Logistics & Introduction

Toni Baroncelli
Haiping Peng
USTC

March 2nd
March 4th

Logistics - 1

Attending a course in English is difficult for young persons who are not too familiar with foreign languages

I understand your difficulty and appreciate your effort
The world of HEP (High Energy Physics) is a world-wide collaboration and English is the standard tool of communication.
Attending these lectures will help you to improve your foreign language skills (… and studying abroad?)

Course will last 16 weeks, Lectures by

- me, (Antonio) Toni Baroncelli — March 2nd[d] to April 22[th]
- Prof. Haiping Peng — from April 27[th] to June 17[th]

Two lectures/week

- Monday, 3 slots, 15:55-18:20
- Wednesday 2 slots, 14:00-15:35

Topic	Weeks	Who	from	→	# lectures
Introduction to basic concepts	2	T.Baroncelli	02/03/26	11/03/26	4
Deep Inelastic Scattering	1	T.Baroncelli	08/03/26	18/03/26	6
Accelerators	1	T.Baroncelli	15/03/26	25/03/26	8
Detectors	1	T.Baroncelli	22/03/26	01/04/26	10
Measurements at Colliders	3	T.Baroncelli	12/04/26	22/04/26	16
				22/04/26	
Standard Model Theory	2	H.Peng	27/04/26	06/05/26	4
CPV theory and experiment (BELLE,BABAR, LHCb)	2	H.Peng	11/05/26	20/05/26	8
Hadron physics (BESIII, STCF)	2	H.Peng	25/05/26	03/06/26	12
Higher Symmetries (GUT, SUSY, Superstrings....)	2	H.Peng	08/06/26	17/06/26	16
	16				

Slides will be made available soon after the lecture at http://cicpi.ustc.edu.cn/indico/.... Will be defined soon

Each Lecture will be preceded by a short recap of the lecture before

Logistic - 2

The course is not historically-organised

Mathematics is used to represent nature, not the opposite

First part

- Overall picture of how we see (today!) the microscopic world;
- How laws and structure of nature can be represented by models / mathematical formalism;
- Little formalism, just main ideas. Much more material can be found in the reference book.

Second part

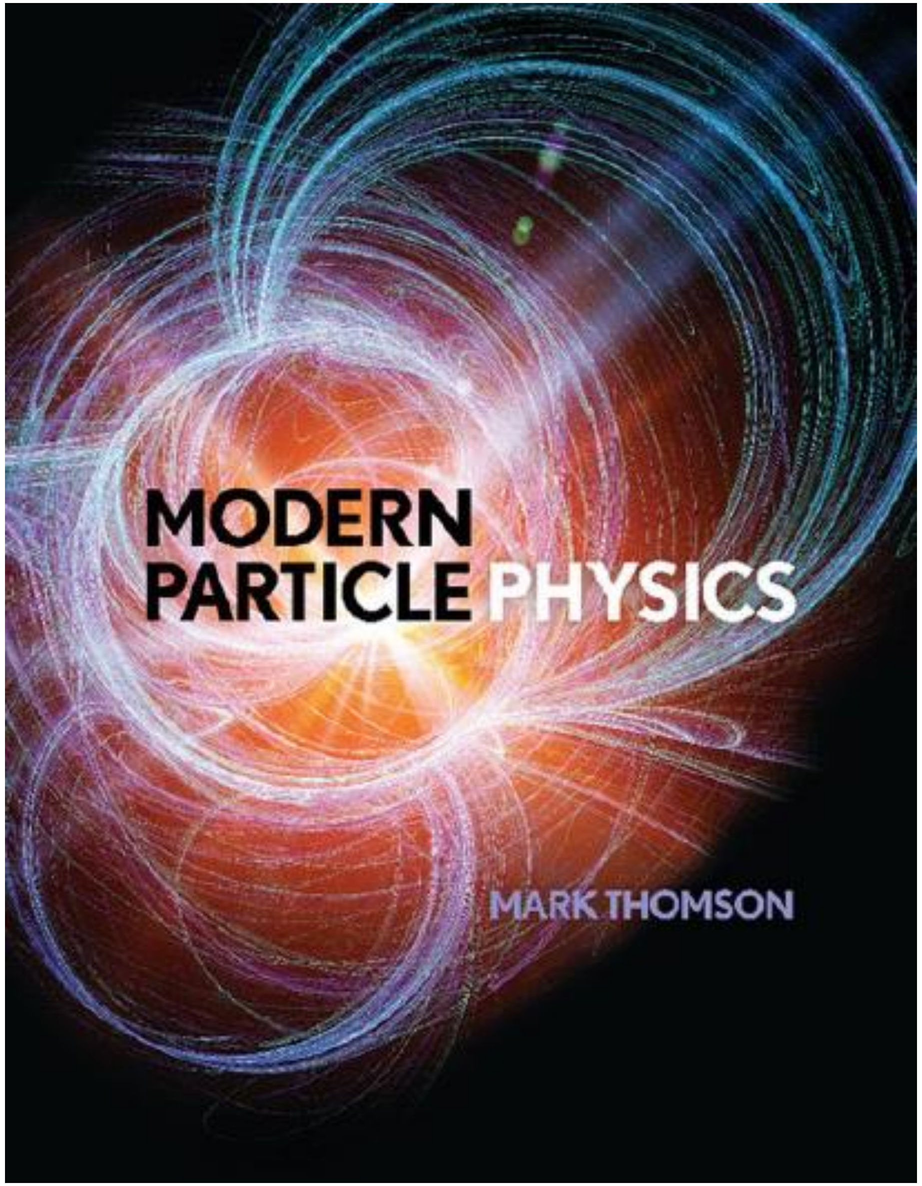
- Instruments and tools of the research in High-Energy Particle Physics (HEP)
  - Accelerators
  - Detectors and Analysis
  - Analysis of discoveries of the past 50 years

Reference Textbook

Lectures of the first part: recent book including much more than in these lectures

- Standard Model
- Discovery of the Higgs Boson
- …

Formalism well documented



Modern Particle Physics

MARK THOMSON
University of Cambridge

Mark Thomson is the present Director of CERN

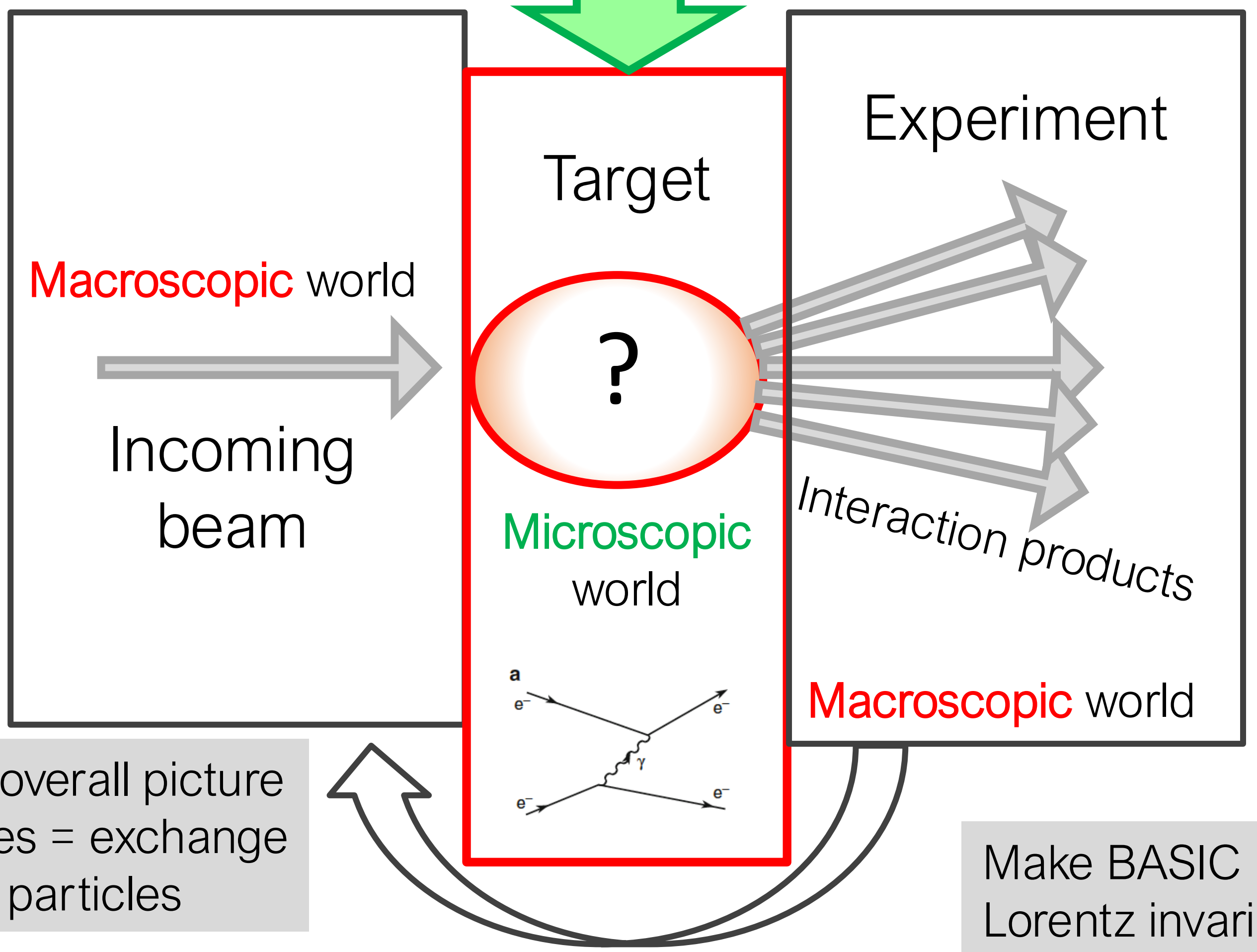
CAMBRIDGE
UNIVERSITY PRESS

Very Basic Ideas

Micro to Macro world

Elementary particles & interactions between them

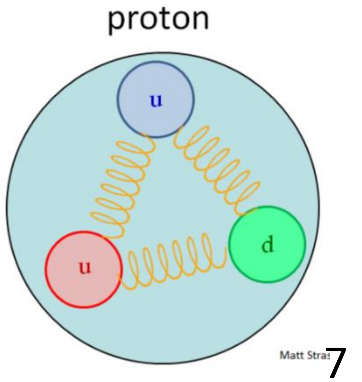
Physics



No way to 'see' what is in the microscopic world → can only see the effect (*look at final state*) of sending a projectile on your target (*initial state*)

'Incoming beam' and 'Target' may be or may be not point-like,

it may have a structure, like a proton (or a nucleus)



→ SM overall picture
→ forces = exchange of particles

Make BASIC assumptions (Einstein special relativity, Lorentz invariance) and use models. Are they OK?

Prologue: Many Order of Magnitude

(Reduced) *Planck*'s Constant ($\hbar = h/2\pi$)

The uncertainty principle: "position x (uncertainty Δx) and momentum p_x (with uncertainty Δp_x) cannot simultaneously be known to better than

$$\Delta x \Delta p_x \sim \hbar/2.$$

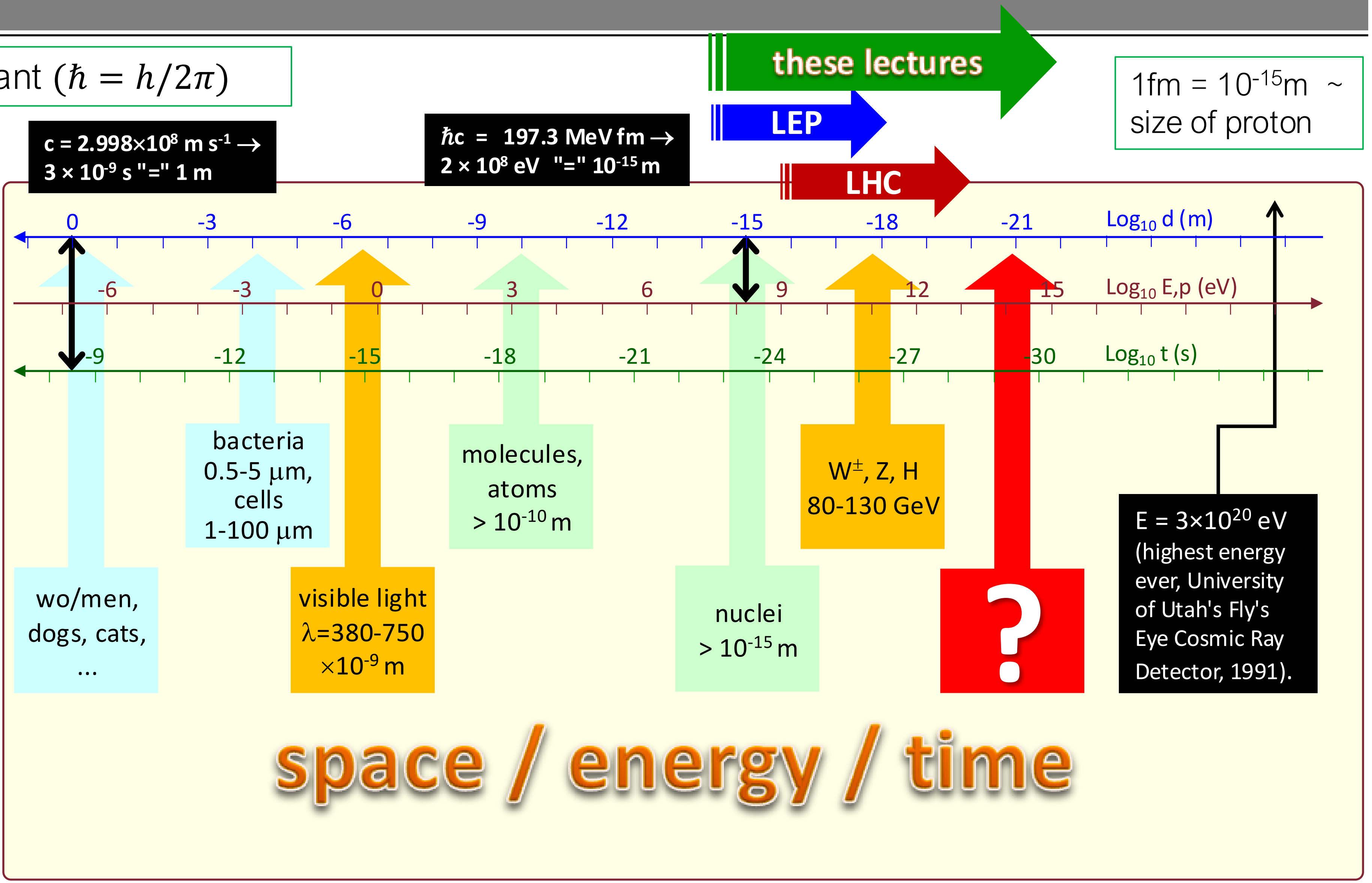
A relation for the energy is obtained by multiplying c,

$$\Delta x \Delta E \sim \frac{\hbar c}{2}$$

which gives numerically,

$$\Delta E(MeV) = \frac{1.973^{-11}(MeV\ cm)}{2\Delta x(cm)}$$

Also $\Delta x = c\Delta t \rightarrow \Delta t \Delta E \sim \frac{\hbar}{2}$



First Part: Preview of Lectures

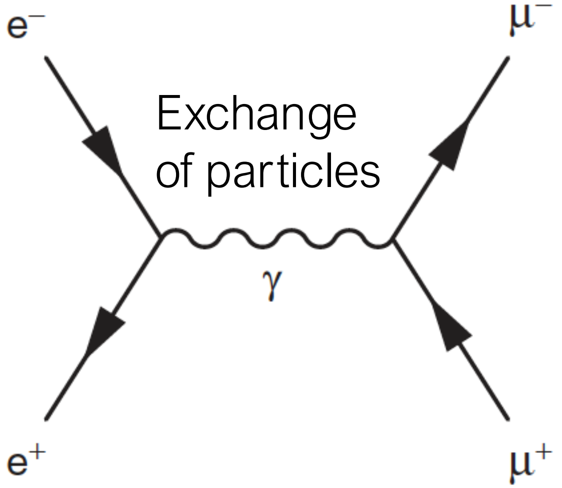
Content of the lectures

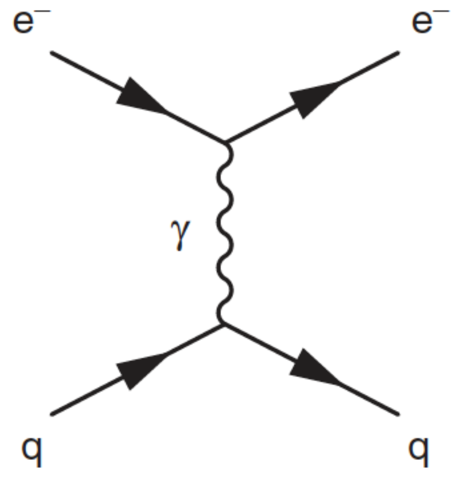
- Calculation of cross sections and decay rates
  - Fermi's golden rule
  - Phase space
- Spin ½ particles (→ Dirac equation)
  - Klein-Gordon equation
  - Dirac equation
  - Antiparticles
  - Spin & helicity
  - Parity of Dirac Particles
- Interaction by particle exchange
  - Perturbation theory
  - Feynman diagrams & virtual particles
  - QED
- Deep Inelastic Scattering
  - Electron-proton scattering
  - Electron-quark scattering
  - PDFs

