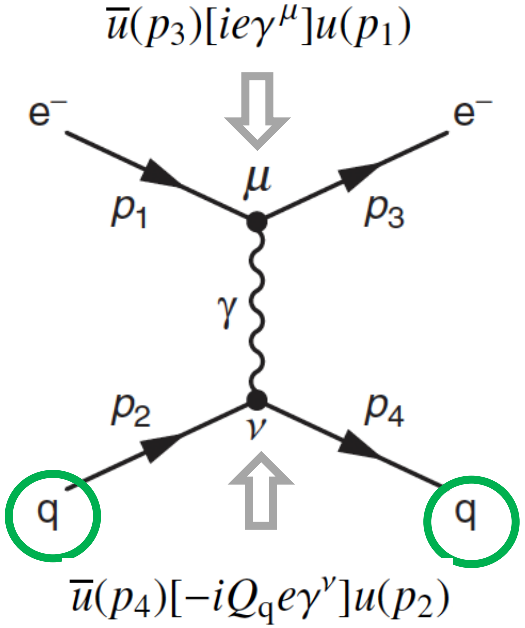
$$\overline{u}(p_3)[ie\gamma^\mu]u(p_1)$$

2. Write $\mathcal{M}$

$$\mathcal{M}_{fi} = \frac{Q_q e^2}{q^2}\left[\overline{u}(p_3)\gamma^\mu u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_4)\gamma^\nu u(p_2)\right]$$

CMS system





$t = q^2$

$s = p_1 + p_2$, $t = p_1 - p_3$ and $u = p_1 - p_4$.

Neglect masses

Result is (look at the textbook): $$\langle |\mathcal{M}_{fi}|^2 \rangle = 2Q_q^2 e^4 \left(\frac{s^2 + u^2}{t^2}\right) = 2Q_q^2 e^4 \frac{(p_1.p_2)^2 + (p_1.p_4)^2}{(p_1.p_3)^2},$$

$eq \rightarrow eq$ Calculation in the CMS System

In the CMS system →

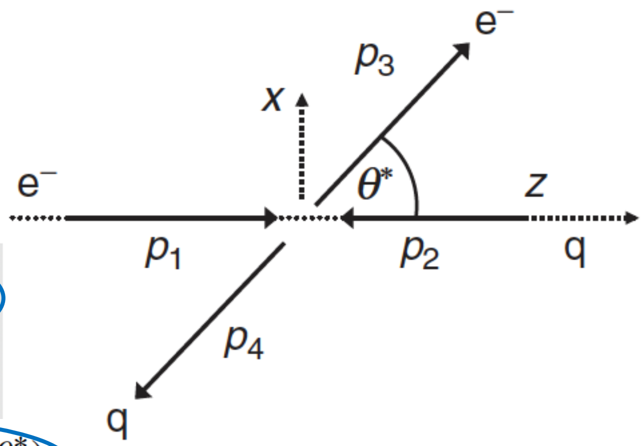
$$energy_{CMS} = \sqrt{s}$$

$$define\ E = \frac{\sqrt{s}}{2}$$

→

$-E_q^* = E_e^* = E$ (in the relativistic region $E = \mathrm{p}$)

$$p_1 = (E, 0, 0, +E), \quad p_3 = (E, +E\sin\theta^*, 0, +E\cos\theta^*),$$
$$p_2 = (E, 0, 0, -E), \quad p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*).$$



$$\langle|\mathcal{M}_{fi}|^2\rangle = 2Q_q^2 e^4 \left(\frac{s^2+u^2}{t^2}\right) = 2Q_q^2 e^4 \frac{(p_1.p_2)^2 + (p_1.p_4)^2}{(p_1.p_3)^2},$$

$p_1 \cdot p_2 = 2E^2$, $p_1 \cdot p_3 = E^2(1-\cos\theta^*)$ and $p_1 \cdot p_4 = E^2(1+\cos\theta^*)$.

$$\langle|M_{fi}|^2\rangle = 2Q_q^2 e^4 \frac{4E^4 + E^4(1+\cos\theta^*)^2}{E^4(1-\cos\theta^*)^2}$$

Spin averaged !

Discussed this formula in early lecture

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}|\mathcal{M}_{fi}|^2,$$

$$\frac{d\sigma}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s}\frac{\left[1+\frac{1}{4}(1+\cos\theta^*)^2\right]}{(1-\cos\theta^*)^2}$$

Spin Allowed States in $eq \to eq$

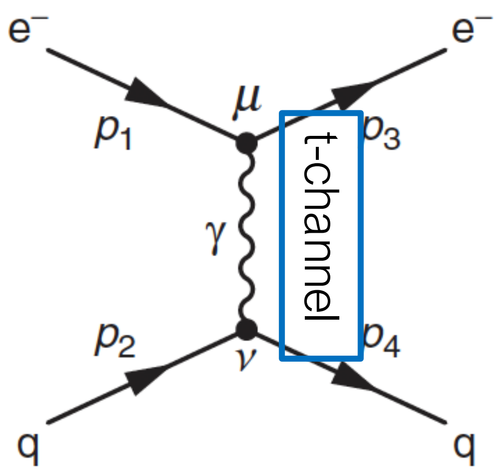
Do we understand these angular terms?

$$\frac{d\sigma}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s} \frac{\left[1 + \frac{1}{4}(1+\cos\theta^*)^2\right]}{(1-\cos\theta^*)^2}$$

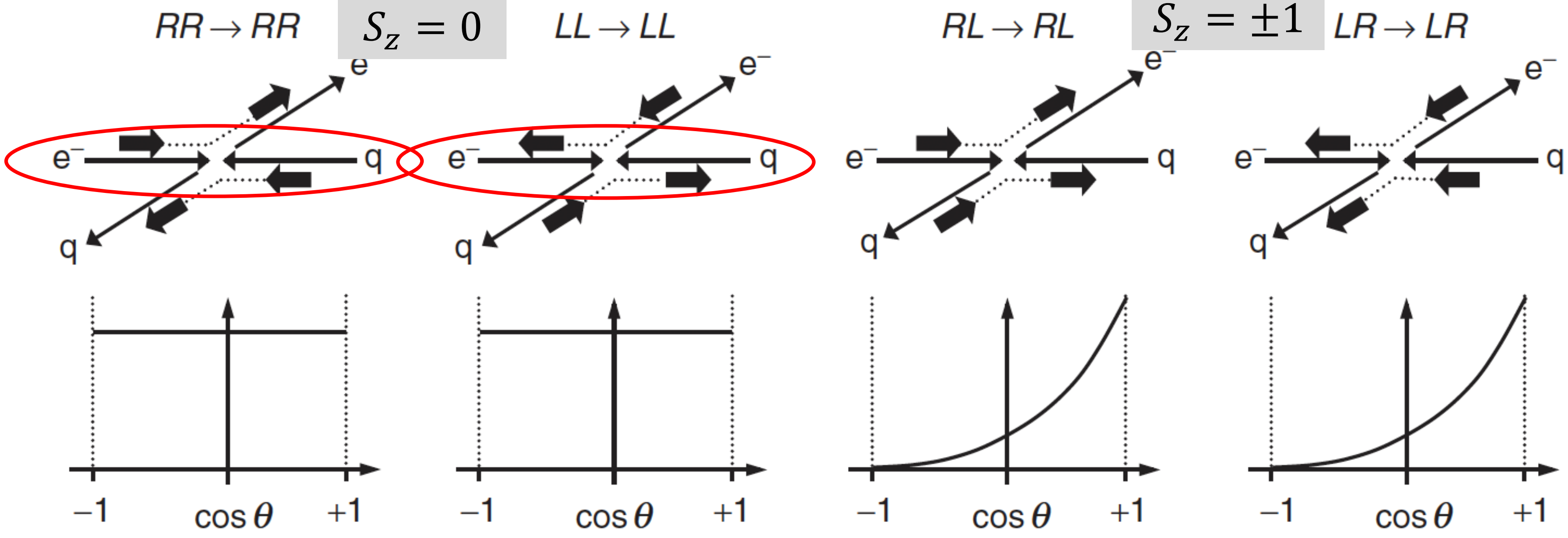
Helicity is conserved in high energy QED interactions

Propagator effect

$$q^2 = t = (p_1 - p_3)^2 \approx -E^2(1-\cos\theta^*)$$



The only terms that are non-zero:

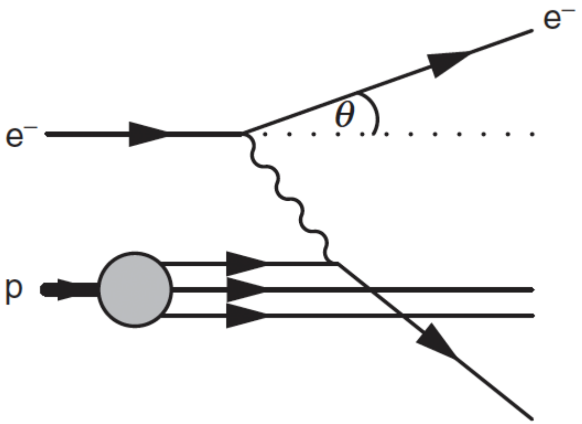


→ no preferred direction

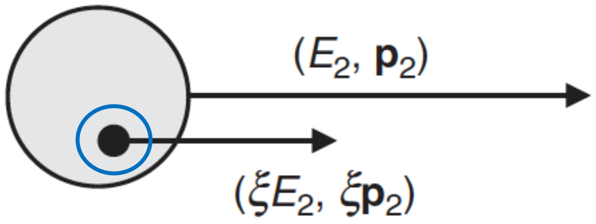
Angular distribution $^1/_4\,(cos\theta^*)^2$

The Quark-Parton Model

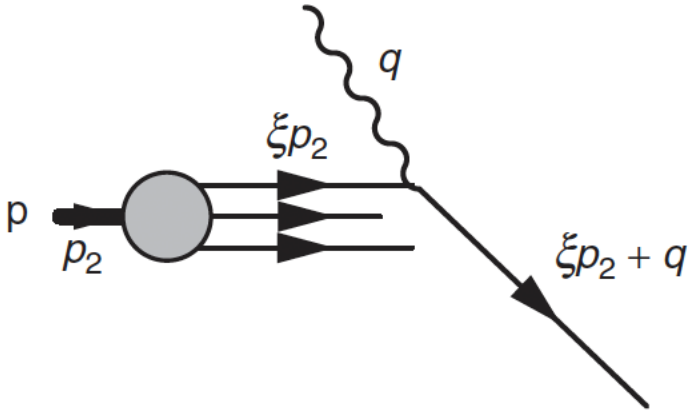
DIS studied in a fast moving frame such that masses can be neglected ("*Infinite momentum frame*")



Proton 4-momentum: $(E_2, \boldsymbol{p_2})$



$$p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2)$$



One 'component' (quark) inside the proton; carries a fraction ξ of the proton energy E_2

$$\boldsymbol{x} = \frac{\boldsymbol{Q^2}}{\boldsymbol{2p_2 \cdot q}}$$

After the interaction the quark

$$\xi p_2 \rightarrow \xi p_2 + q$$

(remember $p^2 = m^2$)

$$\xi = \frac{-q^2}{2p_2 \cdot q} = \frac{Q^2}{2p_2 \cdot q} \equiv x.$$

$$(\xi p_2)^2 = m_q^2$$
$$(\xi p_2 + q)^2 = m_q^2$$

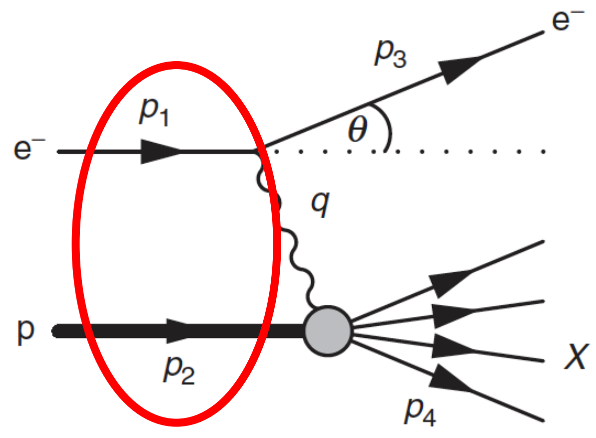
$$(\xi p_2 + q)^2 = \xi^2 p_2^2 + 2\xi p_2 \cdot q + q^2 = m_q^2$$

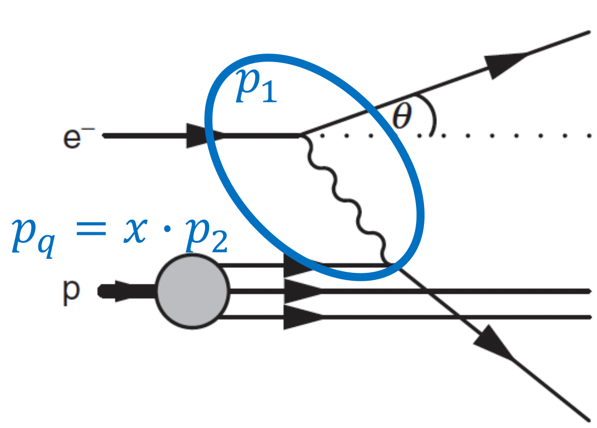
$$\rightarrow 2\xi p_2 \cdot q + q^2 = 0$$

$\boldsymbol{x}$ is the fraction of p_2 carried by the quark

ep scattering vs eq scattering: kinematic variables

Question: how to correlate variables of ep scattering vs eq scattering?
Study DIS variables.





Centre Of Mass energy s of the ep system:

$$s = (p_1 + p_2)^2 \approx 2p_1 \cdot p_2$$

Centre Of Mass energy s_q of the eq system:

$$s_q = (p_1 + x \cdot p_2)^2 \approx 2p_1 \cdot x \cdot p_2 = x \cdot s$$

$$\frac{d\sigma}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s} \frac{\left[1 + \frac{1}{4}(1 + \cos\theta^*)^2\right]}{(1 - \cos\theta^*)^2}$$

Compute x and y in the ep system:

$$y = p_2 \cdot q / p_2 \cdot p_1$$

$$x = Q^2 / 2 \cdot p_2 \cdot q$$

and in the eq system:

$$y_q = p_q \cdot q / p_q \cdot p_1 = p_2 \cdot x \cdot q / p_2 \cdot x \cdot p_1 = y$$

$$x_q = x = 1$$ (elastic scattering)

Rewritten in terms of q^2 and s

eq elastic scattering

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right].$$

Elaborate on eq scattering

ep **elastic scattering**

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1+\left(1+\frac{q^2}{s}\right)^2\right].$$

eq **elastic scattering**

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1+\left(1+\frac{q^2}{s_q}\right)^2\right]$$

$$Q^2 = (s - m_p^2)xy.$$

$$q^2 = -Q^2 = -(s_q - m_q^2)x_q y_q$$

Neglect m_q

$y_q = y$

$x_q = x = 1$

$$\frac{q^2}{s_q} = -x_q y_q = -y.$$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1+(1-y)^2\right]$$

Remember $Q^2 = -q^2$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right],$$

$$\frac{d^2\sigma}{dx\,dQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

$$F_2^{ep}(x,Q^2) = 2xF_1^{ep}(x,Q^2) = x\sum_i Q_i^2 q_i^p(x)$$

Parton Distribution Functions

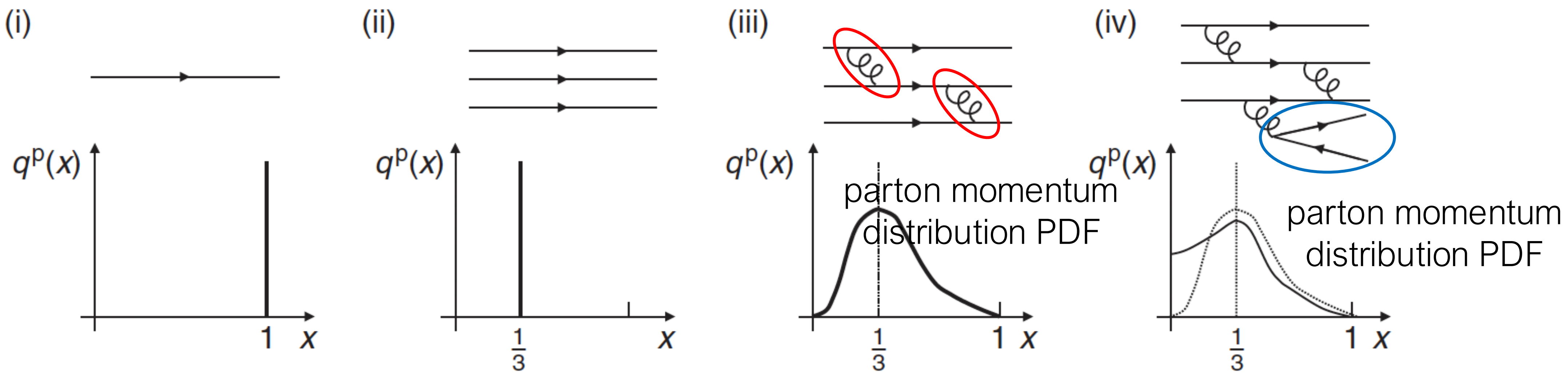
Quarks interact inside the proton by exchanging *gluons*

- Exchange of momentum → distribution of x is smeared
- gluons radiate $q\bar{q}$ pairs (mostly low momentum, propagator $\approx \frac{1}{q^2}$)

$u^p(x)\delta x$ number of up-quarks inside the proton between x and x + δx

Proton

PDFs are not a priori known and have to be obtained from experiment.



a single point-like particle

three static quarks each 1/3 of p_{proton}

three interacting quarks which can exchange momentum

interacting quarks including higher-order diagrams

More on the Parton Model

$q_i^p(x)$: *how many quarks of type i and charge* Q_i *in the interval* $x \to x + \delta x$

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right] \times Q_i^2 q_i^p(x)\delta x$$

Quark i

- charge Q_i and
- # quarks i with momentum fraction $q_i^p(x), \cdot x \cdot \delta x$

- Divide by δx
- Sum over all quark flavours
  $\to ep$ scattering double differential cross section

$$\frac{d^2\sigma^{ep}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]\sum_i Q_i^2 q_i^p(x).$$

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x,Q^2)}{x} + y^2 F_1^{ep}(x,Q^2)\right]$$

Compare with expression containing F_1^{ep} *and* F_2^{ep}

$$F_2^{ep}(x,Q^2) = 2xF_1^{ep}(x,Q^2) = x\sum_i Q_i^2 q_i^p(x).$$

The parton model:

- underlying process is elastic point-like objects: $eq \to$ no (strong) Q^2 dependence expected;
- $F1$ *and* F_2 can be written as functions of x alone: $F_1(x, Q^2) \to F_1(x)$ and $F_2(x, Q^2) \to F_2(x)$.
- $F_1(x) = 2xF_2(x)$: elastic underlying process scattering between spin 1/2 Dirac particles; the quark magnetic moment fixes structure functions are fixed with respect to one another.