~2 weeks

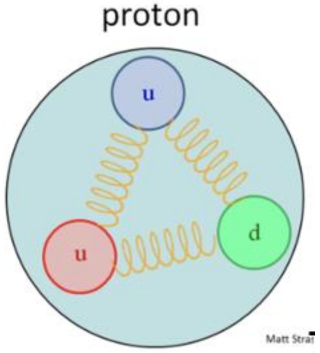
1 week

Feynman diagrams for $e^+e^- \to \mu^+\mu^-$ and $e^-q \to e^-q$ scattering





Probe proton structure

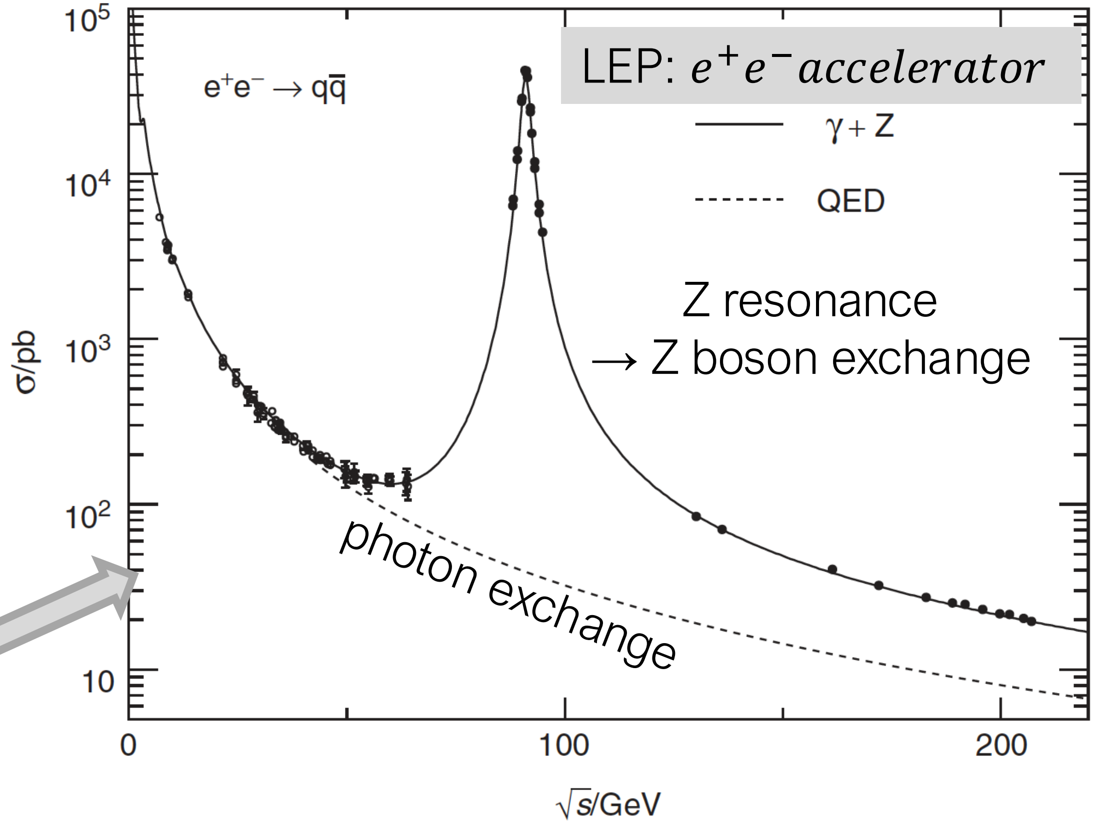


LHC: protons against protons

Second Part: Preview of Lectures

Content of the lectures

- Accelerators (basic ideas, future accelerators?)
- Experiments (no hardware of detectors!)
  - Assembly of detectors $\rightarrow$ Experiments
  - Analysis techniques
- Precision measurements of last ~50 years
  - *Resonances*
  - The discovery of $charm\ and\ bottom$ quarks
  - The discovery of the top quark
  - The $Z - line\ shape$ & number of neutrinos @ LEP (e^+e^- collider at CERN)
  - The discovery of the $Higgs\ boson$ (pp collider at CERN)



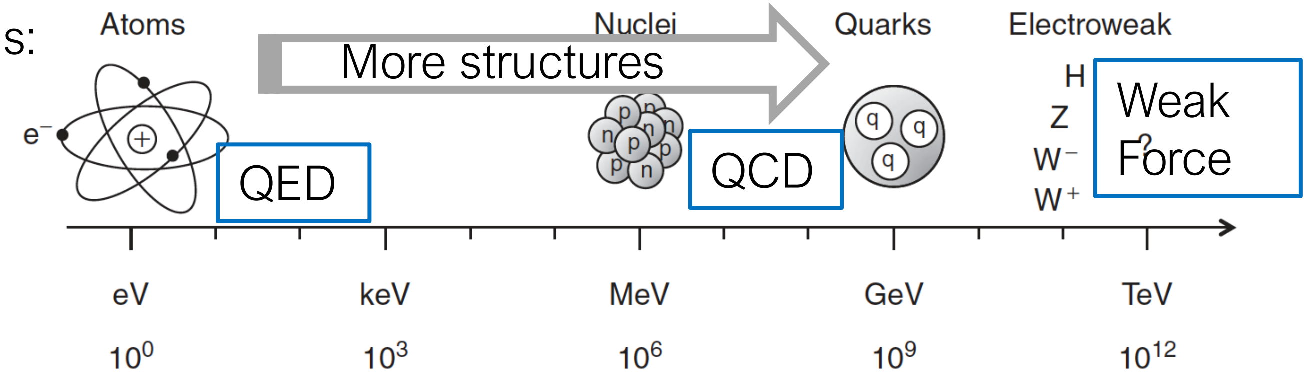
The measurements of the $e^+e^- \rightarrow q\bar{q}$ cross section from LEP close to and above Z resonance. Also shown are the lower-energy measurements from earlier experiments. The dashed line shows the contribution to the cross section from the QED process alone. Adapted from LEP and SLD Collaborations (2006).

Fundamental Particles & Forces

Our world seems to be populated by few objects/particles:

- Atoms p,n & electrons (kept together by em forces)
- Nuclei p,n kept together by strong forces
- Radioactive decays → weak forces
- Gravity → large scale structures in the Universe
- Protons & neutrons → quarks

Elementary particles + Forces → Standard Model



The Universe at different energy scales, from atomic physics to modern particle physics at the TeV scale.

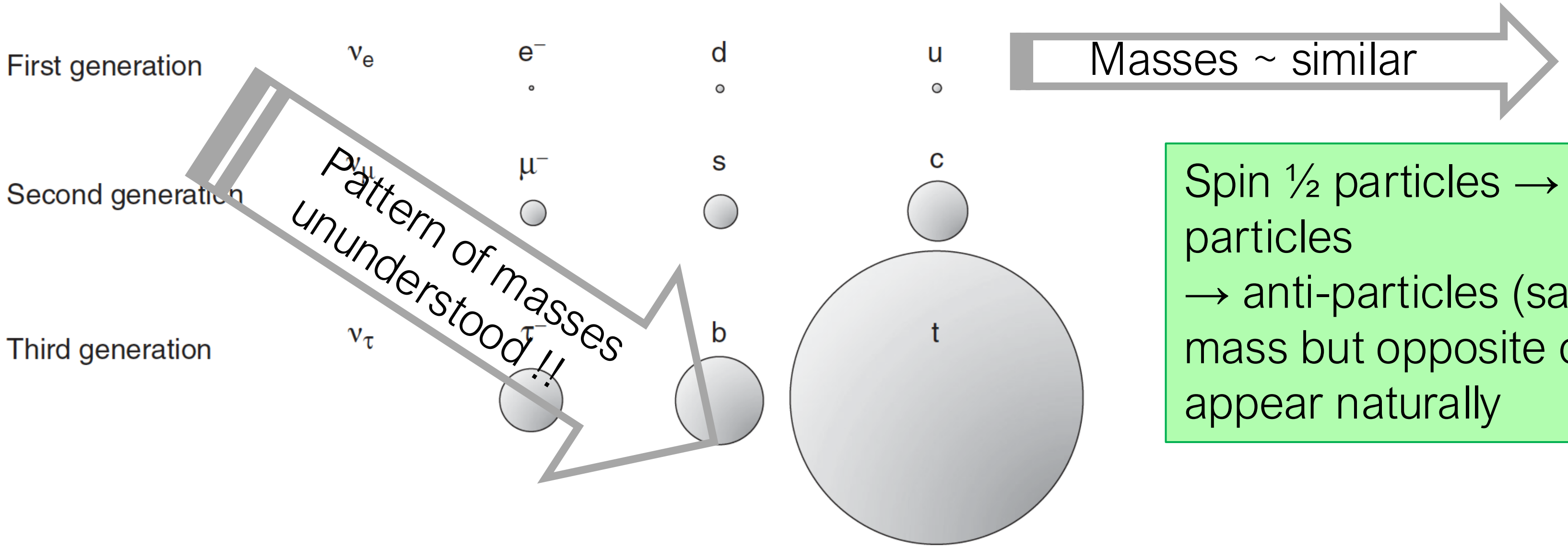
12 particles

Table 1.1 The twelve fundamental fermions divided into quarks and leptons. The masses of the quarks are the current masses.

	Leptons				Quarks			
	Particle		Q	mass/GeV	Particle		Q	mass/GeV
First generation	electron	(e^-)	−1	0.0005	down	(d)	−1/3	0.003
	neutrino	(ν_e)	0	$< 10^{-9}$	up	(u)	+2/3	0.005
Second generation	muon	(μ^-)	−1	0.106	strange	(s)	−1/3	0.1
	neutrino	(ν_μ)	0	$< 10^{-9}$	charm	(c)	+2/3	1.3
Third generation	tau	(τ^-)	−1	1.78	bottom	(b)	−1/3	4.5
	neutrino	(ν_τ)	0	$< 10^{-9}$	top	(t)	+2/3	174

- **Quarks & leptons** are ~point-like, no structure inside; *spin 1/2*
- Organised into **three generations**, differing only in mass, same properties;
- Apparently, **no more generations**;
- → 4 particles x 3 generations

Quarks & Leptons - 2



The particles in the three generations of fundamental fermions with the masses indicated by imagined spherical volumes of constant density. In reality, fundamental particles are believed to be point-like.

Spin ½ particles → Dirac particles
→ anti-particles (same mass but opposite charge) appear naturally

Table 1.2 The forces experienced by different particles.

					strong	electromagnetic	weak
Quarks	down-type	d	s	b	✓	✓	✓
	up-type	u	c	t			
Leptons	charged	e^-	μ^-	τ^-		✓	✓
	neutrinos	ν_e	ν_μ	ν_τ		no charge	✓

4 forces;

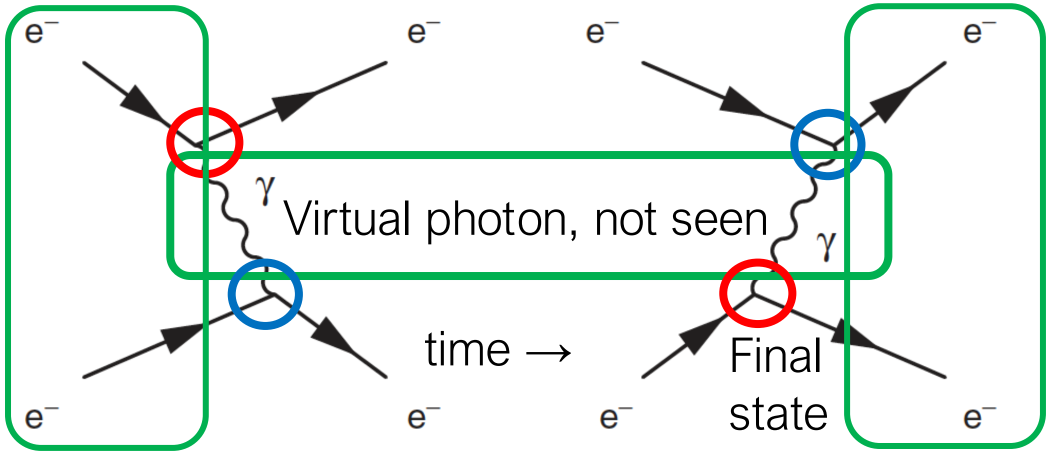
- Gravity (neglect, no role in particle-interactions)
- Weak force
- EM force
- Nuclear (strong) force

Forces: potential?

In Classical Mechanics (EM) forces can be described by means of a scalar potential.
Unsatisfactory! Transfer of momentum without mediating body!

In QFT each force acts via virtual mediators.
No action at a distance!

Transfer of momentum: natural !



Each of the three forces (*not gravity*) is mediated by a

spin-1 force-carrying particle

Relative strength very different (we do not know why …!)

Table 1.3 The four known forces of nature. The relative strengths are approximate indicative values for two fundamental particles at a distance of 1 fm $= 10^{-15}$ m (roughly the radius of a proton).