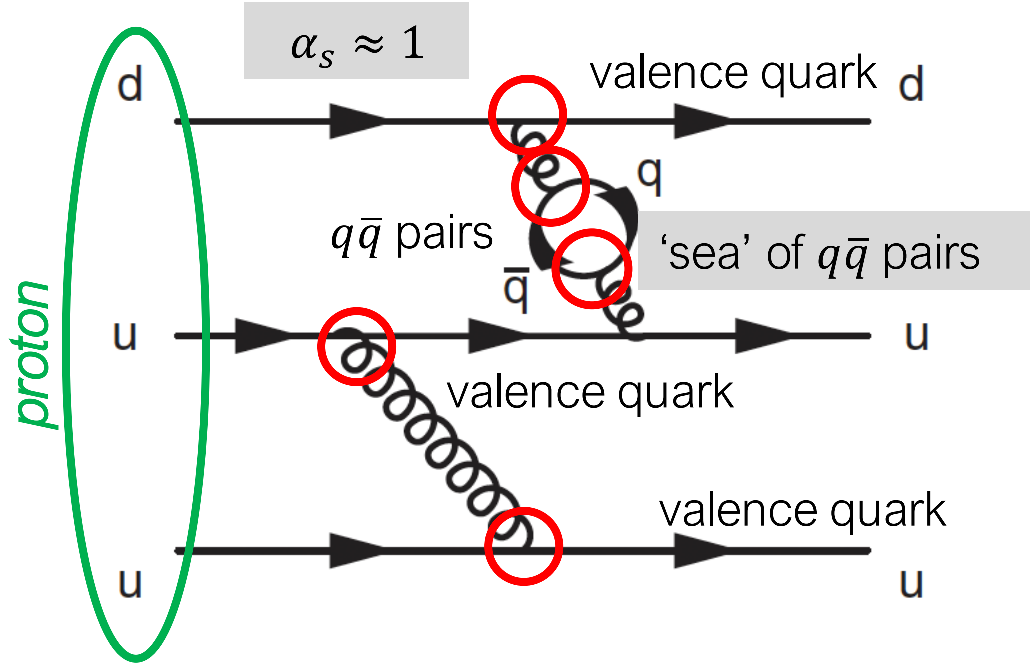
Determining Parton Structure Functions

How to determine Parton Structure Functions?

Problems:

- $\alpha_s \approx 1 \rightarrow$ cannot do a perturbation expansion (coupling constant has to be small);
- The proton consists of components bound together → components interact with each other;
- Gluons couple to quarks → $q\bar{q}$ pairs are created/annihilated

→ Structure Functions have to be measured, cannot get them from *'first principles'*



- $q\bar{q}$ pairs are mostly created at low x values due to $1/q^2$ gluon propagator;
- ep inelastic scattering due to quarks and antiquarks;
- Quarks of all types may contribute but relevance decreases with increasing quark masses → $s\bar{s}$ can already be neglected (small contribution, even smaller from heavier quarks).

Targets in F_2^{ep} and F_2^{en}

$$p = (uud), n = (ddu)$$
$$F_2^{ep} \rightarrow Liquid\ Hydrogen$$
$$F_2^{en} \rightarrow Deuterium\ \ (p + n)$$
$$F_2^{en} \approx 2F_2^{ed} - F_2^{ep}$$

Determining Parton Structure Functions - 2

Write explicitly:

$$F_2^{\mathrm{ep}}(x, Q^2) = 2xF_1^{\mathrm{ep}}(x, Q^2) = x \sum_i Q_i^2 q_i^{\mathrm{p}}(x).$$

Isospin Symmetry see later!

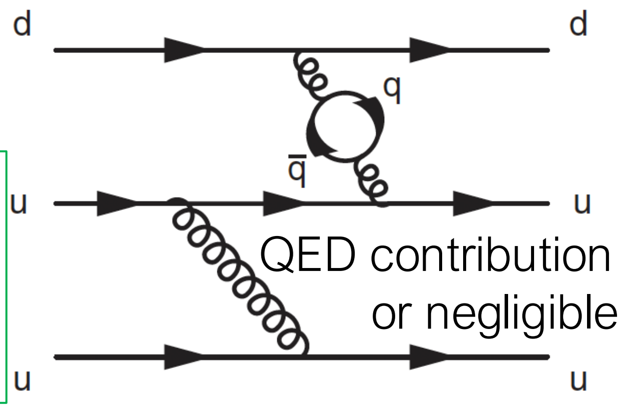
- In principle $q_i^p \neq q_i^n$
- $u^p(x)$ x-distribution of 'up' quarks in the proton (shape + integral)
- $u^p(x)$ = valence quarks + sea quarks
- $\overline{u^p}(x)$ only sea quarks

Charge[2]

$$F_2^{\mathrm{ep}}(x) = x \sum_i Q_i^2 q_i^{\mathrm{p}}(x) \approx x\left(\frac{4}{9}u^{\mathrm{p}}(x) + \frac{1}{9}d^{\mathrm{p}}(x) + \frac{4}{9}\overline{u}^{\mathrm{p}}(x) + \frac{1}{9}\overline{d}^{\mathrm{p}}(x)\right)$$

$$F_2^{\mathrm{en}}(x) = x \sum_i Q_i^2 q_i^{\mathrm{n}}(x) \approx x\left(\frac{4}{9}u^{\mathrm{n}}(x) + \frac{1}{9}d^{\mathrm{n}}(x) + \frac{4}{9}\overline{u}^{\mathrm{n}}(x) + \frac{1}{9}\overline{d}^{\mathrm{n}}(x)\right)$$

Protons and neutrons have very similar characteristics (isospin symmetry)
→ can exchange up and down



Not only gluons, also photons but $\alpha_s \approx 1 \gg \alpha_{EM}$

$$d^{\mathrm{n}}(x) = u^{\mathrm{p}}(x) \equiv u(x) \quad \text{and} \quad u^{\mathrm{n}}(x) = d^{\mathrm{p}}(x) \equiv d(x)$$

$$\overline{d}^{\mathrm{n}}(x) = \overline{u}^{\mathrm{p}}(x) \equiv \overline{u}(x) \quad \text{and} \quad \overline{u}^{\mathrm{n}}(x) = \overline{d}^{\mathrm{p}}(x) \equiv \overline{d}(x)$$

Determining Parton Structure Functions - 3

$$F_2^{ep}(x) = 2xF_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right)$$

$$F_2^{en}(x) = 2xF_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right)$$

$u^n(x) \rightarrow d^p(x)$

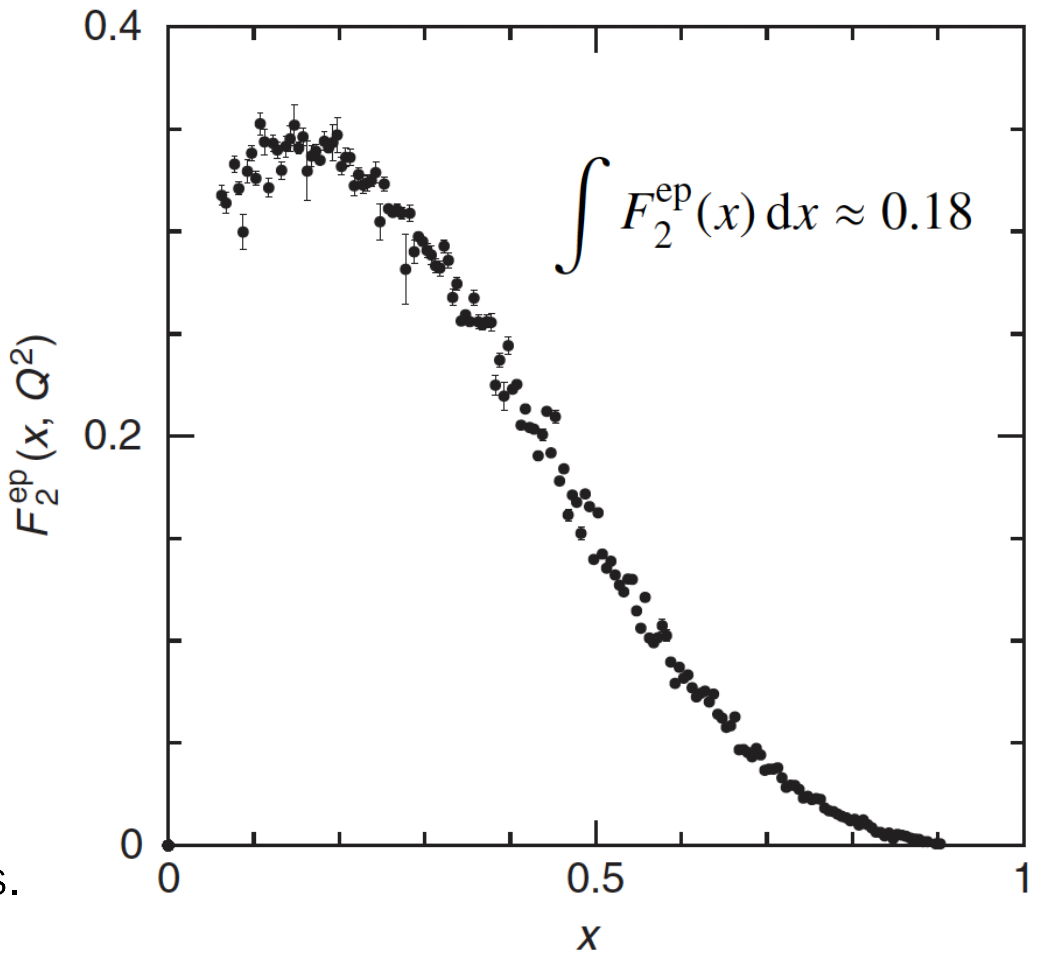
Integral, not shape

$$\int_0^1 F_2^{ep}(x)\,dx = \frac{4}{9}f_u + \frac{1}{9}f_d \quad \text{and} \quad \int_0^1 F_2^{en}(x)\,dx = \frac{4}{9}f_d + \frac{1}{9}f_u$$

charge^2 charge^2

$$f_u = \int_0^1 [xu(x) + x\overline{u}(x)]\,dx \quad \text{and} \quad f_d = \int_0^1 [xd(x) + x\overline{d}(x)]\,dx$$

f_u fraction of proton momentum carried by up- and anti-up quarks.
f_d fraction carried by the down-/anti-down quarks.



f_u and f_d obtained directly from the experimental measurements of the proton and neutron structure functions.

Integral of F_2^{en} and F_2^{ep}

$$\int F_2^{\mathrm{ep}}(x)\,\mathrm{d}x \approx 0.18 \qquad \int F_2^{\mathrm{en}}(x)\,\mathrm{d}x \approx 0.12$$

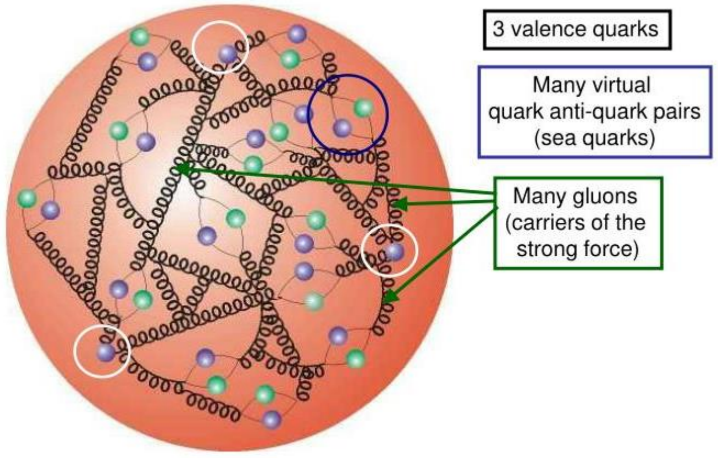
$$\int_0^1 F_2^{\mathrm{ep}}(x)\,\mathrm{d}x = \frac{4}{9}f_{\mathrm{u}} + \frac{1}{9}f_{\mathrm{d}} \quad \text{and} \quad \int_0^1 F_2^{\mathrm{en}}(x)\,\mathrm{d}x = \frac{4}{9}f_{\mathrm{d}} + \frac{1}{9}f_{\mathrm{u}}$$

Using the relation above you get $f_{\mathrm{u}} \approx 0.36$ and $f_{\mathrm{d}} \approx 0.18$ $\quad f_u/f_d \approx 2/1$

proton = two up-quarks and one down-quark not surprising that $f_u/f_d \approx 2/1$

- ***More important***: the total fraction of the momentum of the proton carried by quarks and antiquarks is ~ 50%;
- the rest is carried by the gluons, mediators of the strong interaction;
- Because the gluons are electrically neutral, they do not contribute to the QED process of electron–proton deep inelastic scattering.

Looking into the Proton from a different point of view



Proton very complex structure.

Gluons → $q\bar{q}$ pairs generated/absorbed → two big types of quarks:

- Valence quarks, determine quantum numbers of the proton;
- Sea quarks, produced in pairs

→

$u(x)$ & $d(x)$ [....] are made of valence and sea quarks at the same time;

$$u(x) = u_V(x) + u_S(x) \quad \text{and} \quad d(x) = d_V(x) + d_S(x).$$

$\bar{u}(x)$ & $\bar{d}(x)$ [....] are made of sea quarks only;

$$\bar{u}(x) \equiv \bar{u}_S(x) \quad \text{and} \quad \bar{d}(x) \equiv \bar{d}_S(x)$$

How many valence $u(x)$ & $d(x)$ quarks inside the proton (uud)?

$$\int_0^1 u_V(x)\,dx = 2 \quad \text{and} \quad \int_0^1 d_V(x)\,dx = 1$$

How many $\bar{u}(x)$ & $\bar{d}(x)$ quarks inside the proton (made of valence quarks only uud)?
Since quarks and antiquarks are produced in pairs and since $m_u \approx m_d$ → equally populated of $u_s, d_s, \bar{u}_s, \bar{d}_s$

$$u_S(x) = \bar{u}_S(x) \approx d_S(x) = \bar{d}_S(x) \approx S(x)$$

Qualitative Arguments F_2^{ep}

If we start from the expression for F_2 derived before

$$F_2^{ep}(x) = 2xF_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right)$$

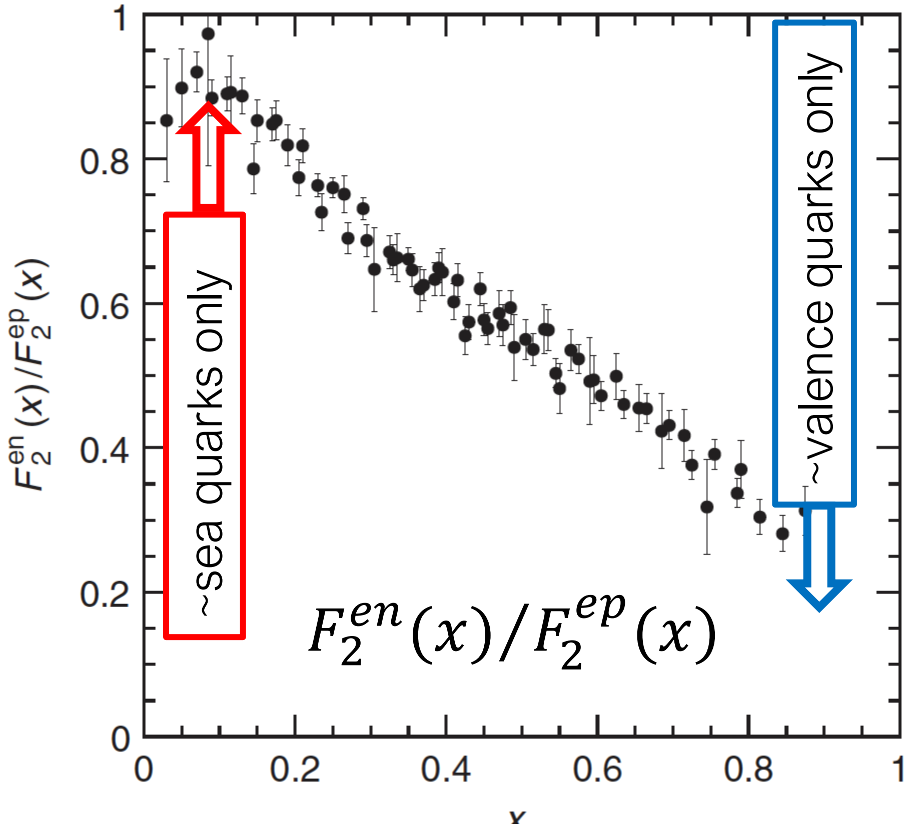
$$F_2^{en}(x) = 2xF_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right)$$

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_V(x) + \frac{1}{9}d_V(x) + \frac{10}{9}S(x)\right)$$

$$F_2^{en}(x) = x\left(\frac{4}{9}d_V(x) + \frac{1}{9}u_V(x) + \frac{10}{9}S(x)\right)$$

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

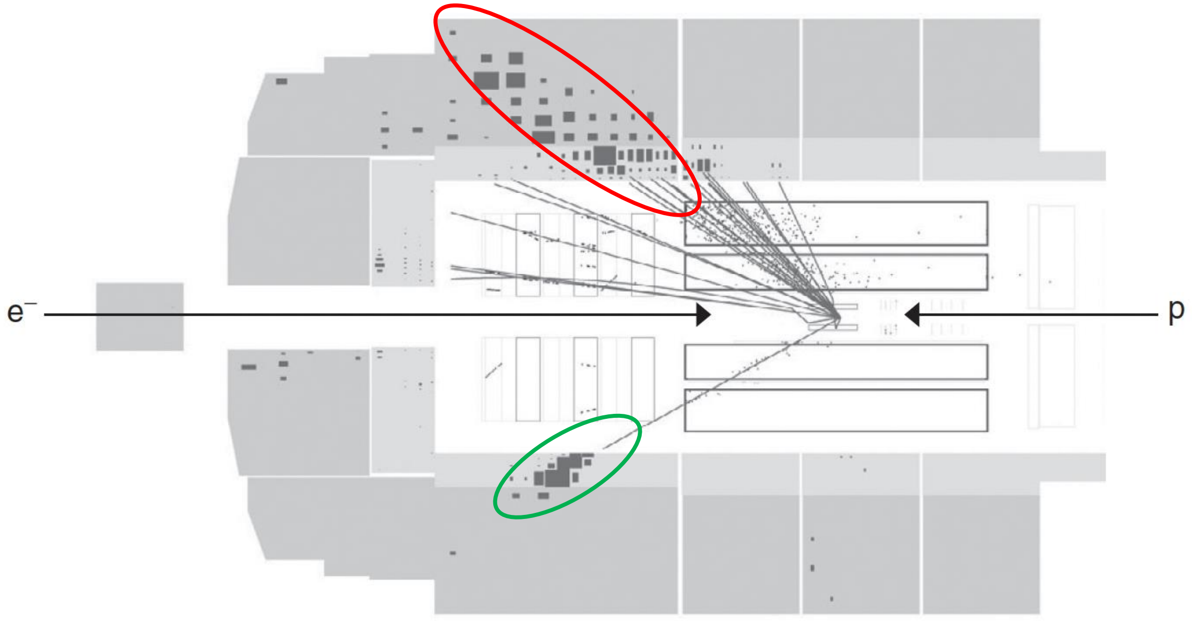
$u_V(x) = 2d_V(x)$



Qualitative arguments:

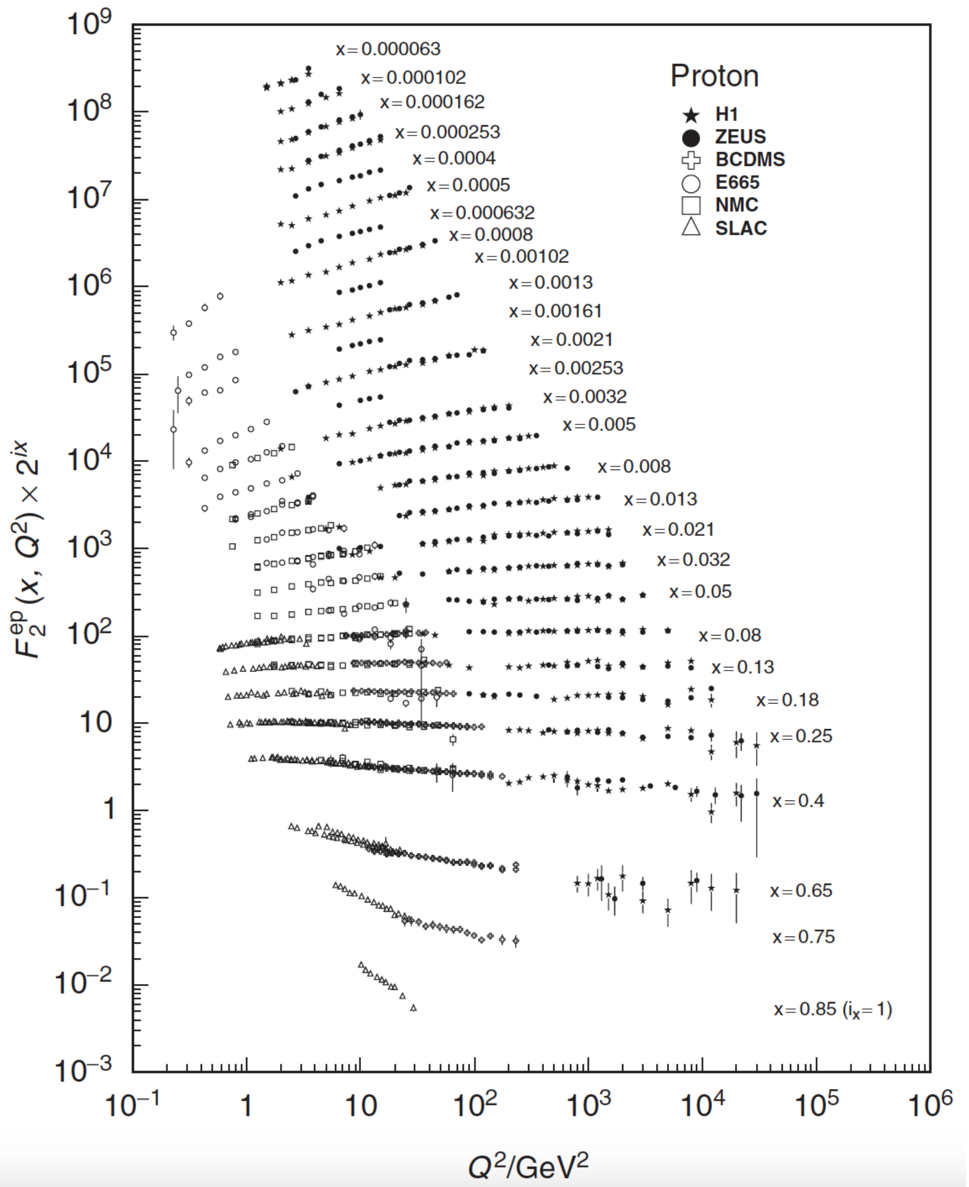
- Sea quarks produced at low x → dominant at x~0 $\qquad \frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 1$ as $x \to 0$. OK!
- Valence quarks mostly at x~1 *($u_V(x) = 2d_V(x)$, (4+2)/8+1) → expect 2/3)* $\qquad \frac{F_2^{en}(x)}{F_2^{ep}(x)} \to \frac{1}{4}$ as $x \to 1$. ?

Structure Functions at Hera

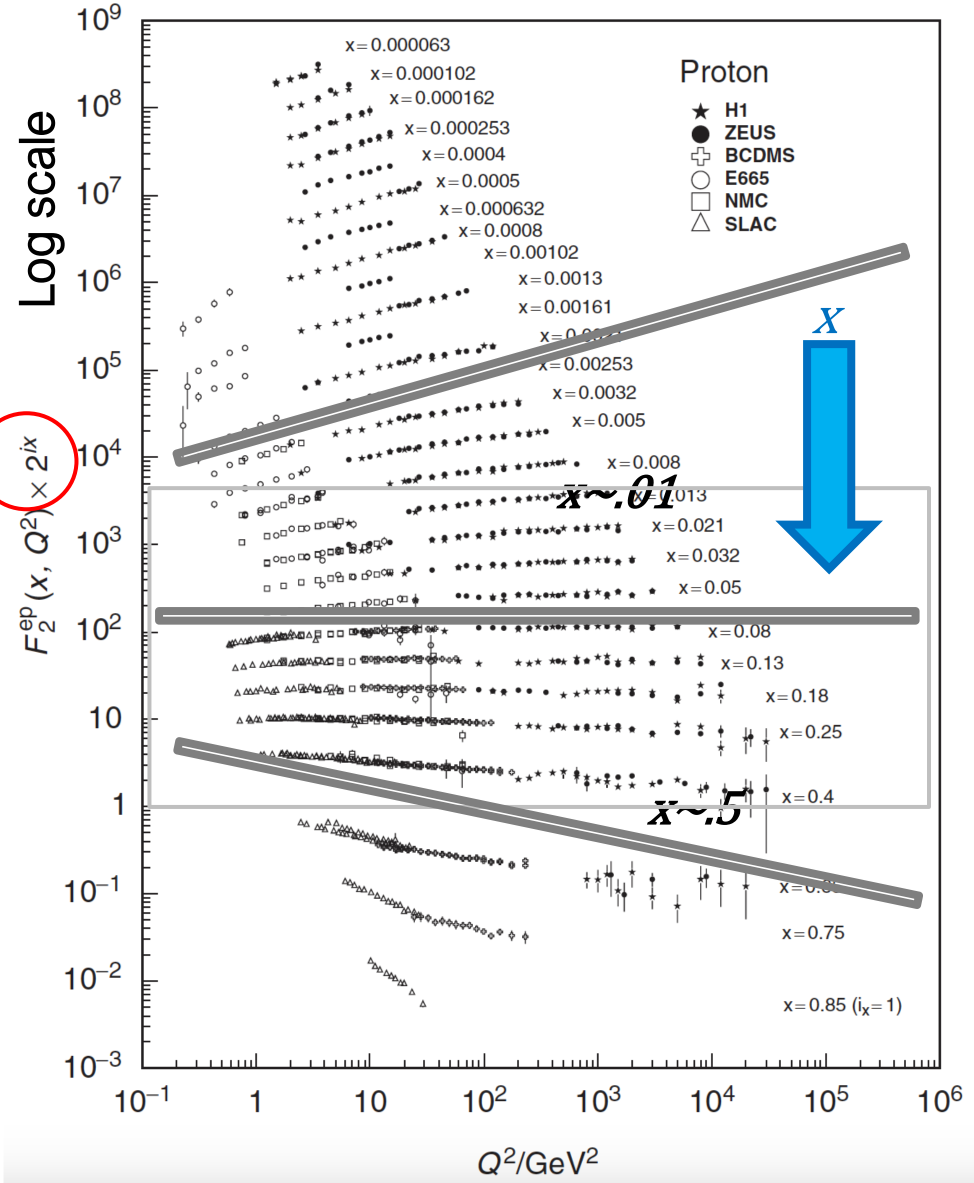


- The final-state hadronic system: a jet of high-energy particles. The energy and direction of this jet of particles is measured badly than
- electron showers;

→ Q^2 and x are determined from the energy and scattering angle of the electron.



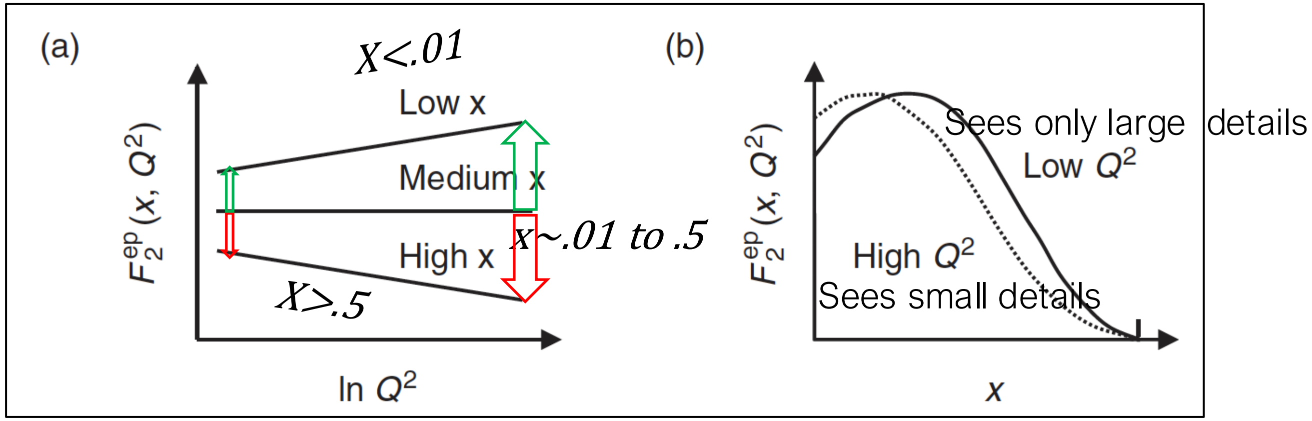
Scaling Violations



We observed that $F_1(x, Q^2) \to F_1(x)$ and $F_2(x, Q^2) \to F_2(x)$.