Force	Strength	Boson		Spin	Mass/GeV
Strong	1	Gluon	g	1	0
Electromagnetism	10^{-3}	Photon	γ	1	0
Weak	10^{-8}	W boson	$W^{\pm}$	1	80.4
		Z boson	Z	1	91.2
Gravity	10^{-37}	Graviton?	G	2	0

The Higgs Boson

Discovered in 2012 by ATLAS & CMS Experiments at the LHC;

- Fundamental fermions: spin ½ particles;
- Gauge bosons: spin-1 particles;

Higgs boson is a spin-0 scalar particle. As conceived in the Standard Model, the Higgs boson is the only fundamental scalar discovered to date.

$$m_{\mathrm{H}} \approx 125\,\mathrm{GeV},$$

The Higgs boson, in the SM of particles, has the role of 'giving mass' to all particles. Whiteout it the Universe would be VERY different, all particles would be massless and would travel at the speed of light

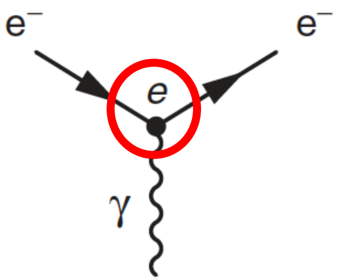
Forces: mediators

Graphical representation of interactions: ***3-point vertex***, one gauge boson + incoming fermion + outgoing fermion

Rule:

Particle couples to a force carrying mediator *only* if it carries the charge of the interaction

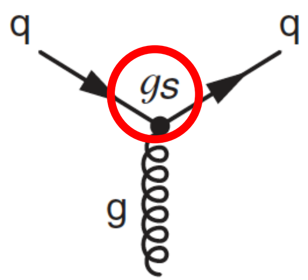
Electromagnetism



All charged particles
Never changes flavour

$\alpha \approx 1/137$

Strong interaction

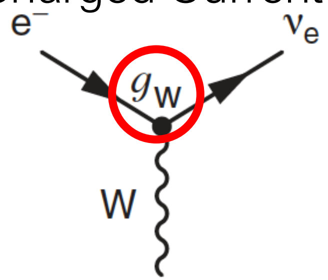


Only quarks
Never changes flavour

$\alpha_S \approx 1$

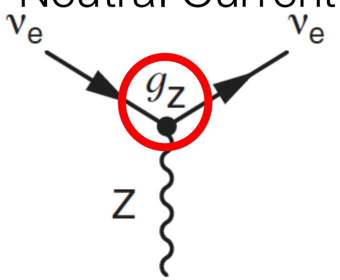
Weak interaction

Charged Current



All fermions
Always changes flavour

Neutral Current



All fermions
Never changes flavour

$\alpha_{W/Z} \approx 1/30$ Mass of W,Z

Change of flavour

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}, \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}, \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$$

Strength of Weak force is greatest for transitions same generation

$$\begin{pmatrix}u\\ d\end{pmatrix}, \begin{pmatrix}u\\ s\end{pmatrix}, \begin{pmatrix}u\\ b\end{pmatrix}, \begin{pmatrix}c\\ d\end{pmatrix}, \begin{pmatrix}c\\ s\end{pmatrix}, \begin{pmatrix}c\\ b\end{pmatrix}, \begin{pmatrix}t\\ d\end{pmatrix}, \begin{pmatrix}t\\ s\end{pmatrix}, \begin{pmatrix}t\\ b\end{pmatrix}$$

- Interaction probability: $\mathcal{M}$ (one state to another)
- Coupling constant: g (probability spin ½ fermion emits or absorbs the interaction boson)

Table 1.2 The forces experienced by different particles.

					strong	electromagnetic	weak
Quarks	down-type	d	s	b	✓	✓	✓
	up-type	u	c	t			
Leptons	charged	e^-	μ^-	τ^-		✓	✓
	neutrinos	ν_e	ν_μ	ν_τ			✓

Weak Charged Current

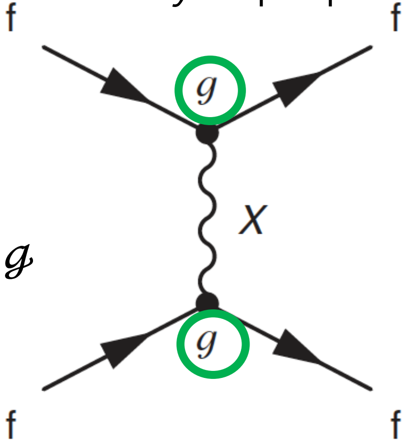
The weak charged-current interaction NOT a usual force: it couples together different flavours fermions. Since the W^+ and W^- bosons have charges of +e and -e

→ in order to conserve electric charge, the weak charged-current interaction only couples together pairs of fundamental fermions that differ ***by one unit of electric charge***. In the case of the leptons, by definition, the weak interaction couples a charged lepton with its corresponding neutrino

Decays due to weak charged current → change of flavour

Interaction probability is proportional to the matrix element $|\mathcal{M}|^2$

Scattering of two fermions via exchange of a vector boson X
Each vertex ffX is described by g



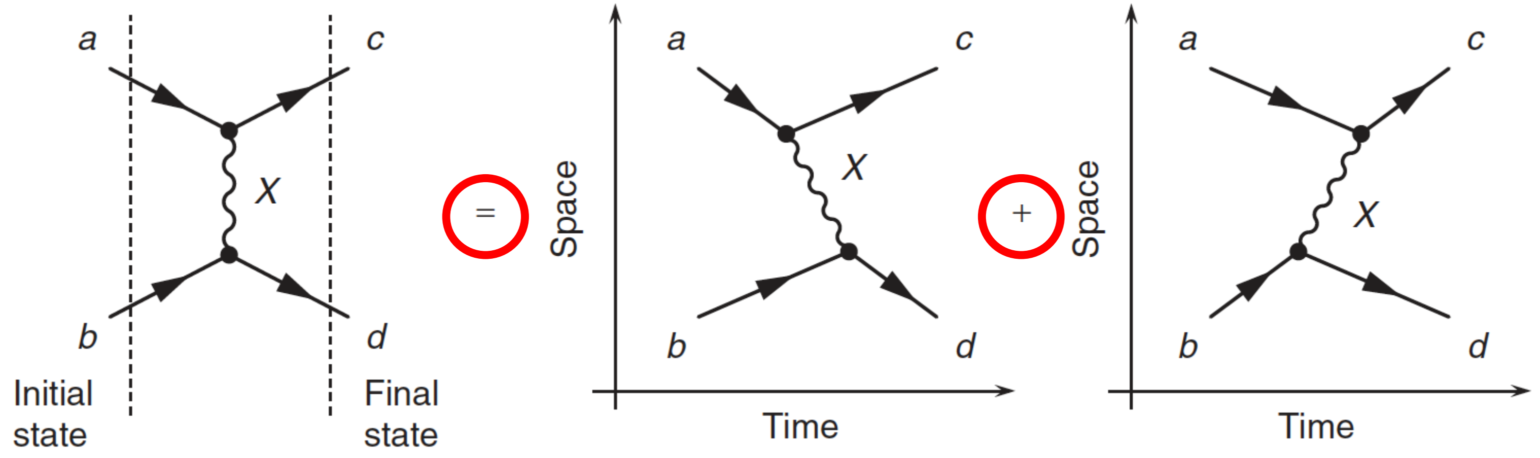
The matrix element $\mathcal{M}$ includes a factor g for each vertex

$$\mathcal{M} \propto g^2$$

→ interaction probability is the square of $\mathcal{M}$

$$|\mathcal{M}|^2 \propto g^4$$

- Feynman diagrams give a graphical representation of an interaction;
- Shows possible time orderings of the interaction
- The interaction is the sum of possible time orderings



The Feynman diagram for the scattering process $a + b \to c + d$ and the two time-ordered processes that it represents.

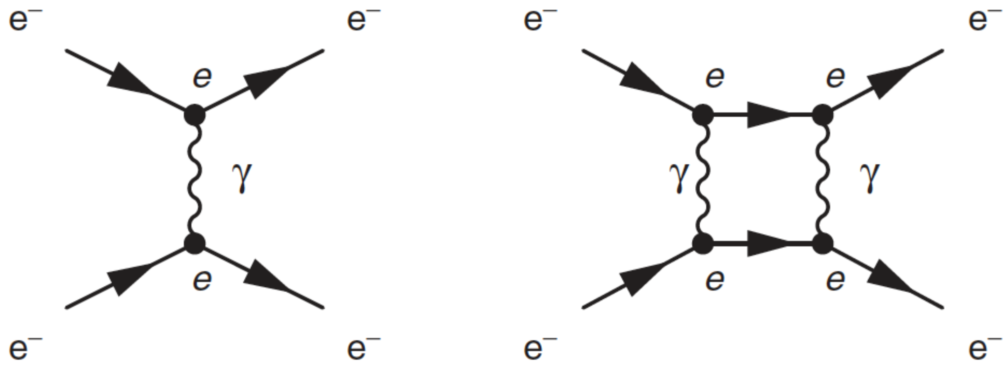
Time ⇒

Feynman Diagrams

One process = superposition of infinite number of Feynman diagrams.

Example: scattering of two electrons via the exchange of one or two photons.

- Same initial and final state;
- Use α (contains e^2) $\approx {}^1/_{137}$
- First diagram (one photon)
  $\mathcal{M} \propto \alpha^2$
- Second diagram (two photons)
  $\mathcal{M} \propto \alpha^4 \rightarrow 2nd\ diagram\ is\ \approx 10^{-4}\ times\ lower\ than\ 1st\ one$

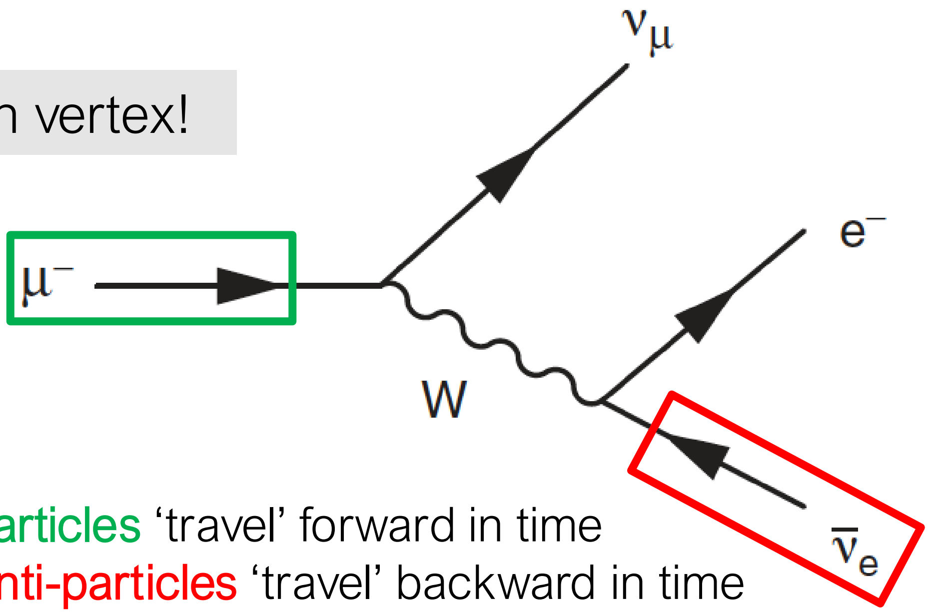


Two Feynman diagrams for $e^-e^- \rightarrow e^-e^-$ scattering.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}.$$

α at each vertex!

- particles and antiparticles created/annihilated only in pairs.
- arrows on the incoming and outgoing fermion in the same sense and flow through the vertex;
- they never both point towards or away from the vertex.



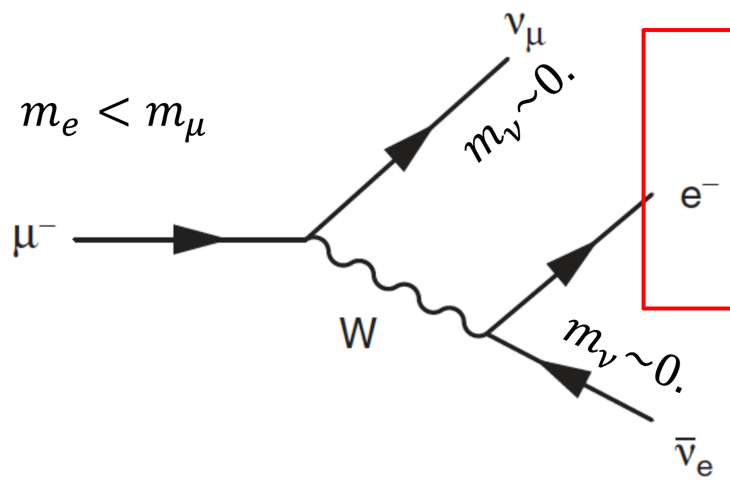
Particles 'travel' forward in time
Anti-particles 'travel' backward in time

Unstable Particles

Most particles decay with a very short lifetime → few long-lived or stable particles detected in experiments

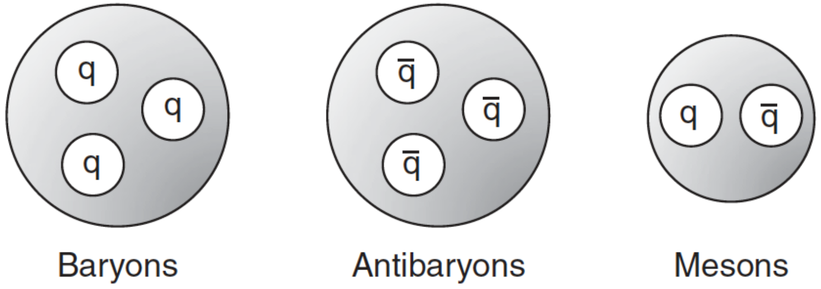
The decay of a particle can always be described in terms of a Feynman diagram

- the decay products must have a rest mass lower than the initial state:
- Weak force: all particles (and change of flavour)
- Coupling Constant increases → Lifetime decreases



Lightest charged lepton → cannot decay

- Hadrons exist as Baryons, Antibaryons, Mesons;
- Strong force, QCD interactions:
  - quarks cannot exist as free particles
  - → only bound states
  - → decays to be interpreted as transitions between bound states

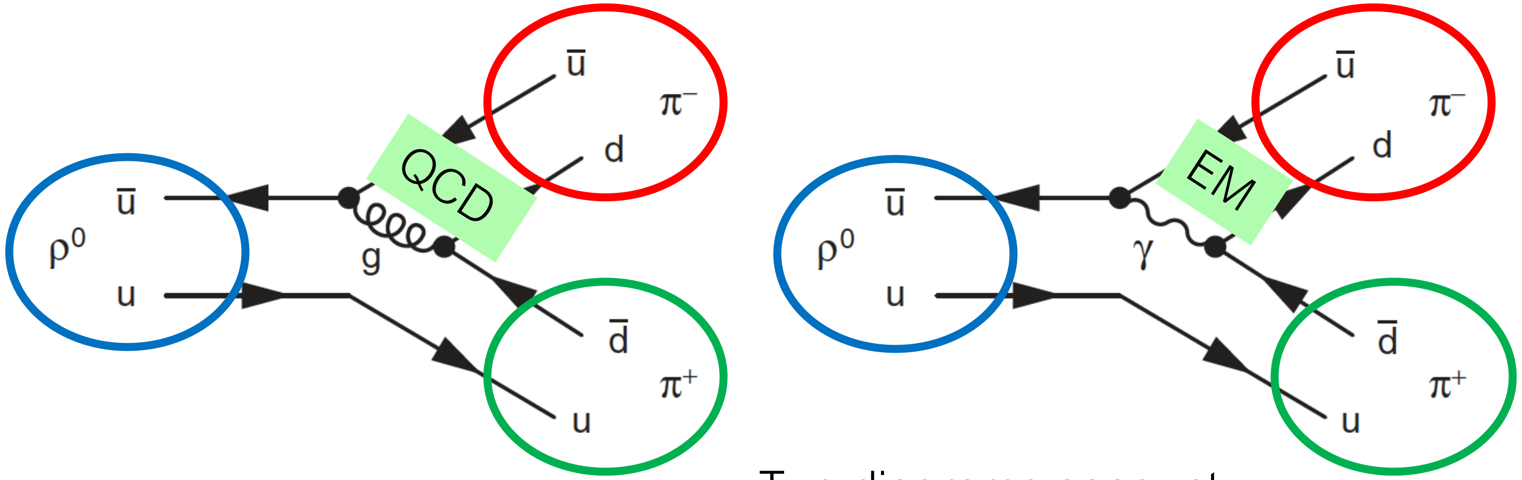


Unstable Particles – continued

$|\mathcal{M}_g|^2 \propto \alpha_s^2 \gg |\mathcal{M}_\gamma|^2 \propto \alpha^2$

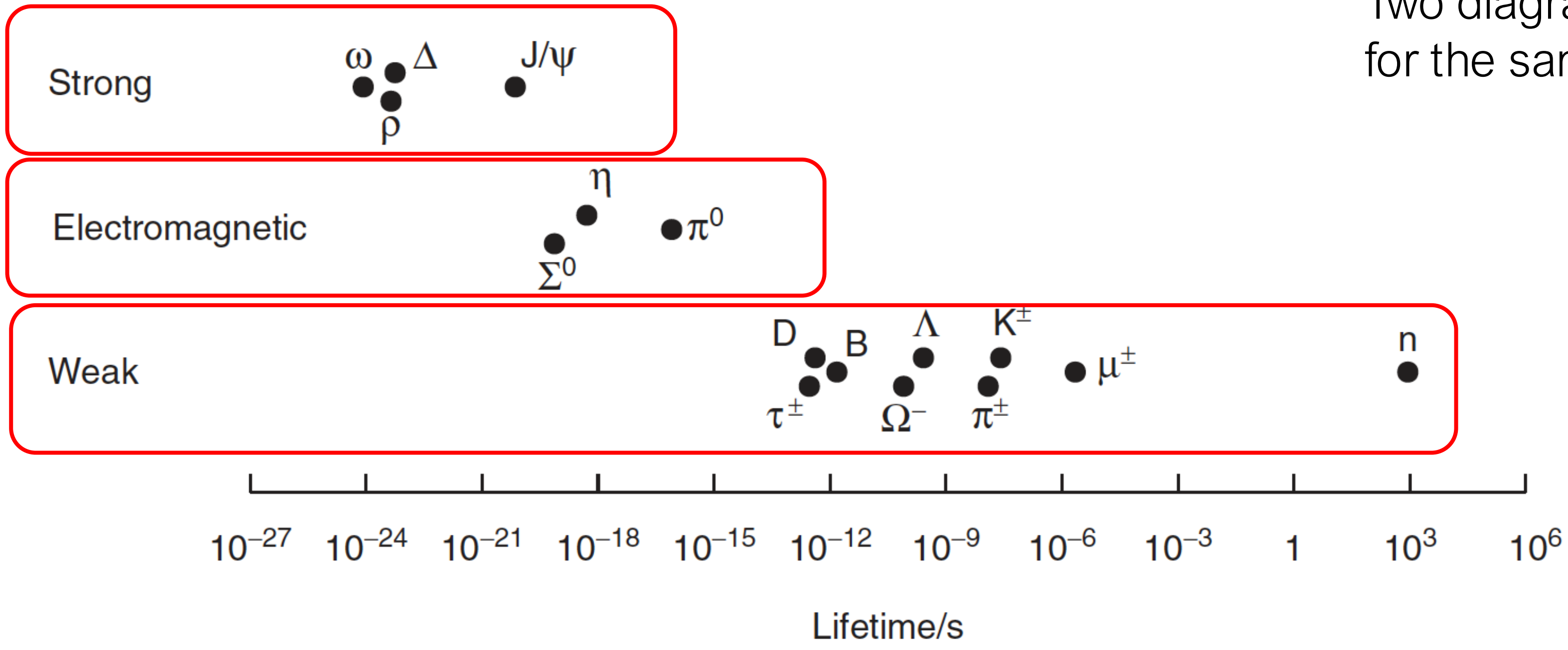
Exchange of gluon dominates

$\tau_{strong} \ll \tau_{EM} \ll \tau_{weak}$



Two diagrams account for the same decay

Strong decays dominates over EM decays
EM decays dominate over weak decays

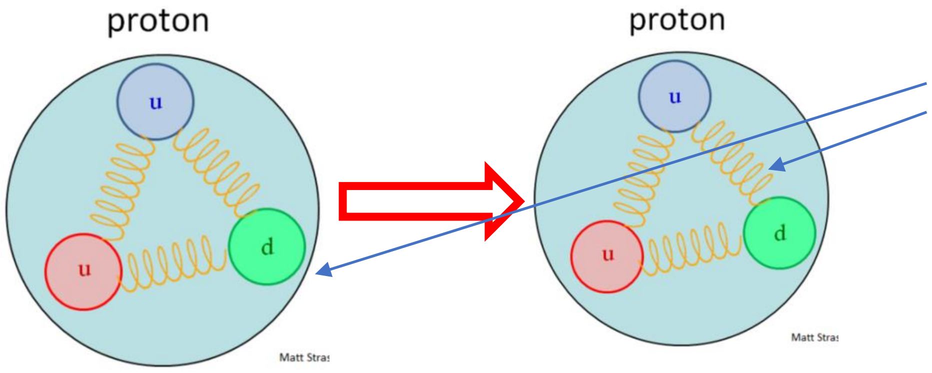


Secondary vertices *may* be detected

Anticipation → why electron scattering?

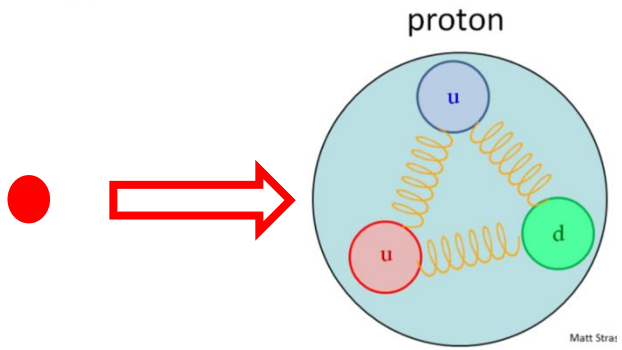
Nuclear sizes and shapes → use scattering technique → use a projectile (accelerated or from radioactivity) that hits a target

Protons are extended and complex objects



nuclear forces between the projectile and the target are complex and complex to describe

Use electrons! Point-like projectiles!



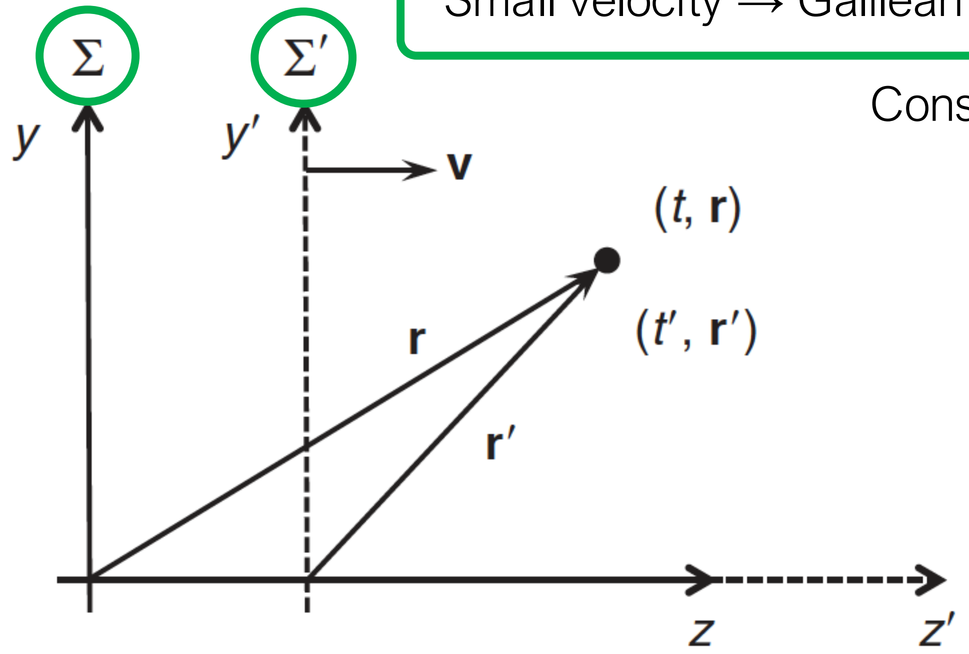
- *The interactions between an electron and a nucleus, nucleon or quark takes place via the exchange of a virtual photon — this may be very accurately calculated in quantum electrodynamics (QED).*
- *These QED processes are in fact manifestations of the well-known electromagnetic interaction, whose coupling constant $\alpha \approx 1/137$ is much less than one. This last means that higher order corrections play only a tiny role*

Kinematics & Co

Reminder: Special Relativity

$\hbar = c = \varepsilon_0 = \mu_0 = 1$

Small velocity → Galilean transformation $t' = t, \quad x' = x, \quad y' = y \quad \text{and} \quad z' = z - vt.$



Consider two inertial frames: Σ *and* Σ', Σ' moving with velocity v *along* z: $\beta = {}^{v}/_{c}$ $\gamma = (1-\beta^2)^{1/2}$

Einstein: r and r' are the same in all systems →

r, r': space-time interval

$$c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2,$$

$$t' = \gamma(t - \beta z), \quad x' = x, \quad y' = y \quad \text{and} \quad z' = \gamma(z - \beta t).$$

When $v \ll c \;\rightarrow\; \beta = 0,\ \gamma = 1$

In matrix form:

$$\underbrace{\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix}}_{\Lambda} \underbrace{\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix}}_{X'}. \qquad \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}}_{\Lambda^{-1}} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix},$$

4-vector $(t, \boldsymbol{x})$, $\boldsymbol{X}' = \boldsymbol{\Lambda X}$, $\boldsymbol{X} = \boldsymbol{\Lambda}^{-1}\boldsymbol{X}'$, $\boldsymbol{\Lambda\Lambda}^{-1} = \boldsymbol{I}$.

Lorence Invariance: why?

Throughout particle physics it is highly desirable to express physical predictions, such as interaction cross sections and decay rates, in an explicitly Lorentz-invariant form that can be applied directly in all inertial frames.

The Lorentz invariance of the space-time interval

$$t^2 - x^2 - y^2 - z^2$$

can be expressed as a *four-vector scalar* product by introducing

- the contravariant space-time four-vector,

$$x^\mu = (t, x, y, z)$$

- the covariant space-time four-vector,

$$x_\mu = (t, -x, -y, -z)$$

→ the Lorentz-invariant space-time interval can be written as the four-vector scalar product

$$x^\mu x_\mu = t^2 - x^2 - y^2 - z^2$$

4-Vectors and Lorentz Invariance

A fundamental idea in Physics is that laws of Nature do not depend on the frame where they are measured.

This is best expressed by

1. Introducing contravariant and covariant 4 vectors and
2. Requiring space-time intervals to be Lorentz invariant

$$x^\mu = (t, x, y, z), \qquad x_\mu = (t, -x, -y, -z).$$

$$x^\mu x_\mu = x^0 x_0 + x^1 x_1 + x^2 x_2 + x^3 x_3 = t^2 - x^2 - y^2 - z^2.$$

$$\begin{pmatrix} t' \\ -x' \\ -y' \\ -z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t \\ -x \\ -y \\ -z \end{pmatrix}.$$

Lorentz transformation of covariant 4-vector $\quad x'^\mu = \Lambda^\mu{}_\nu x^\nu,$

contravariant 4-vector to a covariant 4-vector $\quad x_\mu = g_{\mu\nu} x^\nu,$

$$g_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Only quantities with Lorentz transformation properties are such that $x^\mu x_\mu$ are Lorentz invariant

If a^μ *and* b^μ are contravariant then the scalar product is also invariant $\quad a^\mu b_\mu = a_\mu b^\mu = g_{\mu\nu} a^\mu b^\nu,$

Four Momentum and Four Derivatives (Use $c = 1$)

Relativistic momentum and energy of a particle with mass m

$$E = \gamma m \quad \text{and} \quad \mathbf{p} = \gamma m \boldsymbol{\beta}.$$

$$t' = \gamma(t - \beta z), \quad x' = x, \quad y' = y \quad \text{and} \quad z' = \gamma(z - \beta t)$$

the scalar product

$$p^\mu p_\mu = E^2 - \mathbf{p}^2, \\ = m^2$$

Momentum and energy are conserved separately
→ also 4-momentum scalar product is:

$$p^\mu = (E, p_x, p_y, p_z),$$

Lorentz transformation of a 4-derivative from frame Σ' to Σ

$$\frac{\partial}{\partial z'} = \left(\frac{\partial z}{\partial z'}\right)\frac{\partial}{\partial z} + \left(\frac{\partial t}{\partial z'}\right)\frac{\partial}{\partial t} \quad \text{and} \quad \frac{\partial}{\partial t'} = \left(\frac{\partial z}{\partial t'}\right)\frac{\partial}{\partial z} + \left(\frac{\partial t}{\partial t'}\right)\frac{\partial}{\partial t}.$$

$$\frac{\partial}{\partial z'} = \gamma\frac{\partial}{\partial z} + \gamma\beta\frac{\partial}{\partial t} \quad \text{and} \quad \frac{\partial}{\partial t'} = \gamma\beta\frac{\partial}{\partial z} + \gamma\frac{\partial}{\partial t}.$$

$$\left(\frac{\partial z}{\partial z'}\right) = \gamma, \quad \left(\frac{\partial t}{\partial z'}\right) = +\gamma\beta, \quad \left(\frac{\partial z}{\partial t'}\right) = +\gamma\beta \quad \text{and} \quad \left(\frac{\partial t}{\partial t'}\right) = \gamma,$$

In matrix notation

$$\begin{pmatrix} \partial/\partial t' \\ \partial/\partial x' \\ \partial/\partial y' \\ \partial/\partial z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \partial/\partial t \\ \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix},$$

→ transforms as a covariant four-vector

Laplacian

The corresponding contravariant four-derivative is $\partial^{\mu} = \left(\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right),$

The Laplacian for the four-derivative $\Box = \partial^{\mu}\partial_{\mu} = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}.$

Notations

Quantities written as
x, p
are four-vectors

Quantities written **in bold** as
$\boldsymbol{x}, \boldsymbol{p}$
Are three-vectors

Four-vectors scalar product is $a \cdot b \equiv a^{\mu} b_{\mu} \equiv g_{\mu\nu} a^{\mu} b^{\nu} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3.$

The Einstein energy-momentum relationship → $p^2 = m^2 \; since \; p^2 = p \cdot p = E^2 - \boldsymbol{p}^2$

A quantity in the Center of Mass System (cms) of a group of particles is labelled with a * , example q*

Mendelstam Variables

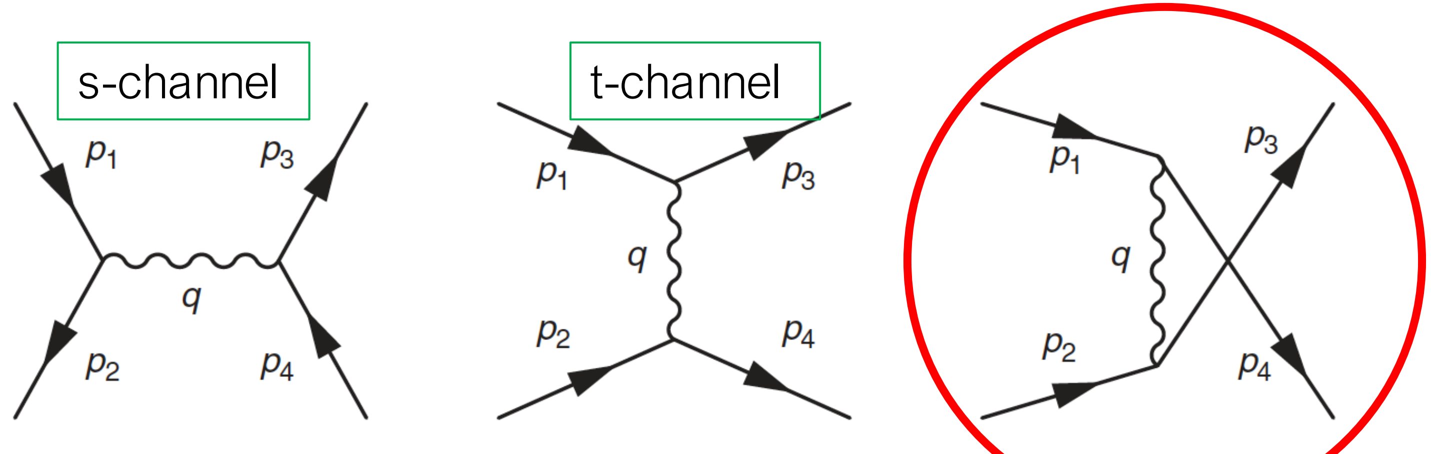
In reaction $1 + 2 \rightarrow 3 + 4$ one mediating particle is emitted/absorbed in different ways

- s-channel: particle 1 emits a mediator absorbed by particle 3
- t-channel: particle 1 emits a mediator absorbed by particle 2

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$\mathrm{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$\mathrm{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$s + u + t = m_1^2 + m_2^2 + m_3^2$$

These variables are equivalent to the four-momentum squared q^2 of the exchanged particle



Only relevant when there are identical particles in the final state

The Feynman diagrams for *s*-channel, *t*-channel and *u*-channel processes. The *u*-channel diagram applies only when there are identical particles in the final state.