OK between *x~.01 to .5* → quark point-like object (small dep. on Q^2)

Outside this interval, data show '*scaling violations*'



high Q^2 the measured structure functions → lower values of x relative to the structure functions at low Q^2

scaling violation

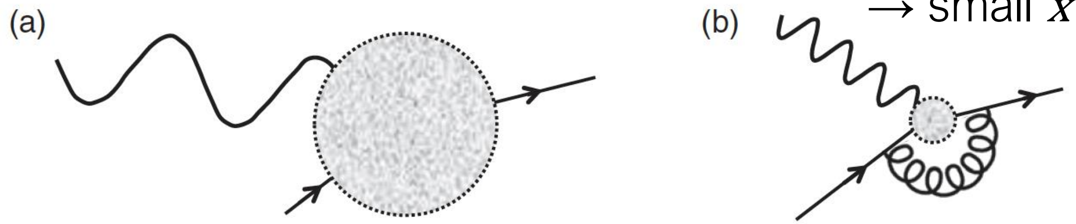
→ at high Q^2, the proton is observed to have a greater fraction of low x quarks

Scaling Violations

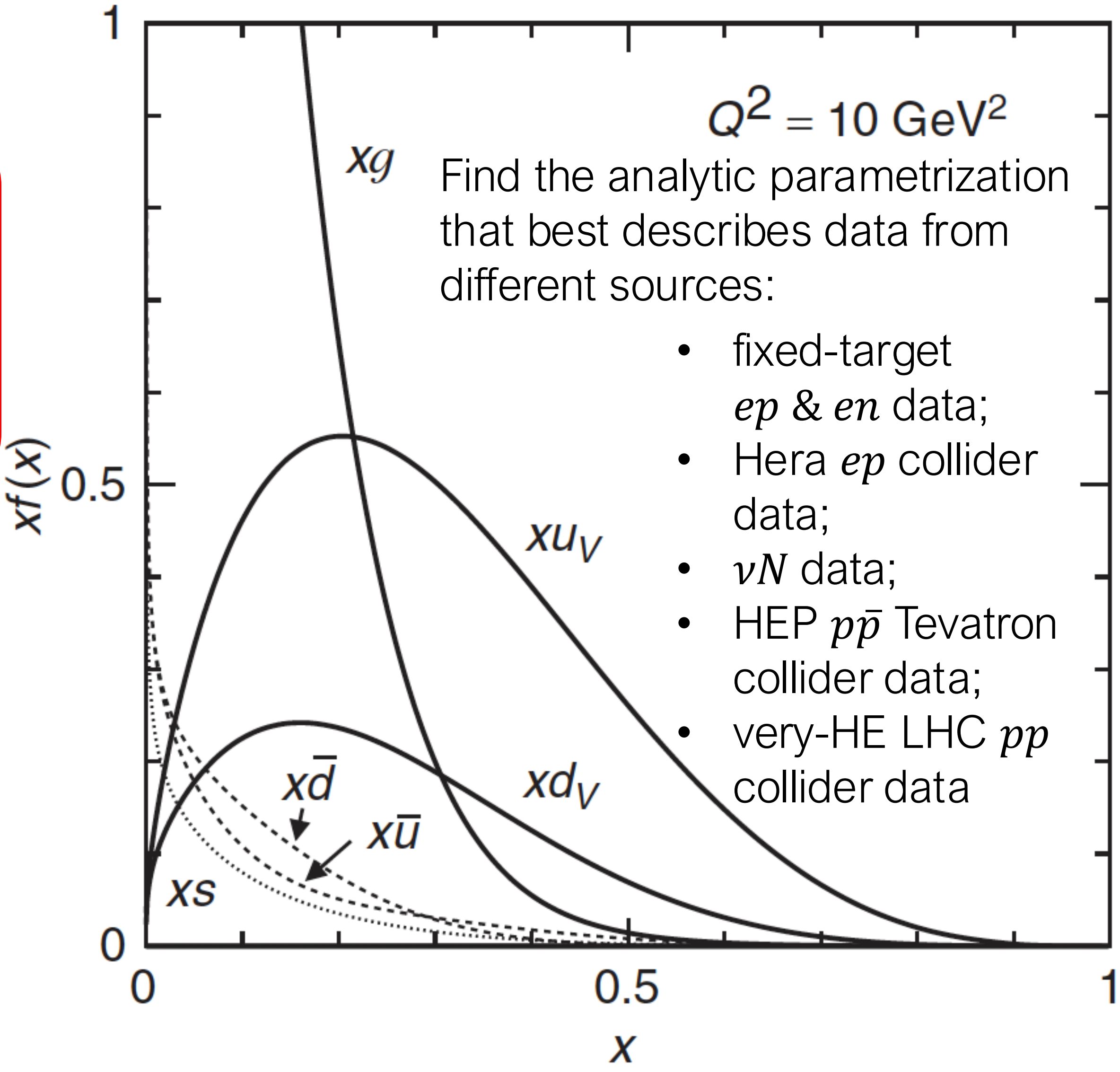
Do we understand why scaling violations?

Low Q^2 photon
→ sees 'little'

High Q^2 photon
→ sees '$q \to q + g$
→ small x



- proton PDFs cannot be calculated from first principles;
- the Q^2 dependence of the PDFs is calculable DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) equations (QCD).
- These equations are based on parton splitting functions for the QCD processes $q \to qg$ and $g \to q\bar{q}$



Find the analytic parametrization that best describes data from different sources:

- fixed-target ep & en data;
- Hera ep collider data;
- νN data;
- HEP $p\bar{p}$ Tevatron collider data;
- very-HE LHC pp collider data

Scaling Violations in DIS

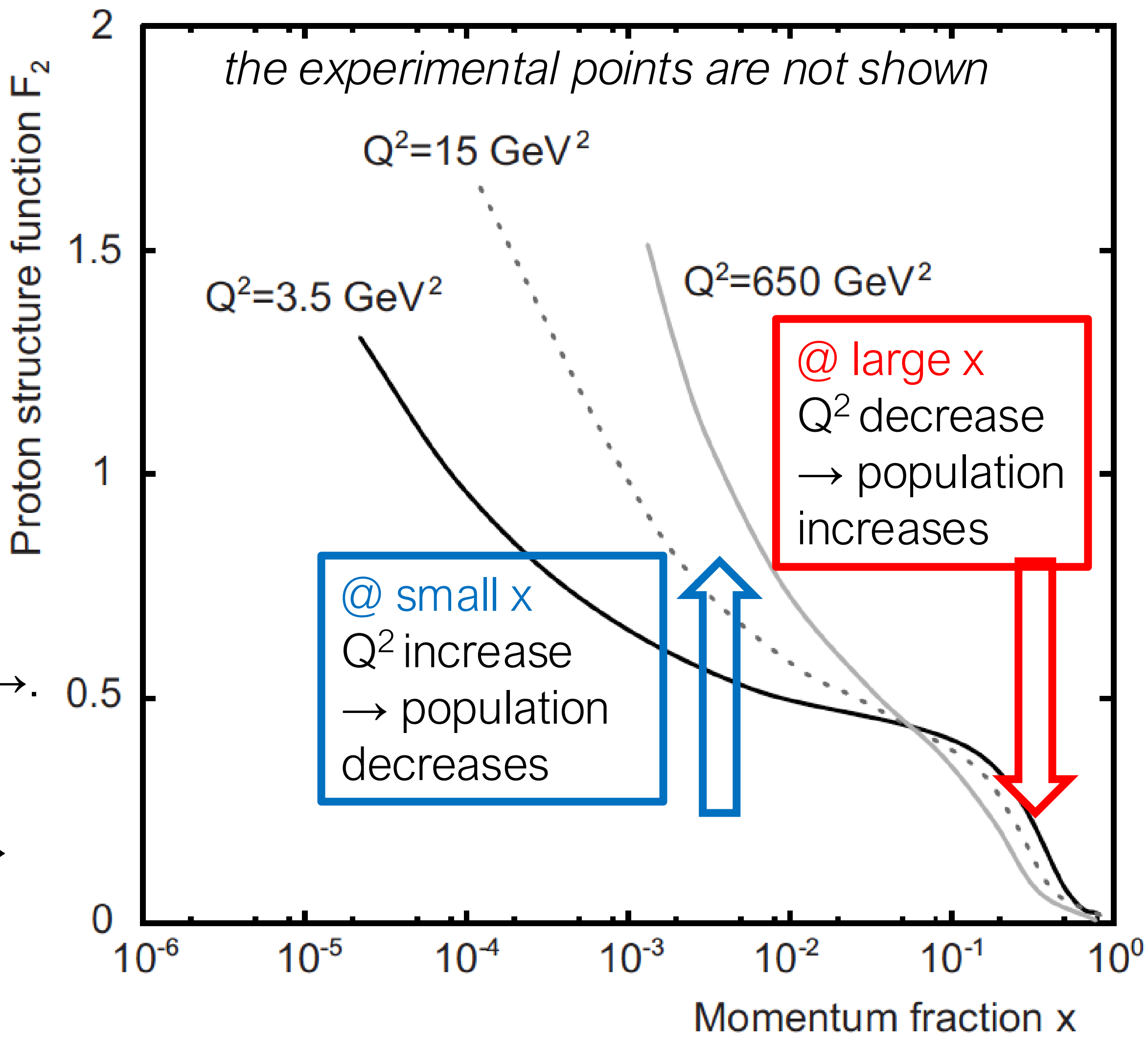
Quarks can emit or absorb gluons, gluons may split into $q\bar{q}$ pairs, or emit gluons themselves. Thus, the momentum distribution between the constituents of the nucleon is changing continuously.

We see that the structure function

- increases with Q^2 at small values of x and
- decreases when Q^2 increases at large values of x .

This behaviour, called **scaling violation**, is sketched in Fig. →.

With increasing values of Q^2 many quarks seen → the momentum of the proton is shared among many partons → there are few quarks with large momentum fractions in the nucleon → quarks with small momentum fractions predominate.



Isospin, a new Quantum Number

- $m_p \approx m_n$; $\sigma_{pp} \approx \sigma_{pn}$
- If you exchange $n \leftrightarrow p$ in Nuclei they remain very similar: $^7Li(3p+4n) \approx {}^7Be(4p+3n)$, $^{13}C(6p+7n) \approx {}^{13}N(7p+6n)$

In ~1930 Heisenberg, Condon and Carren made the hypothesis:
proton and the neutron are *two different states of the nucleon*.

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

→ A new quantum number: the *Isospin*, *I*. The nucleon was chosen to have *I=1/2* → I_3 projections:
the proton (+1/2) and the neutron (-1/2)

baryons	$m(\text{MeV}/c^2)$	B	Q	S	mesons	$m(\text{MeV}/c^2)$	B	Q	S
p	938.272	+1	+1	0	K^+	493.68	0	+1	+1
n	939.565	+1	0	0	K^0	497.65	0	0	+1
Λ	1115.68	+1	0	-1	η	547.7	0	0	0
Σ^+	1189.4	+1	+1	-1	π^+	139.570	0	+1	0
Σ^0	1192.6	+1	0	-1	π^0	134.977	0	0	0
Σ^-	1197.4	+1	-1	-1	π^-	139.570	0	-1	0
Ξ^0	1314.8	+1	0	-2	$\bar{K}^0$	497.65	0	0	-1
Ξ^-	1321.3	+1	-1	-2	K^-	493.68	0	-1	-1

Isospin combines like the spin: for a value *I* of the Isospin you have (2*I* + 1) possible combinations → I_3 values.
0 → singlet
½ → doublet
1 → triplet
The state np (I_3=+1/2-1/2) may belong to a doublet

Isospin and Symmetry of Wave Functions

A rotation in the space of the Isospin does not change the state, Isospin is a conserved quantity in strong interactions (but not in weak and electro-weak). Mass differences are only due to EM interactions

If you have a nucleus with *Z* protons and *N* neutrons the projection of the Isospin will be

$$I_3 = \frac{1}{2}Z - \frac{1}{2}N = \frac{(Z-N)}{2}$$

Considering that the baryon number is B = Z + N and that the charge is Q = Z one can write

$$Q = I_3 + B/2$$

Isospin has to be considered when studying the symmetry of a pair of fermions:

$$\Psi = \psi(Space)\chi(spin)I(Isospin)$$
$$Space \rightarrow -1^L$$
$$Spin \rightarrow -1^{S+1}$$
$$Isospin \rightarrow -1^{I+1}$$

The symmetry of a system with *L, S, I* goes like

$$Symmetry \rightarrow (-1)^L\,(-1)^{S+1}\,(-1)^{I+1}$$

A system with two nucleons has to be anti-symmetric as requested by the Pauli principle

Mesons Isospin

baryons	$m(\mathrm{MeV}/c^2)$	B	Q	S	mesons	$m(\mathrm{MeV}/c^2)$	B	Q	S
p	938.272	+1	+1	0	K^+	493.68	0	+1	+1
n	939.565	+1	0	0	K^0	497.65	0	0	+1
Λ	1115.68	+1	0	-1	η	547.7	0	0	0
Σ^+	1189.4	+1	+1	-1	π^+	139.570	0	+1	0
Σ^0	1192.6	+1	0	-1	π^0	134.977	0	0	0
Σ^-	1197.4	+1	-1	-1	π^-	139.570	0	-1	0
Ξ^0	1314.8	+1	0	-2	$\bar{K}^0$	497.65	0	0	-1
Ξ^-	1321.3	+1	-1	-2	K^-	493.68	0	-1	-1

$$Q = I_3 + B/2$$

Protons and neutrons we saw already, all OK

Pion's masses are close by and may be considered as members of the same triplet with I=1 and I_3=-1,0,1
Also the charges are correctly computed using the standard formula (baryon number=0)

The η has a mass very different from the pion's mass and it is ~isolated → only member of a singlet. Charge is OK, I=0, I_3=0

K^+K^0 are also close in mass, like the pair $K^-\overline{K^0}$, may be assumed to be members of a doublet, I=1/2, I_3=-1/2,1/2. However the Q formula fails.

All is restored if we include S, the *strangeness*, in the charge formula and define a new quantum number, the *Hypercharge*

$Y = B + S$

$$Q = I_3 + \frac{B+S}{2} = I_3 + \frac{Y}{2}$$

Baryon Isospin

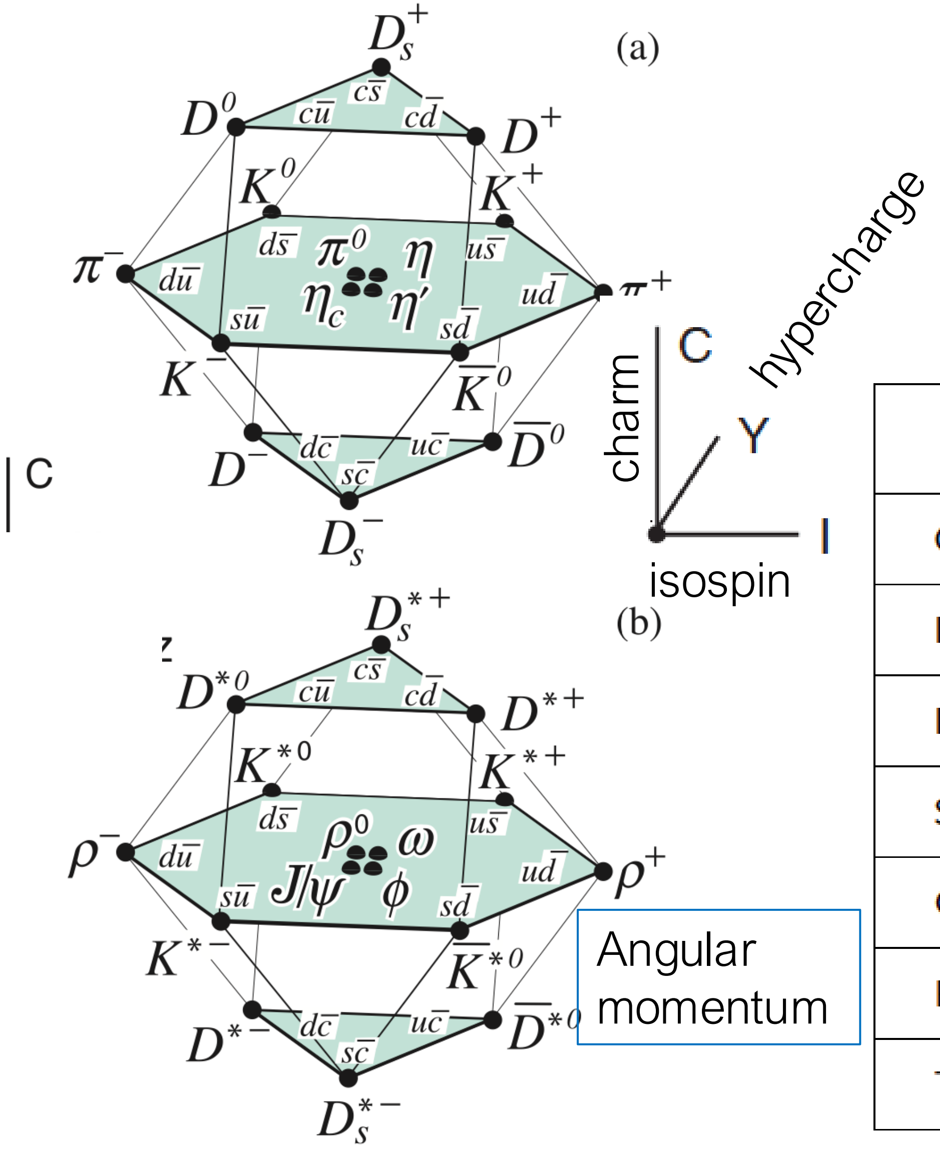
baryons	$m(\mathrm{MeV}/c^2)$	B	Q	S	mesons	$m(\mathrm{MeV}/c^2)$	B	Q	S
p	938.272	+1	+1	0	K^+	493.68	0	+1	+1
n	939.565	+1	0	0	K^0	497.65	0	0	+1
Λ	1115.68	+1	0	-1	η	547.7	0	0	0
Σ^+	1189.4	+1	+1	-1	π^+	139.570	0	+1	0
Σ^0	1192.6	+1	0	-1	π^0	134.977	0	0	0
Σ^-	1197.4	+1	-1	-1	π^-	139.570	0	-1	0
Ξ^0	1314.8	+1	0	-2	$\bar{K}^0$	497.65	0	0	-1
Ξ^-	1321.3	+1	-1	-2	K^-	493.68	0	-1	-1

$$Q = I_3 + \frac{B+S}{2} = I_3 + \frac{Y}{2}$$

Protons and neutrons we saw already, all OK

Baryons seem to be organised into multiplets as mesons: 1 singlet, 2 doublets, 1 triplet.
The charge-formula works well for baryons!

Extension to Other Generations (→c, b, t quarks)



Angular momentum

Generalised charge formula $Q = I_z + \frac{\mathcal{B} + S + C + B + T}{2}$,

Baryon number

Y

Bottomness number

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

The Structure of Hadrons

- deep inelastic scattering may be used as a tool to study the structure and composition of the nucleons.
- the spectroscopy of these particles also gives information about the structure of the hadrons and the forces acting between them.
- By the mid-sixties a large number of apparently different hadrons were known. The quark model was invented to accommodate the 'zoo' of hadrons which had been discovered

		u	d	p (uud)	n (udd)
Charge	z	$+2/3$	$-1/3$	1	0
Isospin	I	$1/2$		$1/2$	
	I_3	$+1/2$	$-1/2$	$+1/2$	$-1/2$
Spin	s	$1/2$	$1/2$	$1/2$	$1/2$

Quantum numbers of u, d quarks and of protons and neutrons

Use information from both

- deep inelastic scattering and
- spectroscopy to extract the properties of the quarks.

Idea: reconstruct the properties of the nucleons (charge, mass, magnetic moment, isospin, etc.) by combining the quantum numbers of these constituents.

Combining Quantum Numbers

		u	d	p (uud)	n (udd)
Charge	z	$+2/3$	$-1/3$	1	0
Isospin	I	$1/2$		$1/2$	
	I_3	$+1/2$	$-1/2$	$+1/2$	$-1/2$
Spin	s	$1/2$	$1/2$	$1/2$	$1/2$

- The quarks have spin ½
- in the quark model, their spins must combine to give the total spin 1/2 of the nucleon → nucleons are built up out of at least 3 quarks. The proton has two u-quarks and one d-quark, while the neutron has two d-quarks and one u-quark.

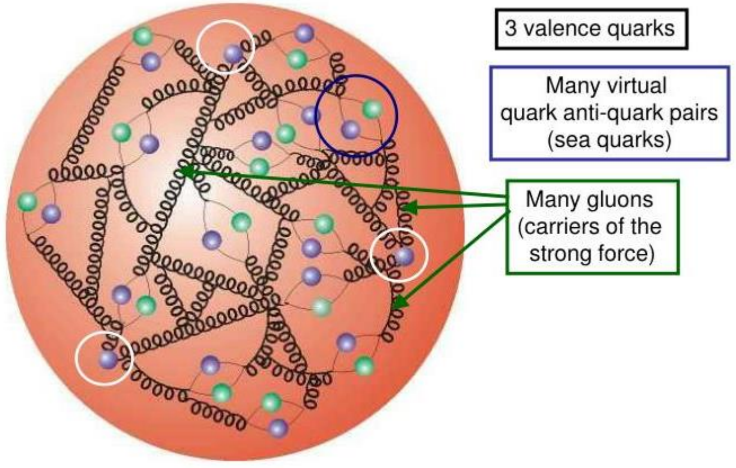
- **u** and **d** quarks form an isospin doublet, it is natural to assume that also the proton and the neutron form an isospin doublet (I = 1/ 2) u-quark and d-quark can be exchanged (isospin symmetry) → proton → neutron.
- The fact that the charges of the quarks *are multiples of 1/3 is derived by* the fact that the maximum positive charge found in hadrons is two (e. g., Δ^{++}), and the maximum negative charge is one (e. g., Δ^{-}). Hence the charges of these hadrons are attributed to 3 u quarks (charge: $3 \cdot \left(\frac{2e}{3}\right) = 2e$)) and 3 d-quarks (charge: $3 \cdot \left(\frac{-1e}{3}\right) = -1e$)) respectively.

Toni Baroncelli: Introduction to Particle Physics

Combining the Quarks (Recap!)

- Nucleons: three *valence quarks* determine the *quantum numbers*
- Virtual quark-antiquark pairs, *sea quarks:*
  - *quantum numbers average out to zero*
  - carry a very small fractions x of the nucleon's momentum.
- There are not only "u" and "d" quarks but also s (strange), c (charm), b (bottom) and t (top). These heavy quarks contribute very little to the 'sea'.
- *Electrically charged*, *sea quarks* → "*visible*" in deep inelastic scattering.

Content of the nucleon



… only quarks and anti-quarks interact with neutrinos

The cross-section for electro-magnetic interactions is proportional charge^2, e_k^2

$$\Longrightarrow \quad F_2(x) = \sum_k e_k^2 \cdot x \cdot f_k(x).$$

- The six quark types can be arranged in doublets (called families or generations), according to their increasing mass :

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Charge 2/3 e
Charge -1/3 e

Very heavy quarks, contribute very little to Deep Inelastic Scattering at ~low or moderate Q^2 . They can be neglected

Exploded View of the Proton & Neutron F_2

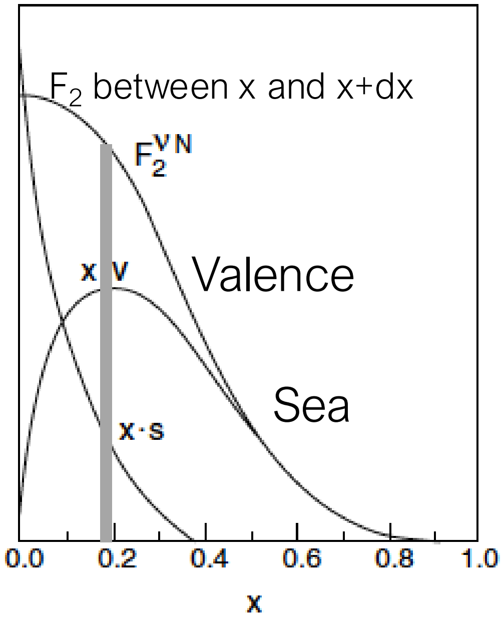
$F_2^{e,p}$ and $F_2^{e,n}$: structure functions of of protons and neutrons;
$d_{v,s}, \overline{d_{v,s}}$: x-distribution of d-valence and of sea quarks (similarly for other quarks)

$$F_2^{\mathrm{e,p}}(x) = x \cdot \left[\frac{1}{9}\left(d_{\mathrm{v}}^{\mathrm{p}} + d_{\mathrm{s}} + \bar{d}_{\mathrm{s}}\right) + \frac{4}{9}\left(u_{\mathrm{v}}^{\mathrm{p}} + u_{\mathrm{s}} + \bar{u}_{\mathrm{s}}\right) + \frac{1}{9}\left(s_{\mathrm{s}} + \bar{s}_{\mathrm{s}}\right)\right]$$

$$F_2^{\mathrm{e,n}}(x) = x \cdot \left[\frac{1}{9}\left(d_{\mathrm{v}}^{\mathrm{n}} + d_{\mathrm{s}} + \bar{d}_{\mathrm{s}}\right) + \frac{4}{9}\left(u_{\mathrm{v}}^{\mathrm{n}} + u_{\mathrm{s}} + \bar{u}_{\mathrm{s}}\right) + \frac{1}{9}\left(s_{\mathrm{s}} + \bar{s}_{\mathrm{s}}\right)\right]$$

Valence quarks

Sea quarks



The proton and the neutron can be interchanged by exchanging d and u quarks (isospin symmetry)

$$u_{\mathrm{v}}^{\mathrm{p}}(x) = d_{\mathrm{v}}^{\mathrm{n}}(x)\,,$$
$$d_{\mathrm{v}}^{\mathrm{p}}(x) = u_{\mathrm{v}}^{\mathrm{n}}(x)\,,$$
$$u_{\mathrm{s}}^{\mathrm{p}}(x) = d_{\mathrm{s}}^{\mathrm{p}}(x) = d_{\mathrm{s}}^{\mathrm{n}}(x) = u_{\mathrm{s}}^{\mathrm{n}}(x)$$

The proton has two u-quarks and one d-quark, the neutron has two d-quarks and one u-quark.

And the 'average' Nucleon structure function can be written as

$$F_2^{\mathrm{e,N}}(x) = \frac{F_2^{\mathrm{e,p}}(x) + F_2^{\mathrm{e,n}}(x)}{2} = \frac{5}{18}\, x \cdot \sum_{q=d,u}\left(q(x) + \bar{q}(x)\right) + \frac{1}{9}\, x \cdot \left[s_{\mathrm{s}}(x) + \bar{s}_{\mathrm{s}}(x)\right]$$

Term with sea quarks only → negligible

5/18 is ~ the mean square charge of u + d quarks

Comparing $F_2^{\upsilon,N}$ and $F_2^{e,N}$

- $\nu, \bar{\nu}$ DIS: same *weak charge for all quarks* no charge factors z_f^2
- Because of charge conservation and helicity, neutrinos and antineutrinos couple differently to the different types of quarks and antiquarks. These differences, however, cancel out when the structure function of an average nucleon is considered. One then obtains:

e,N

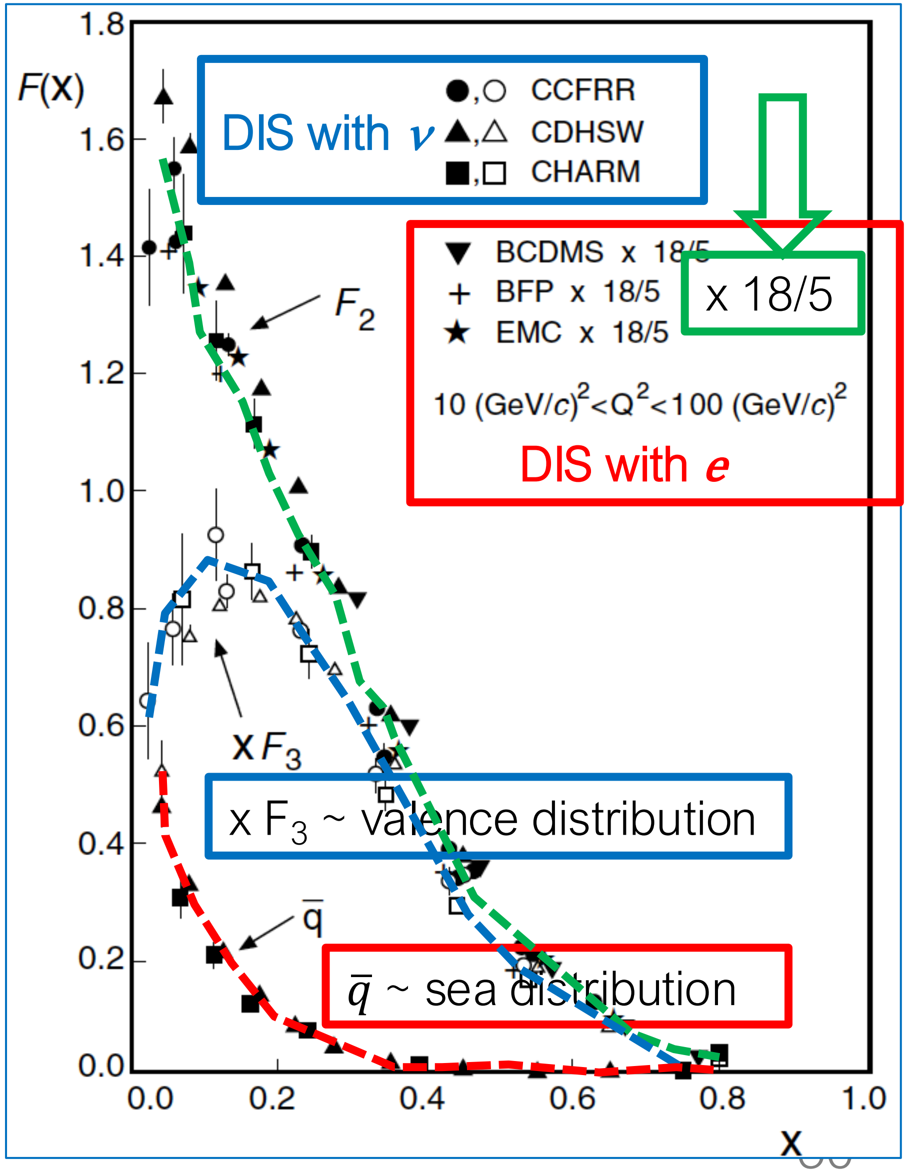
$$F_2^{e,N}(x) = \frac{5}{18} x \cdot \sum_{q=d,u} (q(x) + \bar{q}(x))$$

νN

$$F_2^{\nu,N}(x) = x \cdot \sum_f (q_f(x) + \bar{q}_f(x))$$

Experiments show that $F_2^{\upsilon,N}$ and $F_2^{e,N}$ are identical ((but for the factor 5/18 due to charge) → This means that the charge numbers +2/3 and −1/3 have been correctly attributed to the u- and d-quarks.

- Valence quarks peak at $x \approx 0.17$ and an average value of $\langle x_v \rangle \approx 0.12$
- Sea quark →low x values with an average value of $\langle x_s \rangle \approx 0.04$



Comparing $F_2^{\upsilon,N}$ and $F_2^{e,N}$

WARNING!

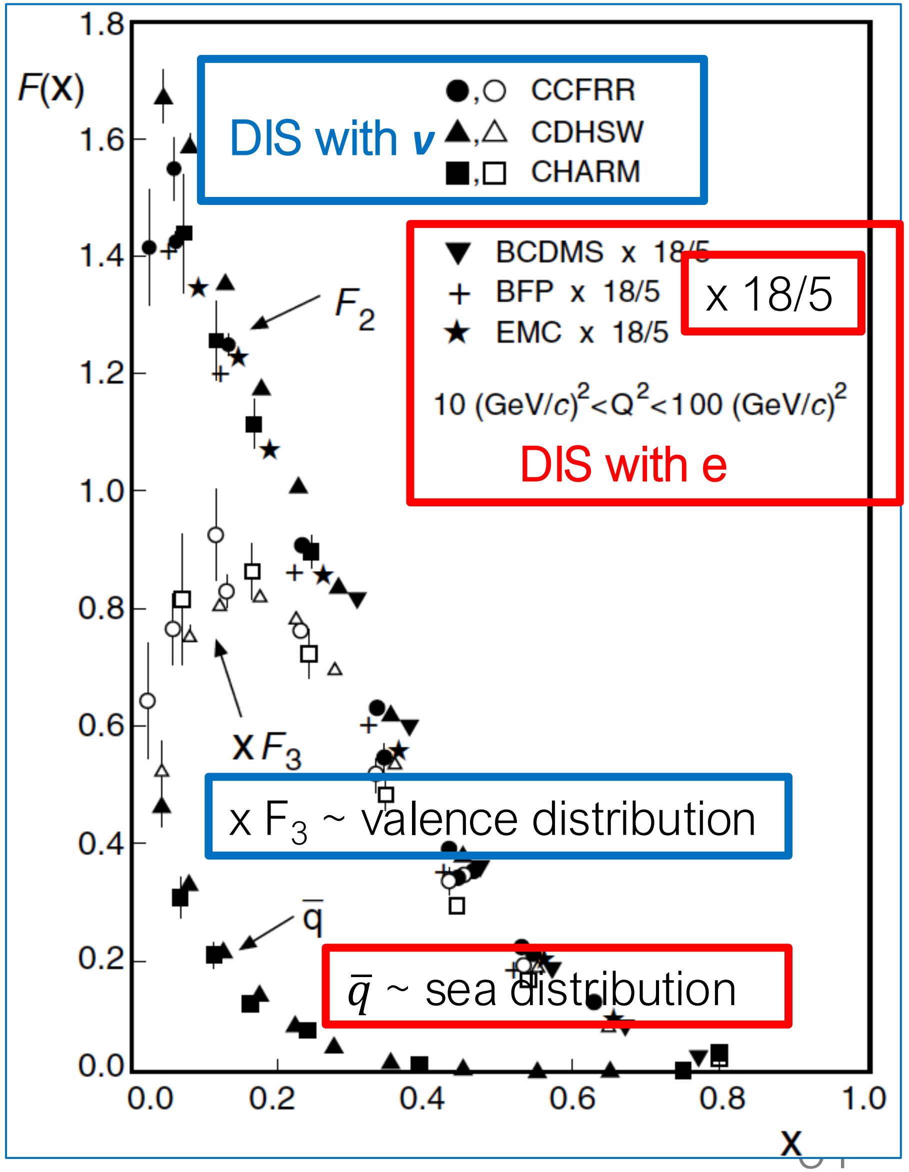
The integral of $F_2^{\upsilon,N}$ and $F_2^{e,N}$ gives about 0.5 → IMPORTANT INFORMATION: half of the momentum of a nucleon is carried by components that are NOT quarks

$$\int_0^1 F_2^{\nu,\mathrm{N}}(x)\,\mathrm{d}x \approx \frac{18}{5}\int_0^1 F_2^{\mathrm{e,N}}(x)\,\mathrm{d}x \approx 0.5$$

This component is not detected in $F_2^{\upsilon,N}$ or $F_2^{e,N}$. This means it is sensible

- neither to electromagnetic interactions
- nor to weak interactions

→ *gluons*



Constituent Quarks and their Masses

- About 1/2 of the momentum of a nucleon is carried by valence and sea quarks.
- Nucleons can be constructed using only the valence quarks.
- *Quark masses cannot really be measured because quarks are never free (will discuss this!).*
- Bare u and d quarks are are (expected to be) small: m_u =1.5 – 5 MeV/c^2 , m_d =3 – 9 MeV/c^2 . These masses are commonly called ***current quark masses***.