In the cms $p_1 = (E_1^*, \boldsymbol{p}^*)$ and $p_2 = (E_2^*, -\boldsymbol{p}^*) \rightarrow s = (p_1 + p_2)^2 = (E_1^* + E_2^*)^2 - (p^* - p^*)^2 = (E_1^* + E_2^*)^2$

$\rightarrow s$ is the total available energy in the cms system

Short reminder, define notations

Non Relativistic Quantum Mechanics

Forward to Dirac Equation

Wave Mechanics & Schrödinger Equation

Non-relativistic QM: Free particles = Fourier superposition of plane waves ($\boldsymbol{k} = \boldsymbol{p}, \omega = E$)

$$\psi(\mathbf{x}, t) \propto \exp\{i(\mathbf{k} \cdot \mathbf{x} - \omega t)\}.$$

Wavelength of a particle in quantum mechanics λ = h/p, or, equivalently, the wave vector k is given by k = p.
The angular frequency of the plane wave describing a particle is given by E = ω.

$$\psi(\mathbf{x}, t) = N \exp\{i(\mathbf{p} \cdot \mathbf{x} - Et)\},$$

Classical physics: energy & momentum of a particle *time dependent* real numbers;

Quantum Mechanics (Schrödinger view):

- Wave function completely defines a state;
- *time-dependent* wavefunction;
- Dynamical variables (energy & momentum): *time-independent* operators acting on the wavefunction:

$$\hat{A}\psi = a\psi$$

Time Dependence and Conserved Quantities

Time dependence of a system $i\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \hat{H}\psi(\mathbf{x},t),$ If $\psi(x,t)$ is eigenstate of $\hat{H}$ with energy E

$$\psi(\mathbf{x},t) = N\exp\{i(\mathbf{p}\cdot\mathbf{x} - Et)\},$$

a free particle with energy E and momentum p, it is reasonable to identify the momentum and energy operators, $\hat{p}$ and $\hat{E}$, as

$$\hat{\mathbf{p}} = -i\nabla \quad \text{and} \quad \hat{E} = i\frac{\partial}{\partial t}.$$

$$i\frac{\partial\psi_i(\mathbf{x},t)}{\partial t} = E_i\psi_i(\mathbf{x},t). \implies \psi_i(\mathbf{x},t) = \phi_i(\mathbf{x})e^{-iE_it}.$$

Consider an observable corresponding to an operator $\hat{A}\psi = a\psi$; is a a conserved quantity? Expectation value

$$\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle = \int \psi^\dagger\hat{A}\psi\, \mathrm{d}^3\mathbf{x},$$

If the Hamiltonian and the operator commute, then the corresponding observable does not change with time

$$\frac{\mathrm{d}\langle\hat{A}\rangle}{\mathrm{d}t} = i\left\langle[\hat{H},\hat{A}]\right\rangle,$$

If two operators commute $[\hat{A},\hat{B}] = 0$ then they can be simultaneously measured

Wave Mechanics

In classical mechanics: total energy = kinetic energy + potential energy (Hamiltonian)

$$E = H = T + V = \frac{\mathbf{p}^2}{2m} + V,$$

$$i\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \hat{H}\psi(\mathbf{x},t), \qquad \hat{H}_{NR} = \frac{\hat{\mathbf{p}}^2}{2m} + \hat{V} = -\frac{1}{2m}\nabla^2 + \hat{V}.$$

Time-dependent Schrödinger Equation

$$i\frac{\partial\psi(\mathbf{x},t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(\mathbf{x},t)}{\partial x^2} + \hat{V}\psi(\mathbf{x},t).$$

Time Dependence and Conserved Quantities

The expectation value of an operator $\hat{A}$ is given by

$$\psi^\dagger = (\psi^*)^T$$

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle = \int \psi^\dagger \hat{A} \psi d^3\boldsymbol{x}$$

$$\frac{\mathrm{d}\langle \hat{A} \rangle}{\mathrm{d}t} = \int \left[\frac{\partial \psi^\dagger}{\partial t} \hat{A} \psi + \psi^\dagger \hat{A} \frac{\partial \psi}{\partial t} \right] \mathrm{d}^3\mathbf{x},$$

$$\begin{aligned} \frac{\mathrm{d}\langle \hat{A} \rangle}{\mathrm{d}t} &= \int \left[\left\{ \tfrac{1}{i} \hat{H} \psi \right\}^\dagger \hat{A} \psi + \psi^\dagger \hat{A} \left\{ \tfrac{1}{i} \hat{H} \psi \right\} \right] \mathrm{d}^3\mathbf{x} \\ &= i \int \left[\psi^\dagger \hat{H}^\dagger \hat{A} \psi - \psi^\dagger \hat{A} \hat{H} \psi \right] \mathrm{d}^3\mathbf{x} \\ &= i \int \psi^\dagger (\hat{H}\hat{A} - \hat{A}\hat{H}) \psi \, \mathrm{d}^3\mathbf{x}. \end{aligned}$$

$$\frac{d\langle \hat{A} \rangle}{dt} = i \langle [\hat{H}, \hat{A}] \rangle$$

If $\hat{H}$ commutes with $\hat{A}$ then the derivative with respect to time is 0 and the corresponding observable does not vary with time → conserved quantity

Matrices, reminder-1

A matrix is a rectangular table with ordered elements. Horizontal sequences are called rows, vertical sequences are columns. Each element has an index identifying the row and the column: $a_{row,column}$. An example of a matrix with *m* columns and *n* rows is shown below.

$$A = \begin{matrix} a_{1,1} & a_{1,2} & .. & a_{1,m} \\ a_{2,1} & a_{2,2} & .. & a_{2,m} \\ .. & .. & .. & .. \\ a_{n,1} & a_{n,2} & .. & a_{n,m} \end{matrix}$$

We may define operations between matrices:

- Sum between two matrices of type $m \times n$: $[A+B]_{i,j} = [A]_{i,j} + [B]_{i,j}$. The sum of two matrices commutes $[A]+[B]=[B]+[A]$
- Multiplication by a scalar: $[cA]_{i,j} = c[A]_{i,j}$
- Product: this operation is defined only between two matrices of type $A = m \times p$ and $B = p \times n$. The result is a matrix of type $C = m \times n$. Each element of C is defined as $[C]_{i,j} = Row_i(A) \times Col_j(B) = [A]_{i,1} \times [B]_{1,j} + [A]_{i,2} \times [B]_{2,j} + \cdots + [A]_{i,p} \times [B]_{p,j}$. The product of two matrices do not commute, in general $A \times B \neq B \times A$

We have properties of matrices:

- $A + 0 = 0 + A = A$
- $A + (-A) = 0$
- $(A+B)+C = A+(B+C) = 0$
- $A + B = B + A$
- $(ab)A = a(bA)$
- $a(A+B) = aA + aB$
- $(AB)C = A(BC)$
- $(A+B)C = AC + BC; C(A+B) = CA + CB$

Matrices, reminder-2

Calculation of the determinant of a square matrix:
Order two:
$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$,

$$\det(A) = |A| = a_{1,1} \cdot a_{2,2} - a_{1,2} \cdot a_{2,1}$$

Order three:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

$$\det(A) = |A| = a_{1,1} \cdot \begin{vmatrix} a_{2,2} & a_{2,3} \\ a_{3,2} & a_{3,3} \end{vmatrix} - a_{1,2} \cdot \begin{vmatrix} a_{2,1} & a_{2,3} \\ a_{3,1} & a_{3,3} \end{vmatrix} + a_{1,3} \cdot \begin{vmatrix} a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{vmatrix}$$

Special matrices:

- Unitary (Identity), it is a square matrix with all diagonal elements equal to 1 while all the other off-diagonal elements are 0;
- Transpose matrix: rows and columns are exchanged (same as reflecting elements of A along its diagonal); indicated as A^T.
- Hermitian matrix $A^\dagger$ is a matrix composed of complex numbers that coincides with its transpose conjugate matrix. This implies that $A = \overline{A^T}$ also $a_{i,j} = \overline{a_{j,i}}$

- $A = \begin{pmatrix} 2 & 4 & 8 \\ 3 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $\quad A^T = \begin{pmatrix} 2 & 3 & 5 & 0 \\ 4 & 2 & 3 & 1 \\ 8 & 0 & 1 & 0 \end{pmatrix}$ $\quad A^\dagger \quad \begin{pmatrix} 3 & 2+i \\ 2-i & 1 \end{pmatrix}$

Schrödinger Equation & Co (Sec.2.3)

Non-relativistic Quantum Mechanics (QM), free particles = superposition of wave-packets (Fourier decomposition)

$$\psi(\boldsymbol{x},t) \propto e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$$

$\psi(\boldsymbol{x},t)$ contains all the information about a state

Use $\lambda = h/p$ or $\boldsymbol{k} = \boldsymbol{p}/\hbar$ and $E = \hbar\omega$ and put $\hbar = 1$

$$\psi(\boldsymbol{x},t) = N \cdot e^{i(\boldsymbol{p}\cdot\boldsymbol{x}-Et)}$$

The result of an observation is the result of an operator $\hat{A}$ on the wavefunction resulting in a *real* eigenvalue a:

$$\hat{A}\psi = a\psi$$

In classical mechanics

$$E = H = T + V = \frac{\boldsymbol{p}^2}{2m} + V$$

We want that $\hat{\boldsymbol{p}}$ and $\hat{E}$ applied on $\psi(\boldsymbol{x},t)$ return p and E →
$\hat{\boldsymbol{p}} = -i\boldsymbol{\nabla}$ and $\hat{E} = i\frac{\partial}{\partial t}$

$$i\frac{\partial\psi(\boldsymbol{x},t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(\boldsymbol{x},t)}{\partial x^2} + \hat{V}\psi(\boldsymbol{x},t)$$

Commutation Relations

In general, any state can be described as a superposition of states

$$|\varphi\rangle = \sum_i c_i|\psi_i\rangle,$$

If at time $t = 0$ the system is in the state

$$|\varphi(\boldsymbol{x},t)\rangle = |\varphi(\boldsymbol{x})\rangle$$

then the evolution of the system is determined by the evolution of the different components.

$$|\varphi(\mathbf{x},t)\rangle = \sum_i c_i\,|\phi_i(\mathbf{x})\rangle e^{-iE_i t}$$

If $[\hat{A},\hat{B}] = 0$ then the observables can be determined at the same time:

$$\hat{A}|\phi\rangle = a|\phi\rangle,$$
$$\hat{A}\hat{B}\,|\phi\rangle = \hat{B}\hat{A}\,|\phi\rangle = a\hat{B}\,|\phi\rangle.$$
$$\hat{B}|\phi\rangle = b|\phi\rangle.$$

If $[\hat{A},\hat{B}] \neq 0$ then the observables cannot be determined at the same time to better than : $\Delta A\,\Delta B \geq \frac{1}{2}\left|\langle i[\hat{A},\hat{B}]\rangle\right|,$

Example: position and momentum: $\hat{x}\psi = x\psi$ and $\hat{p}_x\psi = -i\dfrac{\partial}{\partial x}\psi$

$$[\hat{x},\hat{p}_x]\psi = -ix\frac{\partial}{\partial x}\psi + i\frac{\partial}{\partial x}(x\psi)$$
$$= -ix\frac{\partial\psi}{\partial x} + i\psi + ix\frac{\partial\psi}{\partial x} = +i\psi, \qquad [\hat{x},\hat{p}_x] = +i.$$

$$\boxed{\Delta x\,\Delta p_x \geq \frac{\hbar}{2}.}$$

Cross Sections and Decay Rates

Fermi's Golden Rule

Particle Physics:

- Study of decays (→ measure decay rates, how often $a \to 1 + 2$?);
- Study of cross-sections (→ measure reaction rates, how often $a + b \to 1 + 2$?).

$\sigma \propto |T_{fi}|^2$, Transition Matrix

These processes correspond to transitions between states.

Non-relativistic quantum mechanics, $|i\rangle \to |f\rangle$: Fermi's golden rule
Transition rate $\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$

Non relativistic!

- If interaction potential is known or calculable → compute the cross section
- if T_{fi} is not known one can measure σ and derive T_{fi} from it.

The Golden Rule applies both to scattering and decay processes. In the second case the lifetime of the process will be

$$\tau = \frac{1}{W}$$

- if the lifetime is (can be) measured then T_{fi} can be derived.
- If τ cannot be measured, then the uncertainty principle can be used → we can take $\Delta E = \hbar/\tau$ (resonance shape)

Elaborate technicalities in next slides

The Fermi Golden Rule - continued

According to the (second) Fermi golden rule,

- the *reaction rate* Γ_{fi} from the initial state $|i\rangle$ to a final state $|f\rangle$ is given by

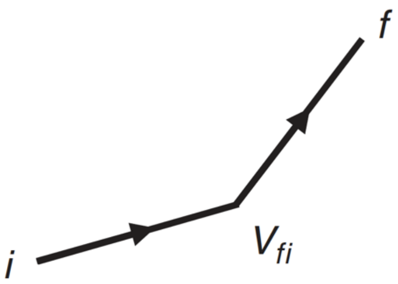
Γ_{fi} = 'transition rate'
Your Experiment

$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$$

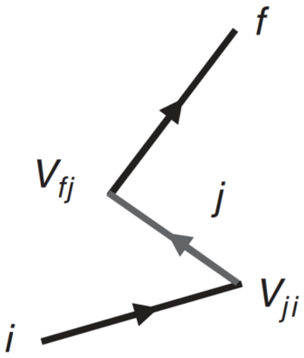
$\rho(E_i)$ = density of states
Kinematics

T_{fi} = Matrix element
Physics

Problem: this expression is NOT Lorentz invariant



$$\mathrm{T}_{fi} = \langle f|\mathcal{H}'|i\rangle + \sum_{j\neq i} \frac{\langle f|\mathcal{H}'|j\rangle\langle j|\mathcal{H}'|f\rangle}{E_i - E_j} +' \, higher\ order\ diagrams'$$



The Density of States

$\rho(E_i) = |\frac{dn}{dE}|_{E_i}$ dn is the number of states in the interval $E \rightarrow E + dE$

in how many ways we can construct the final state (and conserve energy and momentum of the initial state E_i).