- One can assume that there are only three valence quarks, with enlarged masses (~“incorporating sea & gluons”) but unchanged quantum numbers, call them “***constituent quarks***”.
- The ***constituent quark masses*** are much larger (300 MeV/c^2). The ***constituent masses*** must be mainly due
  - the electromagnetic interaction → mass differences of a few MeV;
  - Additional effects must be due to interaction between quark–quark.

- It is often assumed that m_u ~ m_d ~ few MeV and m_s ~ m_u + 150MeV.
- The masses of heavier quarks are m_c ~1.550 MeV and m_b ~ 4.300 MeV.
- Hadrons made of the t quarks cannot be formed because the quark t is free for a very short time.

Quark	Colour	Electr. Charge	Mass $[\mathrm{MeV}/c^2]$ Bare Quark	Mass $[\mathrm{MeV}/c^2]$ Const. Quark
down	b, g, r	$-1/3$	3 – 9	≈ 300
up	b, g, r	$+2/3$	1.5 – 5	≈ 300
strange	b, g, r	$-1/3$	60 – 170	≈ 450
charm	b, g, r	$+2/3$	1 100 – 1 400	
bottom	b, g, r	$-1/3$	4 100 – 4 400	
top	b, g, r	$+2/3$	$168 \cdot 10^3$ – $179 \cdot 10^3$	

Quarks in Hadrons: Baryons and Mesons

Hadrons can be classified in two groups:

1. the baryons , fermions with half-integral spin
2. the mesons, bosons with integral spin.

Baryons.

.

- baryons are composed of three quarks.
- quarks have spin 1/2, baryons have half-integral spin.
- baryons are produced in pairs. Baryon number B = 1 for baryons and B = – 1 to antibaryons → B = +1/ 3 for quarks, and B = – 1/ 3 for antiquarks.
- Experiments indicate that baryon number is conserved in all particle reactions and decays.
- The B(quark) – B(antiquark) number is conserved.
- This would be violated by, e. g., in the hypothetical **decay of the proton: p →** π^0 **+** e^+ . Without baryon number conservation this decay mode would be energetically favoured. Yet, it has not been observed.

Mesons.

- Pions are the lightest mesons ~ 140 MeV/c^2.
- They are found in three different charge states: π^-, π^0 and π^+ .
- Pions have spin 0. It is, therefore, natural to assume that they are composed of a quark and an antiquark: this is the only way to build the three charge states out of quarks.

$$|\boldsymbol{\pi^+}\rangle = |\boldsymbol{u\bar{d}}\rangle \quad |\boldsymbol{\pi^-}\rangle = |\boldsymbol{d\bar{u}}\rangle \quad |\boldsymbol{\pi^0}\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\boldsymbol{u\bar{u}} + \boldsymbol{d\bar{d}}\rangle$$

- The pions are the lightest systems of quarks. Hence, they can only decay into the even lighter leptons or into photons.

- The pion mass is considerably smaller than the constituent quark mass → the interquark interaction energy has a substantial effect on hadron masses.
- The total angular momentum = vector sum of the quark, antiquark spins, integer orbital angular momentum contribution.

- Mesons eventually decay into electrons, neutrinos and/or photons; there is no “meson number conservation (the number of quarks minus the number of antiquarks is zero) → any number of mesons may be produced or annihilated.

Introducing Coloured Quarks and Coloured Gluons

Quarks have another important property called colour.
This is needed to ensure that quarks in hadrons obey the Pauli principle.

Δ^{++} resonance (***baryon***!)

- It is made of three u-quarks, has spin J = 3/2 and positive parity; it is the lightest baryon with $J^P = 3/2^+$ → we therefore can assume that its ***orbital angular momentum*** is = 0;
- it has a symmetric spatial wave function. In order to yield total angular momentum 3/2, the spins of all three quarks have to be parallel:

$$|\Delta^{++}\rangle = |u^\uparrow u^\uparrow u^\uparrow\rangle$$

- Thus, the spin wave function is also symmetric.
- The wave function of this system is furthermore symmetric under the interchange of any two quarks, as only quarks of the same flavour are present.
- The total wave function is symmetric, in violation of the Pauli principle.

To fulfil the Pauli principle the colour, a kind of quark charge, has to be introduced.
HP: The colour quantum number can assume three values, which may be called red, blue and green.
Accordingly, antiquarks carry the anti-colours anti-red, anti-blue , and anti-green.

The strong interaction binds quarks into a hadron → mediated by force carriers → gluons.
... And gluons? Do they carry colour?

Gluons and the QCD

The gluons carry simultaneously colour and anti-colour → 3 colors x 3 anti-colors → 9 combinations.

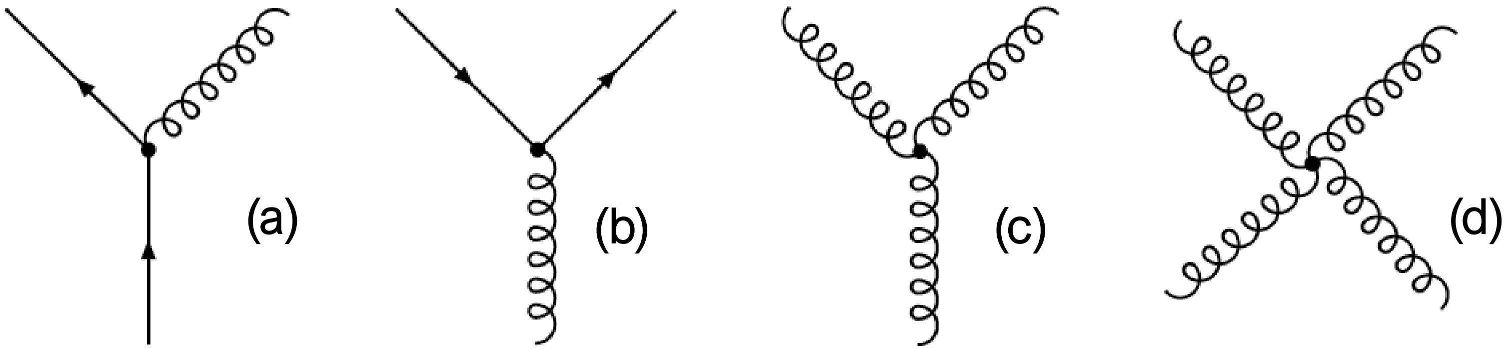
Colour forms combinations that may be organised in multiplets of states: a singlet and an octet. One possible choice is (others exist):

Octet $\quad r\bar{g}, \quad r\bar{b}, \quad g\bar{b}, \quad g\bar{r}, \quad b\bar{r}, \quad b\bar{g}, \quad \sqrt{1/2}\,(r\bar{r} - g\bar{g}), \quad \sqrt{1/6}\,(r\bar{r} + g\bar{g} - 2b\bar{b})$

Singlet $\quad \sqrt{1/3}\,(r\bar{r} + g\bar{g} + b\bar{b})$ $\quad$ Net colour of singlet = 0 → do not mediate QCD

Exchange of the eight gluons mediate the interaction between particles carrying colour charge, i.e., not only the quarks but also the gluons themselves.

→ This is an important difference to the electromagnetic interaction, where the photon field quanta have no charge, and therefore cannot couple with each other.



The fundamental interaction diagrams of the strong interaction: emission of a gluon by a quark **(a)**, splitting of a gluon into a quark–antiquark pair **(b)** and "self-coupling" of gluons **(c, d)**.

Colour Carriers

	Quarks	Anti-quarks	Gluon	Photon
Charge	✅	✅		
Colour	✅	✅	✅	

Hadrons and the Colour-Neutrality

In principle each hadron might exist in many different colours (the colours of the constituent quarks involved), would

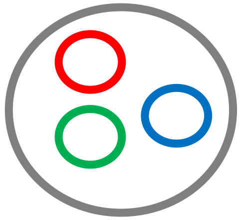
- have different total (net) colours
- but would be equal in all other respects.

In practice only one type of each hadron is observed (one π^-, p, Δ^0 etc.)

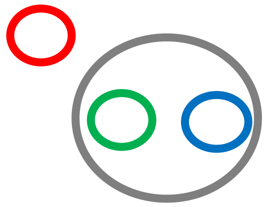
additional condition: only colourless particles can exist as free particles ⟶
Hadrons as colour-neutral objects.

- colour + anti-colour = "white" (= white objects!)
- Three different colours = "white" as well.

➢ This is why quarks are not observed as free particles. Breaking one hadron into quarks would produce at least two objects carrying colour: the quark, and the rest of the hadron. This would be a violation of the hadron colour-neutrality.



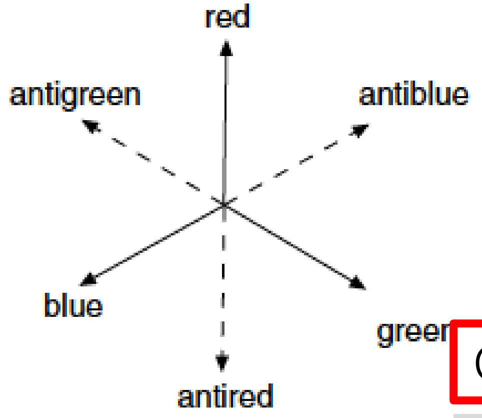
This phenomenon is, therefore, called confinement.

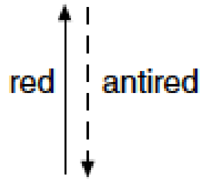


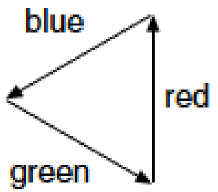
➢ This implies that the potential acting on a quark increases with increasing separation—in sharp contrast to the Coulomb potential. This phenomenon is due to the inter-gluonic interactions.

Colourless –White- Hadrons

Graphically: three vectors in a plane symbolising the three colours, rotated by 120°







$$|\pi^+\rangle = \begin{cases} |u_r \bar{d}_{\bar{r}}\rangle \\ |u_b \bar{d}_{\bar{b}}\rangle \\ |u_g \bar{d}_{\bar{g}}\rangle \end{cases}$$

The pion π^+ is a superposition of these states

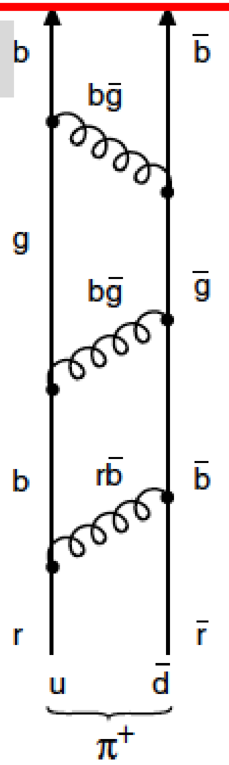
Combination of 2/3 colours giving white: colour+anti-colour or r+b+g

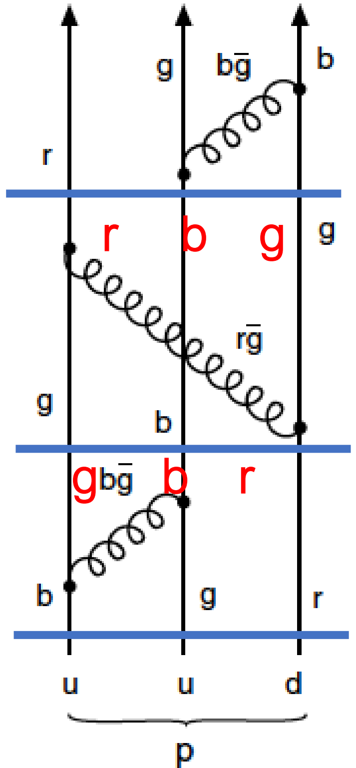
Gluons are not white: they carry colour and anti-colour

- Due to exchange of gluons the colour combination of hadrons continuously changes; **but the net-colour "white" remains.**
- to obtain a colour neutral baryon, each quark must have a different colour. The proton is a mixture of such states:

$$|p\rangle = \begin{cases} |u_b u_r d_g\rangle \\ |u_r u_g d_b\rangle \\ \vdots \end{cases}$$

- From this argument, it also becomes clear why no hadrons exist which are $|qq\rangle$ or $|qq\bar{q}\rangle$ combinations, or similar combinations. These states would not be colour neutral



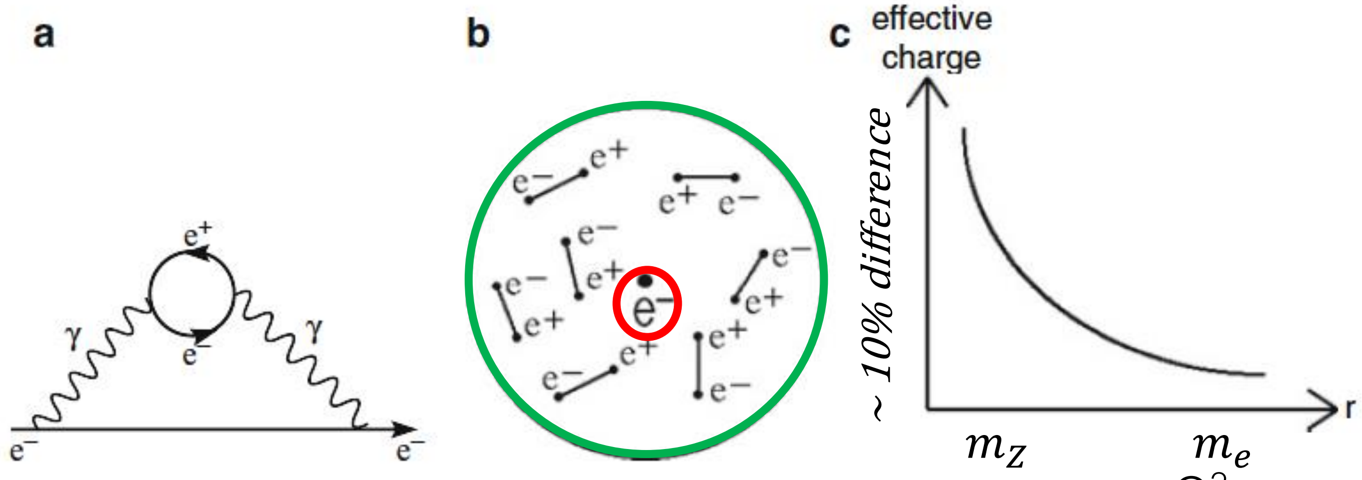


QED: Running $\alpha(Q^2)$: visible charge of electrons

Virtual pairs of e^+e^- in *em* interactions have the effect of screening the real e^- charge.

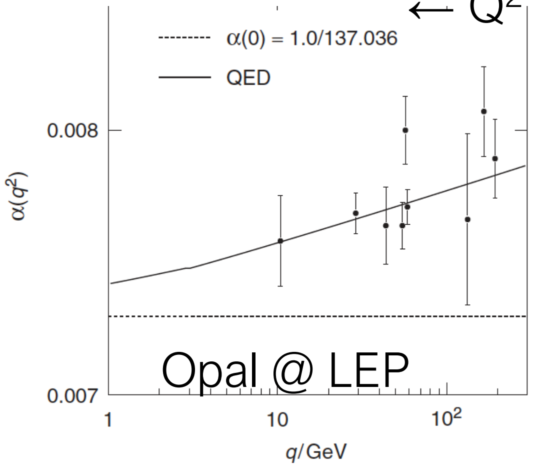
At low Q^2 only large objects are visible→

- ➢ the virtual photon sees a cloud of charges
- ➢ the effective charge of the interacting particles decreases:
- ➢ the coupling constant is small.



At high Q^2 small objects are visible →

- ➢ the virtual photon sees the individual charge
- ➢ the effective charge of the interacting particles increases:
- ➢ the coupling constant is large.



A parametrization describing the variation of α with Q^2 is given here and it is defined at a given scale μ^2.