Alternative: counting of **all possible final states** but imposing the energy conservation by means of a δ function:

$$\rho(E_i) = |\frac{dn}{dE}|_{E_i} = \int \frac{dn}{dE} \delta(E - E_i)\, dE$$

Giving a new expression for the Fermi Golden Rule

$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i) \rightarrow$$

$$\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \delta(E_i - E_n) dn$$

Transition matrix depends on

- $|T_{fi}|$ this term contains physics;
- $\rho(E_i)$ this tern describes the kinematics of the event

Normalisation of States (nonrelativistic)

Example: two body decay of particle a

$$a \to 1 + 2$$

$$\Gamma_{fi} = \langle \psi_1 \psi_2 | \widehat{H'} | \psi_a \rangle = \int_V \psi_1^* \psi_2^* \widehat{H'} \psi_a d^3\boldsymbol{x}$$

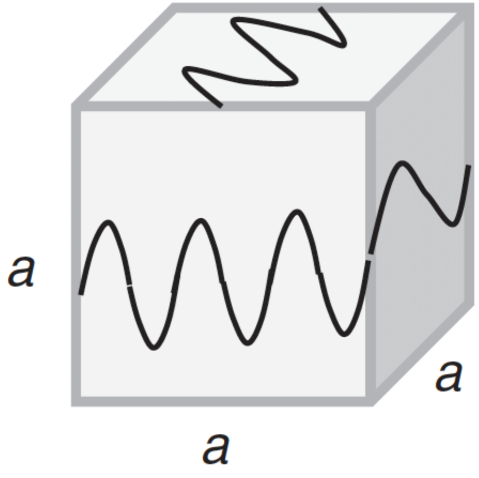
In the Born approximation & perturbation is small

$$\psi_a(\boldsymbol{x}, t) = A e^{i(\boldsymbol{p}\cdot\boldsymbol{x} - Et)} \quad A \to normalisation$$

$$\int_0^a \int_0^a \int_0^a \psi^* \psi \, dx \, dy \, dz = 1$$

A …

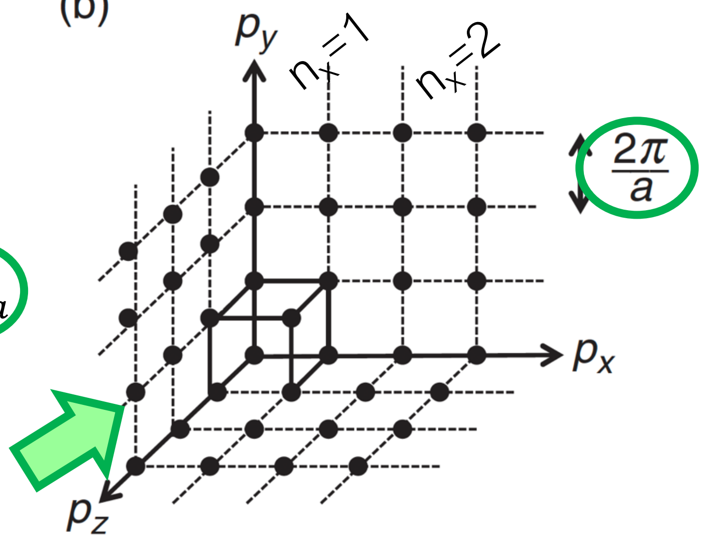
A …



1 particle in a cube of side a →

$$A^2 = {}^1/_{a^3} = {}^1/_V$$

(b)



The normalisation of one particle in a volume a^3 implies periodic conditions (wave function is zero at boundaries)

$$\psi(x + a, y, z) = \psi(x, y, z) \ \to e^{i(p_x x)} = e^{i(p_x(x+a))} \ \to (p_x, p_y, p_z) = (n_x, n_y, n_z)\, {}^{2\pi}/_a$$

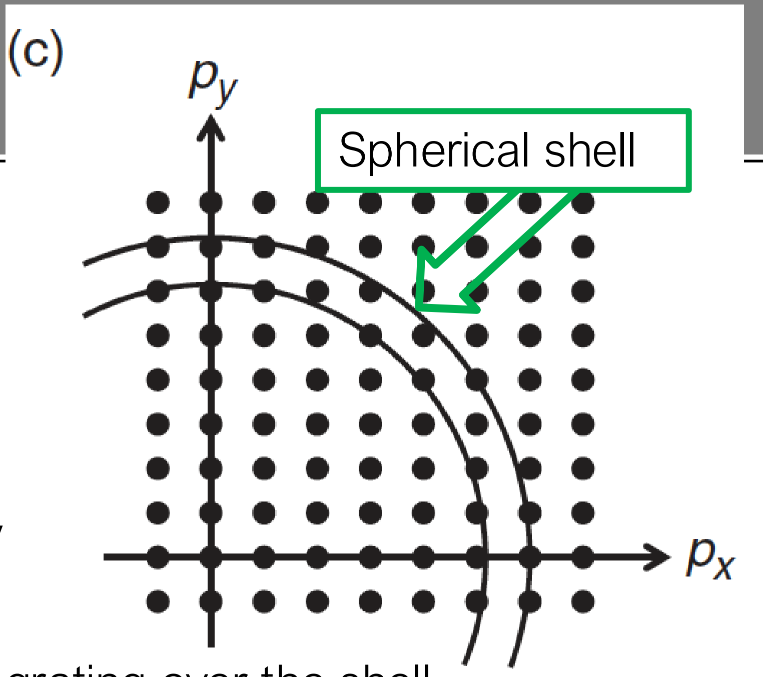
Where n_x, n_y, n_z are integers → momenta are quantised as shown in the figure here

Normalisation of States - 2

Each state occupies a cubic volume

$$d^3\boldsymbol{p} = dp_x dp_y dp_z = \left(\frac{2\pi}{a}\right)^3 = \frac{(2\pi)^3}{V}$$

Density of States: how many states can I put inside this normalisation volume?



The number of available states dn in the momentum interval $p \rightarrow p + dp$ is given by

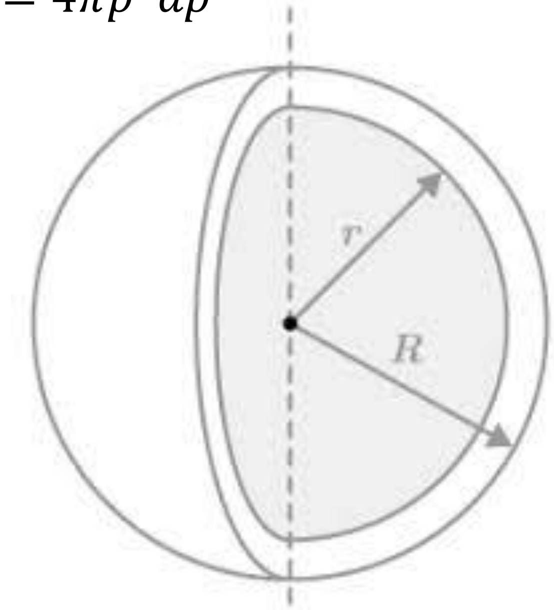
$$\frac{dn}{dp} = \frac{4\pi p^2}{(2\pi)^3} V$$

$$p = \beta E \quad \frac{dp}{dE} = \beta$$

Integrating over the shell

$$d^3p = 4\pi p^2 dp$$

$$\frac{volume\ of\ spherical\ shell}{(2\pi)^3/V} \rightarrow \rho(E) = \frac{dn}{dE} = \frac{dn}{dp}\frac{dp}{dE} = \frac{4\pi p^2}{(2\pi)^3} \cdot \beta$$



All above for ONE particle!

Comment: *"V"* appears in $\frac{dn}{dp}$ but will cancel with wavefunction normalisation → use V=1

Normalisation of a System with N Particles

$$d^3\boldsymbol{p} = dp_x dp_y dp_z = \left(\frac{2\pi}{a}\right)^3 = \frac{(2\pi)^3}{V} \Rightarrow \mathrm{d}n_i = \frac{\mathrm{d}^3\mathbf{p}_i}{(2\pi)^3} \Rightarrow \mathrm{d}n = \prod_{i=1}^{N-1} \mathrm{d}n_i = \prod_{i=1}^{N-1} \frac{\mathrm{d}^3\mathbf{p}_i}{(2\pi)^3}.$$

We have put V=1

Always non-relativistic case!

- Decay to two particles $a \to 1 + 2$ the phase space is determined by one particle, the other is constrained by $\boldsymbol{p}$ conservation → $\boldsymbol{\delta}$ function;
- When there are more than two particles, *N* particles → *N-1* are 'free'

$$\mathrm{d}n = \prod_{i=1}^{N-1} \frac{\mathrm{d}^3\mathbf{p}_i}{(2\pi)^3} \delta^3\left(\mathbf{p}_a - \sum_{i=1}^{N}\mathbf{p}_i\right) \mathrm{d}^3\mathbf{p}_N \to \mathrm{d}n = (2\pi)^3 \prod_{i=1}^{N} \frac{\mathrm{d}^3\mathbf{p}_i}{(2\pi)^3} \delta^3\left(\mathbf{p}_a - \sum_{i=1}^{N}\mathbf{p}_i\right).$$

δ Function → Momentum conservation

Nth particle

Make Golden Rule Lorentz Invariant

We have to transform the Fermi Golden rule into a Lorentz invariant form:

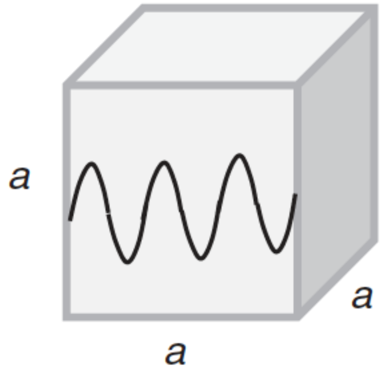
- Phase space (kinematics)
- T_{fi} (Physics)

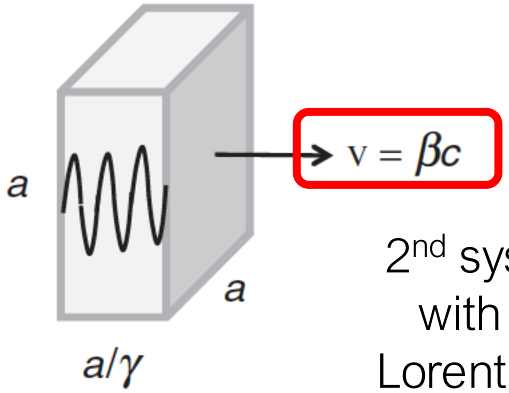
$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$$

Lorentz Invariant Normalisation

Normalise V to one particle per unit volume → V=1? Unsatisfactory…
The decay/interaction rate = f(Physics) doesn't depend on normalisation volume

Lorentz Invariance!





2^{nd} system moving with velocity v → Lorentz contracted

ψ normalised to 1 particle in volume V

$$\int_V \psi^* \psi \, \mathrm{d}^3\mathbf{x} = 1.$$

ψ' normalised to 2E particle in volume V

$$\int_V \psi'^* \psi' \mathrm{d}^3\mathbf{x} = 2E,$$

(factor '2' for convenience)

$$\psi' = (2E)^{1/2}\psi.$$

T_{fi} : *Fermi's golden rule matrix element*

For a process $a + b \to 1 + 2 + \cdots$

$$\mathcal{M}_{fi} = \langle \psi_1' \psi_2' \cdots | \hat{H}' | \psi_a' \psi_b' \cdots \rangle = (2E_1 \cdot 2E_2 \cdots 2E_a \cdot 2E_b \cdots)^{1/2} T_{fi},$$

Lorentz Invariance

$\mathcal{M}_{fi}$: *Lorentz invariant matrix element*

Decays and Cross Sections

Recap Without Formalities: what we did

1. Start from Fermi's Golden Rule $\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$
2. Rewrite it using a δ function $\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \delta(E_i - E_n) dn$ (energy conservation)
3. Compute number of states $\rho(E) = \frac{dn}{dE} = \frac{dn}{dp}\frac{dp}{dE} = \frac{4\pi p^2}{(2\pi)^3} \cdot \beta$
4. Rewrite $\rho(E)$ by introducing a δ function

$$dn = (2\pi)^3 \prod_{i=1}^{N} \frac{d^3\mathbf{p}_i}{(2\pi)^3} \delta^3\left(\mathbf{p}_a - \sum_{i=1}^{N} \mathbf{p}_i\right).$$

p conservation

$\rho(E) = dn/dE$

5. Introduce Lorentz invariant normalization

$$\mathcal{M}_{fi} = \langle \psi_1' \psi_2' \cdots | \hat{H}' | \psi_a' \psi_b' \cdots \rangle = (2E_1 \cdot 2E_2 \cdots 2E_a \cdot 2E_b \cdots)^{1/2} T_{fi},$$

$$\Gamma_{fi} = \int_V \psi_1^* \psi_2^* \widehat{H'} \psi_a d^3\boldsymbol{x}$$

$$T_{fi}^2 = M_{fi}^2 / (2E_1 \cdot 2E_1 \ldots)$$

2 Body Particle Decays $a \to 1 + 2$

For a decay process $a \to 1 + 2$

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}_{fi}|^2 \delta(E_a - E_1 - E_2)\delta^3(\mathbf{p}_a - \mathbf{p}_1 - \mathbf{p}_2) \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2},$$

$\mathcal{M}_{fi}$: *Lorentz invariant matrix element*: contains physics and has to be computed for each type of process

The phase space is the same for all types of processes, depends only on the number of particles

Since Γ_{fi} is Lorentz invariant → can be computed in any reference system → cms where

$$E_a = m_a, \qquad \boldsymbol{p_2} = -\boldsymbol{p_1} \qquad E_2^2 = (m_2^2 + \mathrm{p}_1^2)$$

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a} \int |\mathcal{M}_{fi}|^2 \frac{1}{4E_1 E_2} \delta(m_a - E_1 - E_2)\, d^3\mathbf{p}_1,$$

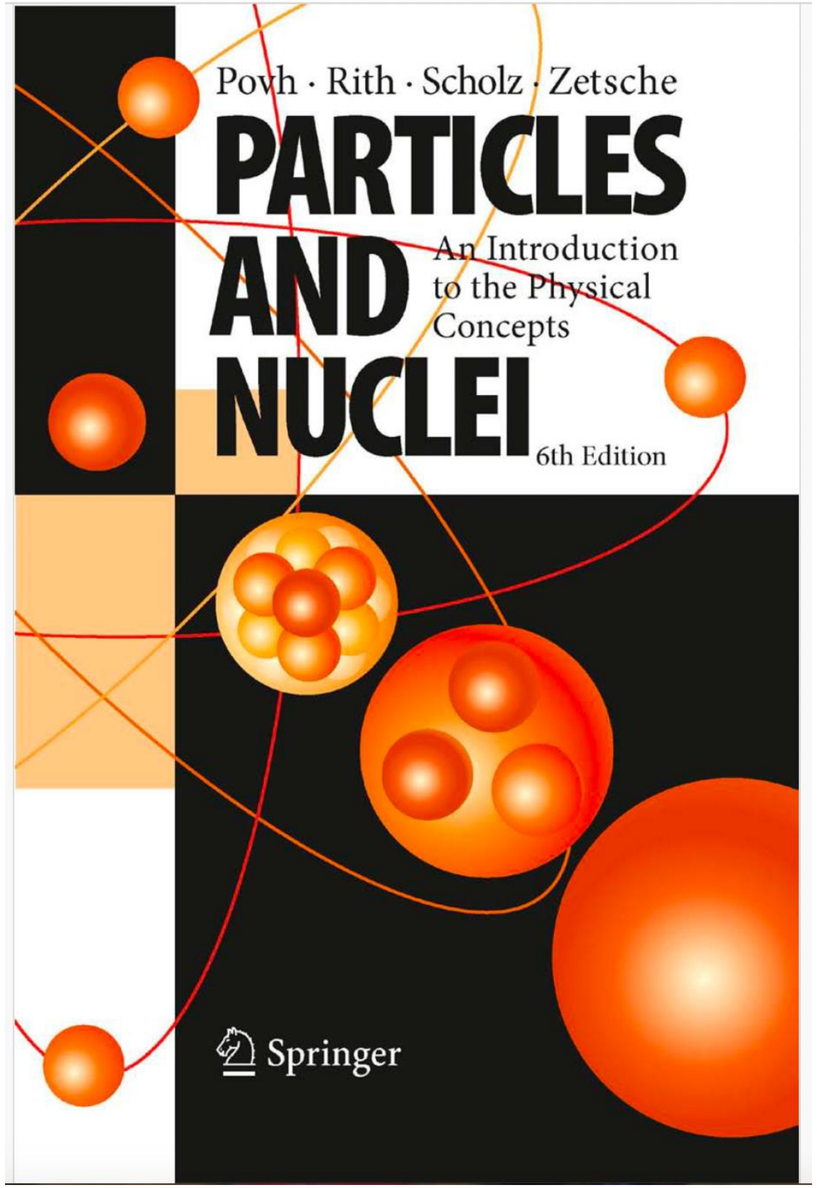
If we use polar coordinates

$$d^3\mathbf{p}_1 = \mathrm{p}_1^2 d\mathrm{p}_1 \sin\theta\, d\theta\, d\phi = \mathrm{p}_1^2\, d\mathrm{p}_1 d\Omega, \quad \mathrm{p}^* = \frac{1}{2m_a}\sqrt{\left[(m_a^2 - (m_1 + m_2)^2\right]\left[m_a^2 - (m_1 - m_2)^2\right]}.$$