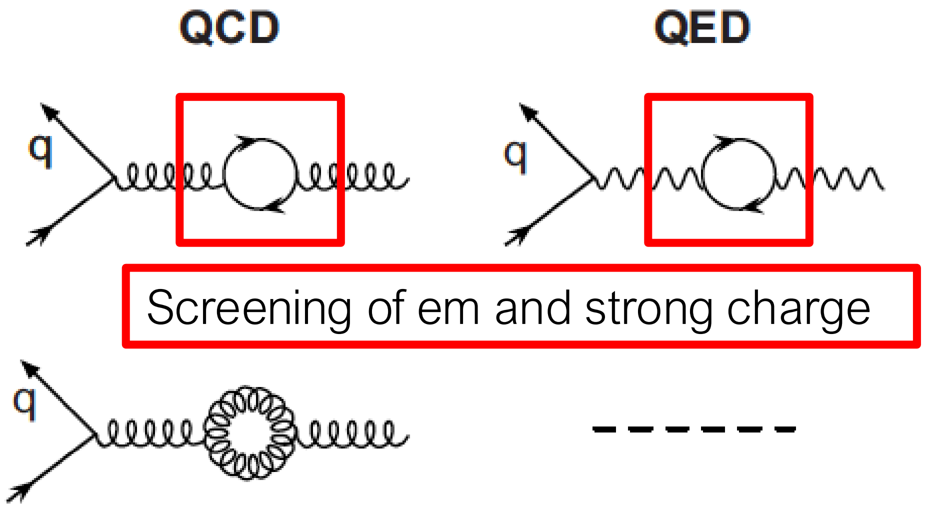
$\alpha(m_e) = 1/137 \quad \alpha(m_Z) = 1/128$ *~ 10% difference*

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

The Running Coupling Constant α_s

- The coupling "constant" α_s describing the strength of the hadronic interaction between two particles depends on Q^2.
- While in the *em* interaction α_{em} depends weakly on Q^2 , in the strong interaction, however, it is very strong.

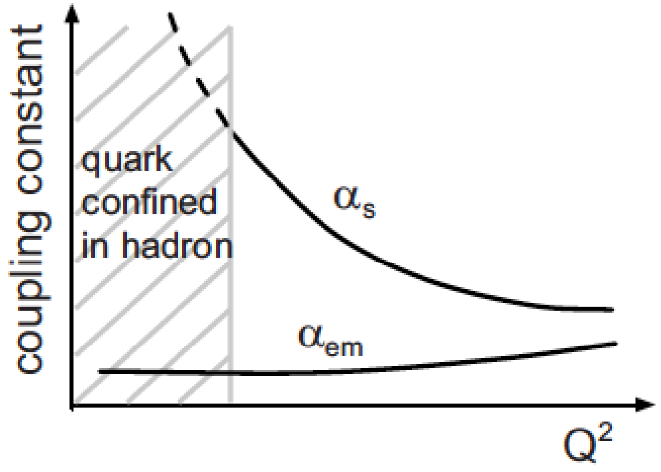
Why?



The fluctuation of the photon into a electron-positron pair
and
The fluctuation of the gluon into the quark-antiquark pair
generates a

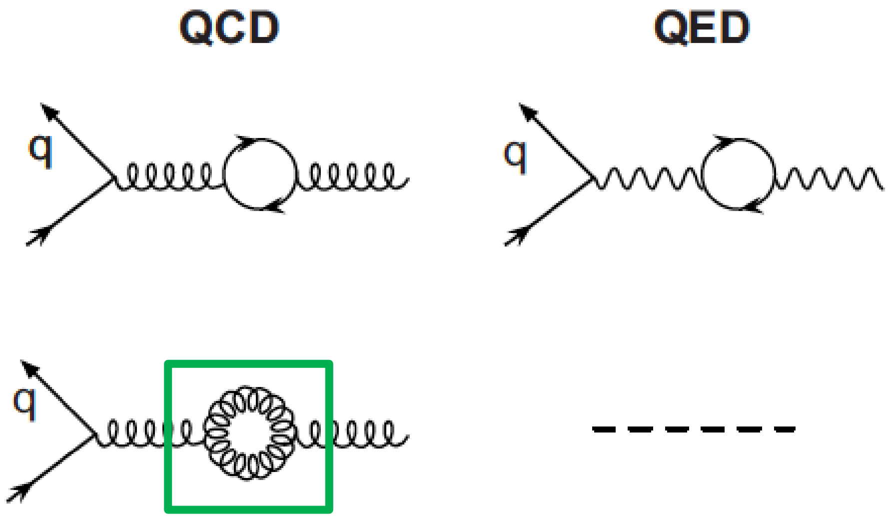
- ➢ repulsive force between two quarks of the same colour (same charge) and
- ➢ the attractive force between quarks with (opposite charge) colour and anticolour

Generates screening of the electric and strong charge.



The Running Coupling Constant α_s

Gluons couple with gluons (photons do NOT couple to photons)!

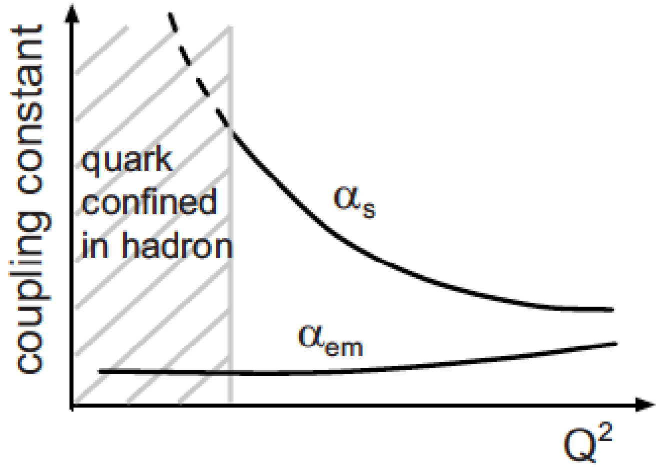


Anti-Screening of strong charge

Different colours may give rise to an attractive force if the quantum state is antisymmetric, and a repulsive force if it is symmetric under the interchange of quarks.

This means that the favourite state of three quarks is the state with three quarks of different colours, q_r q_b q_g, that is, the colourless state of baryons.

The higher Q^2 is, the smaller are the *visible* distances between the interacting particles; effective charge of the interacting particles increases: the coupling constant increases.



Gluons can fluctuate into gluons → this can be shown to give anti-screening. The closer the interacting particles are, the smaller is the charge they see.

α_s decreases with increasing Q^2.

Confinement and Asymptotic Freedom

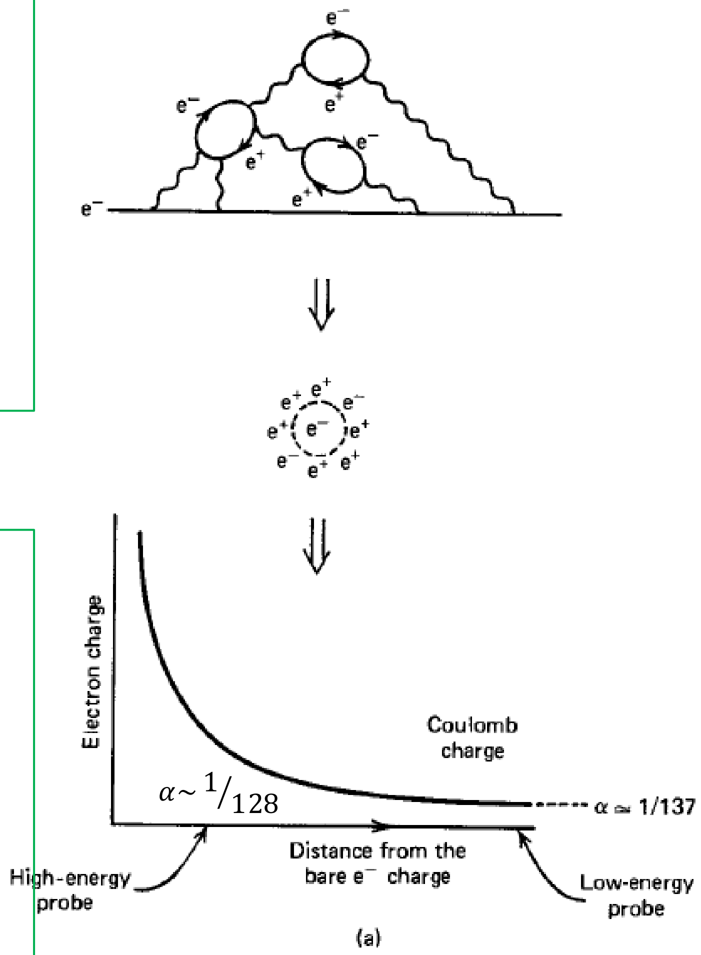
At low Q^2, the the distances between the interacting particles are (appear to be) large →

- the virtual photon sees a cloud of charges
- the effective charge of the interacting particles decreases:
- the coupling constant is small.

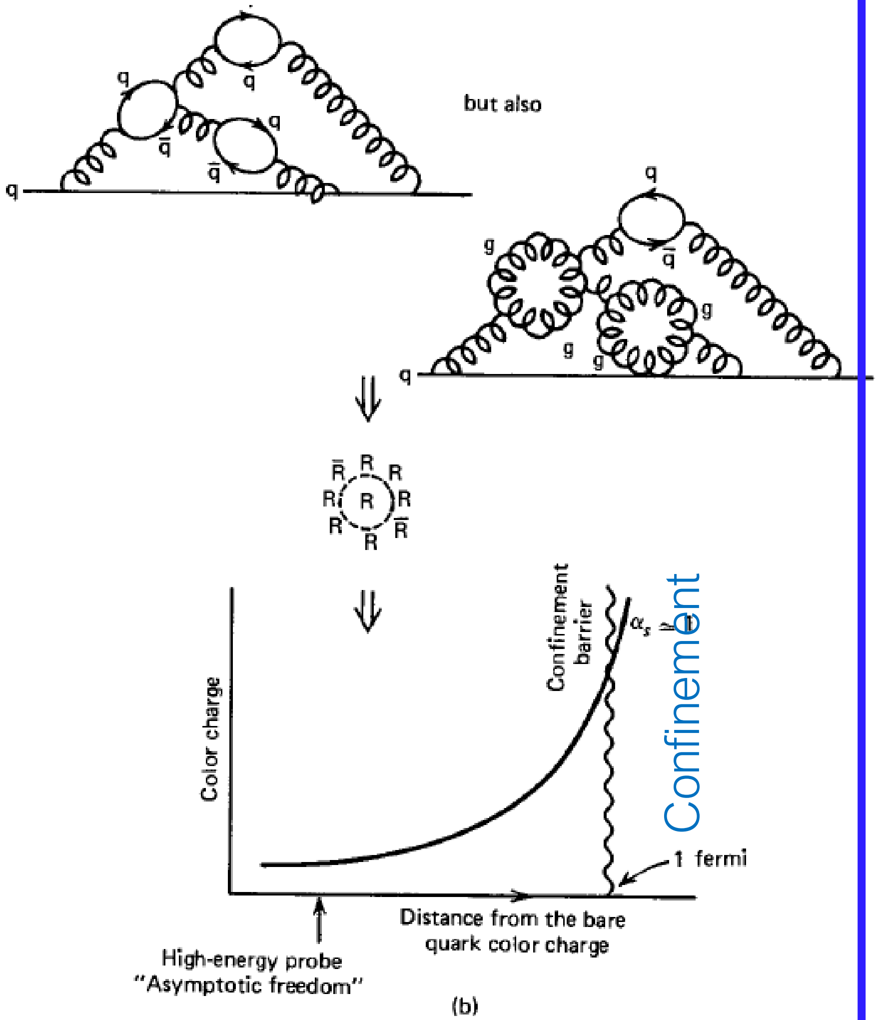
At high Q^2 is, the the distances between the interacting particles are small →

- the virtual photon sees the individual charge
- the effective charge of the interacting particles increases:
- the coupling constant is large.

Quantum electrodynamics



Quantum chromodynamics



Asymptotic Freedom and Confinement

In the case of gluons the anti-screening is far stronger than the screening. A first-order perturbation calculation in QCD gives:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\cdot \ln(Q^2/\Lambda^2)}$$

Colour neutral!

n_f number of flavours that contribute to the interaction

$Q^2 \rightarrow$ separation among different components

Λ parameter of the function determined from data

$33 = 11 \times N_c$

A heavy virtual quark–antiquark pair has a very short lifetime and range, it can be resolved only at very high Q^2. This means that n_f varies with Q^2 between $n_f \approx$ 3–6 → when Q^2 increases n_f increases too.
The parameter Λ is the only free parameter of QCD. It was found to be $\Lambda \approx$ 250 MeV/c by comparing the prediction with the experimental data. The application of perturbative expansion procedures in QCD is valid only if $\alpha_s \ll 1$. This is satisfied for $Q^2 \gg \Lambda^2 \approx 0.06\ (\mathrm{GeV}/c)^2$.

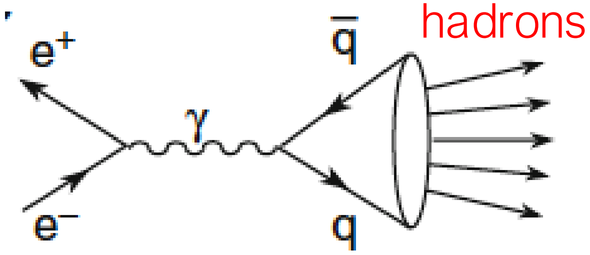
The formula indicates two regions:

- For very small distances (high values of Q^2) ” α_s decreases, vanishing asymptotically. In the limit $Q^2 \rightarrow \infty$, quarks can be considered “free”, this is called asymptotic freedom.
- At large distances, (low values of Q^2) α_s increases so strongly that it is impossible to separate individual quarks inside hadrons (confinement).

Measuring the Number of Colours

Study the production of
- $q\bar{q}$ pairs and of
- $\mu^+\mu^-$ pairs

in e^+e^- interactions



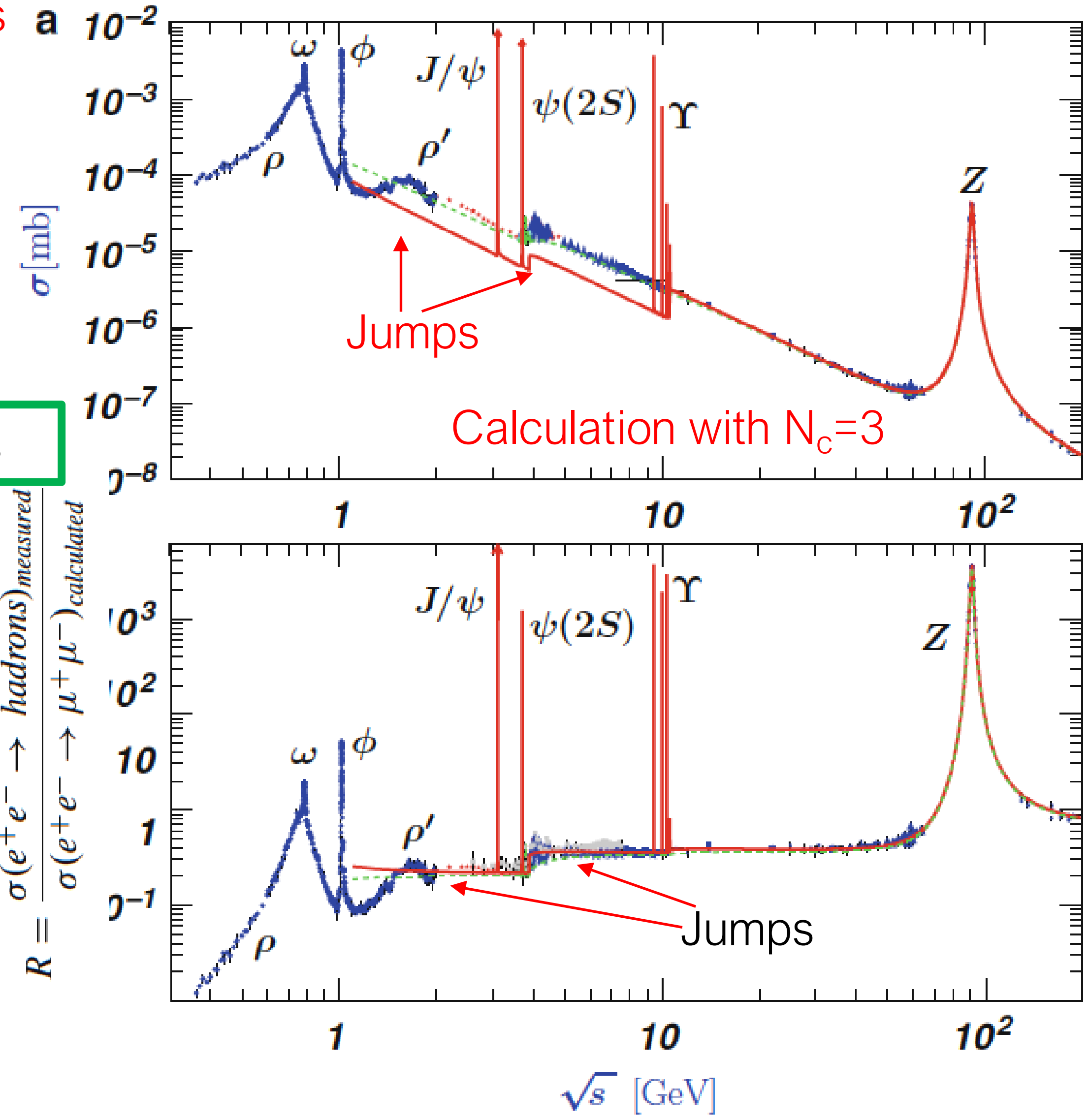
The two cross-sections are due to the exchange of one photon and are described by the two expressions below

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha_{EM}^2(\hbar c)^2}{3s}$$

$$\sigma(e^+e^- \to \gamma \to q\bar{q} \to hadrons) = N_C \frac{4\pi\alpha_{EM}^2}{3}\frac{(\hbar c)^2}{s}\sum_{n=1}^{N_f} Q_n^2$$

N_f # accessible quarks

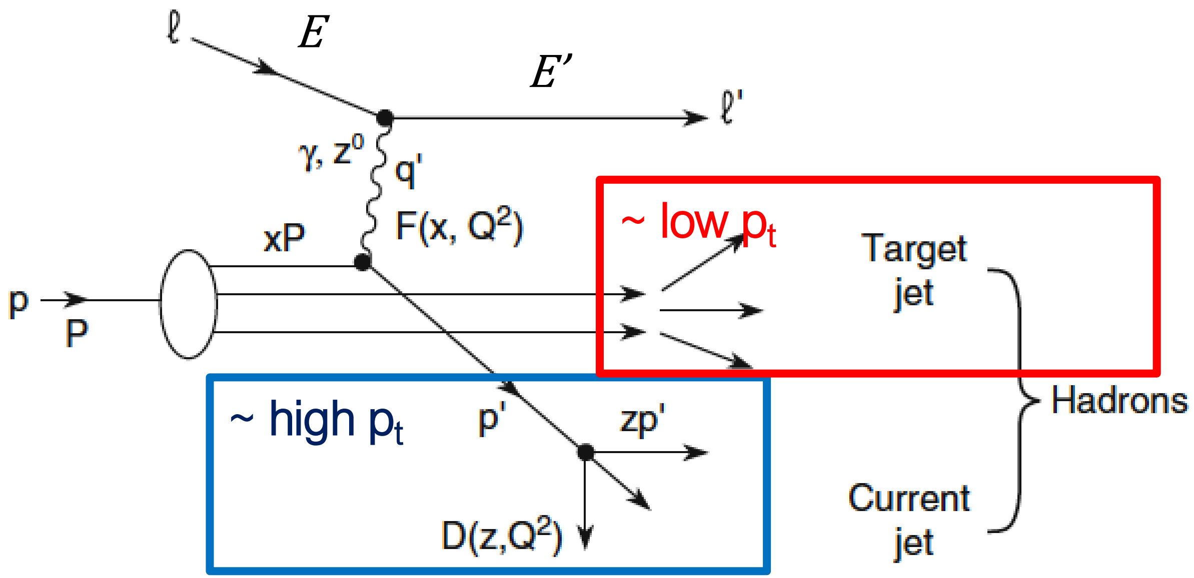
Q_q^2 *charge of quarks involved*



Jumps are understood with the opening of kinematical windows as soon as $\sqrt{s} > m_q + m_{\bar{q}}$

A factor $N_C = 3$ had to be introduced to account for the number of different coloured hadrons

Fragmentation of quarks into hadrons



The second stage of the DIS process is the parton fragmentation into two jets of hadrons (also called hadronisation). This is a strong interaction process, which "dresses" naked quarks to form hadrons in the final state.

The fragmentation function,
$D(z;Q^2)$
gives the energy distribution of hadrons from the interacting parton.

$D(z;Q^2)$ gives the probability that a given hadron carries a fraction z of the interacting parton energy. This energy is not experimentally measurable and must be estimated. In this second stage, the gluons play an important role since the strong interaction amongst constituents modifies the structure function, making it dependent on Q^2.

- The virtual γ-parton collision of the first phase occurs within a time $\Delta t_1 \approx \frac{\hbar}{\upsilon}, \upsilon = E - E'$.
- The quark hadronisation (or quark dressing) is characterized by a time $\Delta t_2 \approx \frac{\hbar}{m_p^2} \approx 10^{-24} s$ (m_p = proton mass).

If $\upsilon \gg m_p$, one has $\Delta t_1 \gg \Delta t_2$ and the two subprocesses can be considered as distinct.

The Fragmentation Process

1. Basic (EW) Interaction
2. The quark or the antiquark can radiate a gluon, which can radiate another gluon, or produce a $q\bar{q}$ pair.
3. The coloured partons (quarks and gluons) fragment (hadronise) in colourless hadrons. The process cannot be treated with perturbation methods; → *models*
4. the hadronic resonances decay to hadrons

- The virtual γ-parton collision of the first phase occurs within a time $\Delta t_1 \approx \frac{\hbar}{\upsilon}, \upsilon = E - E'$.
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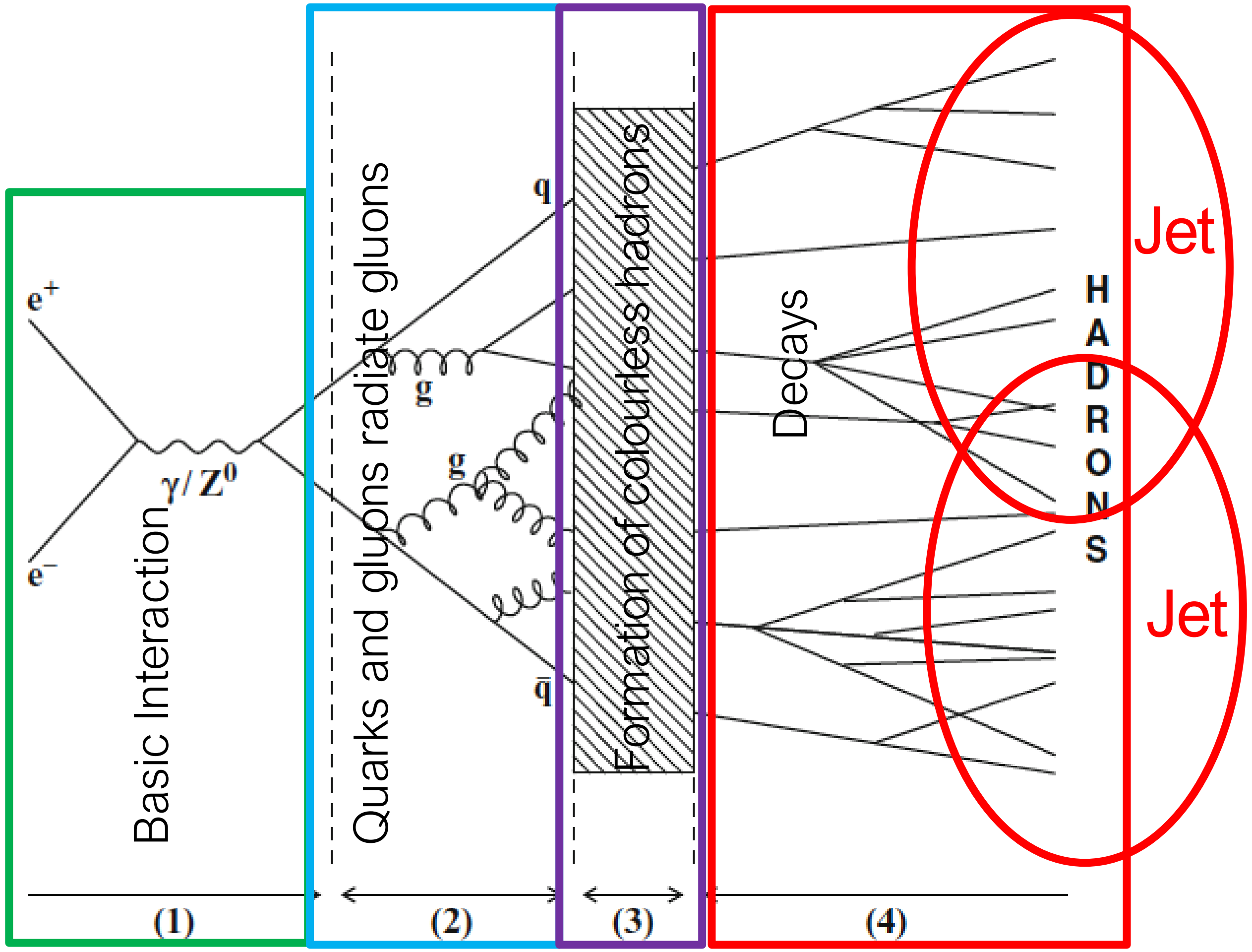
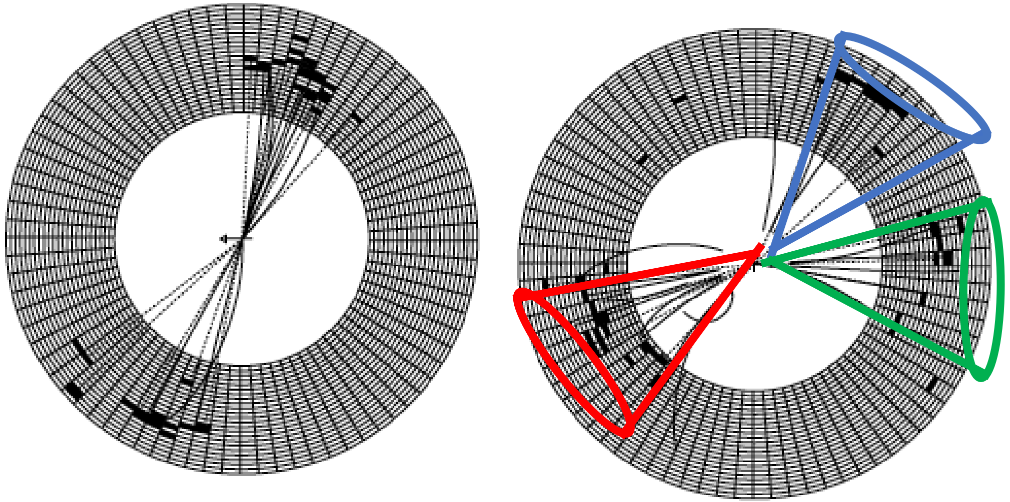
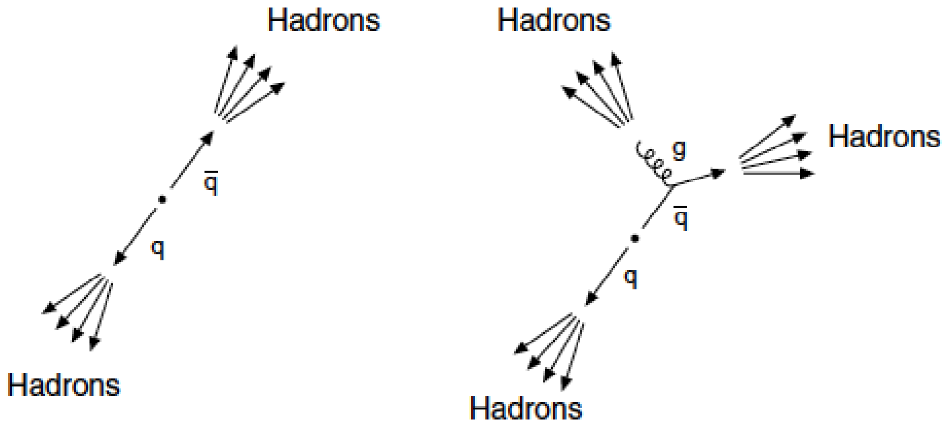


Measuring $\alpha_s(Q^2)$ at different Q^2

Jet production in $pp, p\overline{p}$ interactions

- At high energies, hadrons are typically produced in two jets, emitted in opposite directions.
- These jets are produced in the hadronization of the primary quarks and antiquarks.
- In addition to simple qq production, higher-order processes can happen. For example, a high-energy ("hard") gluon can be emitted, which can then manifest itself as a third jet of hadrons.





This is ~ to the emission of a γ in em bremsstrahlung. The em coupling constant α is small → emission of a hard photon is a relatively rare process.
The probability of gluon bremsstrahlung (right part of the Figure) is given by the coupling constant α_s .

A comparison of the 3- and 2-jet event rates → α_s.

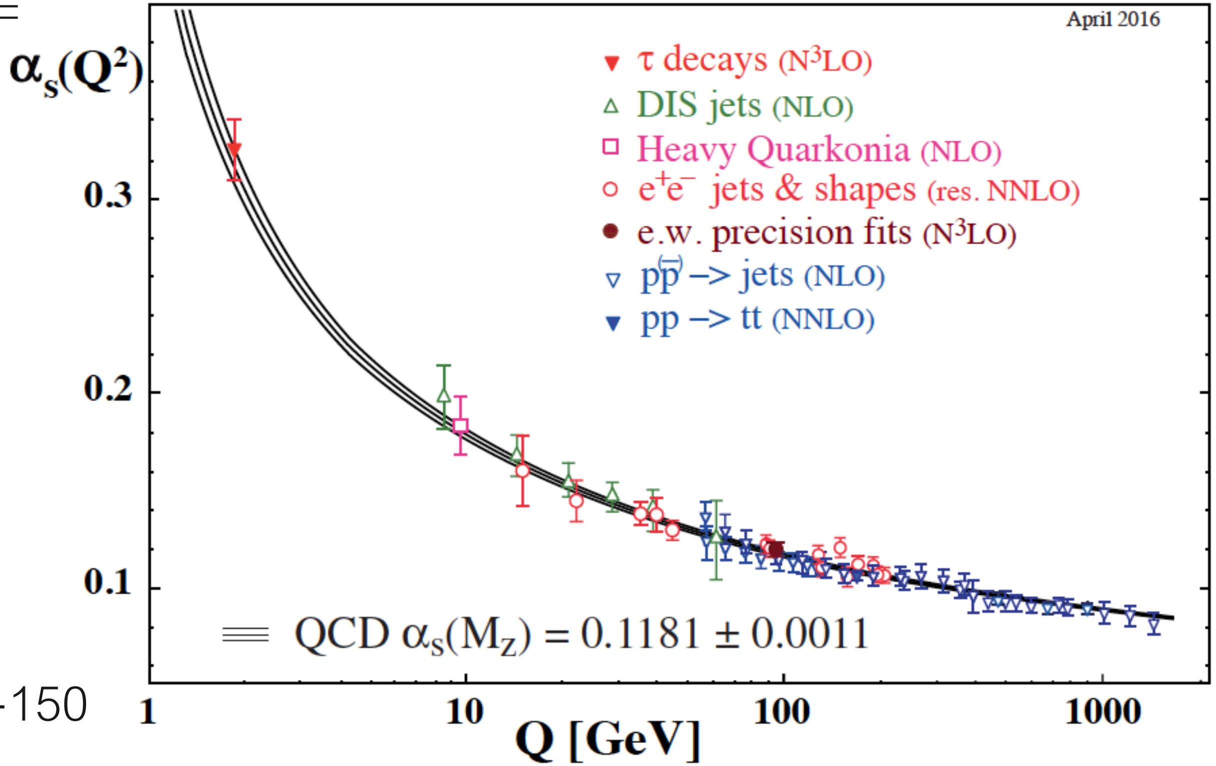
Measurements at different energies show that α_s decreases with increasing Q^2 as predicted by

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \cdot \ln(Q^2/\Lambda^2)}$$

More Ways of Measuring $\alpha_s(Q^2)$

Review of Particles Properties 2018 edition: http://pdg.lbl.gov/2018/reviews/rpp2018-rev-qcd.pdf

- Hadronic decays of the τ lepton: $\tau \rightarrow \upsilon_\tau + hadrons$ (Q=1.77GeV)
- Evolution of the nucleon structure functions measured in inelastic scattering of e,μ,ν on nucleons (Q=2 ÷ 50 GeV)
- Jet production in the inelastic scattering $ep \rightarrow eX$ (Q= 2 ÷ 50 GeV)
- Analyses of the energy levels of bound states $q\bar{q}$ (quarkonium) (Q = 1.5 ÷ 5 GeV)
- Decays of the vector mesons ϒ (Q = 5 GeV)
- Hadronic cross-section of the annihilation $e^+e^- \rightarrow hadrons$ (Q = 10 ÷ 200 GeV)
- Fragmentation function of jets produced in $e^+e^- \rightarrow hadrons$ (Q = 10 ÷ 200 GeV)
- *Hadronic decays of the Z^0 boson (Q = 91 GeV)*
- *Jet production in $pp, p\bar{p}$ interactions (Q = 50 ÷1000 GeV)*
- Photon production in in $pp, p\bar{p}$ interactions (Q = 30 ÷150 GeV)



Backup