We arrive (few steps in the book) to an expression valid for all two bodies decays

$$\Gamma_{fi} = \frac{\mathrm{p}^*}{32\pi^2 m_a^2} \int |\mathcal{M}_{fi}|^2\, d\Omega.$$

Povh & Co: Particles and Nuclei



Bogdan Povh · Klaus Rith · Christoph Scholz · Frank Zetsche

Particles and Nuclei

An Introduction to the Physical Concepts

Translated by Martin Lavelle

Springer

Cross Section Measurement

- *What to measure in an experiment?*
- *How??*

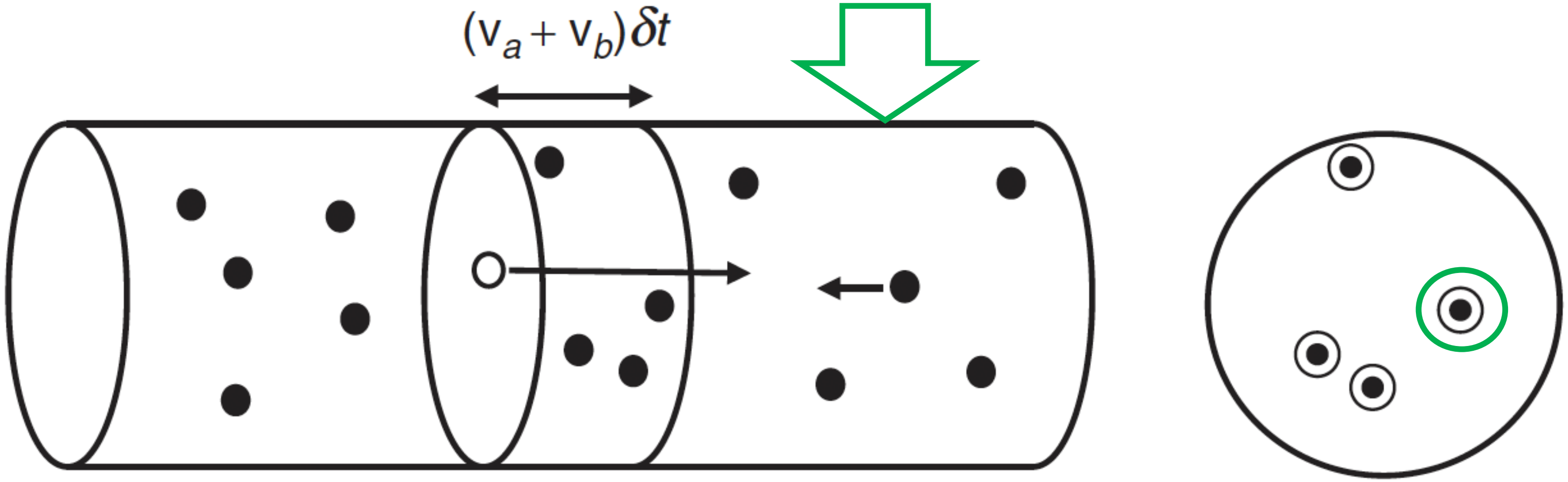
Interaction rate ⟺ Cross Section

The cross section can be defined through the relation

Physics: σ

Experiment: r_b $$r_b = \sigma\phi_a$$ Technique: ϕ_a

- Slightly more complicated than the calculation of decay rates: account for the flux of incoming particles hitting a target with Nb scattering centres;
- In modern experiments two beams colliding against each other;



- One may think of σ as an "*effective area*";
- Rarely it is the case (scattering of 'big' objects);
- More correct to think of σ as a Quantum Mechanics observable associated to the interaction probability.

(Geometric) Cross Sections

Calculation of interaction rates more complex: account for flux of incoming particles. You cannot do $a + b \rightarrow 1 + 2 + \cdots$
You do beam on target or beam against beam

Structure of the matter is studied with scattering experiments. Energetic projectiles → small equivalent wave length

$$\lambda = \hbar / p$$

Ideal Simplified Experiment:

Beam particles **a** bombard scattering centres b.

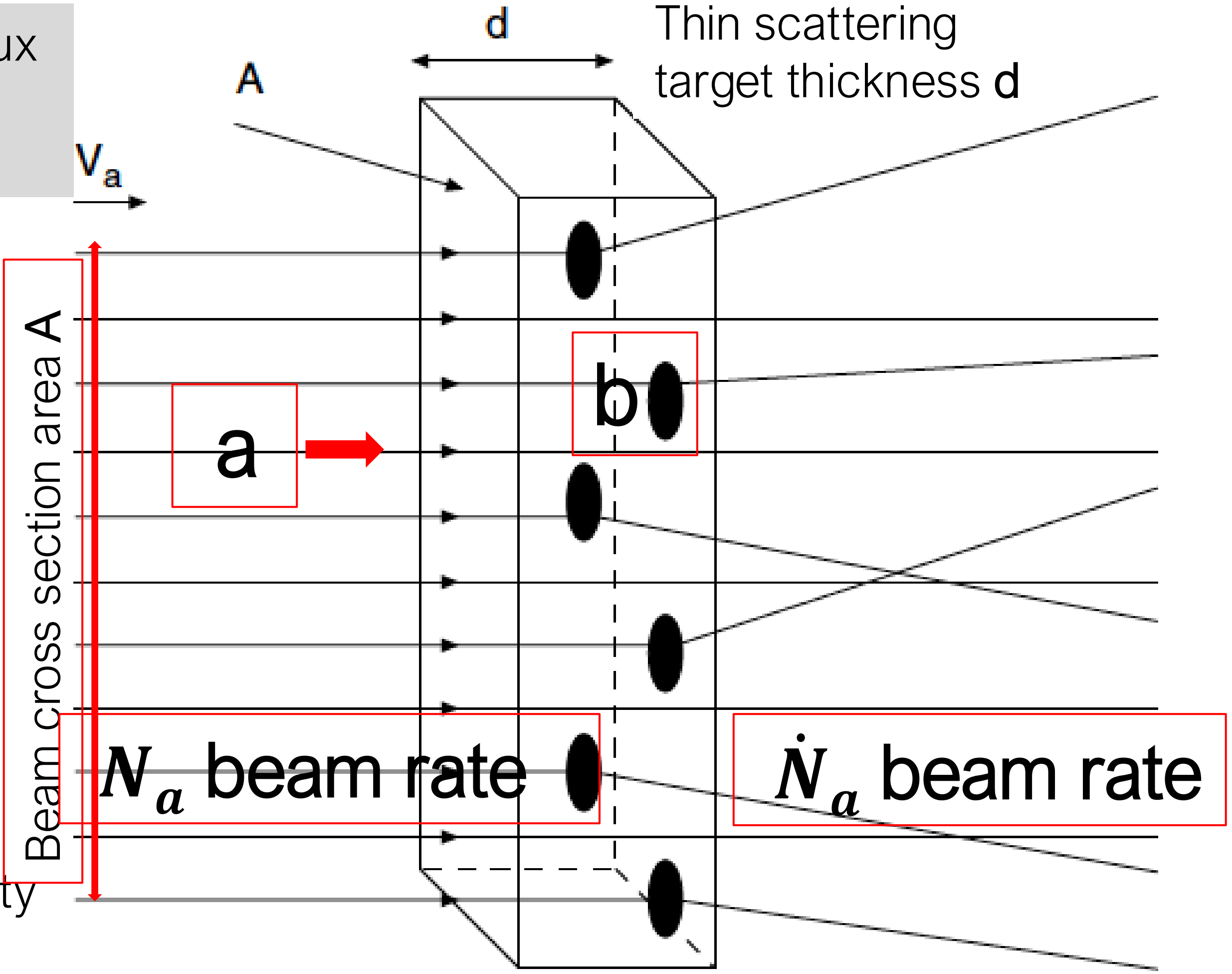
- reaction occurred when **a** hits **b**.
- The beam particle **a** disappears after the interaction

$$a + b \rightarrow anything$$

Particle beam a coming from left with density n_a and velocity v_a The corresponding flux is

$$\phi_a = n_a \times v_a$$

Target with N_b scattering centres **b** and particle density $\mathbf{n_b}$



$$N_b = A\, d\, n_b$$
= (density x Volume)$_{target}$

(Geometric) Cross Sections

Ideal Simplified Experiment:

After the interaction beam particles disappear (we do not distinguish different final topologies, we sum elastic + inelastic cross sections). Reaction rate is

$$\dot{N} = \boldsymbol{N_a} - \dot{\boldsymbol{N}}_{\boldsymbol{a}}$$

Particle beam **a** coming from left with density n_a and velocity v_a The corresponding flux is

$$\phi_a = n_a \times v_a = \frac{\boldsymbol{N_a}}{\boldsymbol{A}} (\boldsymbol{area} \times \boldsymbol{time})^{-1}$$

Target with N_b scattering centres **b** and particle density n_b. Target particles within the beam area A are

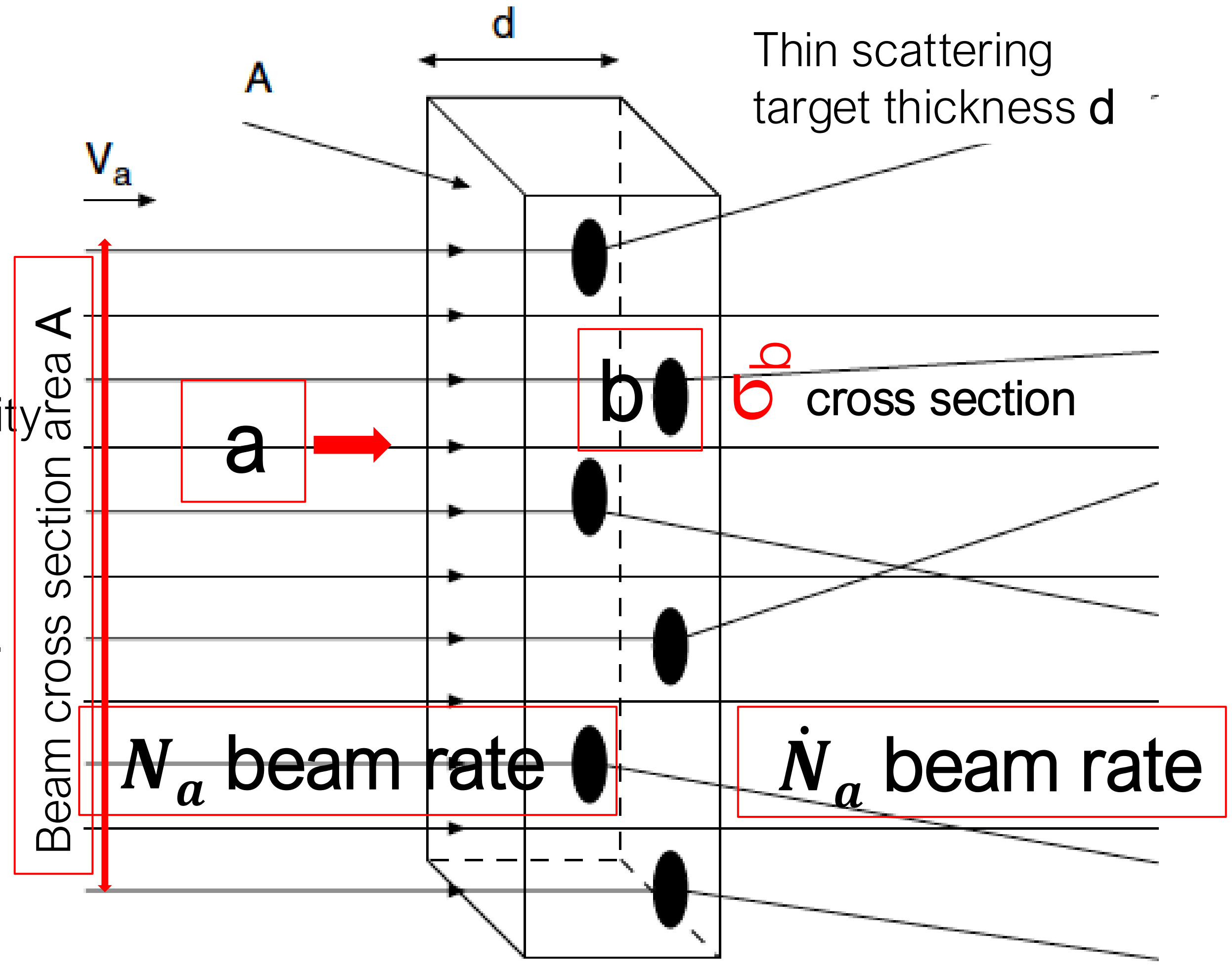
$$N_b = A \times d \times n_b$$

→ the reaction rate $\dot{N}$ is

$$\dot{N} = \phi_a \times N_b \times \boldsymbol{\sigma_b}$$

$$\boldsymbol{\sigma_b} = \frac{\dot{N}}{\phi_a \times N_b}$$

$$= \frac{\boldsymbol{number\ or\ reactions\ per\ unit\ time}}{\boldsymbol{beam\ particles\ per\ unit\ time\ per\ unit\ area} \times \boldsymbol{scattering\ centres}}$$



Limitations: HP, scattering centres do not overlap + only one scattering

(Geometric) Cross Sections - 3 (Povh…)

If beam is not uniform

$$\sigma_b = \frac{\dot{N}}{\phi_a \times N_b} = \frac{number\ or\ reactions\ per\ unit\ time}{(beam\ particles\ per\ unit\ time\ \times scattering\ centres)\ per\ unit\ area}$$

In the expression

$$\sigma_b = \frac{\dot{N}\ \text{Physics!}}{\phi_a \times N_b\ \ \text{Experiment}}$$

$(\phi_a \times N_b)$ = Luminosity, $\mathcal{L}$ in this case

- Energy dependence
- Particle types..

$\dot{N} = \mathcal{L} \times \sigma_b$ *Remember!!!*

The total cross section σ_{tot} is as the sum of elastic and inelastic cross section

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel}$$

and has dimensions of area. a common unit to define cross sections is the *barn*

σ_{pp}(10 GeV) ~ 40 mb, $\sigma_{\nu p}$(10 GeV) ~ 70 fb (ratio is → 10^{-12})

Unit	Symbol	m^2	cm^2
megabarn	Mb	10^{-22}	10^{-18}
kilobarn	kb	10^{-25}	10^{-21}
barn	b	10^{-28}	10^{-24}
millibarn	mb	10^{-31}	10^{-27}
microbarn	µb	10^{-34}	10^{-30}
nanobarn	nb	10^{-37}	10^{-33}
picobarn	pb	10^{-40}	10^{-36}
femtobarn	fb	10^{-43}	10^{-39}
attobarn	ab	10^{-46}	10^{-42}
zeptobarn	zb	10^{-49}	10^{-45}
yoctobarn	yb	10^{-52}	10^{-48}

The Luminosity (~ Technology, not Physics)

$$\mathcal{L} = \phi_a \cdot N_b$$

Beam on a target

Luminosity : [(area x time)$^{-1}$]. From $\phi_a = n_a \times v_a$ and $N_b = n_b \cdot d \cdot A$ we have

$$\mathcal{L} = \varphi_a \cdot N_b = \boxed{\dot{N}_a \cdot n_b \cdot d} = \boxed{n_a \cdot v_a \cdot N_b}$$

Luminosity → defined as one of two products below

1. number of incoming beam particles per unit time N_a, the target particle density in the scattering material n_b, and the target's thickness d;
2. beam particle density n_a, their velocity v_a and the number of target particles N_b exposed to the beam.

j packets with N_a or N_b particles, a ring of circumference U. velocity v ~ c in opposite directions and cross at an interaction point

two beams in a storage ring.

The luminosity is then:

$$\mathcal{L} = \frac{N_a \cdot N_b \cdot j \cdot v/U}{A}$$

A = beam cross-section at the collision point. For a Gaussian distribution of the beams (σ_x and σ_y respectively),

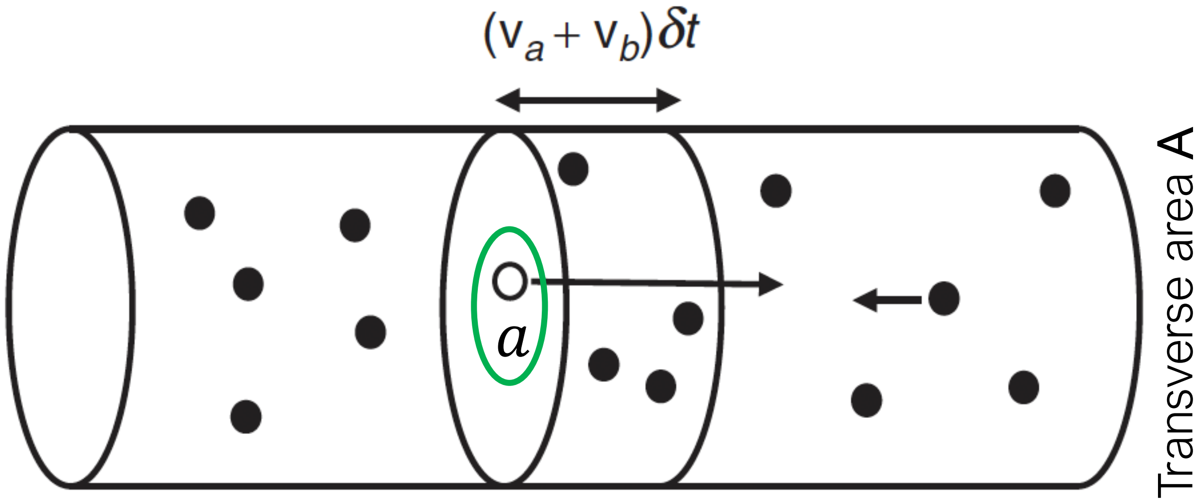
$$A = 4\pi\sigma_x\sigma_y \ .$$

… and have to be well aligned:
LHC ~27Km circumference!

55

→ beams must be focused at the interaction point into the smallest possible area possible. Typical beam diameters are of the order of tenths of millimetres or less.

A Simple Experiment



Simplified Experiment: *ONE* particle 'a' travels in a medium with a particle density n_b of type 'b'

- v_a, v_b velocities of particles of type a, b respectively;
- a, b travel opposite to each other;
-

In time δt, traverses a volume with $\delta N = n_b(v_a + v_b)\delta t A$;
The interaction probability will be,

$$\delta P = \frac{\delta N\,\sigma}{A} = \frac{n_b(v_a + v_b)A\,\sigma\delta t}{A} = n_b v \sigma \delta t,$$

where σ can be considered as the 'effective area' of the particle

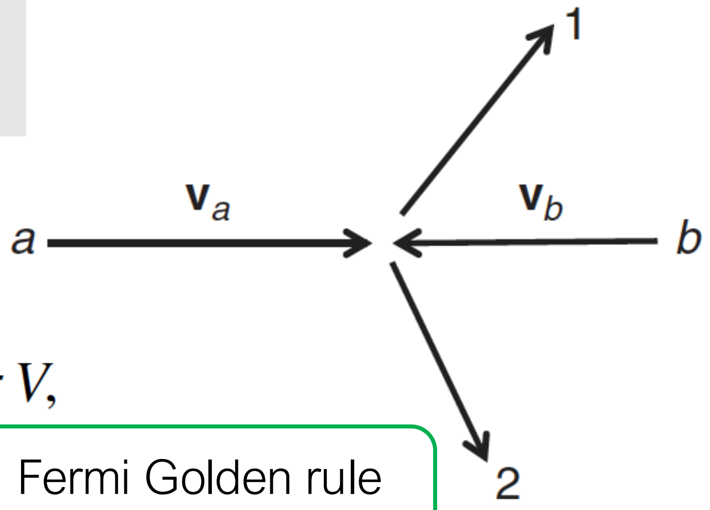
$$\frac{d\sigma}{d\Omega} = \frac{\text{number of particles scattered into } d\Omega \text{ per unit time per target particle}}{\text{incident flux}}.$$

Interaction Cross Section

$$\sigma = \frac{\text{number of interactions per unit time per target particle}}{\text{incident flux}}.$$

Consider the process $a + b \to 1 + 2$; observed in a rest frame with

- $v_a,\ v_b$ velocities;
- $n_a,\ n_b$ particle densities
- Cross section σ
- Normalized in a volume V



$$\phi_a = n_a(\mathrm{v}_a + \mathrm{v}_b).$$

$$\text{rate} = \phi_a\, n_b V \sigma = (\mathrm{v}_a + \mathrm{v}_b)\, n_a n_b\, \sigma\, V,$$

$$\sigma = \frac{(2\pi)^4}{(\mathrm{v}_a + \mathrm{v}_b)} \int |T_{fi}|^2 \delta(E_a + E_b - E_1 - E_2)\delta^3(\mathbf{p}_a + \mathbf{p}_b - \mathbf{p}_1 - \mathbf{p}_2)\frac{d^3\mathbf{p}_1}{(2\pi)^3}\frac{d^3\mathbf{p}_2}{(2\pi)^3}$$

Fermi Golden rule (non relativistic)

The cross section can be expressed in a Lorentz invariant form as

$$\sigma = \frac{(2\pi)^{-2}}{4\, E_a E_b(\mathrm{v}_a + \mathrm{v}_b)} \int |\mathcal{M}_{fi}|^2 \delta(E_a + E_b - E_1 - E_2)\delta^3(\mathbf{p}_a + \mathbf{p}_b - \mathbf{p}_1 - \mathbf{p}_2)\frac{d^3\mathbf{p}_1}{2E_1}\frac{d^3\mathbf{p}_2}{2E_2}.$$

$F = 4E_aE_b(\mathrm{v}_a + \mathrm{v}_b)$

Lorentz invariant flux

The most convenient way is to express the cross section in the centre of mass system

- $\boldsymbol{p_a} = -\boldsymbol{p_b} = \boldsymbol{p_i^*} \quad \boldsymbol{p_1} = -\boldsymbol{p_2} = \boldsymbol{p_f^*}$
- $\sqrt{s} = (E_a^* + E_b^*)$

In cms:

$F = \sqrt{(p_a p_b)^2 - (m_a m_b)^2}$

$$\sigma = \frac{1}{(2\pi)^2}\frac{1}{4\mathrm{p}_i^*\sqrt{s}} \int |\mathcal{M}_{fi}|^2 \delta\left(\sqrt{s} - E_1 - E_2\right)\delta^3(\mathbf{p}_1 + \mathbf{p}_2)\frac{d^3\mathbf{p}_1}{2E_1}\frac{d^3\mathbf{p}_2}{2E_2}.$$

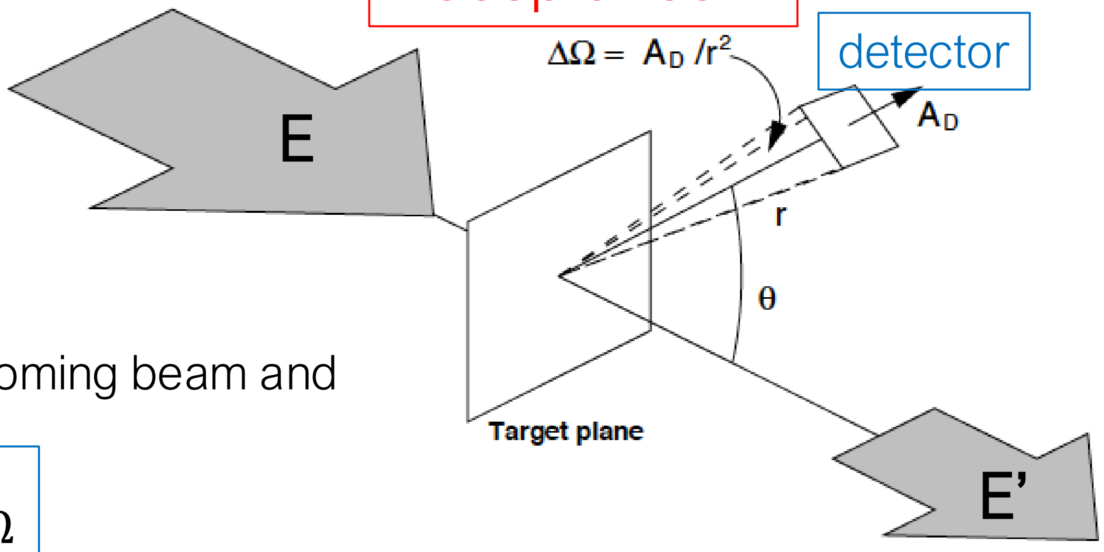
It may be shown to give

$$\sigma = \frac{1}{64\pi^2 s}\frac{\mathrm{p}_f^*}{\mathrm{p}_i^*}\int |\mathcal{M}_{fi}|^2 d\Omega^*.$$

Differential and Doubly-Differential Cross Sections

Real life: In all experiments only a fraction of all reactions are measured or accessible because of limited acceptance of the experimental set-up.

Acceptance!



Detector of area A_D at a distance r and at an angle θ, it covers a solid angle equal to $\Delta\Omega = A_D/r^2$.

The reaction rate (assumed to depend on the energy of the incoming beam and on the angle θ) will be:

$$\dot{N}(E,\theta,\Delta\Omega) = \mathcal{L}\frac{d\sigma(E,\vartheta)}{d\Omega}\Delta\Omega$$

If the energy & direction of the products is measured then the doubly differential cross section is also measured $d^2\sigma(E,E',\theta)/d\Omega dE'$. The total cross section, in this case, will be the integral over the solid angle and over the scattering energies

$$\sigma_{tot}(E) = \int_{E_{min}}^{E_{max}}\int_{\theta_{min}}^{\theta_{max}}\frac{d^2\sigma(E,E',\theta)}{d\Omega dE}\,d\Omega dE'$$

Only a part of $\sigma_{tot}(E)$ measured due to acceptance

Additional complication: detector efficiency!

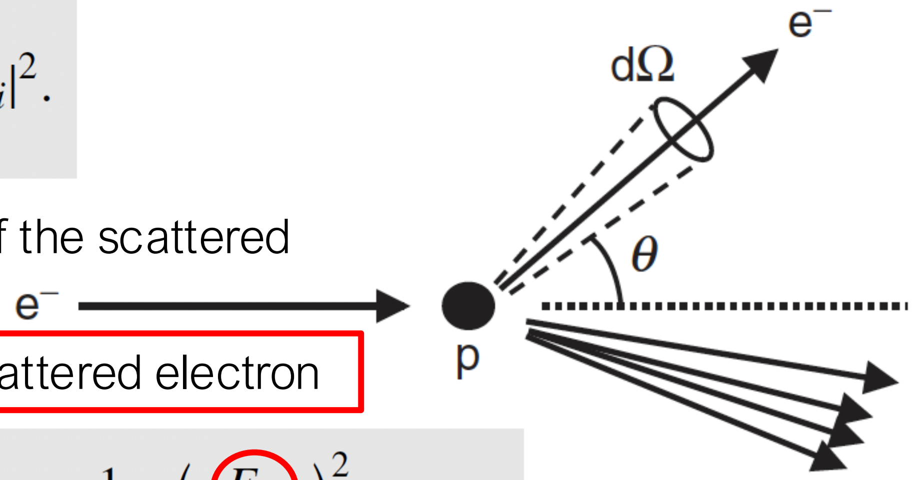
Differential Cross Sections

It may be important to measure the distribution of kinematic quantities, like angle and/or energy
→ derive information on the nature of the interaction

You measure the reaction rate in a solid angle element: but not only directions: single (E) or double differential cross-section (E & direction) $\frac{d\sigma}{dE}$ or $\frac{d^2\sigma}{dE d\Omega}$.

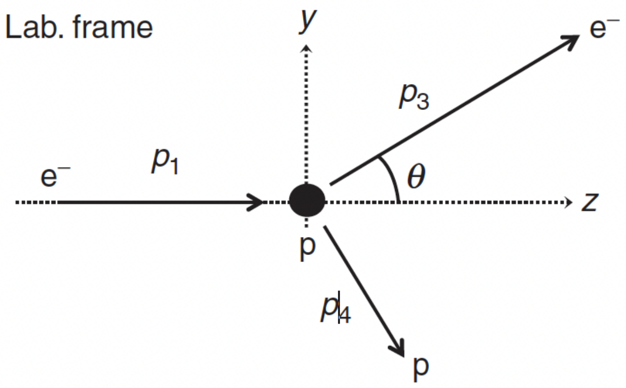
$$\sigma = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} \int |\mathcal{M}_{fi}|^2 d\Omega^*.$$

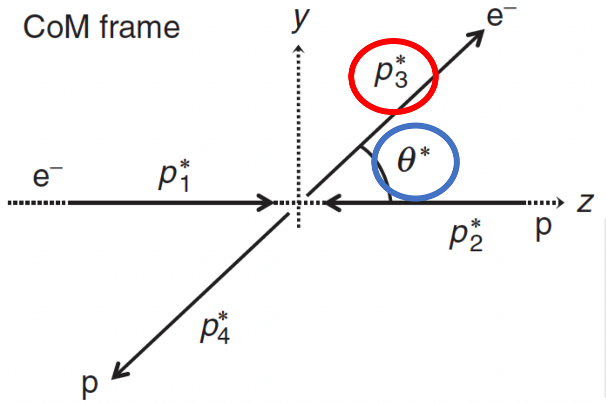
$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} |\mathcal{M}_{fi}|^2.$$



Example: Electron on a proton → Measure the direction (energy?) of the scattered electron (in the laboratory frame)

Measure angle or energy of the scattered electron





$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{E_3}{m_p E_1} \right)^2 |\mathcal{M}_{fi}|^2.$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{1}{m_p + E_1 - E_1 \cos\theta} \right)^2 |\mathcal{M}_{fi}|^2.$$